Supplementary Appendix for
"Hidden Talents: Entrepreneurship and Pareto-Improving Private Information"
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Claim concerning U^{CI}. U^{CI} is strictly increasing in λ.
Proof of Claim. Recall that $U^{CI} \equiv \lambda^2 u_{HH} + \lambda(1 - \lambda)u_{HL} + (1 - \lambda)\lambda u_{LH} + (1 - \lambda)^2 u_{LL}$, where $u_{HH} > u_{HL} > u_{LH} > u_{LL}$. Alternatively, $U^{CI} = \lambda[\lambda u_{HH} + (1 - \lambda)u_{HL}] + (1 - \lambda)[\lambda u_{LH} + (1 - \lambda)u_{LL}]$. Since the expressions u_{rs} (for r, s = H, L) are independent of λ,

$$\partial U^{CI}/\partial\lambda = [\lambda u_{HH} + (1 - \lambda)u_{HL}] - [\lambda u_{LH} + (1 - \lambda)u_{LL}] + \lambda[u_{HH} - u_{HL}] + (1 - \lambda)[u_{LH} - u_{LL}]$$

$$= \lambda[u_{HH} - u_{LH}] + (1 - \lambda)[u_{HL} - u_{LL}] + \lambda[u_{HH} - u_{HL}] + (1 - \lambda)[u_{LH} - u_{LL}] > 0,$$

where the final inequality follows since each term in brackets is positive. QED

Claim concerning $\Lambda(t_H, t_L)$: The critical value $\Lambda(t_H, t_L) \equiv t_L(2 - (t_H + t_L))/((t_H + t_L)(t_H - t_L))$ is strictly increasing in t_L for fixed t_H.

Proof of Claim. Differentiation implies that $\partial\Lambda(t_H, t_L)/\partial t_L = [(2 - t_H)(t_H^2 + t_L^2) - 2t_L t_H^2]/(t_H^2 - t_L^2)^2$. The numerator is positive as long as $w(t_L) \equiv 2 - t_H - 2t_L t_H^2/(t_H^2 + t_L^2) > 0$. We will show that $w(t_L) > 0$ for all $t_L \in (0, t_H)$. To see this, note that $w(0) = 2 - t_H > 0$ and that $w'(t_L) = - 2t_H^2(t_H^2 - t_L^2)/(t_H^2 + t_L^2)^2 < 0$. Thus, the "worst-case scenario" involves the limit as t_L approaches t_H. But $w(t_H) = 2 - 2t_H > 0$ for all $t_H < 1$. Thus, $w(t_L)$ starts out positive and decreases as t_L increases, approaching a positive limit (from above) as t_L approaches t_H. Thus, $\partial\Lambda(t_H, t_L)/\partial t_L > 0$ for all $t_L \in (0, t_H)$. QED

Sole-entrepreneur model

As we have noted in the text, in single-agent signaling models it is typical that (a) the separating equilibrium depends only on the support of the distribution, not the prior over that support; and (b) the agent is worse off under incomplete information than under complete information because, although the L-type chooses her complete information action, the H-type's equilibrium action is typically distorted. These features apply to a sole-producer version of our model as well.

Suppose that the product requires only one worker's effort, with the value of the product equal to the worker's productivity (t + e). Using the notation from the text, under complete information, the type-dependent payoff functions are:

$$u(e_r, t_r, t_r) \equiv (t_r + e_r) - (e_r)^2/(4t_r), \quad r = H, L.$$

Note that the true type and the perceived type are the same under complete information. The complete-information optimal effort levels are given by $e_r = 2t_r$, r = H, L.

In the case of incomplete information, the worker's payoff is:

$$u(e, t, \tilde{t}) \equiv (\tilde{t} + e) - (e)^2/(4t),$$

where the true type t and the perceived type $\tilde{t}$ need not be the same. The incentive compatibility constraints now become:

$$(t_H + e_H) - (1/4t_H)(e_H)^2 \geq (t_L + 2t_H) - (1/4t_H)(2t_H)^2,$$

which is satisfied for all e_H in the closed interval:

$$[\hat{e}^-, \hat{e}^+] = [2t_H - 2[t_H(t_H - t_L)]^{1/2}, 2t_H + 2[t_H(t_H - t_L)]^{1/2}];$$

and:

$$(t_L + 2t_L) - (1/4t_L)(2t_L)^2 \geq (t_H + e_H) - (1/4t_L)(e_H)^2,$$

which is satisfied for all e_H <u>not</u> in the open interval:

$$(\tilde{e}^-, \tilde{e}^+) = (2t_L - 2[t_L(t_H - t_L)]^{1/2}, 2t_L + 2[t_L(t_H - t_L)]^{1/2}).$$

The following are candidates for the H-type's equilibrium strategy (respecting the requirement of no mimicry): $e_H \in [\hat{e}^-, \tilde{e}^-] \cup [\tilde{e}^+, \hat{e}^+]$. The interval $[\hat{e}^-, \tilde{e}^-]$ can be shown to be empty, while the interval $[\tilde{e}^+, \hat{e}^+]$ is always non-empty. The interval $[\tilde{e}^+, \hat{e}^+]$ may contain the H-type's complete-information optimal effort $e = 2t_H$; in this case, the H-type can deter mimicry without distorting her effort (that is, analogous to the NPBE equilibrium in the main text when there are two partners). Otherwise she will distort her effort to the least extent necessary to deter mimicry; that is, to $e = \tilde{e}^+$ (analogous to the DPBE equilibrium in the main text). Thus, the H-type's separating equilibrium effort level is given by $e_H^* = 2t_H$ if $t_L \leq t_H/2$ and $e_H^* = 2t_L + 2[t_L(t_H - t_L)]^{1/2}$ if $t_L > t_H/2$. Note that $2t_L + 2[t_L(t_H - t_L)]^{1/2} > 2t_H$, so that when $t_L > t_H/2$, the equilibrium requires the H-type to distort effort to a level in excess of the complete-information level, which means that the H-type chooses an effort level where profits for such a type are falling in effort. The L-type's separating equilibrium effort level is given by $e_L^* = 2t_L$. Note that the separating equilibrium effort levels depend on the support of the prior distribution, but not on λ. Moreover, when $t_L > t_H/2$ the H-type distorts her effort level upward from her monopoly level; thus the worker's *ex ante* expected payoff under incomplete information is always lower than under complete information.

<u>Two-Entrepreneur Additive-Productivity Model</u>

If the value of the product is equal to the sum of the entrepreneurs' productivities (that is, $V = P_i + P_j = t_i + e_i + t_j + e_j$), then there is no strategic interaction between the entrepreneurs' effort choices. Rather, the two entrepreneurs simply signal separately to potential buyers. In this case, we find that incomplete information yields higher *ex ante* expected equilibrium payoffs whenever the H-type entrepreneur needs to distort her effort in order to signal her type. To see this, note that the payoff function for entrepreneur i is:

$$u_i(e_i, t_i, \tilde{t}_i | e_j, \tilde{t}_j) \equiv (\tilde{t}_i + e_i + \tilde{t}_j + e_j)/2 - (e_i)^2/(4t_i), \qquad i, j = 1, 2, j \neq i$$

where $\tilde{t}_i$ and $\tilde{t}_j$ denote entrepreneur i's and entrepreneur j's perceived types. Under complete information, an entrepreneur type r has a dominant strategy to choose $e_r = t_r$, r = H, L. Following the same procedure as above for the sole-producer model, it is straightforward to show that under incomplete information the L-type entrepreneur will choose her complete-information optimal effort ($e_L^* = t_L$), while the H-type entrepreneur will choose $e_H^* = t_H$ if $t_L \leq t_H/3$ and $e_H^* = t_L + [2t_L(t_H - t_L)]^{1/2}$ if $t_L > t_H/3$. Finally, we note for future use that the joint-profit-maximizing complete-information effort for an entrepreneur of type r is $e_r = 2t_r$, r = H, L.

Thus, if $t_L \leq t_H/3$ then there is no difference between the *ex ante* expected equilibrium payoffs under complete versus incomplete information (since the H-type need not distort her effort to signal her type). On the other hand, if $t_L > t_H/3$ then the H-type distorts her effort upwards, but to a level that does not exceed the joint-profit-maximizing complete-information effort level. That is, $e_H^* = t_L + [2t_L(t_H - t_L)]^{1/2} \in (t_H, 2t_H)$. It is tedious but straightforward to show that when distortion is needed to signal type H, then each entrepreneur's *ex ante* expected equilibrium payoff under incomplete information is higher than the *ex ante* expected equilibrium payoff when strategies are chosen under complete information.