Six of fifteen outstanding problems in sediment transport science

Angel Abbott, Peter Almond, **Erika Anderson**, **Ashley Ball**, Francesco Ballio, Shawn Chartrand, **Elise Childs**, Nakul Deshpande, **Tyler Doane**, **Siobhan Fathel**, David Furbish, Rachel Glade, Harrison Gray, Peter Haff (Glorfindel), Douglas Jerolmack, **Ben Kahn**, Amanda Keen-Zebert, Daniel Morgan, Allesio Radice, **Andrew Roberts**, Joshua Roering, **John Roseberry**, Danica Roth, Mark Schmeeckle (Gandalf), Rina Schumer, **Mitchell Stocky**, **Tim Watkins**, **Sarah Williams**, **Stacey Worman**

1) Dynamically correct Knudsen number (think continuum versus rarefied)
2) Suitable macroscopic state variables (think equation of state)
3) Sediment phase transitions (e.g. entrainment and deposition)
4) Ensemble descriptions of transport (versus individual realizations)
5) Mechanics of particle diffusion (versus kinematics)
6) Transient surface transport (with emphasis on post-fire erosion)

These are problems in statistical physics…

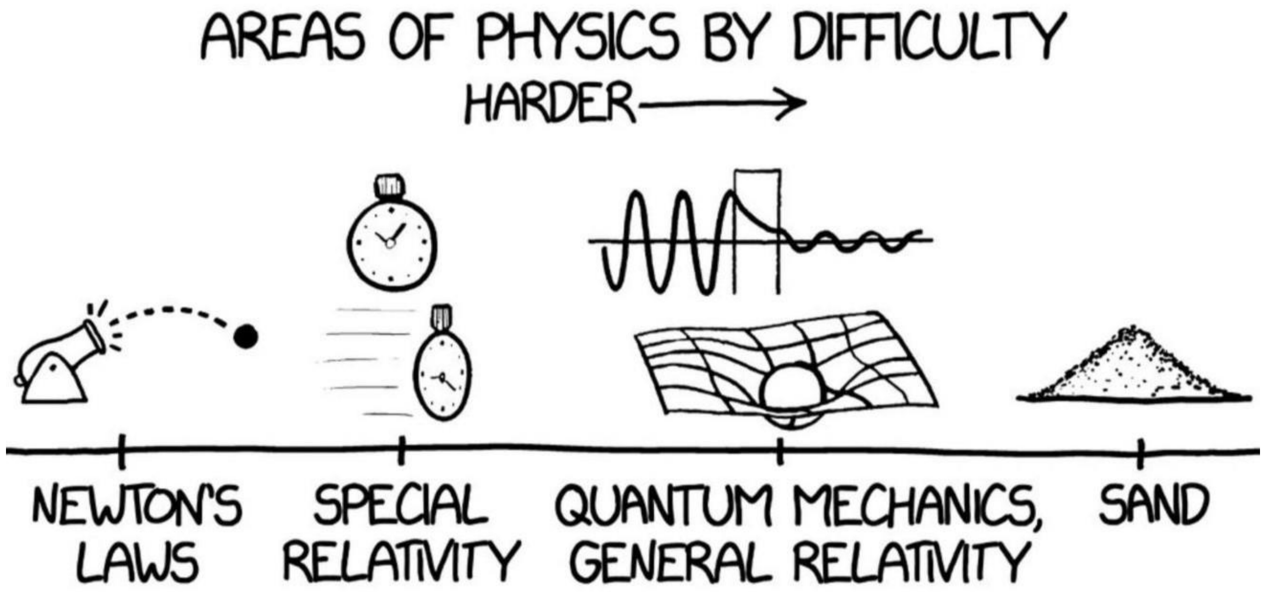
…in full view of our tepid – if not dismissive – relationship with probability.

“we can, in the best case, predict the range of **dynamical possibilities**, and **their probabilities**”

Eric De Giuli, on the significance of the work on complex systems by Nobel laureate Giorgio Parisi

“The objective therefore is to aim at probabilistic descriptions of sediment particle motions and transport that lean on the *style of thinking* of statistical mechanics, recognizing that this endeavor is not simply about adopting established theory or methods “off the shelf.” Rather, such efforts involve tailoring descriptions of transport to the process, the scales of interest and the techniques of observation and measurement used.” (Furbish and Doane, 2021)

1) Dynamically correct Knudsen number (or suitable analogue)



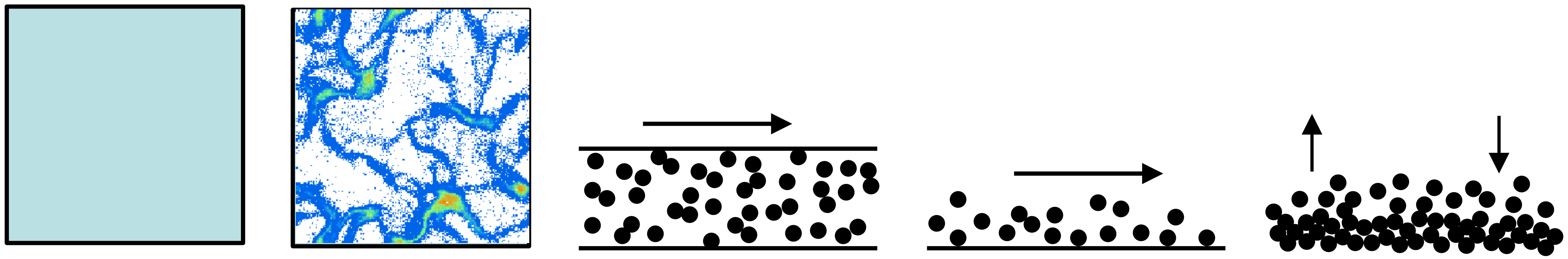
R. Munroe, 09 Nov 2020, *New York Times*

- H. Einstein had it right, but he/we could not move his ideas forward until recently
- Legacy from continuum mechanics; nearly a century based on the incorrect starting point
- Continuum-like divergence forms of Exner are problematic, except maybe for soils
- *Kn* is dynamical, representing local relaxation to equilibrium Maxwell-Boltzmann distribution
- Suitable analogues may not have same form

Knudsen number (increasingly rarefied)

Harder →

$$Kn = \frac{\lambda}{L} = \frac{\text{mean free path}}{\text{characteristic length scale}}$$



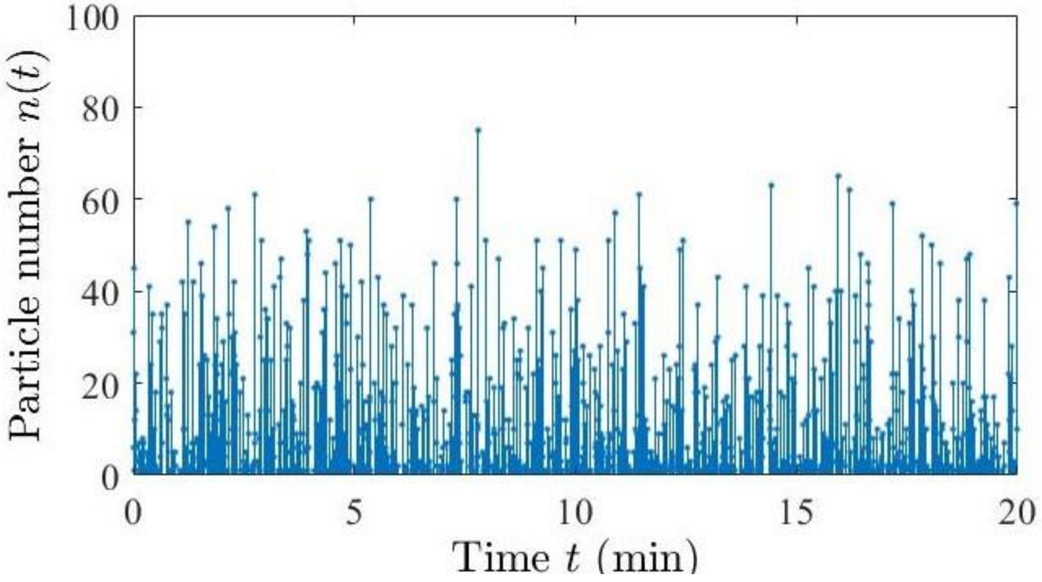
ordinary gas (p,T) | granular gas | sheared granular gas | highly rarefied | with phase transitions

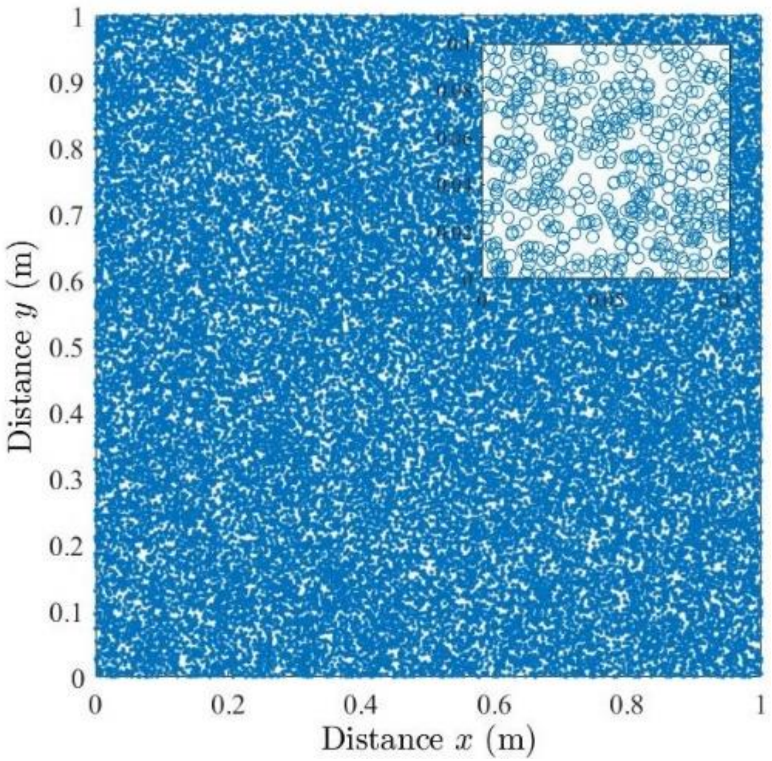
$$q = \gamma\overline{u_p}$$

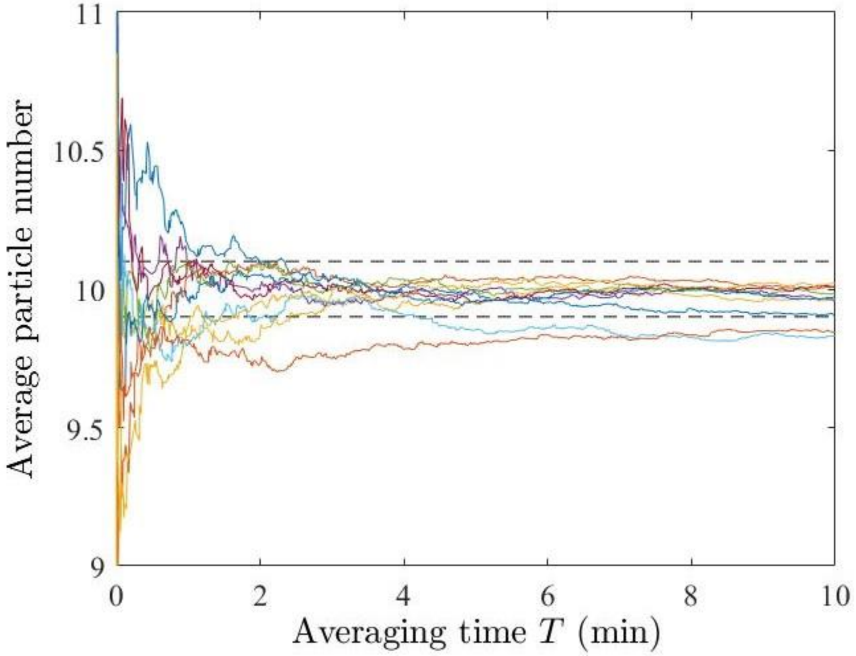
$$q = \langle\gamma\rangle\langle u_p\rangle - \frac{\partial}{\partial x}(\langle\gamma\rangle\langle\kappa_x\rangle)$$

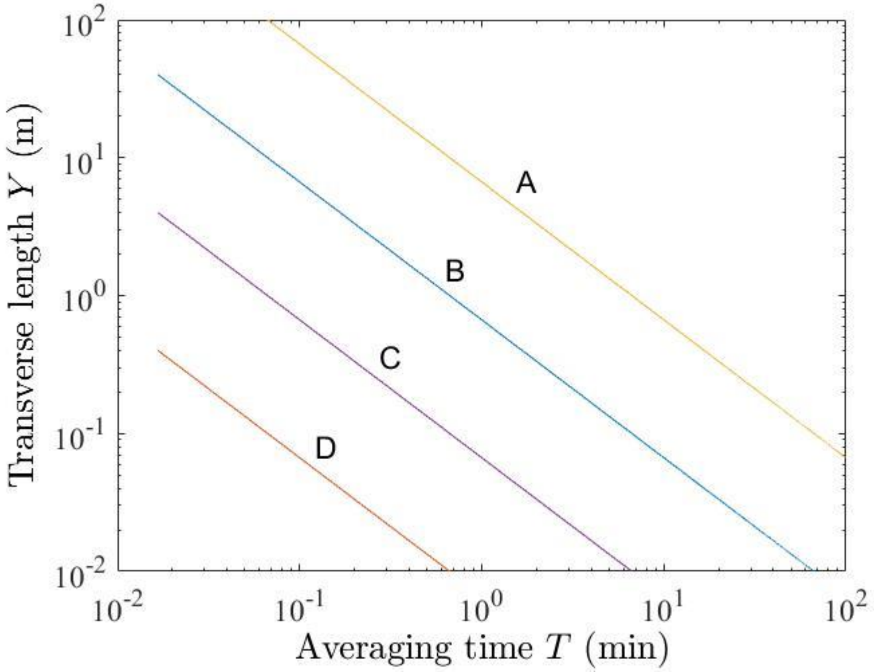
$$q = E\overline{L_x}$$

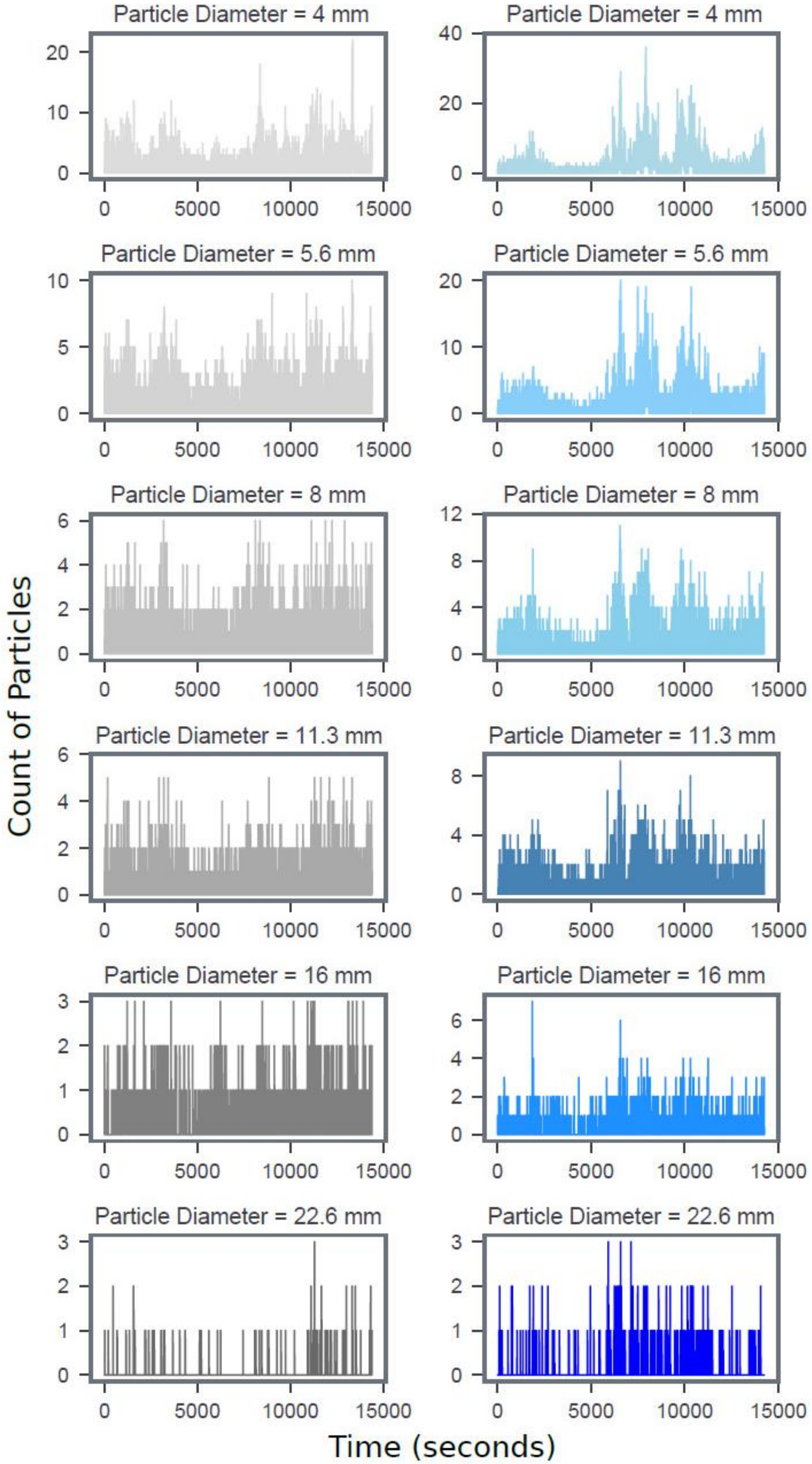
$$q = \langle E\rangle\langle L_x\rangle - \frac{\partial}{\partial x}(\langle E\rangle\langle L_x^2\rangle)$$





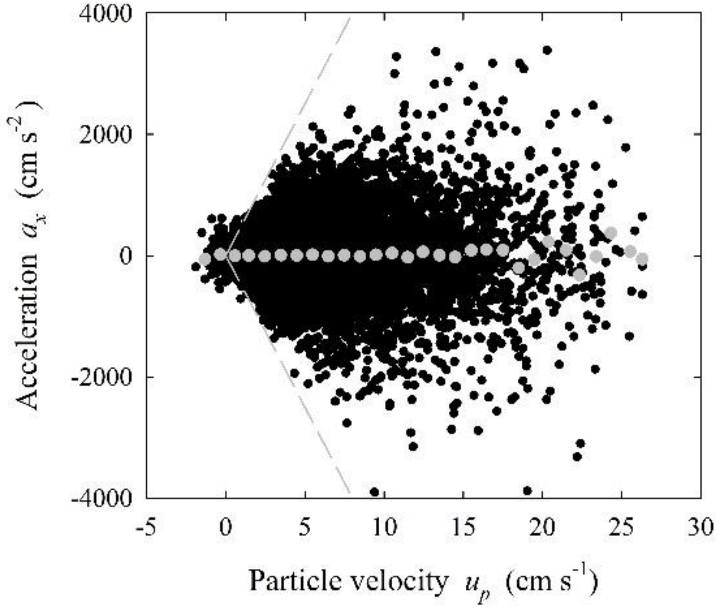






2) Suitable macroscopic state variables

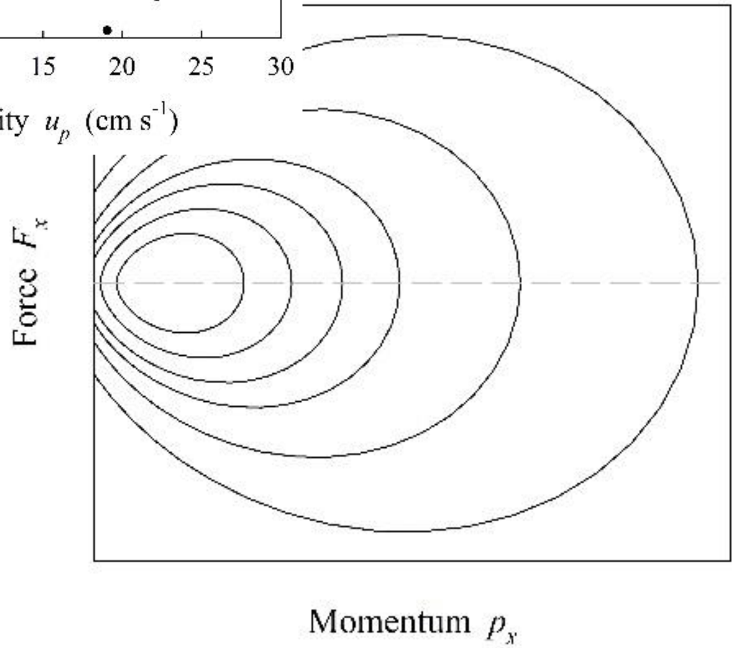
We have not yet identified suitable macroscopic state variables – if they exist – for sediment transport. The macroscopic fluid-imposed bed stress in both laminar and turbulent flows is an empirical heuristic and demonstrably incorrect for many of the things we expect of it. The beginning: **What is equilibrium transport?**



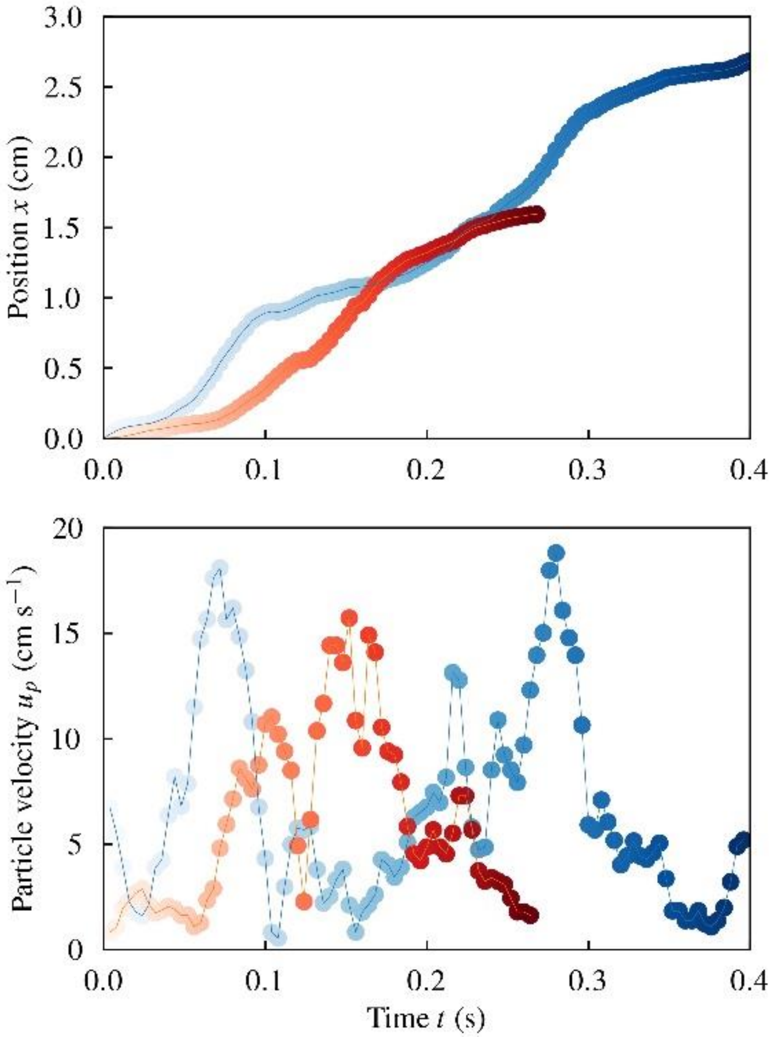
$$u_p = \frac{\mathrm{d}x}{\mathrm{d}t}$$

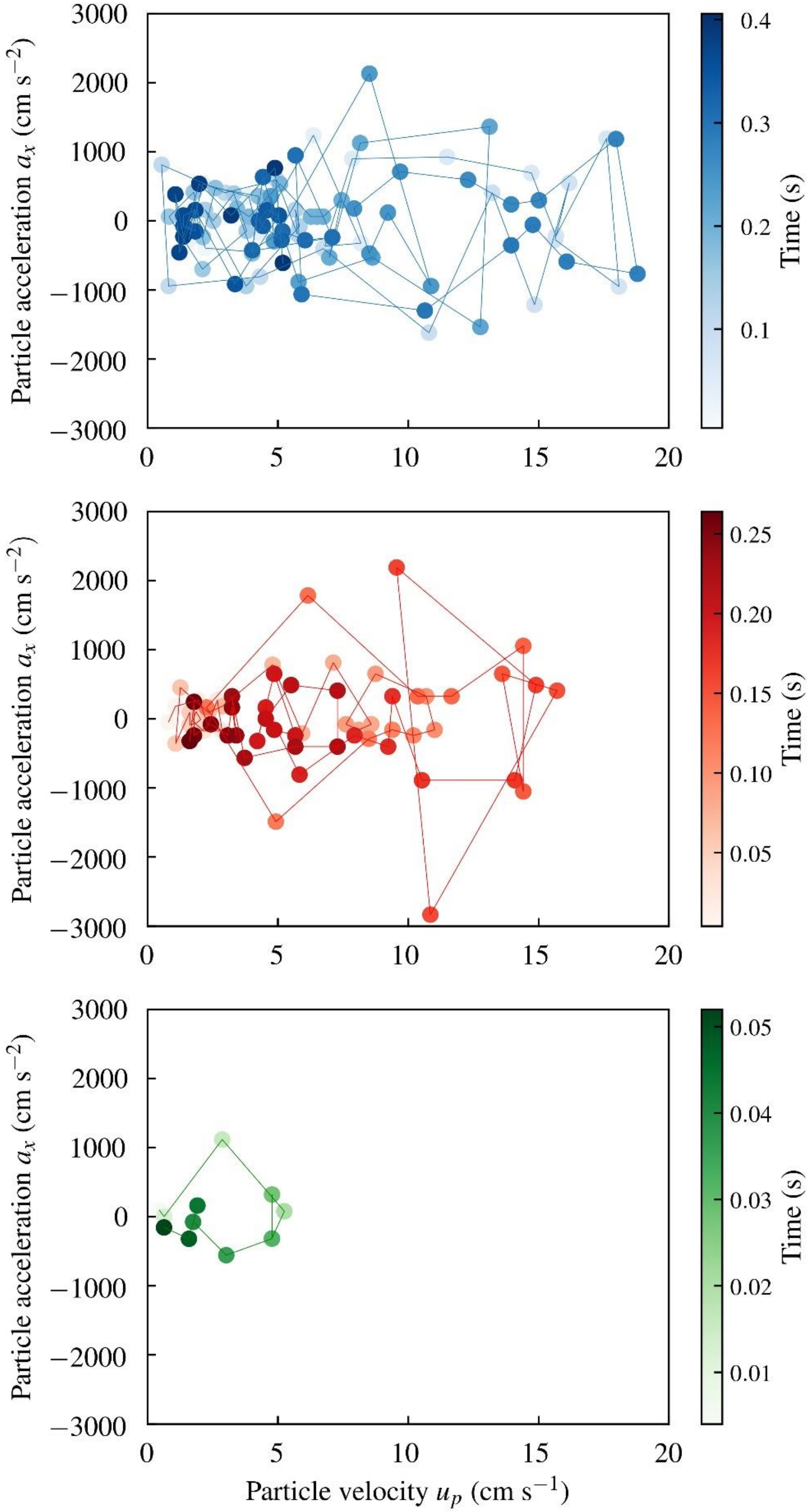
$$a_x = \frac{\mathrm{d}u_p}{\mathrm{d}t}$$

$$j_x = \frac{\mathrm{d}a_x}{\mathrm{d}t}$$



Clockwise circulation of probability





3) Sediment phase transitions

- Significant work on disentrainment is continuing
- Much needed work on **entrainment** and the coupling with turbulence fluctuations
- Shear stress is incorrect starting point
- Moving beyond initial motion problem due to steady fluid forces: collective entrainment
- Role of biota, other processes of entrainment on hillslopes

gravitational heating

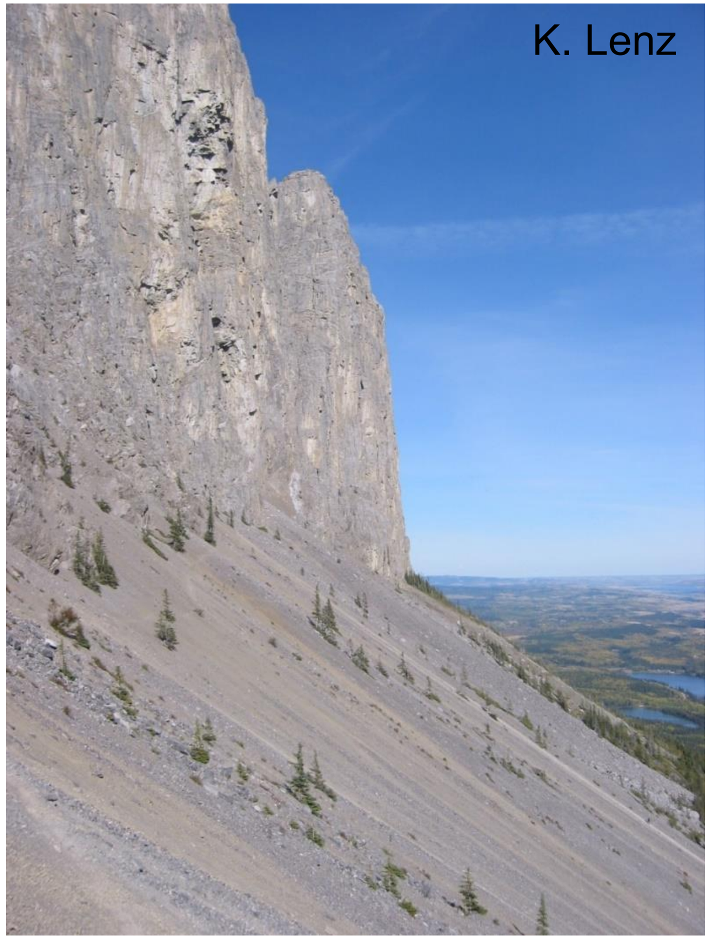
frictional cooling by collisions NOT Coulomb friction

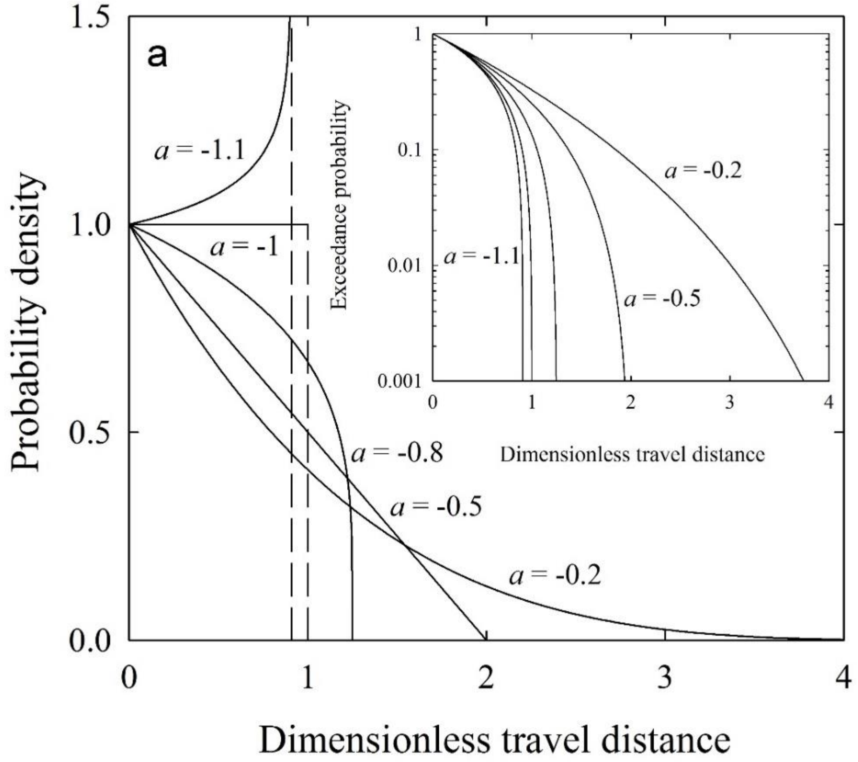
apparent heating with deposition

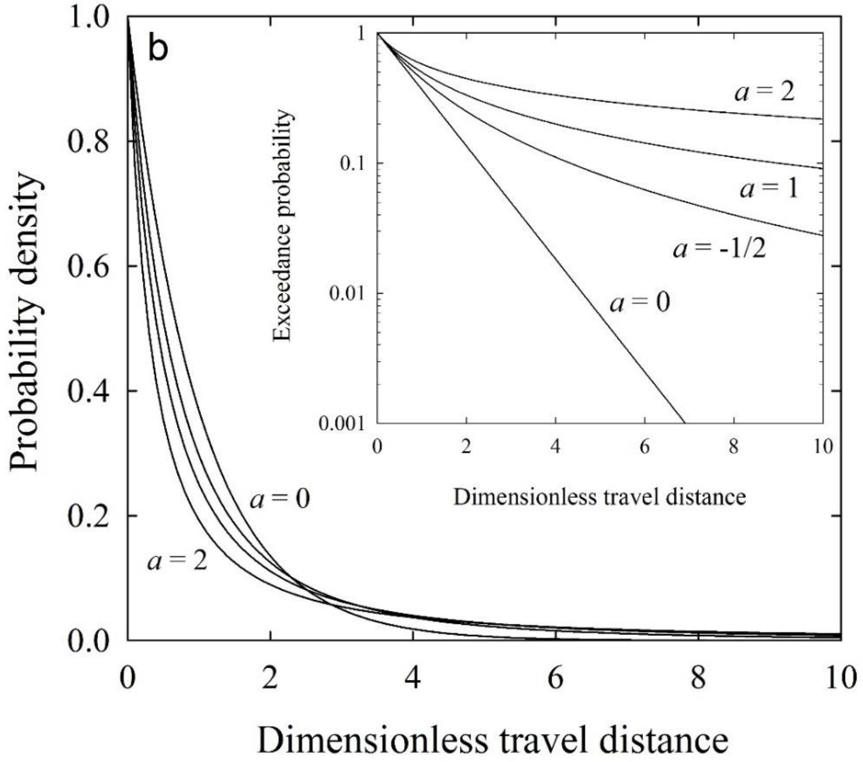
$$\frac{\mathrm{d}E_a(x)}{\mathrm{d}x} = mg\sin\theta - mg\mu\cos\theta + \frac{mg\mu\cos\theta}{\alpha}\left(\frac{E_a}{E_h} - 1\right)$$

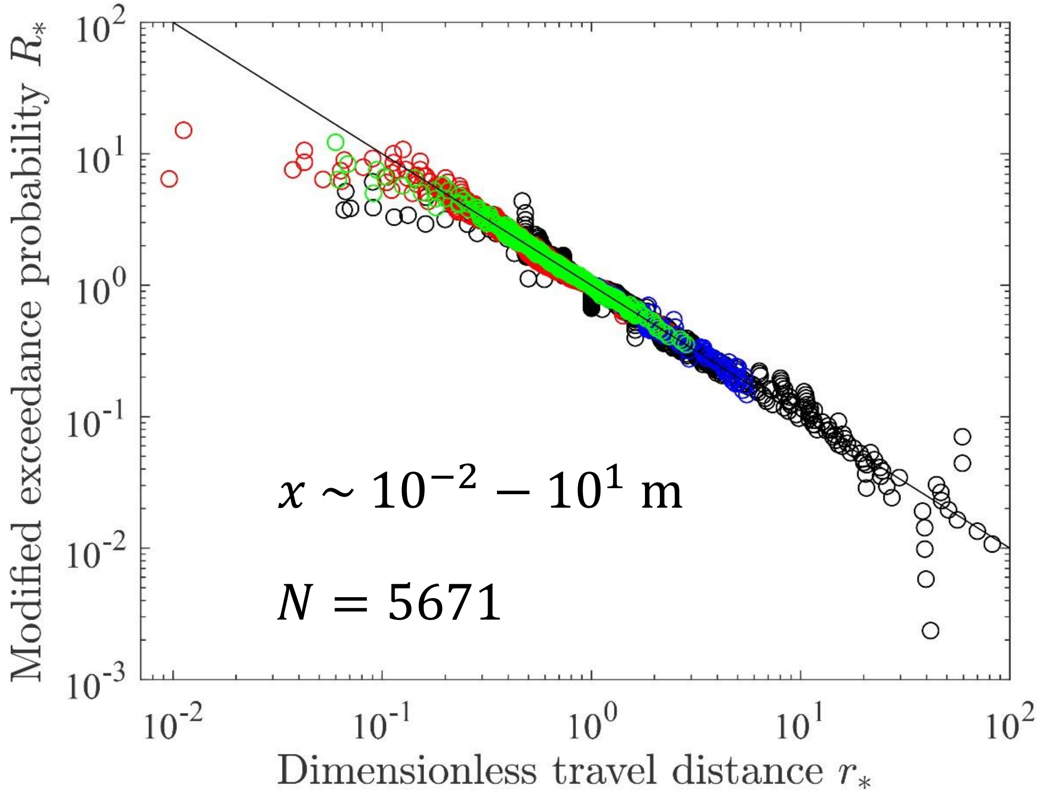
$$\frac{\mathrm{d}N(x)}{\mathrm{d}x} = \frac{Nmg\mu\cos\theta}{\alpha}\frac{\gamma}{E_a}$$

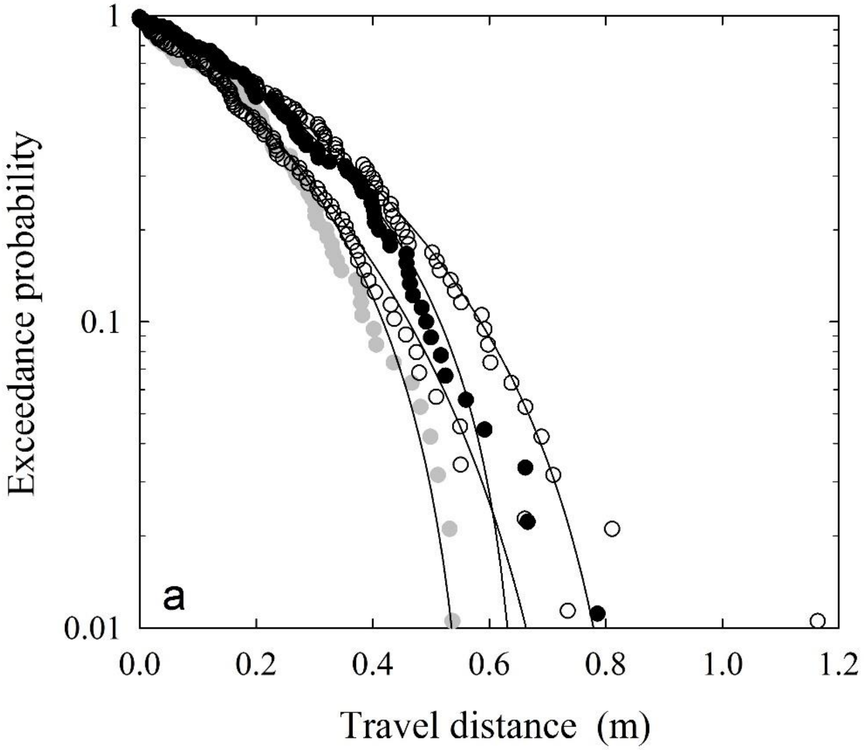
inhomogeneous Poisson disentrainment with NO THRESHOLDS

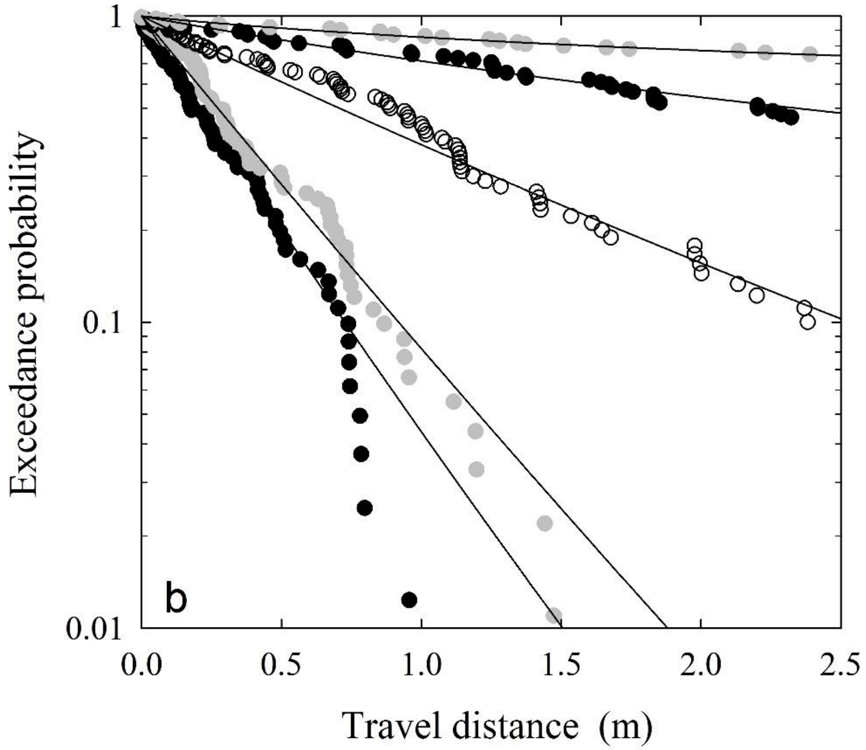


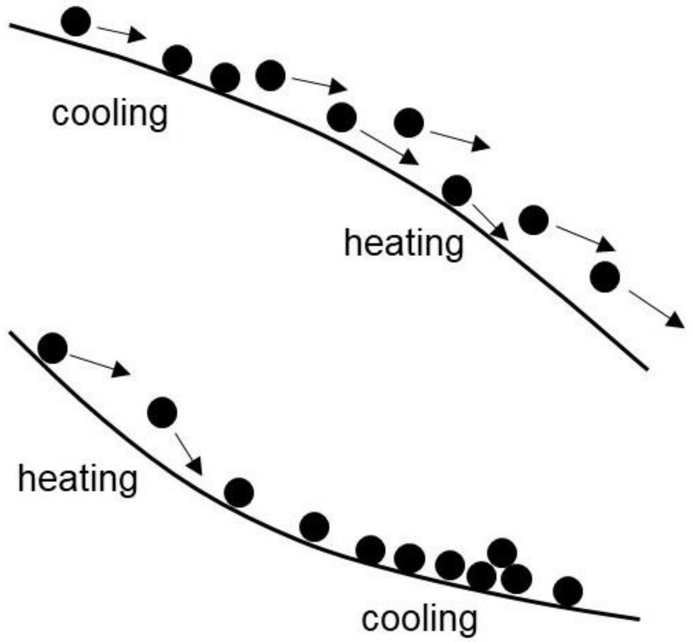

a
Probability density
Dimensionless travel distance
a = -1.1
a = -1
a = -0.8
a = -0.5
a = -0.2
Exceedance probability
Dimensionless travel distance
a = -1.1
a = -0.5
a = -0.2

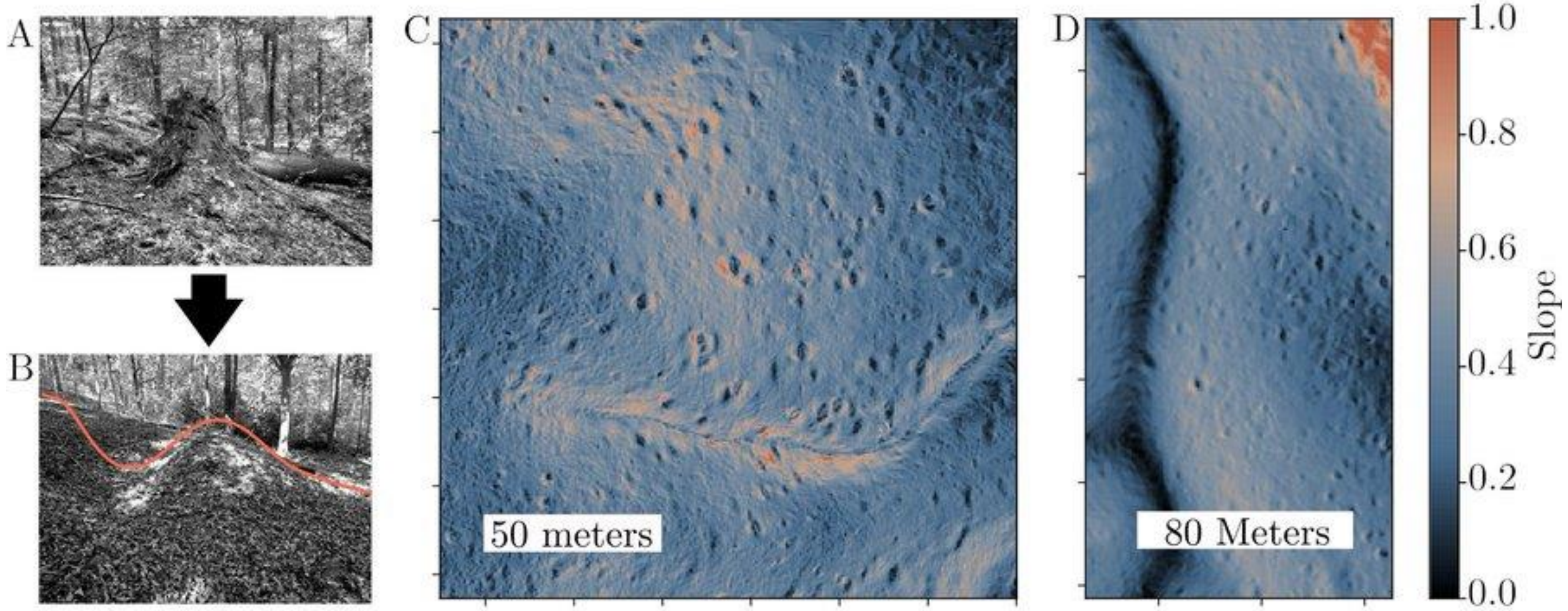

b
Probability density
Dimensionless travel distance
a = 0
a = 2
Exceedance probability
Dimensionless travel distance
a = 2
a = 1
a = -1/2
a = 0

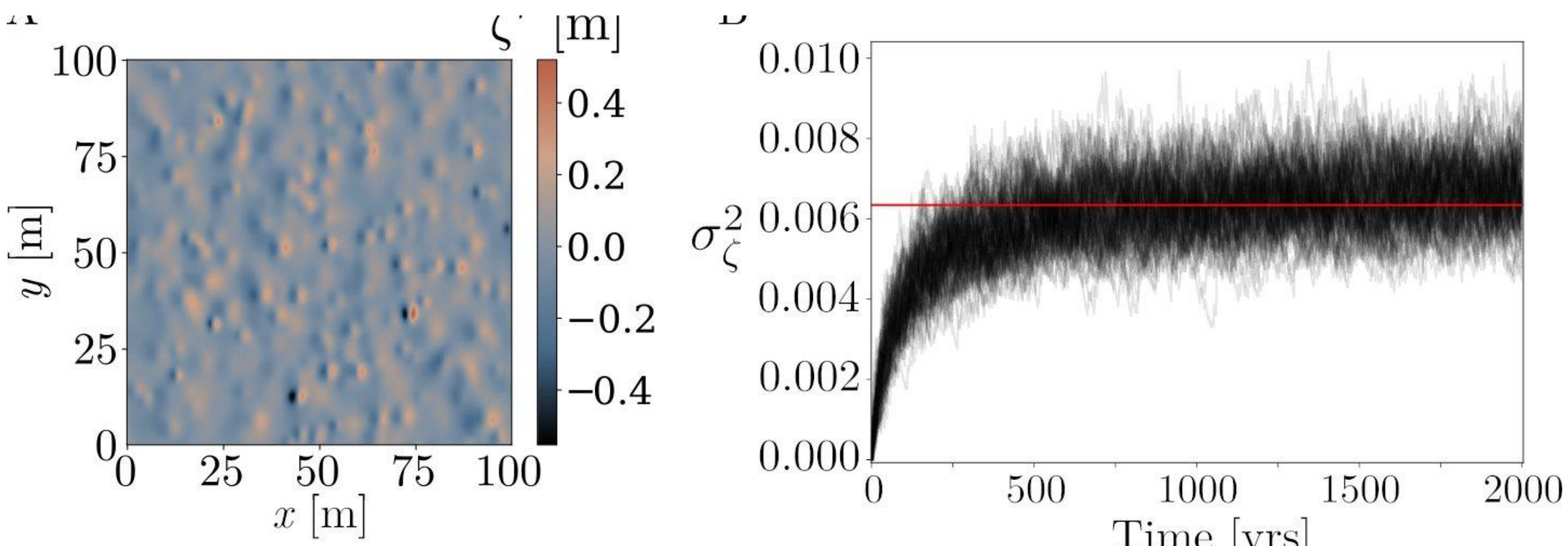

Modified exceedance probability R_*
Dimensionless travel distance r_*
$x \sim 10^{-2} - 10^{1}$ m
$N = 5671$


Exceedance probability
Travel distance (m)
a

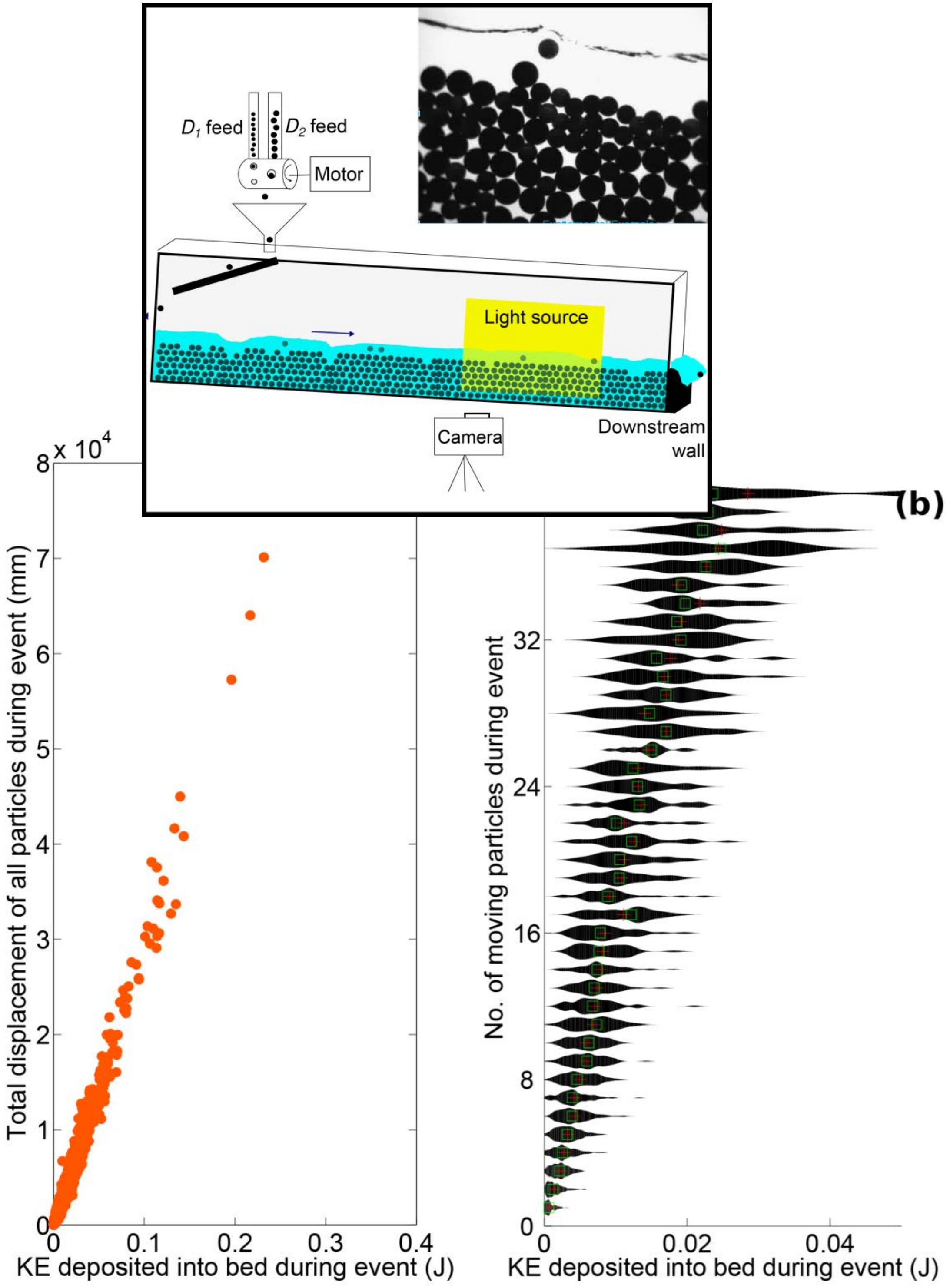

Exceedance probability
Travel distance (m)
b


cooling
heating
heating
cooling





Doane et al. (2021)

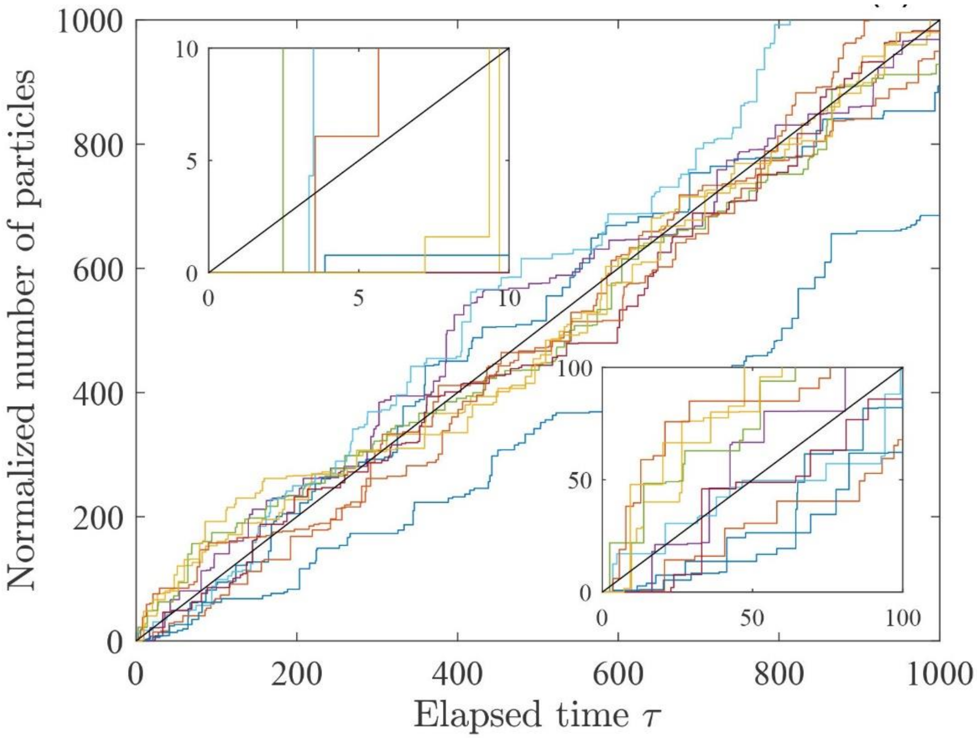


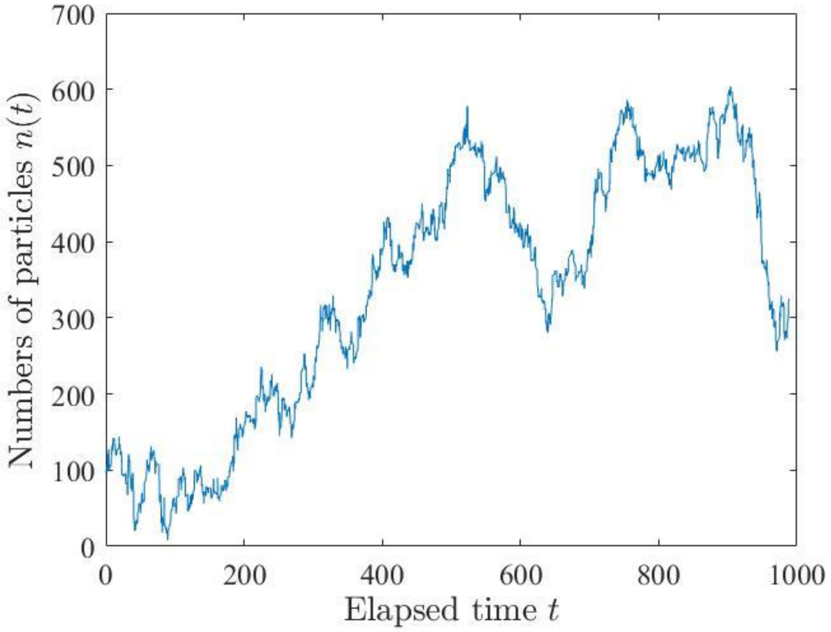
Lee and Jerolmack (2018)

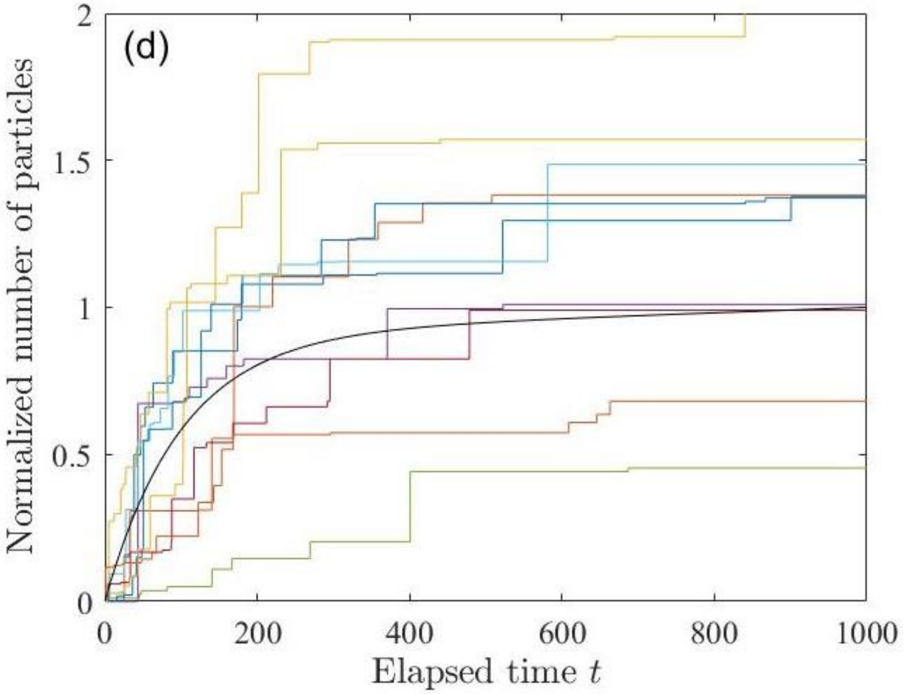
4) Ensemble descriptions of transport

divergence of Poisson fluxes with equal in/out rate constants

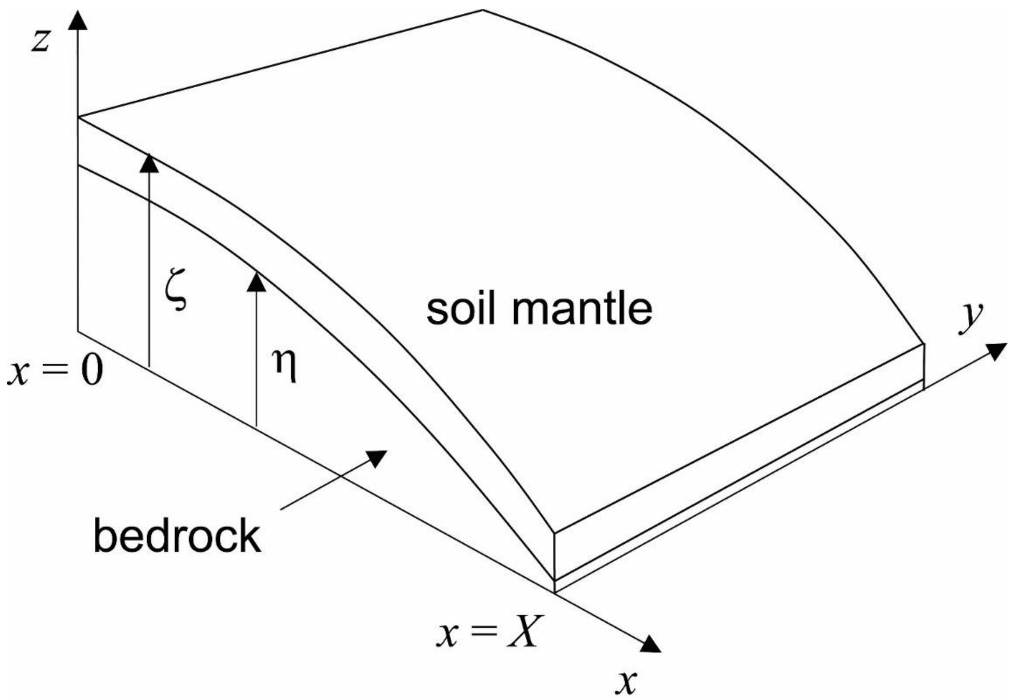
Particularly with rarefied transport conditions – the rule rather than the exception on Earth's Surface – **variations in the flux and its divergence are just as important as the mean or modal values in determining system behavior**. We must examine the relation between ensemble descriptions of transport and descriptions involving deterministic – nominally mean – quantities without neglecting covariances.

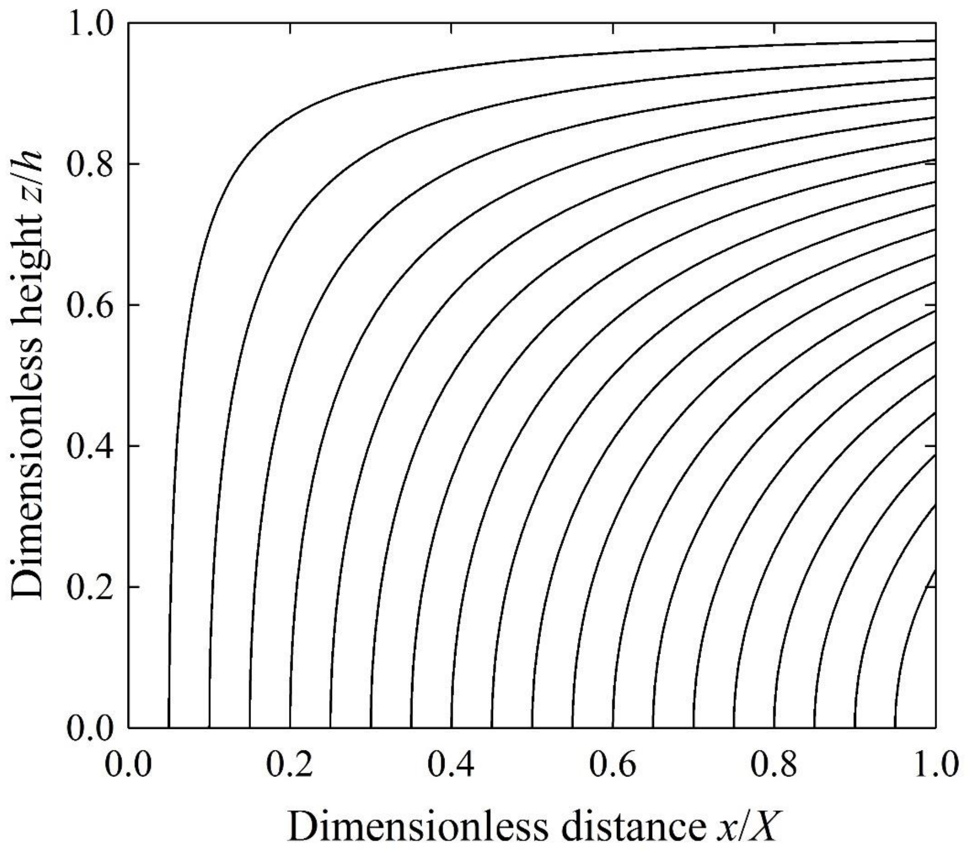


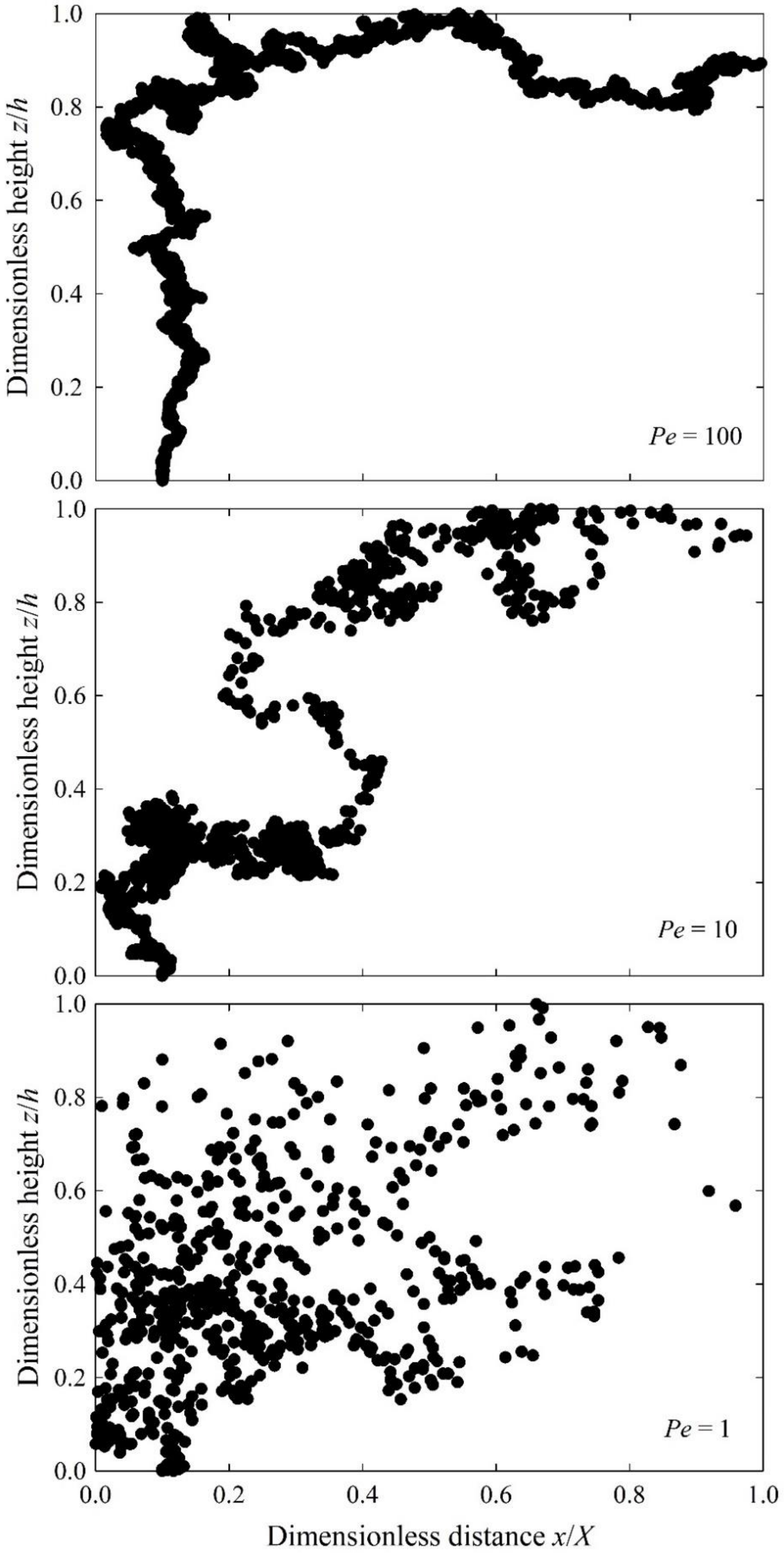


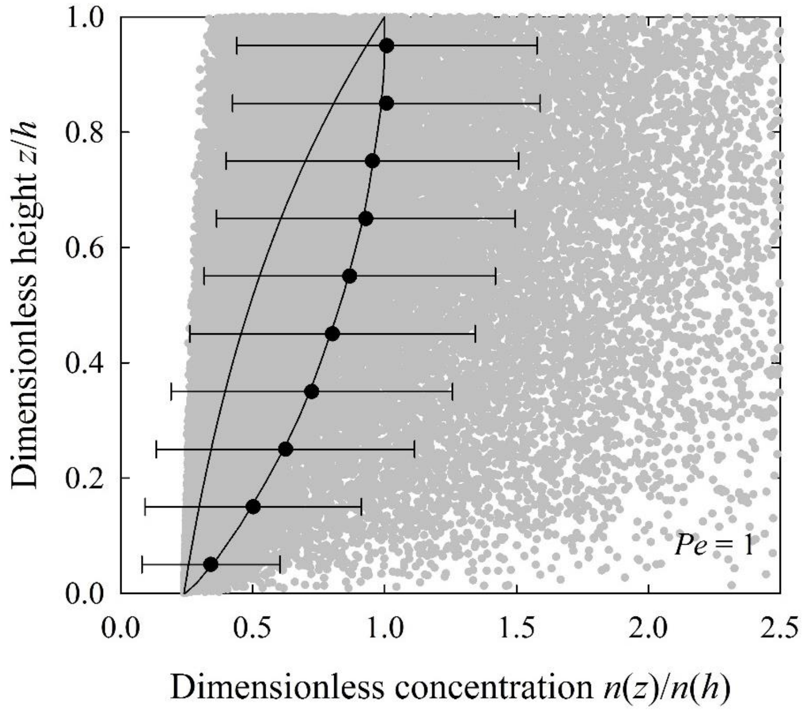


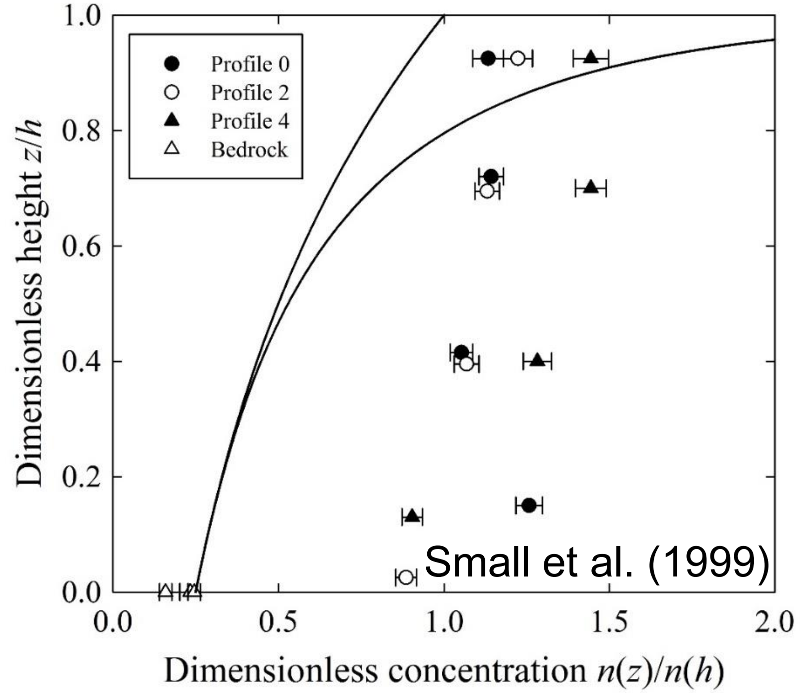
Sediment transport systems are dissipative, but they do not necessarily involve Ornstein-Uhlenbeck (mean reverting) processes







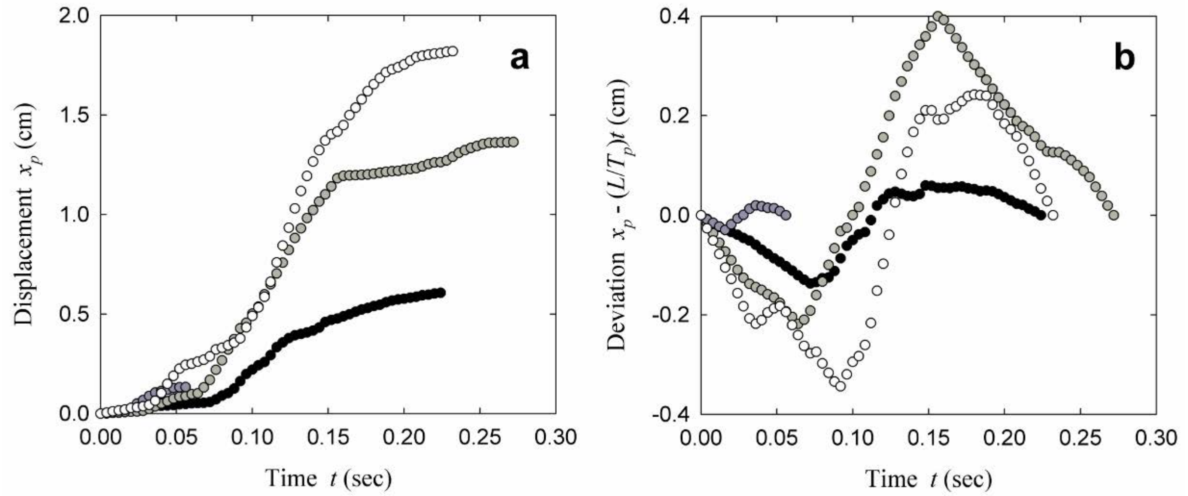


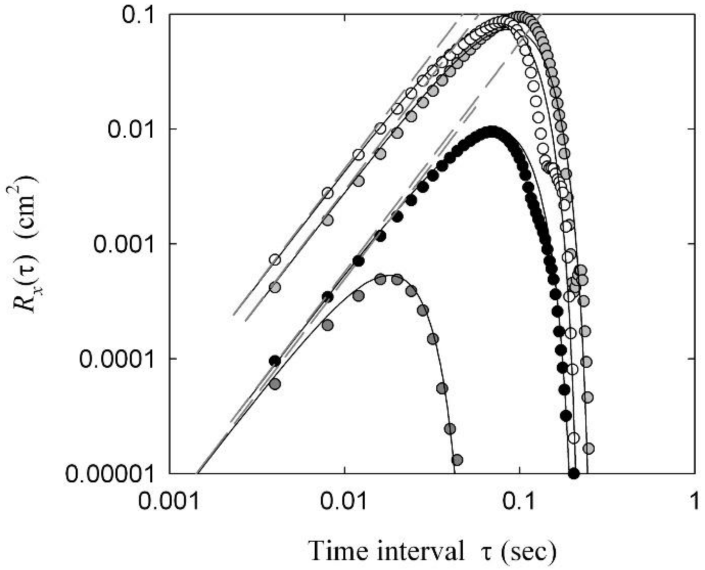


cm^3: $n \sim 10^1 - 10^2$

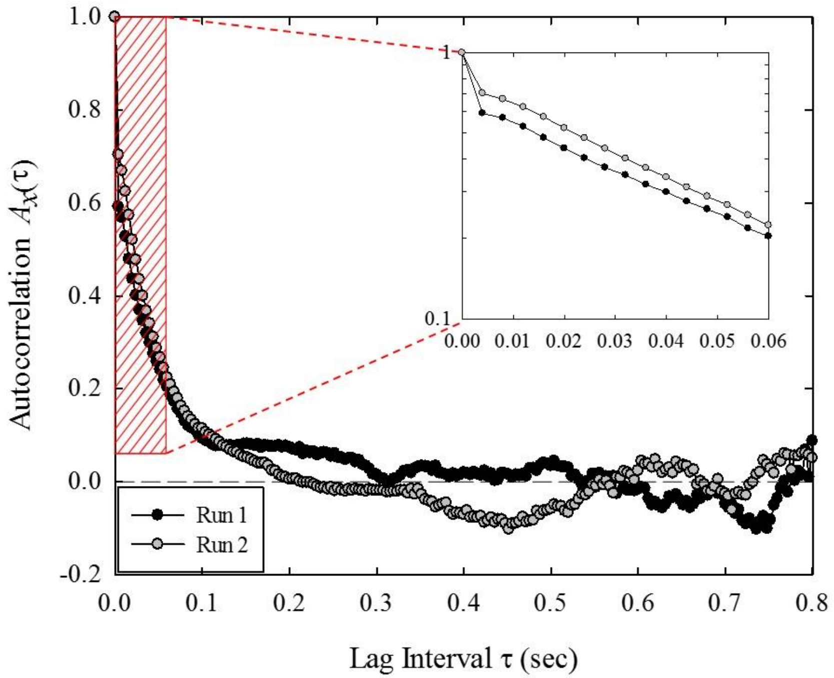
5) Mechanics of particle diffusion

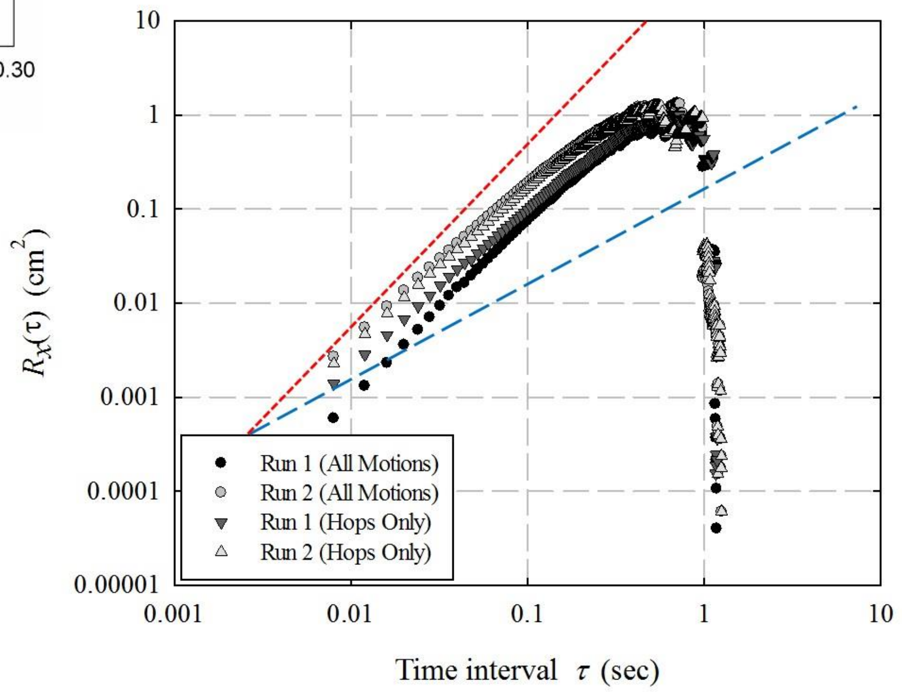
- Previous focus on kinematics, adopting descriptions from kinetic theory
- Non-ergodic behavior
- Bottom-up and top-down controls
- Collision mechanics with right blurring of eyes



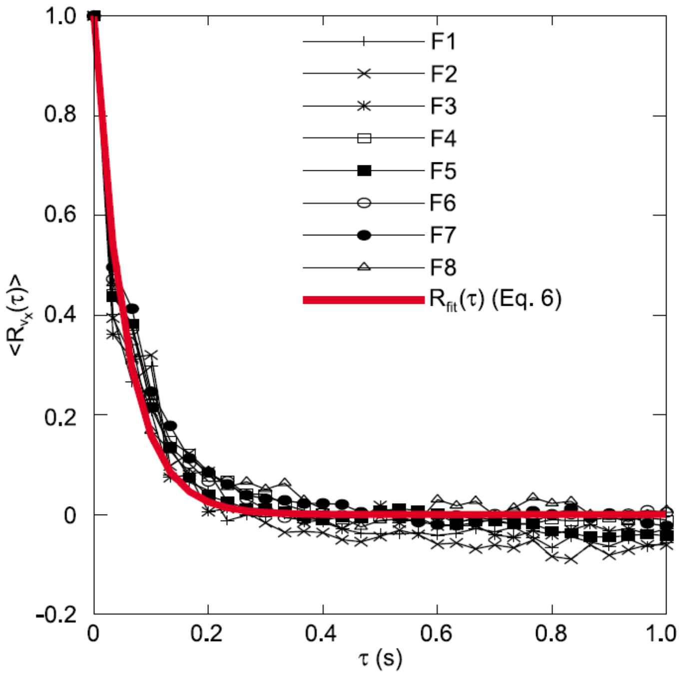


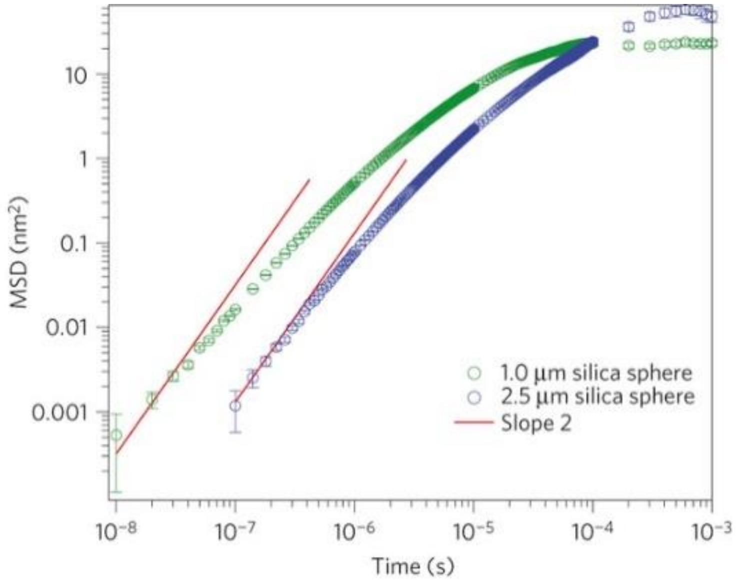
Fathel et al (2016)



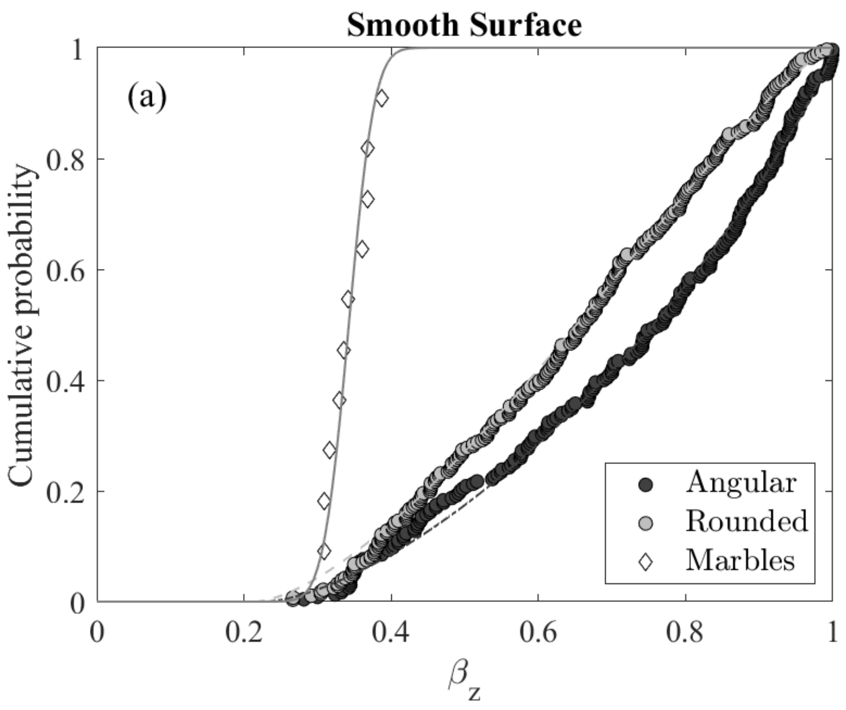


Martin and Jerolmack (2012)

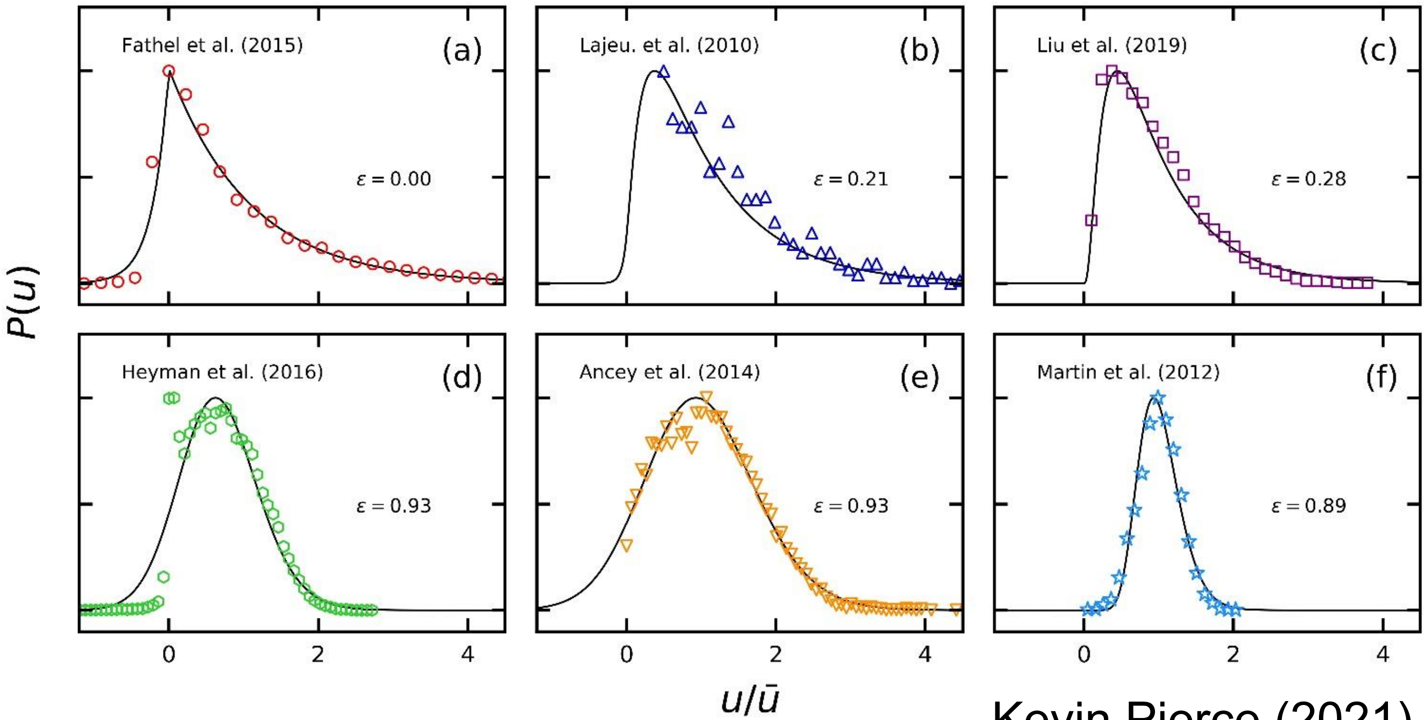




Li et al. (2010)

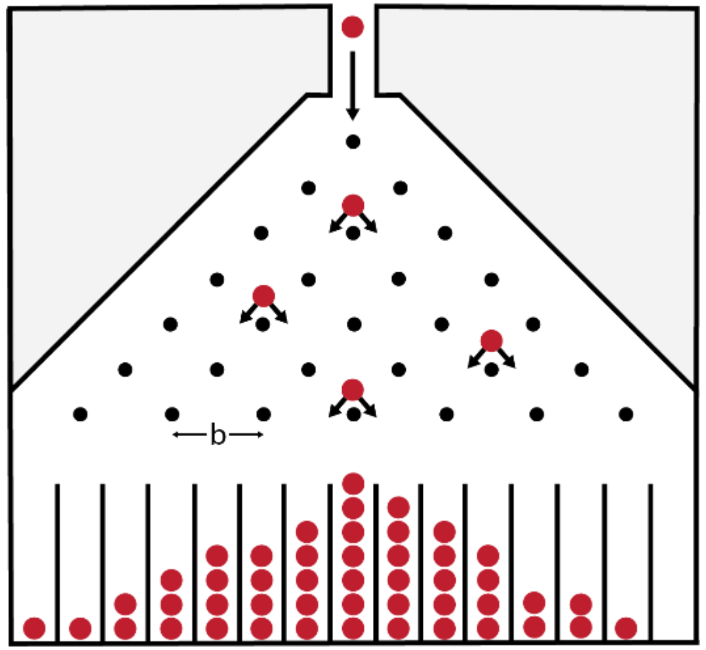


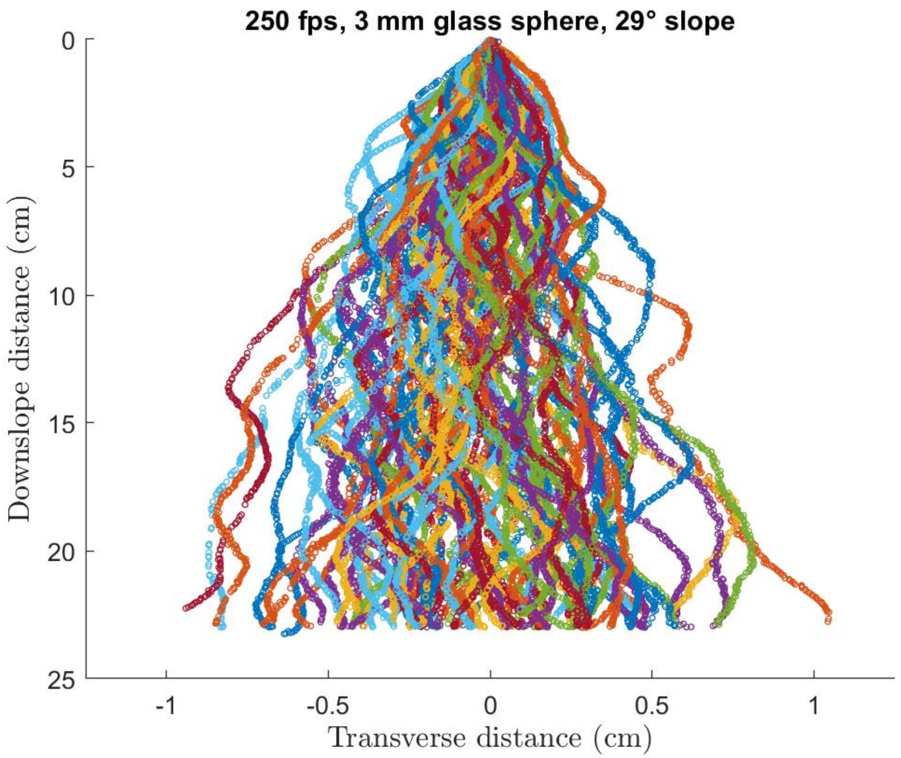
$$\beta_z = -\frac{\Delta E_p}{E_p}$$

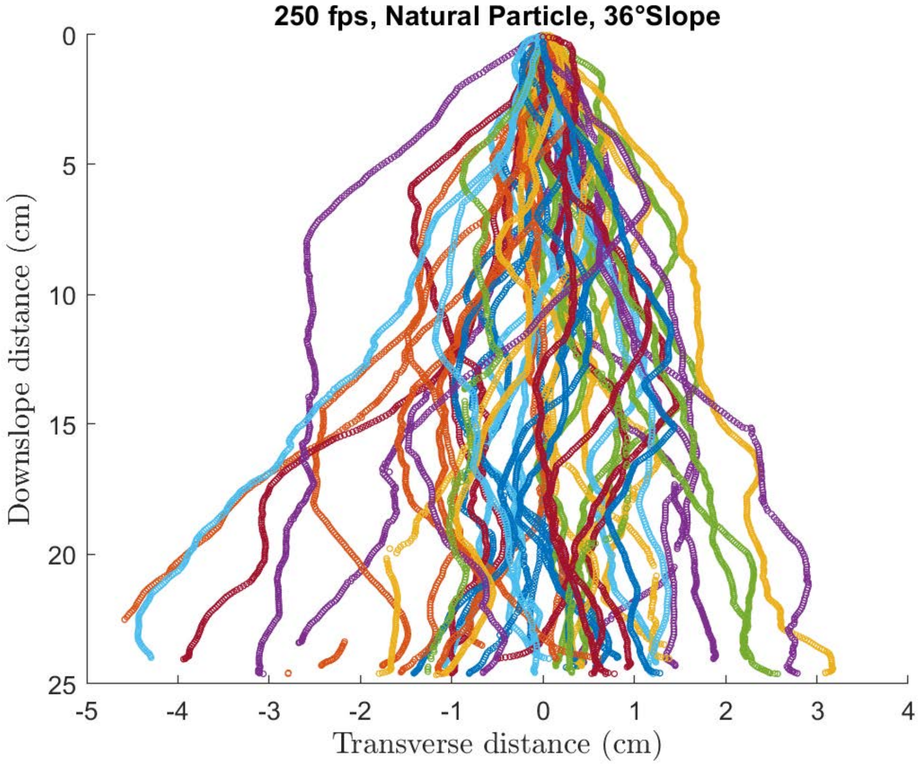


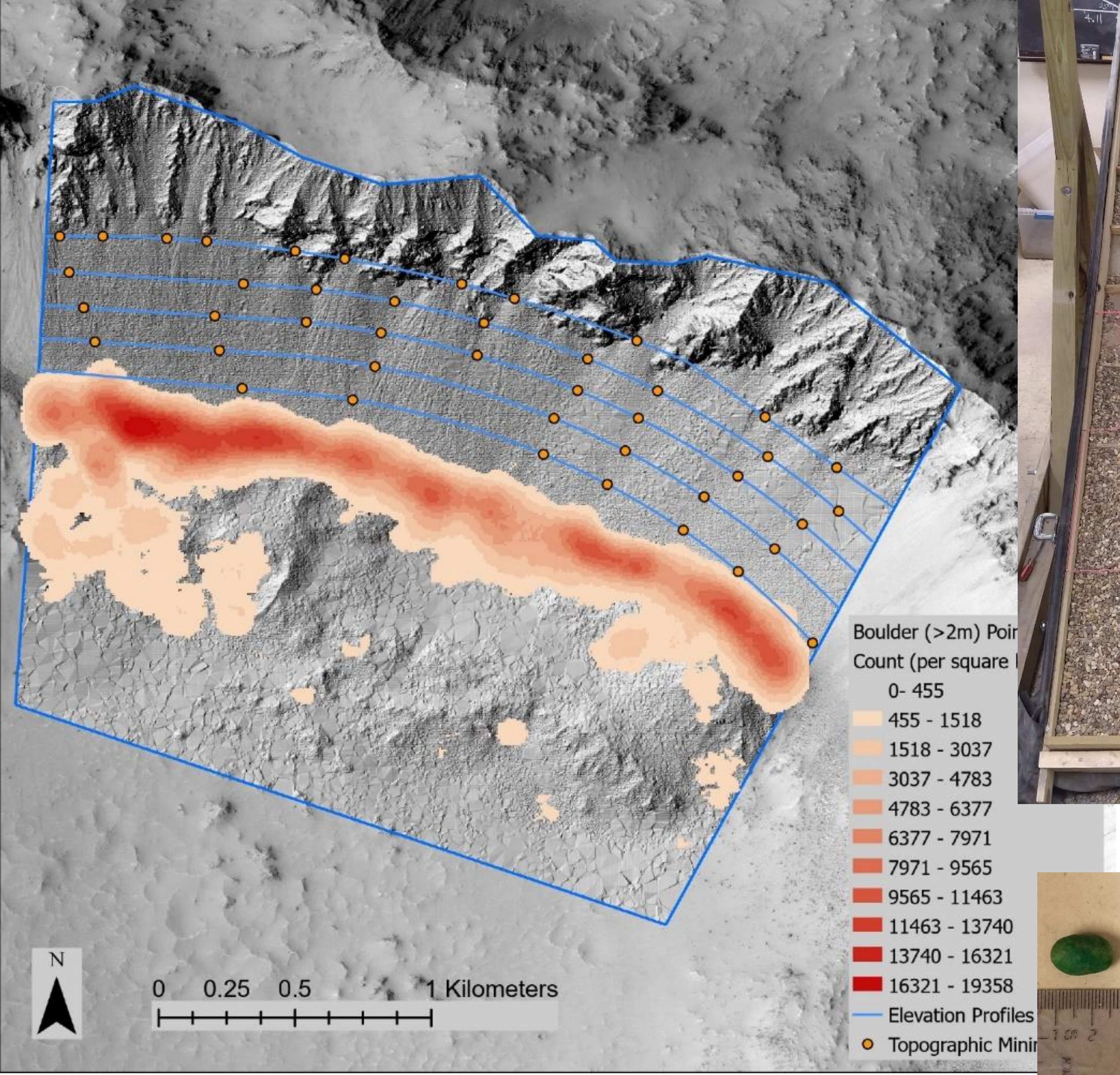
Sarah Williams (in prep)

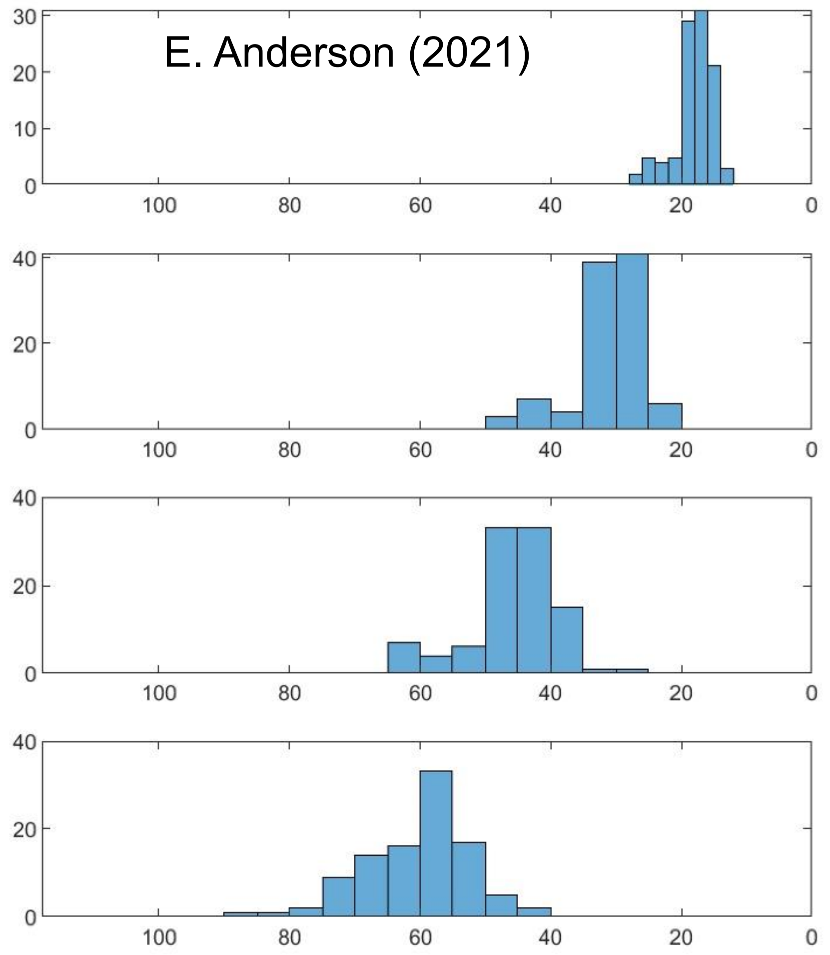
Kevin Pierce (2021)

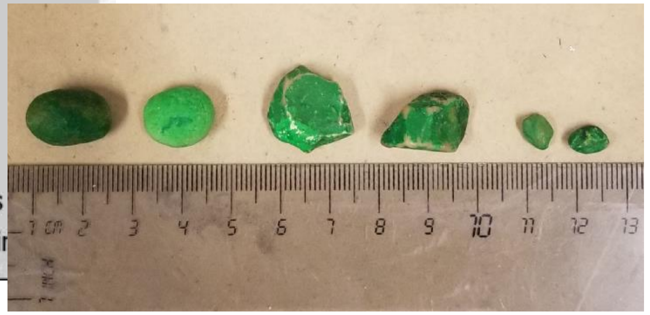




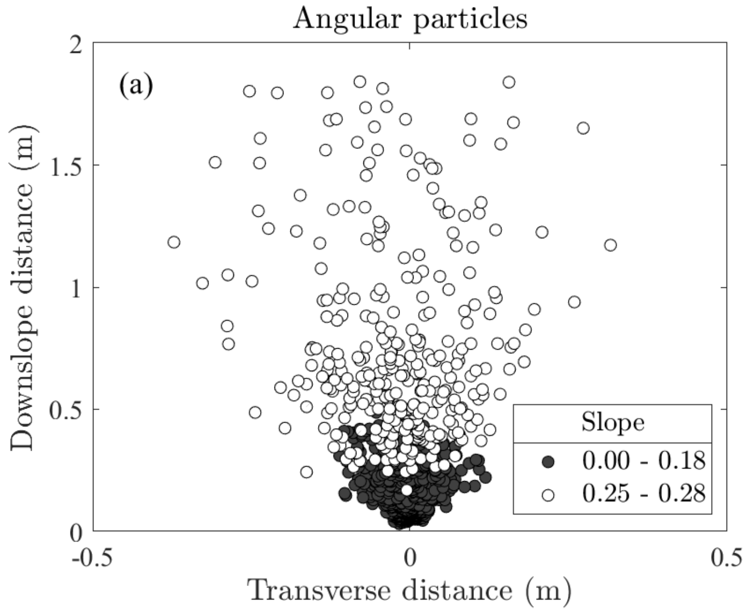






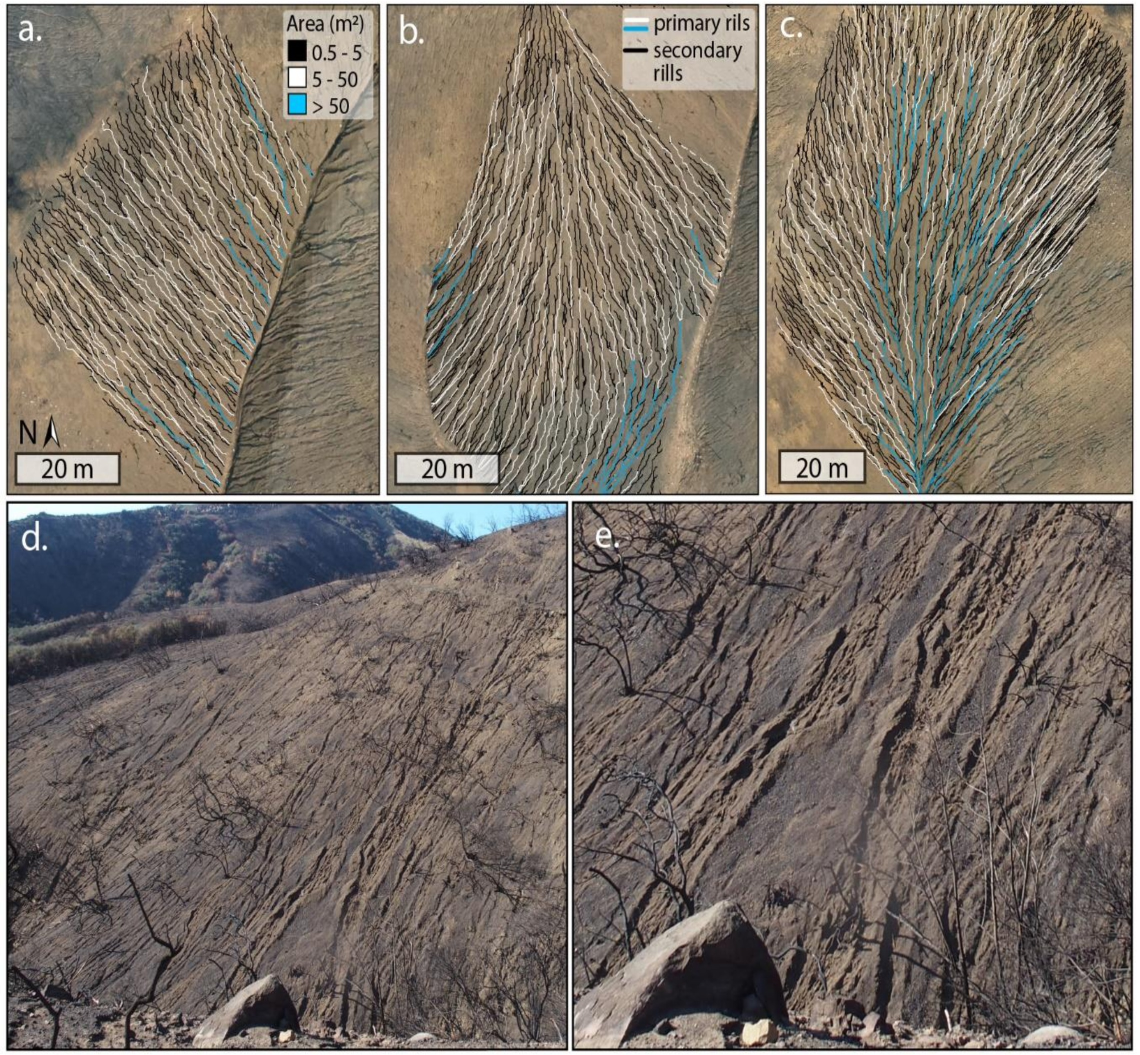


Williams & Furbish (2021)

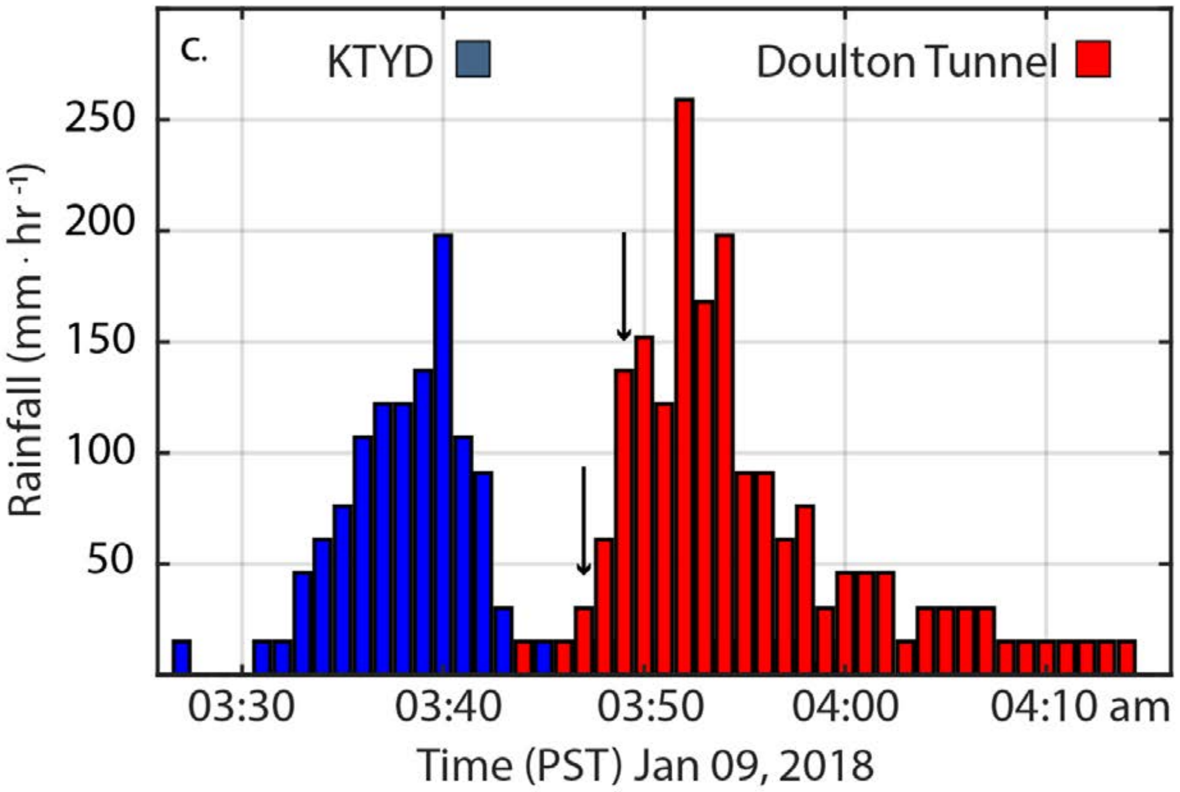


6) Transient surface transport

- We learned from Robert Horton, then seemed to move on
- Much of Earth's terrestrial surface changes due to surface flows and transport
- A transient problem; with extension of time scales of interest, a probabilistic problem



Images: Tom Dunne



$$c_b \frac{\partial \zeta(x,t)}{\partial t} = -E(x,t) + \int_{-\infty}^{x} E(x',t) f_r(x-x';x',t)\mathrm{d}x'$$

- Stability analysis with raindrop induced flow transport
- Patchy, ephemeral flow conditions over long time scales

Statistical physics framework

Overview
- Probabilistic versus deterministic descriptions
- Rarefied versus continuum conditions
- Uncertainty with growing scales

Systems and ensembles

Characteristics of particle motions

Conservation: the Master and Fokker-Planck equations
- Differential forms
- Entrainment form

Advection and diffusion
- Kinematics
- Mechanics

Ergodicity and ensemble behavior
- Statistical equilibrium
- Entropy

Desiderata

Adding the “mechanics” to “statistical mechanics”

Proper dynamics based Knudsen number(s)

Relating ensemble expected behavior to deterministic counterparts

Consequences of ensemble behavior (weather) versus expected behavior (climate)

Extending entropy concepts to larger systems

Mixtures of particle sizes, shapes

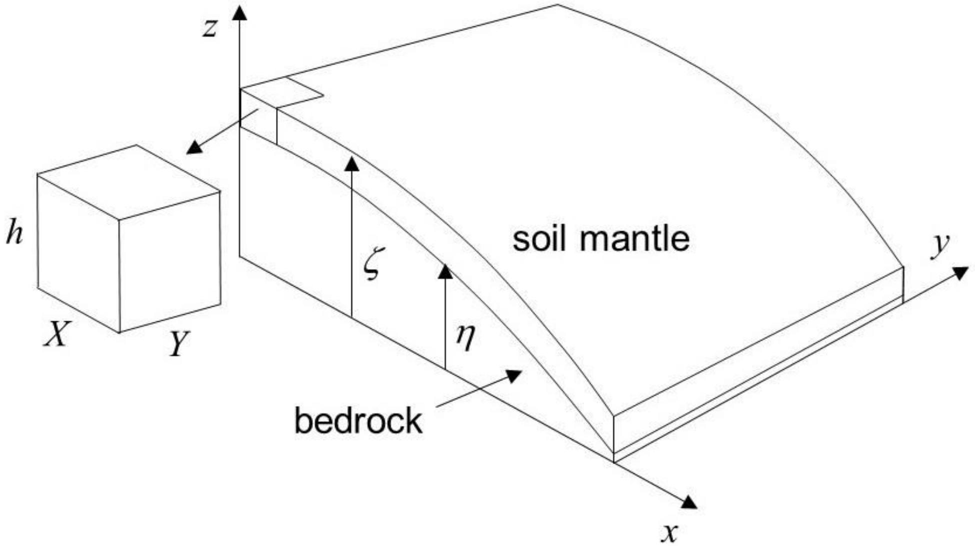
Learning how to formally blur our eyes appropriate to the scales of interest

Further elucidating the common language of transport in hillslope and river systems

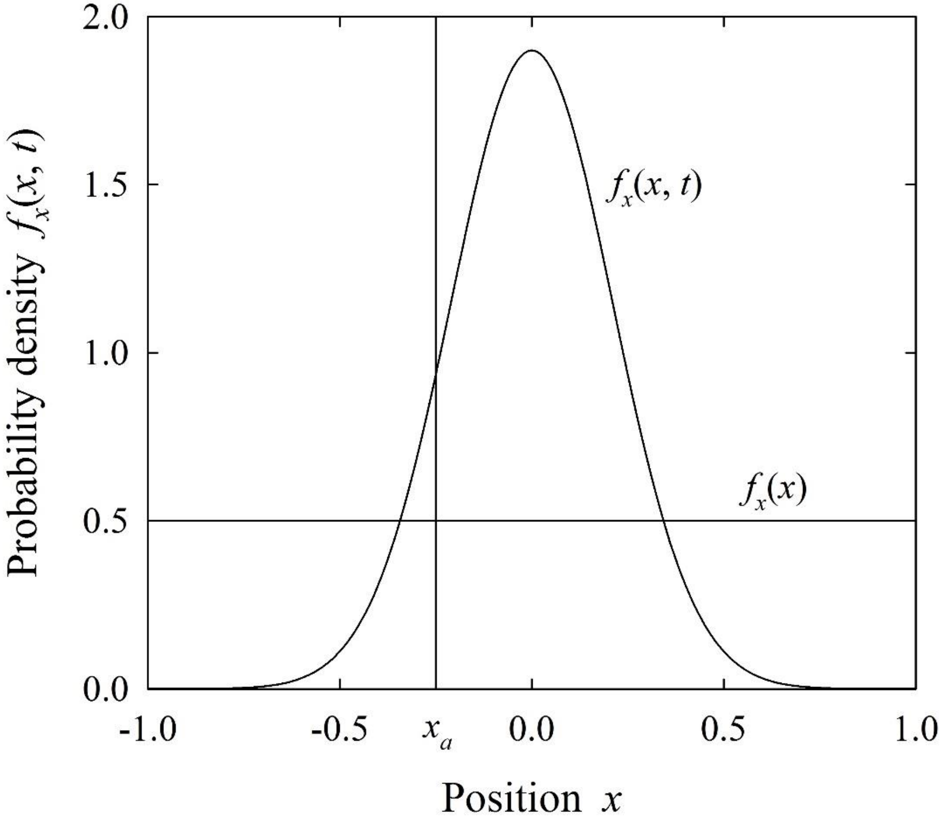
“Aristotle said a bunch of stuff that was wrong. Galileo and Newton fixed things up. Then Einstein broke everything again. Now, we’ve basically got it all worked out, except for small stuff, big stuff, hot stuff, cold stuff, fast stuff, heavy stuff, dark stuff, turbulence, and the concept of time.”

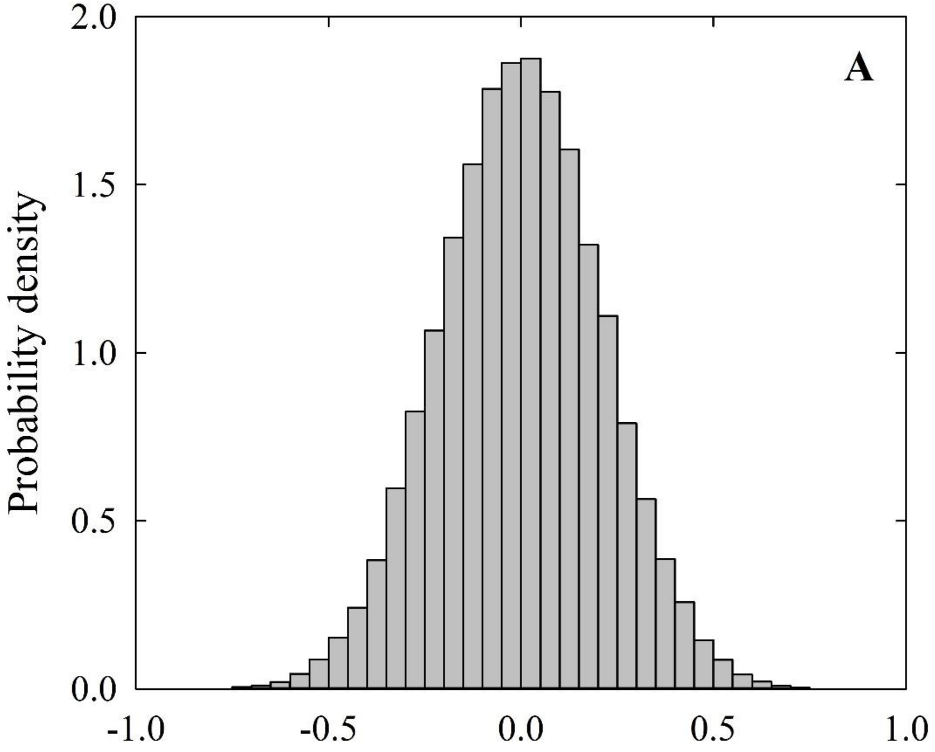
Zach Weinersmith, *Science: Abridged Beyond the Point of Usefulness*

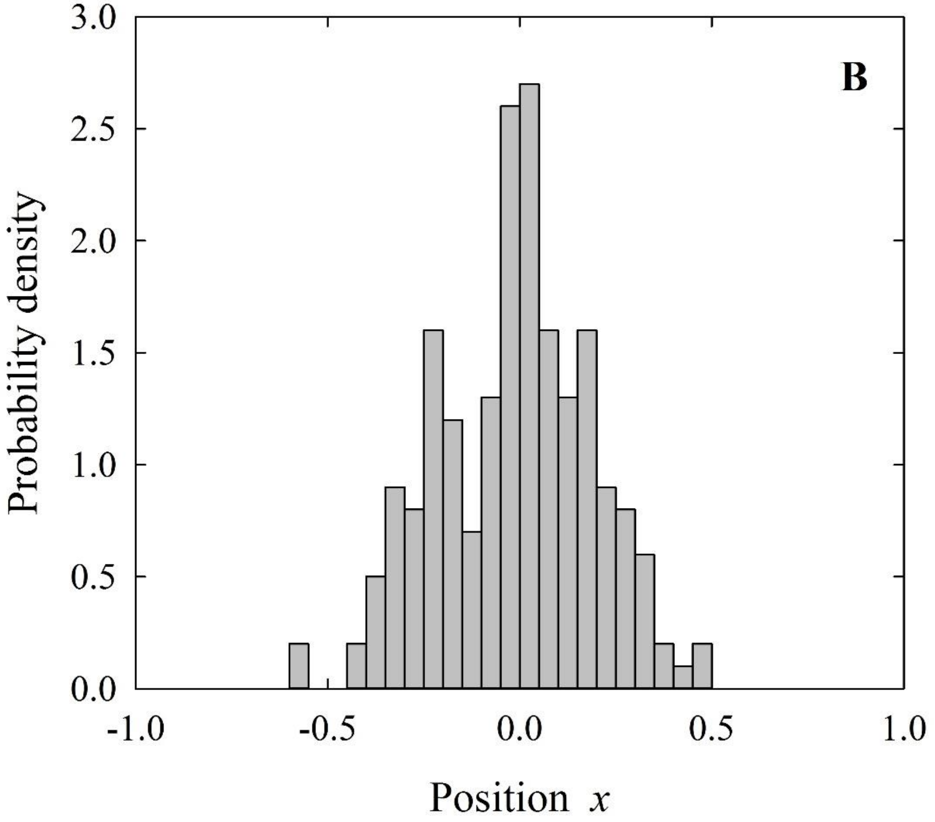
NASA

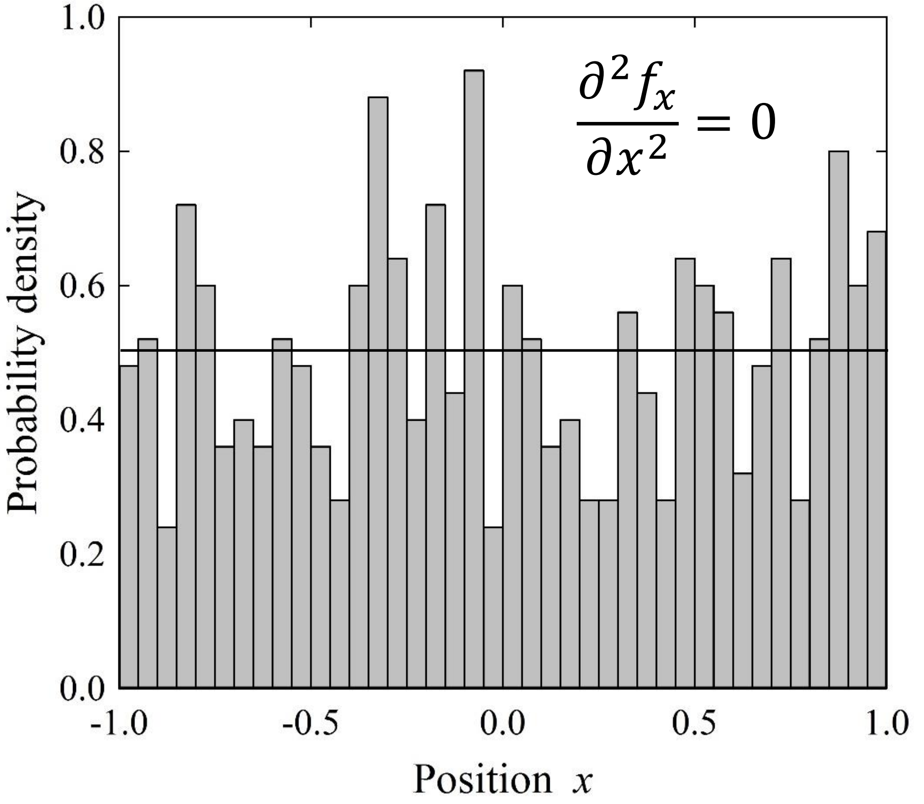


$$\frac{\partial f_x(x,t)}{\partial t} = \kappa_x \frac{\partial^2 f_x}{\partial x^2}$$









Distinction:

continuum

versus

continuum-like