Communicating quality: a unified model of disclosure and signaling

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ABSTRACT

Firms communicate product quality to consumers through a variety of channels. Economic models of such communication take two alternative forms when quality is exogenous: 1) disclosure of quality through a credible direct claim; 2) signaling of quality via producer actions that influence buyers’ beliefs about quality. In general, these two literatures have ignored one-another. We argue that firms should be viewed as choosing which means of communication they will employ. We show that integration of these two alternatives leads to new implications about disclosure, signaling, firm preferences over type, and the social efficiency of the channel of communication employed.

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1. Introduction

For many products, consumers are unable to observe quality directly prior to purchase, but a firm knows the quality of the product it provides (its “type”). Firms communicate product quality attributes to consumers through a variety of channels, such as pricing, advertising, releases of research reports and test results, or warranties and returns policies. The modeling of such communication takes on one of two alternative forms when quality is exogenous: 1) disclosure of quality through a credible direct claim; 2) signaling of quality via producer actions that influence buyers’ beliefs about quality. Examples of the first form of communication would involve disclosure via the use of an independent auditing process with public announcements of what quality was found to exist, or advertising in the presence of truth-in-advertising laws with high penalties for misrepresentation. Examples of the second form of communication include posting prices that consumers might use to infer quality, or advertising in unregulated environments where lost future sales due to misrepresentation can provide incentives for truthfulness.

Both approaches have generated extensive literatures that deal with firms selling products whose quality is determined exogenously, known to the firms themselves, but not observable to consumers prior to purchase. Remarkably, there has been little communication between these two approaches and yet they are intimately related. In this paper we argue that disclosure and signaling are two sides of a coin and that a firm should be viewed as choosing which means of communication it will employ. Moreover, we show that integration of these two alternatives leads to a number of new implications about disclosure, signaling, firm preferences over type, and the social efficiency of the channel of communication employed.

Why have these approaches remained so distinct? The disclosure literature invariably assumes that marginal cost is independent of quality, which renders separation via signaling
impossible. Thus, for these models, non-disclosure is consistent with all non-disclosing types charging the same price; these types pool. On the other hand, few signaling models of product quality assume that products of different quality are equally costly to produce. Most signaling models assume that higher-quality products are more costly to produce, though some are agnostic on the issue, allowing higher-quality products to be either more or less costly. This difference in costs typically allows the price chosen by the firm to reveal its product’s quality.

We argue that the alternative to disclosure should not be viewed as “non-disclosure,” but rather as revealing type via other channels such as price; this modification of perspective alters a number of previously-developed results. We focus on the case wherein higher quality is associated with higher marginal costs of production, and we analyze a continuum–of-types model. We show that a firm with the lowest-quality product will not disclose for any positive disclosure cost, since it obtains its full-information profits in a separating signaling equilibrium. A firm with a higher-quality product will need to distort its price in order to signal its true quality (and this distortion increases with an increase in the true quality of the product) and thus, in a separating equilibrium, its signaling equilibrium profits will be less than those under full information. We then show that there is a level of the cost of disclosure that induces a sufficiently high-quality firm to choose disclosure over signaling. Therefore, for this intermediate level of disclosure cost, some types reveal quality via disclosure while other types reveal quality via signaling. We show that overall profits, as a function of type, are “U-shaped” in that both the lowest-quality and highest-quality types of firm have higher profits than those for intermediate types. Moreover, if disclosure costs are “moderate” (to be made precise below), then the highest-quality types make the highest profits; this partly resolves a dilemma in much of the price-quality signaling literature wherein higher profits are
associated with lower quality, suggesting that firms would prefer to produce lower-quality products.\textsuperscript{7}

A welfare analysis of voluntary disclosure therefore focuses \textit{not} on how much information is ultimately revealed, \textit{but whether it is revealed through the socially-optimal channel}. In a separating signaling equilibrium each type of firm (except the lowest) charges a higher price and sells less output than it would under full information (\textit{a fortiori}, this is less than the socially-optimal output). Since the firm considers its own profit increase from disclosure, but not the value of the additional output to consumers, there will be a range of disclosure costs for which the firm inefficiently chooses to signal rather than disclose. A mandatory disclosure rule may be surplus-increasing as it results in a price reduction and an increase in output produced, but it involves more disclosure than is socially-efficient. We describe a simple, decentralized subsidy policy that induces socially-efficient voluntary disclosure (given equilibrium pricing) at the lowest cost to society. Finally, while the monopolist in our analysis engages in insufficient disclosure, the classical disclosure analysis finds that the monopolist engages in excessive disclosure. A modified version of our model allows us to understand what assumptions drive this result.

**Related literature.** In this subsection, we briefly describe how this paper relates to its closest antecedents.\textsuperscript{8} Several previous papers develop models in which price signals product quality; such a model will appear as part of our analysis, but it will be augmented with a disclosure decision. Bagwell and Riordan (1991) provide a two-type monopoly model wherein high quality is more costly to produce.\textsuperscript{9} The signaling portion of our current paper is based on the model in Daughety and Reinganum (1995), which linked liability to the relationship between a firm’s full marginal cost (i.e., production plus liability costs) and safety. Daughety and Reinganum (2005) provide a model
with a continuum of types, unit demand and marginal costs that increase in quality; in that paper, the firm can commit to a disclosure policy before it learns its type, but cannot not make a disclosure after learning its type (it signals quality via price). In the current paper, we model the disclosure decision as occurring after the firm has learned its type, consistent with the usual timing in the disclosure literature.\textsuperscript{10}

Some of the earliest product-quality disclosure literature (e.g., Grossman, 1981; and Milgrom, 1981) assumed that disclosure was costless. In this case the unique equilibrium involves complete disclosure. To see why, notice that incomplete disclosure pools multiple firm types, and consequently consumers’ willingness to pay is based on the average quality in the pool. If disclosure is costless, then the highest-quality type in the pool can increase consumers’ willingness to pay, and hence its profits, by defecting to disclosure; this thought experiment continues until all types are disclosed (this is often referred to as “unraveling”). Other models (e.g., Viscusi, 1978; Jovanovic, 1982; and Levin, Peck and Ye, 2005) have assumed positive disclosure costs. These models find that high-quality types engage in disclosure, as do we. However, in these models disclosure is typically socially excessive, while we find that voluntary disclosure is insufficient.

One reason that the classical disclosure models find excessive disclosure is because those models assume that consumers have unit demand with a common (and known) reservation price, in which case disclosure is essentially redistributive; while there may be a private value associated with disclosure, there is no social value. In Section 4, we compute a version of our model in which costs are unresponsive to quality (so that only a pooling equilibrium exists among non-disclosing firms) in order to determine circumstances under which disclosure is socially excessive or insufficient. When demand is downward-sloping and costs are unresponsive to quality, then non-disclosing firms
charge a common “pooled” price. Disclosure by a firm with relatively high quality would expand that firm’s demand (for any given price) and allow it to raise both its price and its output. This is privately beneficial, but it need not be socially beneficial, for two reasons. First, consumer’s surplus may fall; second, those firm types remaining in the pool are adversely affected by the consumer’s revised beliefs about the expected quality in the pool. We characterize circumstances under which this model specification results in excessive (or insufficient) disclosure. We further show that disclosure is excessive for a larger portion of the parameter space when demand is less elastic. Thus, the classical model’s typical finding of excessive disclosure is driven by both effects: price pooling and inelastic demand. In our main model, with downward-sloping demand and quality-sensitive costs, disclosure by a relatively high-quality type allows that type to lower its price and expand its output without affecting the non-disclosing types’ profits. Since the social gain from this increase in output exceeds the private gain, some types inefficiently fail to disclosure.

The only published paper of which we are aware that involves both disclosure and signaling of quality is Fishman and Hagerty (2003). However, in their model disclosure and signaling are not substitutes (as they are in our model), but rather are complements due to an externality between different types of consumers. They assume two different types of (unit demand) consumer; one type becomes “informed” about quality when a disclosure is made, while the other remains “uninformed” about actual quality, but is aware that a disclosure has been made. Suppose that most consumers are capable of becoming “informed.” Then a firm which makes a disclosure and charges a high price will only do so if it is a high-quality firm, for if it were a low-quality firm charging a high price, it would alienate all of the informed consumers. So an uninformed consumer who knows that a disclosure was made (but not its content) can infer high quality from a high price. Thus, signaling
does not accompany non-disclosure in their model (because they maintain the crucial assumption that marginal costs are the same for high- and low-quality products); it can only accompany disclosure. Our model is very different from theirs in that we assume only one type of consumer (all of our consumers become informed about quality when the firm discloses it); moreover, our consumers have downward-sloping demand. Our firm has a marginal cost that is increasing in quality. Thus, if a firm does not disclose its quality directly, it reveals it through its price: disclosure and signaling are substitutes.

Finally, two tangentially-related papers also address the choice of signaling versus disclosure. Cai, Riley and Ye (2007) examine an auction wherein a seller’s (privately-known) value is correlated with the bidders’ values. In the working paper version, the authors show that (assuming costly credible disclosure) higher-value sellers will disclose while lower-value sellers will signal via the reserve price. In Bernhardt and Leblanc (1995) a firm with an investment project can either signal its value to the capital market through its debt contract or disclose this value directly. Here the “cost” of disclosure is that it reveals not just the project’s value, but sufficient information for a competitor to enter the relevant market and undermine the project’s value to the disclosing firm.

**Plan of the paper.** In Section 2, we describe the notation and provide the results of the analysis when firms can choose between disclosure via a (costly) credible statement and signaling via pricing. We also show that the firm with access to both disclosure and signaling has “U-shaped” profits with respect to its type, and discuss the implications of this result. In Section 3 we characterize socially-optimal disclosure, assuming the firm retains control of the pricing decision; we find that insufficient disclosure occurs in equilibrium. We describe a decentralized subsidy
scheme that induces socially efficient use of communication channels by the firm, but leaves the firm’s payoff unchanged. Section 4 modifies our model to show that the classical disclosure literature’s excessive-disclosure result occurs because of a combination of two assumptions: 1) that the marginal cost of production is independent of the level of quality and 2) that demand is modeled as sufficiently inelastic. Section 5 summarizes our results and suggests extensions.

2. Equilibrium disclosure and signaling

Model setup.

Quality. We assume that a single firm produces a product whose quality, $\theta$, is unobservable to consumers prior to purchase. Quality here is the probability that the consumer is completely satisfied with a unit of the product. We capture this by assuming that $\theta \in [\underline{\theta}, \bar{\theta}]$, with $0 < \underline{\theta} < \bar{\theta} < 1$; thus, the lowest type is $\underline{\theta}$ while the highest type is $\bar{\theta}$. Further, assume that the consumer’s prior belief about $\theta$ is that it is distributed according to a continuously differentiable distribution function, $G(\bullet)$, with positive density, $g(\bullet)$, on the interval $[\underline{\theta}, \bar{\theta}]$. Thus, any type of product may be completely satisfactory in a given instance of its consumption, but any type of product may also disappoint in a given instance; the key attribute is that higher-quality products are more likely to be satisfactory. This “consumer satisfaction” interpretation of quality is useful in that “satisfaction” is non-verifiable, and thus the firm cannot offer a warranty of the form: “your money will be refunded if you are not completely satisfied with the product,” as this would introduce moral hazard on the part of the consumer, who would always claim, *ex post* of consuming the product, that she was disappointed.12

Consumers. All types of product provide utility to the consumer, but a unit that is completely
satisfactory provides greater utility. In particular, we assume that the consumer’s utility is quadratic in the quantity consumed of the product of interest, with the coefficient on the quadratic term denoted $\beta$, and the coefficient on the linear term denoted $\alpha$ in the case of a satisfactory unit and $\alpha - \delta$ in the case of a disappointing unit, where $\beta > 0$ and $\alpha > \delta > 0$. The consumer is unable to observe directly the product’s quality before purchase. Let the perceived quality of the good be denoted $\tilde{\theta}$.

If the firm discloses the quality before purchase, then $\tilde{\theta} = \theta$; on the other hand, if the product’s quality is not disclosed, then these perceptions will be determined as part of a perfect Bayesian equilibrium wherein the firm’s strategy is its price.

The consumer’s utility function is quasi-linear in all other goods; thus, if the price of the product is $p$, the consumer’s income is $I$, and she consumes $q$ units of perceived quality $\tilde{\theta}$, then her utility is given by:

$$\begin{align*}
U(p, q, \tilde{\theta}) &= (\alpha - (1 - \tilde{\theta})\delta)q - \beta(q)^2/2 + I - pq.
\end{align*}$$

Thus, the consumer’s demand for the product of perceived quality $\tilde{\theta}$ is given by:

$$q(p, \tilde{\theta}) = (\alpha - (1 - \tilde{\theta})\delta - p)/\beta.$$
associated with buying multiple units on one trip outweigh the value of updating associated with experimentation with individual units. Examples of goods with these features include light bulbs, courses in a restaurant meal, food items such as melons or meats, clothing items, wine, and investment recommendations from a financial advisor.

Alternatively, a downward-sloping aggregate demand curve can also be generated by accumulating unit demand functions of consumers with heterogeneous reservation prices. To see this, assume that there is a continuum of heterogeneous consumers of measure $N$, each with unit demand, but with reservation prices distributed uniformly on $[0, V]$. A consumer with reservation price $v$ will buy the product if $v > p + (1 - \bar{\theta})\delta$. Thus aggregate demand can be written as $Q(p, \bar{\theta}) = (N/V)(V - (1 - \bar{\theta})\delta - p)$, which is exactly of the same form as that derived for the representative consumer above. Thus a demand model as shown above (either characterizing a representative buyer or an aggregate of individuals) reasonably captures the essence of the problem at hand.

The firm. The firm of type $\theta$ manufactures units of the product at a constant unit cost of $k\theta$, with $k > 0$, so that marginal cost is increasing in $\theta$. The gross profits for the firm depend on its true price-cost margin and consumer demand, which depends on perceived quality:

$$\pi(p, \theta, \bar{\theta}) = (p - k\theta) (\alpha - (1 - \bar{\theta})\delta - p)/\beta.$$ 

The firm of type $\theta$ can affect its perceived quality in two ways, through its disclosure policy and through its price. We model the choice of disclosure policy and price as being simultaneous, once the firm has learned its true type. Further, we assume that disclosure requires an expenditure of the
amount $D > 0$ and that if the firm elects to disclose its quality then, in keeping with the disclosure literature, the disclosure is truthful so that $\bar{\theta} = \theta$. For example, $D$ might reflect the cost of obtaining, from an independent third party, a verification of the firm’s type (e.g., this could be achieved by testing a sample of units). Note that we are assuming that it is the firm’s type which is verifiable at a cost, not the consumer’s satisfaction with an individual unit of the product. If the firm elects not to disclose its quality, consumers will base their perceptions of quality on the posted price.

Finally, the following parameter restrictions will be maintained throughout the paper.

**Assumption 1.** $\alpha - (1 - \theta)\delta - k\theta$ is increasing in $\theta$.

**Assumption 2.** $\alpha - (1 - \bar{\theta})\delta > k\bar{\theta}$.

Assumption 1 implies that higher-quality products are socially preferred to lower-quality products (even though the high-quality product is more costly to produce). This reduces to the assumption that $\delta > k$: the marginal gain in reduced consumer dissatisfaction exceeds the marginal cost of its provision. Assumption 2 implies that there is a price in the interval $(k\bar{\theta}, \alpha - (1 - \bar{\theta})\delta)$ at which any firm type will have positive demand and a positive price-cost margin, thereby guaranteeing that each type can make positive profits whether they are correctly-perceived or mis-perceived as the worst possible type. Another implication of Assumption 2 is that, while consumers might (in principle) over-pay for a unit, no product type generates negative surplus overall.

**Analysis of equilibrium pricing.** We first characterize the equilibrium pricing behavior that
accompanies a decision to disclose quality directly. Next, we characterize equilibrium pricing behavior when the disclosure cost $D$ is prohibitively high; this involves solving a relatively straightforward signaling model in which price reveals quality. Finally, we lower the disclosure cost to determine which types, if any, will defect from signaling to the outside option of direct disclosure.

Note that any firm type that discloses can (and will) charge its full-information monopoly price (that is, the price it would charge if consumers could observe quality directly). Let $P^f(\theta)$ denote the full-information monopoly price for a firm producing a product of type $\theta$, and let $\Pi^f(\theta)$ denote the corresponding full-information monopoly profits; then $P^f(\theta) = \left(\alpha - (1 - \theta)\delta + k\theta\right)/2$ and $\Pi^f(\theta) = \left(\alpha - (1 - \theta)\delta + k\theta\right)^2/4\beta$, both of which are increasing in $\theta$. Since disclosure is costly, but the pricing game accompanying disclosure is one of full information, the equilibrium price and payoff for a disclosing firm of type $\theta$ are simply $P^f(\theta)$ and $\Pi^f(\theta) - D$ for $\theta \in [\underline{\theta}, \bar{\theta}]$.

Now suppose that the disclosure cost $D$ is prohibitively high, so it is common knowledge that no firm will choose disclosure. Then consumers will try to infer product quality from the price that is being charged. We characterize a separating perfect Bayesian equilibrium in which price serves as a signal of quality. In a two-type version of this model, the Intuitive Criterion or D1 (Cho and Kreps, 1987) eliminates pooling equilibria. Ramey (1996) provides sufficient conditions for D1 to select the separating equilibrium with a continuum of types, but our model fails to satisfy the single-crossing condition (which requires that $-q/(p - k\theta)$ be strictly decreasing in $\theta$) for $q = 0$, which is a feasible response for the consumer (though no firm type would ever choose to induce this response in equilibrium). The single-crossing condition is satisfied for all $q > 0$.

Let $B(p)$ be the belief function that relates the firm’s price to the consumer’s perceived quality; thus, if the firm charges the price $p$, then it is inferred to have quality $B(p) \in [\underline{\theta}, \bar{\theta}]$. A firm
charging price p, with true quality θ and perceived quality \( \tilde{\theta} = B(p) \), obtains profit:

$$\pi(p, \theta, B(p)) = (p - k\theta)(\alpha - (1 - B(p))\delta - p)/\beta.$$ 

In addition to incentive compatibility constraints that ensure separation, a separating perfect Bayesian equilibrium requires that consumers infer correctly the firm’s type from its price; that is, the beliefs must be consistent with equilibrium play. This is formalized in the following definition.

**Definition 1.** Suppose that \( D \) is prohibitively high, so no firm type discloses. A **separating perfect Bayesian equilibrium** in prices consists of a price function, \( P^*(\theta) \), and beliefs, \( B^*(p) \), such that for all \( \theta \in [\underline{\theta}, \bar{\theta}] \):

(i) \( \pi(P^*(\theta), \theta, \tilde{\theta}) \geq \max_p \pi(p, \theta, B^*(p)) \); 

(ii) \( B^*(P^*(\theta)) = \theta \).

We can employ Mailath’s (1987) sufficient conditions for the characterization of a unique separating equilibrium. To do so, we need two further restrictions. First, we restrict the strategy space to \( p \in [P'(\bar{\theta}), \infty) \); this will turn out to be satisfied in equilibrium, so it is without loss of generality. We also need to ensure that \( p - k\theta > 0 \) for all \( (p, \theta) \in [P'(\bar{\theta}), \infty) \times [\underline{\theta}, \bar{\theta}] \). This is accomplished by a slight strengthening of Assumption 2.\(^{14}\)

**Assumption A3.** \( \alpha - (1 - \theta)\delta > k\tilde{\theta} + k(\bar{\theta} - \theta) \).
Then the unique separating equilibrium $P^*(\theta)$ is given by the increasing solution to the following differential equation,\textsuperscript{15} through the boundary condition $P^*(\theta) = P'(\theta)$:

\[ \frac{dp}{d\theta} = \frac{\delta(p - \theta k)(2p - (\alpha - (1 - \theta)\delta) - \theta k)}{2p - (\alpha - (1 - \theta)\delta + k\theta)}. \] (1)

Notice that the numerator is positive, while the denominator is positive if and only if $p > P'(\theta) = (\alpha - (1 - \theta)\delta + k\theta)/2$. Thus, the separating equilibrium price function is an increasing function of $\theta$ that lies above the full-information price function for $\theta > \hat{\theta}$.

The solution $P^*(\theta)$ to equation (1) that maximizes profit is described (implicitly) by:\textsuperscript{16}

\[ (\alpha - (1 - \theta)\delta - p)(\delta - k)(2p - k\theta + k(\alpha - \delta)/(\delta - k))^\delta = K, \] (2)

where $K$ is a constant found by using the boundary condition in equation (2) above. As was discussed in a related version of this problem in Daughety and Reinganum (1995),\textsuperscript{17} the solution to the implicit representation (2) is a hyperbola in $(p, \theta)$-space; this holds because equation (2) is multiplicatively separable into two linear functions of $p$ and $\theta$. Therefore the relevant portion is strictly increasing and concave. Proposition 1 summarizes these results and provides out-of-equilibrium beliefs that support the separating equilibrium; Figure 1 illustrates the separating equilibrium and full-information price functions.

**Proposition 1.** There is a unique separating perfect Bayesian equilibrium.

(i) The lowest-type firm always charges its full-information price: $P^*(\theta) = P'(\theta)$. 

(ii) $P^*(\theta), \theta \in [\bar{\theta}, \tilde{\theta}]$ is the solution to the implicit equation (2) using the boundary condition from part (i). $P^*(\theta) > P^i(\theta)$ for all $\theta \in (\bar{\theta}, \tilde{\theta})$ and $P^*(\theta)$ is strictly increasing and concave.

(iii) $B^*(p) = (P^i)^{-1}(p)$ for $p \in [P^*(\bar{\theta}), P^*(\tilde{\theta})]$; $B^*(p) = \bar{\theta}$ for $p < P^*(\bar{\theta})$; $B^*(p) = \tilde{\theta}$ for $p > P^*(\tilde{\theta})$.

To see how firm profits in the signaling equilibrium depend upon $\theta$, we substitute the solution $P^*(\theta)$ into the profit function to obtain the reduced-form profits:

$$\Pi^s(\theta) = \pi(P^*(\theta), \theta, B^*(P^*(\theta))) = \left( P^*(\theta) - k\bar{\theta} \right) (\alpha - (1 - B^*(P^*(\theta))\delta) - P^* (\theta)) / \beta;$$

since $P^*(\bar{\theta}) = P^i(\bar{\theta})$, then $\Pi^s(\bar{\theta}) = \Pi^i(\bar{\theta})$. Differentiating $\Pi^s(\theta)$ with respect to $\theta$, employing the envelope theorem and the earlier-stated requirement that beliefs are correct in equilibrium, yields:

$$\frac{d\Pi^s(\theta)}{d\theta} = -k(\alpha - (1 - \theta)\delta - P^*(\theta)) / \beta, \text{ for all } \theta \in [\bar{\theta}, \tilde{\theta}].$$

This derivative is strictly negative since $(\alpha - (1 - \theta)\delta - P^*(\theta)) / \beta$ is simply the quantity demanded at price $P^*(\theta)$ and, by Assumption 2, there is always a price at which any firm type can obtain positive demand (and therefore it does so at its equilibrium price). This means that (in the separating equilibrium) profits are declining in quality; this is due to the need to distort price.

In order to decide between disclosure and signaling the firm compares the difference between
the full information and signaling profits with the cost of disclosure. The following proposition characterizes this profit difference as a function of quality.

Proposition 2. The difference $II(\theta) - IF(\theta)$ is strictly increasing in $\theta$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

Remark. To examine the robustness of Proposition 2 to changes in model assumptions, we briefly discuss what would happen if $\delta < k$ (that is, Assumption 1 is reversed), or the constant marginal cost was an increasing and convex function of quality, denoted $c(\theta)$. It is straightforward to verify that if $\delta > c'(\theta)$ for all $\theta$ then all of the previously-described properties of the price and profit functions continue to hold, as does Proposition 2. If $\delta < c'(\theta)$ for all $\theta$, then the signaling equilibrium profits are still decreasing in quality (as can be seen from equation (3), with $k$ there being replaced by $c'(\theta)$), but now so are the full-information profits. Finally, we can write $d(II(\theta) - IF(\theta))/d\theta = (\delta - c'(\theta))Q'(\theta) + c'(\theta)Q'(\theta)$, where $Q'(\theta)$ and $Q'(\theta)$ are the full-information and separating equilibrium quantities, respectively. Since $Q'(\theta) > Q'(\theta)$, the expression for $d(II(\theta) - IF(\theta))/d\theta$ involves one positive and one negative term. However, since none of the relevant expressions depends on $\bar{\theta}$, and $Q'(\theta) = Q'(\theta)$, we can conclude that Proposition 2 continues to hold provided that $\bar{\theta}$ is sufficiently close to $\underline{\theta}$.

Private incentives for voluntarily disclosure. Proposition 2 implies that there exists a range of disclosure costs, $D$, such that some types will signal and others will disclose. That is, for $D$ such that $0 < D < II(\bar{\theta}) - IF(\bar{\theta})$, there is a marginal voluntarily-disclosing type $\theta^* \in (\underline{\theta}, \bar{\theta})$ with the property that:
\[ \Pi_f(\theta) - D (\ge, =, \le) \Pi_s(\theta) \text{ as } \theta (\ge, =, \le) \theta'. \]

Those types below \( \theta' \) choose to signal, while those types above \( \theta' \) choose to disclose. The marginal type, \( \theta' \), is indifferent between signaling and disclosing; we assume that this type discloses so as to be consistent with the classical disclosure literature wherein the lowest type discloses when \( D = 0 \).

This produces a pricing function as shown in Figure 2 below. When \( D > \Pi_f(\tilde{\theta}) - \Pi_s(\tilde{\theta}) \), then signaling is the least-costly means of communicating quality, while if \( D = 0 \), then all types can credibly disclose and are able to avoid using distortionary pricing as a means for signaling quality. On the other hand, when \( 0 < D < \Pi_f(\tilde{\theta}) - \Pi_s(\tilde{\theta}) \), then disclosure is “from above;” that is, the higher quality types pay the cost \( D \) and credibly disclose their types. This allows them to lower their prices from the (distorted) monopoly signaling prices to the (undistorted) monopoly full-information prices, thereby increasing their profits. As the gap between the full-information and signaling profits declines, we find the marginal type that is disclosing voluntarily; all types below this marginal type would obtain full information profits (net of disclosure costs) that are below what they can achieve via signaling, so they choose to communicate quality via a distorted price. Notice that non-

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Place Figure 2 about here

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disclosure accompanied by a price \( p \ge P(\theta') \) is an out-of-equilibrium event; we assume that consumers treat such a price as coming from the type \( B^*(p) \ge \theta' \) (as described in Proposition 1). That is, the consumer responds as if type \( B^*(p) \), who should have disclosed in equilibrium, “trembled” and signaled instead. The incentive compatibility constraints guarantee that the resulting
profit along the signaling price line would be no greater than what the non-disclosing type would have obtained if it had correctly signaled its type, which (for types above $\theta^v$) is lower than the full-information profit minus the disclosure cost. Thus, the beliefs specified in Proposition 1 are enough to deter this sort of defection.\footnote{20}

Denote the overall equilibrium profits incorporating the disclosure-signaling choice as $\Pi(\theta)$, so that $\Pi(\theta) = \Pi^s(\theta)$ for $\theta \in [\theta, \theta^v)$ and $\Pi(\theta) = \Pi^f(\theta) - D$ for $\theta \in [\theta^v, \tilde{\theta}]$. By construction, $\Pi(\theta)$ is continuous everywhere and twice differentiable everywhere except at $\theta^v$. From equation (3) and from the fact that $\delta < dP^s/d\theta$ (see footnote 15), it follows that $\frac{d^2\Pi^s(\theta)}{d\theta^2} = -k(\delta - dP^s/d\theta)/\beta > 0$. It is also straightforward to show that $\Pi^f(\theta)$ is strictly increasing and convex in $\theta$. Thus, Figure 3 illustrates the case of greatest interest, wherein $0 < D < \Pi^f(\tilde{\theta}) - \Pi^f(\tilde{\theta})$.

The equilibrium profit function, $\Pi(\theta)$, is “U-shaped” (with a kink at the bottom) but a little more can be observed. Let $D^{mod} = \Pi^f(\tilde{\theta}) - \Pi^f(\tilde{\theta})$, something that is very easy to compute; $(0, D^{mod})$ defines a set of “moderate” disclosure costs. If $0 < D < D^{mod}$ (that is, if the disclosure cost is moderate in magnitude), then $\Pi(\tilde{\theta}) > \Pi(\tilde{\theta})$, so that the “U-shape” of $\Pi(\theta)$ involves the right end being higher than the left end; it is better to be a high-quality producer than a low-quality producer, when disclosure is incorporated in the equilibrium. In this case, the firm most prefers to be a high-quality producer, next prefers to be a low-quality producer, and least prefers to be a producer of middle-quality products. Alternatively, for $D^{mod} < D < \Pi^f(\tilde{\theta}) - \Pi^f(\tilde{\theta})$, some types disclose, but now $\Pi(\tilde{\theta}) < \Pi(\tilde{\theta})$, so it is better to be a low-quality producer than a high-quality producer.
Thus, the opportunity to disclose at a cost that is moderate ($D$ such that $0 < D < D^{mod}$) partly resolves a problem endemic to many price-quality signaling models wherein high-quality types have reduced-form profits that are less than those of the low-quality firms because higher types must distort in order to separate. Here, disclosure at moderate cost changes this picture as it results in the highest-quality types achieving payoffs higher than those of the lowest-quality types. In Section 5 we discuss what would happen in an extension wherein the firm first engages in R&D in order to influence its type and then either discloses or signals.

3. Disclosure, signaling, and social efficiency

Since the firm only considers its profit when making a decision to disclose, the decision to disclose or to signal is socially inefficient. To see this we construct total surplus and therefore require consumer’s surplus to be added to profits. Let $CS(\theta) = (\alpha - (1 - \theta)\delta - P'(\theta))^2/2\beta$, $i = f, s$, be the consumer’s surplus enjoyed from a transaction at price $P'(\theta)$; here, as usual, the superscript $s$ indicates signaling and the superscript $f$ indicates full-information. Clearly, $CS(\theta)$ is strictly increasing in $\theta$, for $\theta < \theta < \theta$. Furthermore, since $\delta < dP'/d\theta$, then $CS(\theta)$ is strictly decreasing in $\theta$, for $\theta < \theta < \theta$. Here, since $P'(\theta) = P'(\theta)$, it follows that $CS'(\theta) = CS'(\theta)$. The following proposition is straightforward to verify.$^{21}$

**Proposition 3.** The difference $CS'(\theta) - CS'(\theta)$ is strictly increasing in $\theta$, for $\theta < \theta < \theta$.

This follows immediately from the monotonicity of $CS'(\theta)$ and $CS'(\theta)$.

Now let $W(\hat{\theta})$ be the total surplus associated with any policy wherein types in $[\theta, \hat{\theta})$ employ
signaling while types in \([\tilde{\theta}, \bar{\theta}]\) pay the disclosure cost \(D\), disclose type, and use the appropriate full-information price; we assume that \(D \in (0, \bar{\Pi}(\bar{\theta}) - \bar{\Pi}(\tilde{\theta}))\), so at least some types would disclose voluntarily.\(^{22}\) Then total surplus is:

\[
W(\hat{\theta}) = \int_{\hat{\theta}}^A (CS'(\theta) + \bar{\Pi}'(\theta))dG(\theta) + \int_{\hat{\theta}}^B (CS'(\theta) + \bar{\Pi}'(\theta) - D)dG(\theta),
\]

(4)

where \(A = [\bar{\theta}, \hat{\theta}]\) and \(B = [\hat{\theta}, \bar{\theta}]\). Notice that:

\[
W'(\hat{\theta}) = [CS'(\hat{\theta}) + \bar{\Pi}'(\hat{\theta}) - CS'(\hat{\theta}) - \bar{\Pi}'(\hat{\theta}) + D]g(\hat{\theta}).
\]

(5)

Observe that the term in brackets is positive at \(\bar{\theta}\) (since it reduces to simply \(D\)) and negative at \(\theta^V\) (where it reduces to \(CS'(\theta^V) - CS'(\theta^V) < 0\)). Moreover, using Propositions 2 and 3, the term in brackets is decreasing in \(\hat{\theta}\). Thus there is a unique \(\hat{\theta}\) in the interval \((\bar{\theta}, \theta^V)\), which we denote as \(\hat{\theta}^W\), which maximizes \(W(\hat{\theta})\) over \(\hat{\theta} \in [\bar{\theta}, \bar{\theta}]\). In other words, the equilibrium level of disclosure is socially inefficient, as social optimality would entail disclosure by all types in the interval \([\hat{\theta}^W, \bar{\theta}]\), but the firm only discloses for types in the interval \([\theta^V, \hat{\theta}^W]\), where \(\theta^W < \theta^V\).

**Mandatory disclosure.** The effect of mandatory disclosure is clear: all types disclose, but now those below \(\theta^V\) all bear the cost of disclosure while some (those in \([\bar{\theta}, \theta^W]\)) should not. Of course, mandatory disclosure yields a benefit to consumers, since the price falls to the full-information line from the price-signaling line.
Subsidized disclosure. While much of the extant literature focuses on mandatory versus purely voluntary disclosure, a more-nuanced alternative to mandatory disclosure could be to use, for example, a subsidy to achieve the socially optimal level of disclosure. We assume that consumers do not observe the subsidy policy offered to the firm, as otherwise the policy itself may affect the consumer’s beliefs. Consequently, consumers can maintain the beliefs \( B^*(p) \). If consumers maintain the beliefs \( B^*(p) \), the best the social planner can do is to induce types in \([\theta^W, \bar{\theta}]\) to disclose. Moreover, disclosure by these types is consistent with the maintained beliefs \( B^*(p) \). The case in which consumers observe the subsidy policy is more complicated and beyond the scope of this paper.

In what follows we consider a subsidy scheme that induces voluntary disclosure only by those types for whom disclosure is socially efficient. We will see that this subsidy scheme leaves the firm’s overall profits unchanged and therefore results in the same U-shaped overall profits for the firm as indicated in the discussion above. As such this means that if there is a public cost of funds so as to provide such a subsidy, then this is the most efficient subsidy that achieves socially optimal disclosure voluntarily.

Consider the following subsidy function:

\[
s(\theta; D) = IT(\theta) - (IT(\bar{\theta}) - D) \quad \text{for } \theta \in [\theta^W, \theta^V] \text{ and zero elsewhere.}
\]

It is straightforward to show that \( s(\theta^W; D) = CS(\theta^W) - CS(\theta^W) > 0 \), that \( s(\theta^V; D) = 0 \) and that \( s(\theta; D) \) is decreasing in \( \theta \) for \( \theta^W \leq \theta < \theta^V \). Now consider a social planner who chooses to make this subsidy available to any type that discloses. As before, assume that the firm has access to a competitive
market of credible auditors and can pay $D$ for a credible report as to its type, so that a type $\theta$ wishing to receive the subsidy pays $D$ and obtains a report which it then submits to the planner, receiving $s(\theta; D)$. The subsidy function makes all types in the interval $[\theta^w, \theta^v)$ indifferent between disclosing and signaling, so (as earlier) we assume that they disclose. Those types at $\theta^v$ and above do not actually receive any subsidy if they provide an auditor’s report, but they voluntarily disclose quality as discussed earlier. Furthermore, if a type below $\theta^w$ were to pay for an audit and submit the auditor’s report, this type would not qualify for the subsidy; thus, these types continue to signal via their prices. Therefore, under the subsidy function, disclosure and signaling cause all types to be revealed, some via one channel and some via the other channel, but now the efficient channel is always employed voluntarily. All the gains in surplus from inducing additional disclosure accrue to the consumers. The expected payments under the subsidy policy are: $\int_{B1}^{} s(\theta; D)dG(\theta)$, where $B1 = [\theta^w, \theta^v]$, which we assume comes from general tax revenues for the economy. Note that if there is a cost of public funds (due to tax-related distortions or administrative costs), then the subsidy function $s(\theta; D)$ is the most efficient decentralized subsidy policy that achieves socially optimal disclosure voluntarily. This is a decentralized scheme since the planner simply announces the policy and any firm that seeks the subsidy must first pay for an audit that credibly discloses its type.

4. Why voluntary disclosure is excessive in the classical disclosure analysis

In Section 3 we found that $\theta^w < \theta^v$: voluntary disclosure is socially insufficient. However, to our knowledge, the classical model in the disclosure literature finds that there is too much disclosure by the monopolist. Is this difference between the two analyses a result of the use of
pooling as the result of non-disclosure in those models and signaling in our model? We see below that this only partly explains the welfare result; the rest of the explanation appears to emanate from the assumption by the classical models of unit demand with common (and known) reservation price for the product.

We modify the model presented above by assuming that \( k = 0 \), making the cost of production the same for all \( \theta \in [\tilde{\theta}, \bar{\theta}] \). Now the only equilibrium in prices following non-disclosure involves pooling by all types. If \( D \) is positive but not too large,\(^2\) then there will be some marginal type, \( \theta^p \), such that all types less than this type do not disclose and all charge the same price. To make things computable, assume that \( [\tilde{\theta}, \bar{\theta}] = [0, 1] \), and that \( G \), the distribution over types, is the uniform distribution. Then \( E[\theta | \theta < \theta^p] = \theta^p/2 \): each type in the pool is treated as the mean of the types in the pool. In what follows we will use a superscript \( P \) to denote any pooling results, such as prices or profits. Thus, the types that pool set a price \( P^p(\theta^p) = P^p(\theta^p/2) \) and make profits \( I^p(\theta^p) = I^p(\theta^p/2) \), so the equilibrium marginal type that voluntarily discloses when the disclosure cost is \( D \), denoted as \( \theta^{pv} \), is found by solving \( I^p(\theta^{pv}) = I^p(\theta^{pv}) - D \). Now the socially efficient marginal type, \( \theta^{ew} \), is found by maximizing:

\[
\int_{A}(U(P^p(\theta^p), q(P^p(\theta^p), \theta^p), \theta) + I^p(\theta^p))dG(\theta) + \int_{B}(CS(\theta) + IT(\theta) - D)dG(\theta), \tag{6}
\]

where \( A = [\tilde{\theta}, \theta^p] \) and \( B = [\theta^p, \bar{\theta}] \). Equation (6) above is the surplus computation for the pooling case that corresponds to the computation in the separating case, as shown in equation (4) above. Using similar methods to those employed in Section 3, it is straightforward to prove the following proposition.
Proposition 4. Assume that $0 \leq D < \Pi f(1) - \Pi P(1)$, so that there is an interior type $\theta^{PV}$.

(i) $D = 0$ implies that $\theta^{PV} = \theta^{PW}$.

(ii) $D > 0$ implies that $\theta^{PV} > \theta^{PW}$ as $D < 2(\alpha - \delta)^2/3\beta$.

Thus, if $D = 0$, full disclosure is socially optimal and it is also the equilibrium. If $D$ is “low” ($0 < D < \min \{2(\alpha - \delta)^2/3\beta, \Pi f(1) - \Pi P(1)\}$) then there is excessive disclosure since $\theta^{PV}$ is to the left of $\theta^{PW}$. If $D$ is “high” (that is, $2(\alpha - \delta)^2/3\beta < D < \Pi f(1) - \Pi P(1)$) then there will be insufficient disclosure, since now $\theta^{PV}$ is to the right of $\theta^{PW}$. Finally, notice that if $2(\alpha - \delta)^2/3\beta > \Pi f(1) - \Pi P(1)$, then there are no values of $D$ that yield insufficient disclosure. While pooling does imply ranges of disclosure costs that result in excessive disclosure for the monopolist, for some parameters there are sufficiently high values of $D$ such that disclosure is insufficient.

However, notice that if $\alpha$ is increased, then the set of disclosure costs that result in excessive disclosure increases, since both parts of $\min \{2(\alpha - \delta)^2/3\beta, \Pi f(1) - \Pi P(1)\}$ are increasing in $\alpha$. It is straightforward to show that the own-price elasticity of demand for our model is $-p/(\alpha - (1 - \theta)\delta - p)$, and that increasing $\alpha$ makes the demand curve more inelastic, so that a more inelastic demand function is associated with an increase in the set of $D$-values that result in excessive disclosure. This suggests that the other key assumption of the classical disclosure model that leads to the excessive-disclosure result is that aggregate demand is inelastic. Thus, we view the excessive-disclosure result as being quite non-robust because of its reliance on both of these rather special assumptions.

5. Summary, conclusions and possible extensions

In this paper we model the firm as being able to choose either to signal quality (via price)
or to disclose quality (via paying a cost that guarantees credible disclosure; e.g., by employing an outside auditor). If the disclosure cost is sufficiently high the firm will always signal the quality of the product via the price it sets; in the unique separating equilibrium, consumers use the price to infer the quality of the product and buy accordingly. If the cost of disclosure were zero, then all types of the firm would choose to disclose, and the firm would post its type-specific full-information price. Again, consumers would react to the price, knowing the quality of the product, and buy accordingly. In this model full-information profits are increasing in quality, but signaling profits are decreasing in quality, and the lowest possible type’s profits are the same in the two informational settings. Thus, the gap between full-information and signaling profits is increasing in quality. Therefore, for disclosure costs that are positive but not prohibitively high, there is a marginal type of firm that is just indifferent between disclosing and signaling; all types below choose to signal and all types above choose to disclose. We then show that the overall profits of the firm (as a function of quality) are first decreasing (due to signaling) and then increasing (due to disclosure), so that these profits are “U-shaped.” If disclosure costs are moderate then this means that profits for the firm are highest for the highest-quality type, lower for the lowest-quality type and lower yet for types “in the middle.”

In this paper, the firm’s type space $[\theta, \bar{\theta}]$ is given exogenously, but previous work allows us to engage in informed speculation about what would happen in an extension wherein the firm first conducts R&D in order to influence its type and then either discloses or signals as discussed above. In Daughety and Reinganum (1995) we model the R&D process as sequential sampling from a distribution defined over $[\theta, \bar{\theta}]$. That is, a firm pays a sampling cost and draws a quality level from $[\theta, \bar{\theta}]$; it then decides whether to stop sampling and produce this quality of product, or pay another
sampling cost and take another draw. The firm’s problem is one of optimal stopping, and there will be a subset of \([\theta, \bar{\theta}]\) such that the firm will stop if it draws a quality level from within this set, and will otherwise sample again. Assuming the consumer knows the firm’s sampling cost then, in equilibrium, the consumer correctly conjectures the stopping set. Therefore the firm’s ultimate “type space” is determined endogenously as a subset of \([\theta, \bar{\theta}]\).

We can speculate about the results of doing this same exercise in the current model, which allows the firm to choose between signaling and disclosure. When disclosure costs are very high, then only signaling will occur; since signaling profits are decreasing in quality, the firm will stop sampling when it obtains a sufficiently low level of quality; the endogenously-determined type space will be of the form \([\theta, \theta_L]\), with \(\theta_L < \bar{\theta}\). When disclosure costs are negligible, then almost all firm types will engage in disclosure; since full-information profits are increasing in quality, the firm will stop sampling when it obtains a sufficiently high level of quality; the endogenously-determined type space will be of the form \([\theta_H, \theta_G]\), with \(\theta_H > \theta\). Finally, when disclosure costs are neither very high nor very low, then some firm types will signal and others will disclose, resulting in overall firm profits that are U-shaped. In this case, there will be an intermediate range of disclosure costs for which the firm’s optimal stopping set will be the union of two disconnected intervals; the firm will stop sampling either when it draws a sufficiently low quality level or a sufficiently high quality level, but it will reject intermediate levels of quality in favor of sampling again. Here the endogenously-determined type space will be of the form \([\theta, \theta_L] \cup [\theta_H, \bar{\theta}]\), with \(\theta < \theta_L < \theta_H < \bar{\theta}\).

In the base model, information about quality is always revealed, but this may involve the use by the firm of an inefficient means of revelation. Since the firm’s choice between signaling and disclosure is based on its profits under signaling versus the full-information profits net of the
disclosure cost, some types will inefficiently choose to signal when social welfare would be maximized by those types disclosing instead, as the ensuing reduction in price and expansion in output increase overall surplus. We provide a decentralized subsidy scheme that addresses this inefficiency at the lowest possible cost to the government. The subsidy function specifies a payment based on the firm having elected to pay the disclosure cost and obtain third-party certification by a credible auditor. A firm that obtains this certification receives a subsidy (in the amount specified by the rule) based on its certified quality. The only firm types that receive a positive payment are those who would otherwise inefficiently choose to signal. This subsidy scheme results in each type of firm choosing the socially-efficient channel through which to communicate quality to consumers and maintains the same overall profits as would arise without the subsidy.

Finally, our model finds that voluntary disclosure is socially insufficient while the classical analysis finds that disclosure is socially excessive. We modify our model to examine this contrast by making production cost independent of quality, which means that non-disclosure must involve pooling. We find that the combined assumptions of equal marginal cost and sufficiently inelastic aggregate demand generate the prediction of excessive disclosure.

Possible extensions. In this paper, we adhere closely to the classical price-quality signaling and disclosure models, in which, respectively, no consumer is informed \textit{ex ante} about quality and a consumer learns product quality with certainty from a firm’s disclosure. However, other “hybrid” models are possible. For instance, instead of modeling disclosure as a perfect signal of quality, it could be a “noisy” signal. A noisy signal which simply allows a consumer to update the distribution of quality, while maintaining the same support, would have no effect on the separating signaling
equilibrium (which depends only on the support and not on the distribution over that support). Thus, no firm type would be willing to pay a positive amount for this kind of noisy signal. A noisy signal that excludes some types from the support could be valuable; our model can be viewed as a special case wherein all false types are excluded.

Since our goal was to integrate the traditional disclosure model (wherein quality is disclosed truthfully) with the potential for price signaling that arises with quality-dependent costs, we modeled the third-party auditor as being non-strategic. Lizzeri (1999) examines the case of a strategic intermediary who can observe a seller’s quality (type) costlessly. When trade is always efficient, the equilibrium involves the intermediary disclosing nothing, and charging the seller the difference between the average price based on the prior type distribution and the worst type’s full-information price; all seller types use the intermediary. Thus the intermediary provides no information but appropriates all the information rents. However, if there are many competing intermediaries, then intermediaries set fees to just cover their costs and adopt a policy of full disclosure, which is consistent with how we have modeled the auditor.

Another variation could include a fraction of consumers who are informed *ex ante* about quality, as in Bagwell and Riordan (1991). If this fraction is small the firm still engages in signaling but if this fraction is sufficiently large, then the firm abandons signaling and prices at its full-information monopoly price. Finally, one could assume that not all consumers are informed by a firm’s disclosure as in Caldieraro, Shin and Stivers (2007). This would result in a signaling subgame like that of Bagwell and Riordan (1991), since *ex post* of disclosure there will be a fraction of informed consumers and a fraction who remain uninformed.

We allow only one instrument through which the firm could signal quality: price. As
indicated in Section 1, a number of other signaling instruments have been investigated, sometimes in lieu of price but sometimes in conjunction with price (see, for example, Milgrom and Roberts, 1986a, wherein advertising augments price as a signal of quality). Expanding the model to allow for a richer strategy space in this sense would likely lead to a relaxation of Assumption 2 and may readily improve the profits of a firm engaged in signaling, thereby increasing the portion of the overall parameter space wherein a separating equilibrium can exist. Note also that strategies that might augment price and result in increased signaling profits will result in an increase in the marginal disclosing type ($\theta^i$).

Our model considers a monopolist; the extension to multiple firms is important but quite complex. The complexity arises due to the fact that the incentive compatibility constraints for each firm also are best response functions with respect to the expected price of the firm’s rival (see Daughety and Reinganum, 2007a and 2008a, for examples in the pure signaling context with two types for each firm). This means that the price-signaling function also must satisfy a fixed-point property (so that best responses yield an equilibrium), which makes the analysis considerably more difficult. In the case at hand, one would further need to allow each firm to choose whether to signal or to disclose, and this would also influence the overall pricing equilibrium. The payoff to this exercise would be a better understanding of equilibrium pricing and profits in an oligopoly setting when both signaling and disclosure are possible firm strategies.
References


Mailath, G.J. “Incentive Compatibility in Signaling Games with a Continuum of Types.”


Endnotes

1. A third distinct approach to unobservable quality involves the idea of a “quality-guaranteeing price” (beginning with Klein and Leffler, 1981; see Bester, 1998, for a recent example and further references). In this literature, quality is endogenously-chosen by the firm after it posts its price.


3. But see Hertzendorf and Overgaard (2001a,b) for models that do make this assumption.


5. See, e.g., Milgrom and Roberts (1986a) and Daughety and Reinganum (1995 and 2008b).

6. In a companion paper focused on safety and legal liability considerations (Daughety and Reinganum, 2008b), we examine a two-type model in which marginal costs may be increasing or decreasing with safety, depending upon the liability regime.

7. See, for example, Bagwell and Riordan (1991), Bagwell (1993), and Daughety and Reinganum (1995 and 2008a).

8. More distant literature includes papers using as signaling instruments: 1) advertising, such as Kihlstrom and Riordan (1984), Milgrom and Roberts (1986a), Hertzendorf and Overgaard (2001b), and Fluet and Garella (2002); and 2) warranties, such as Spence (1977), Gal-Or (1989) and Lutz (1989).

9. Daughety and Reinganum (2007a and 2008a) employ a two-type model to analyze price-quality signaling in oligopolies under the assumption that marginal cost is increasing in quality.

10. One exception is Levin, Peck and Ye (forthcoming), who consider both alternatives: (1) the disclosure decision is made before the firm learns its product quality; and (2) the disclosure decision is made after the firm learns its product quality.

11. Since completing this paper, we have become aware of a working paper by Caldieraro, Shin and Stivers (2007) that assumes one high-quality and one low-quality firm. Only some consumers observe a firm’s disclosure, leading to both disclosure and signaling by the firms.

12. We trace this interpretation of quality, and the argument that no such warranty can be offered, to Milgrom and Roberts (1986a); this interpretation is also used in Shieh (1993).

13. We assume the firm has access to a competitive market of credible auditors; see Lizzeri (1999), which is discussed further in Section 5.
14. We employ Mailath’s conditions (1) - (6); Assumption A3 is used to ensure that his condition (2) holds for all \((p, \theta) \in [P^*(\theta), \infty) \times [\theta, \tilde{\theta}]\). Mailath’s conditions are sufficient, but not necessary, to obtain this characterization. A direct proof without Assumption A3 can be found in Daughety and Reinganum (2007b).

15. This equation is found by differentiating \(\pi(p, \theta, B(p))\) with respect to \(p\), which yields the first-order condition \(d\pi/dp = (\alpha - (1 - B(p))\delta - p) + (p - k\theta)(B'(p)\delta - 1) = 0\), and then substituting \(B'(p) = 1/p'(\theta)\) and \(B(p(\theta)) = \theta\) (consistency of beliefs). Note that this implies \(p'(\theta) > \delta\).

16. This solution is found by reducing equation (1) to a homogeneous equation and then to a separable equation via changes in variables, solving the resulting ordinary differential equation, and using the earlier substitutions to recover the desired solution (for these general procedures, see, for example, Hildebrand, 1962, p. 37).

17. There liability was of interest, but the same general differential equation arises, and the structure of the equilibrium signaling function is found in the same manner as is discussed therein.

18. The model continues to satisfy Mailath’s conditions, so the separating equilibrium is the increasing solution to \(dp/d\theta = \delta(p - c(\theta))/(2p - (\alpha - (1 - \theta)\delta) - c(\theta))\) through \(P^*(\theta) = P^*(\theta)\). We are not prepared to speculate about results under yet more general demand and cost structures.

19. The second derivative \(d^2(II'(\theta) - IT'(\theta))/d\theta^2\) also involves multiple terms with conflicting signs, but indicates that \(d(II'(\theta) - IT'(\theta))/d\theta\) is positive but falls off steeply for \(\theta\) near \(\tilde{\theta}\). Thus we cannot guarantee that Proposition 2 continues to hold for arbitrary \(\tilde{\theta}\).

20. Harsher out-of-equilibrium beliefs, that is, \(B(p) < B^*(p)\), would also support this signaling and disclosure equilibrium. Such harsher beliefs would arise if the consumer believes that an out-of-equilibrium price is coming from a type in \([\theta, \theta']\) who made the correct disclosure decision but trembled on the price.

21. As in the Remark following Proposition 2, this result is somewhat more general in that it can accommodate a more general cost function \(c(\theta)\) and a reversal of Assumption 1 (provided that \(\tilde{\theta}\) is sufficiently close to \(\theta\)).

22. For \(D \in [II'(\tilde{\theta}) - IT'(\tilde{\theta}), CS'(\tilde{\theta}) + IT'(\tilde{\theta}) - CS'(\tilde{\theta}) - IT'(\tilde{\theta})\) no type would disclose voluntarily but social optimality requires that some types do disclose. Considering this case adds complexity (boundary considerations) without adding insight, so we abstract from it.

23. In order for there to be an interior marginal type, we need \(0 < D < II'(\tilde{\theta}) - IT'(\tilde{\theta})\).

24. Note that \(CS'(\theta) = U(P^*(\theta), q(P^*(\theta), \theta, \theta))\).

25. When trade with some seller types is inefficient, the intermediary discloses only whether or not trade is efficient and only efficient seller types seek certification. Albano and Lizzeri (2001) re-examine the behavior of a strategic intermediary under the assumption that the seller chooses the quality of his product.
Figure 1: Pricing Under Signaling and Under Full Information
Figure 2: Pricing with an Intermediate Disclosure Cost
Figure 3: Payoff Relationships with Voluntary Disclosure