Technical Appendix for: Daughety and Reinganum, "Exploiting future settlements: a signaling model of most-favored-nation clauses in settlement bargaining," RAND Journal of Economics, 35(3), Autumn 2004

This Technical Appendix contains the proofs of Propositions 1, 4 and 6. We also provide a more detailed discussion of why it will be an equilibrium for P_1 to always use an MFN.

Proof of Proposition 1. To verify that these strategies and beliefs provide a revealing equilibrium, we show that (1.A) $r_0^*(S)$ is an optimal strategy for D, given the beliefs $b_0^*(S)$; (1.B) $S_0^*(x)$ is an optimal strategy for P, given $r_0^*(S)$; and (1.C) the beliefs are correct: $b_0^*(S_0^*(x)) = x$ for $x \in [\underline{x}, \bar{x}]$.

Proof of (1.A). Given the beliefs $b_0^*(S)$, upon observing the demand $S \in [\underline{S}, \bar{S}]$, D expects to pay $b_0^*(S) + k^D = S$ if he rejects the demand S and D expects to pay S if he accepts the demand S, so he is indifferent. Hence he is willing to randomize as specified by $r_0^*(S)$. A demand $S > \bar{S}$ is believed to have come from type $\bar{x}$ so it is optimal for D to reject it (and pay $\bar{x} + k^D$ at trial) rather than to accept it (and pay $S > \bar{S} = \bar{x} + k^D$ in settlement). Finally, a demand $S < \underline{S}$ is believed to have come from type $\underline{x}$ so it is optimal for D to accept it (and pay $S < \underline{S} = \underline{x} + k^D$ in settlement) rather than to reject it (and pay $\underline{x} + k^D$ at trial).

Proof of (1.B). Given the strategy $r_0^*(S)$, a P of type x demanding S anticipates a payoff of $\pi^P(S) = r_0^*(S)(x - k^P) + (1 - r_0^*(S))S$. First note that any strategy $S < \underline{S}$ is dominated by $S = \underline{S}$ since both are accepted for sure. Moreover, any strategy $S > \bar{S}$ is dominated by $S = \bar{S}$ since the former generates a payoff of $x - k^P$ for sure, while the latter generates a convex combination of $\bar{S} > x - k^P$ and $x - k^P$. Thus, the optimal demand must belong to $[\underline{S}, \bar{S}]$. Maximizing the expression $r_0^*(S)(x - k^P) + (1 - r_0^*(S))S$ with respect to S yields the first-order condition: $-\{[S - x + k^P]/K\}\exp\{-(S - \underline{S})/K + \exp\{-(S - \underline{S})/K = 0$, which has the unique solution $S_0^*(x) = x + k^D$. To see that this is a local maximum, note that the second-order condition for a maximum is $\{[S - x + k^P - 2K]/K^2\}\exp\{-(S - \underline{S})/K \leq 0$, which is satisfied at $S_0^*(x) = x + k^D$. If another maximum were to exist on the boundary (that is, at $\underline{S}$ or $\bar{S}$), there would have to be a local minimum between it and $S_0^*(x)$, but no other interior stationary point exists, since $S_0^*(x)$ is the unique interior solution to the first-order condition. Thus $S_0^*(x)$ provides the global maximum to P's payoff.

Proof of (1.C). Substitution yields $b_0^*(S_0^*(x)) = S_0^*(x) - k^D = x + k^D - k^D = x$ for $x \in [\underline{x}, \bar{x}]$; thus the beliefs are correct in equilibrium. Moreover, the equilibrium strategies are robust to arbitrary out-of-equilibrium beliefs. QED

Proof of Proposition 4. Recall the definition of $h(S_1; x_2)$, namely that $h(S_1; x_2) \equiv (x_2 + k^D - S_1)(S_1 + K + k^P - x_2)/4K$. In what follows, for the exponential distribution, take $\bar{x}$ to be infinite. From the definitions of $\hat{r}_1(x_1)$ and $\hat{r}_0(x_1)$, it is clear that $\hat{r}_1(\underline{x}) = \hat{r}_0(\underline{x})$, and that $\hat{r}_1(x_1) < \hat{r}_0(x_1)$ for all $x_1 \in (\underline{x}, \bar{x}]$ if and only if

$$\int_{[\underline{x}, x_1]} (1 + g'(S_1^*(x)))/(K + g(S_1^*(x)))dx < \int_{[\underline{x}, x_1]} (1/K)dx \quad \text{for all } x_1 \in (\underline{x}, \bar{x}].$$

If $(1 + g'(S_1^*(x)))/(K + g(S_1^*(x))) < 1/K$ (or, equivalently, $g'(S_1^*(x)) < g(S_1^*(x))/K$) for all $x \in [\underline{x}, \bar{x})$ then the inequality in the displayed equation above holds for all $x_1 \in (\underline{x}, \bar{x}]$.

Since $g'(S_1) = \int[\partial h(S_1; x_2)/\partial S_1 - h(S_1; x_2)/K]\exp\{-(S_1 - \underline{S})/K\}f(x_2)dx_2$, it follows that $g'(S_1^*(x)) < g(S_1^*(x))/K$ if and only if:

$$H(x) \equiv \int[\partial h(S_1^*(x); x_2)/\partial S_1) - 2h(S_1^*(x); x_2)/K]\exp\{-(x - \underline{x})/K\}f(x_2)dx_2 < 0,$$

where the integral is taken over $x_2 \in T(S_1^*(x)) = [x, \min\{\bar{x}, x + 2K\}]$. Note that $T(S_1^*(x))$ is non-degenerate whenever $S_1^*(x) < \bar{x} + k^D$; that is, whenever $x < \bar{x}$.

i) One sufficient condition for $H(x) < 0$ for all $x \in [\underline{x}, \bar{x})$ is that $F(\bullet)$ is the uniform distribution. To see this, substitute $f(x_2) = 1/(\bar{x} - \underline{x})$, integrate and simplify to obtain:

$$H(x) = [\exp\{- (x - \underline{x})/K\}/2K^2(\bar{x} - \underline{x})]Y\{Y^2/3 - YK/2 - K^2\}, \text{ where } Y \equiv \min\{\bar{x} - x, 2K\}.$$

The term in square brackets is positive, as is Y itself, so $H(x) < 0$ so long as $M(Y) \equiv Y^2/3 - YK/2 - K^2 < 0$ for all $Y \in (0, 2K]$, which can be shown.

ii) A sufficient condition for $H(x) < 0$ for all $x \in [\underline{x}, \infty)$ is that $F(\bullet)$ is the exponential distribution. To see this, substitute $f(x_2) = \lambda\exp\{-\lambda(x_2 - \underline{x})\}$, integrate and simplify to obtain:

$$H(x) = -[\exp\{- \lambda(x - \underline{x})\}/2K^2\lambda^2][(\exp\{-2K\lambda\})(K\lambda + 1) + (K\lambda - 1)][K\lambda + 2].$$

The term in each square bracket is positive, for all $K > 0$ and $\lambda > 0$, so $H(x) < 0$ for all $x \in [\underline{x}, \infty)$.

(iii) A sufficient condition for $H(x) < 0$ for all $x \in [\underline{x}, \bar{x})$ when $F(\bullet)$ is arbitrary is that:

$$[\partial h(S_1^*(x); x_2)/\partial S_1) - 2h(S_1^*(x); x_2)/K] < 0 \text{ for all } x_2 \geq x \text{ and for all } x \in [\underline{x}, \bar{x}).$$

This integrand can be shown to be $[(x_2 - x)^2 - K(x_2 - x) - K^2]/2K^2$, which implies the result. QED.

Proof of Proposition 6. We fix a value of x_1 and argue that, if $F(\bullet)$ satisfies (i), (ii) or (iii), then the expected trial costs for the second period are lower under an MFN. Since this will be shown to be true for all values of x_1 (except $x_1 = \bar{x}$, in which case the MFN never binds for any x_2), the expected trial costs for the second period are lower under an MFN.

Recall that $\hat{r}_2(x_2; x_1) = 1 - \{[x_1 + 2K - x_2]/2K\}\exp\{- (x_1 - \underline{x})/K\}$ for $x_2 \in [x_1, \min\{\bar{x}, x_1 + 2K\}]$, while $\hat{r}_2(x_2; x_1) = 1$ for $x_2 \in (\min\{\bar{x}, x_1 + 2K\}, \bar{x}]$. Without an MFN, $\hat{r}_0(x_2) = 1 - \exp\{- (x_2 - \underline{x})/K\}$ for all $x_2 \in [x_1, \bar{x}]$. Thus, multiplying by the cost per trial and taking the expectation over $x_2 \in [x_1, \bar{x}]$ (where the probability of a trial differs with and without an MFN) yields:

$$\begin{aligned} ETC_2(x_1) - ETC_0 &= K\int_{[x_1, \bar{x}]} \hat{r}_2(x_2; x_1)f(x_2)dx_2 - K\int_{[x_1, \bar{x}]} \hat{r}_0(x_2)f(x_2)dx_2 \\ &= K\int_{[x_1, \bar{x}]} \exp\{- (x_2 - \underline{x})/K\}f(x_2)dx_2 \\ &\quad - K\exp\{- (x_1 - \underline{x})/K\}\int_{[x_1, \min\{\bar{x}, x_1 + 2K\}]} [(x_1 + 2K - x_2)/2K]f(x_2)dx_2. \end{aligned}$$

Clearly, $ETC_2(\bar{x}) - ETC_0 = 0$, since the domains of integration are then degenerate. Thus, consider values of $x_1 < \bar{x}$ in the remainder of the proof.

i) Consider $F(\bullet)$ to be the uniform distribution; substitute $f(x_2) = 1/(\bar{x} - \underline{x})$, integrate and simplify to obtain:

$$\begin{aligned} ETC_2(x_1) - ETC_0 &= [K/(\bar{x} - \underline{x})]\exp\{- (x_1 - \underline{x})/K\} \\ &\quad \times [K(1 - \exp\{- (\bar{x} - x_1)/K\}) - (Z - x_1)(x_1 + 4K - Z)/4K], \end{aligned}$$

where $Z = \min\{\bar{x}, x_1 + 2K\}$. There are two cases to consider.

Case 1. Assume that $x_1 + 2K < \bar{x}$, so $Z = x_1 + 2K$. Substituting and simplifying yields

$$ETC_2(x_1) - ETC_0 = [K/(\bar{x} - \underline{x})]\exp\{-(x_1 - \underline{x})/K\}[K(1 - \exp\{-(\bar{x} - x_1)/K\}) - K],$$

which is clearly negative. So $ETC_2(x_1) - ETC_0 < 0$ for all $x_1 < \bar{x} - 2K$.

Case 2. Assume that $x_1 + 2K \geq \bar{x}$, so $Z = \bar{x}$. Substituting and simplifying yields

$$ETC_2(x_1) - ETC_0 = [K/(\bar{x} - \underline{x})]\exp\{-(x_1 - \underline{x})/K\}$$

$$\times [K(1 - \exp\{-(\bar{x} - x_1)/K\}) - (\bar{x} - x_1)(x_1 + 4K - \bar{x})/4K].$$

Let $v \equiv \bar{x} - x_1$. Then $\text{sgn}\{ETC_2(x_1) - ETC_0\} = \text{sgn}\{[K(1 - \exp\{-v/K\}) - v(4K - v)/4K]\}$. Since $x_1 \in [\bar{x} - 2K, \bar{x})$ implies that $v \in (0,2K]$, we need only verify that $K(1 - \exp\{-v/K\}) - v(4K - v)/4K < 0$ for all $v \in (0,2K]$. This inequality holds for the specified values of v. Thus, $ETC_2(x_1) - ETC_0 < 0$ for $x_1 \in [\bar{x} - 2K, \bar{x})$, as claimed.

ii) Consider F(•) to be the exponential distribution; substitute $f(x_2) = \lambda\exp\{-\lambda(x_2 - \underline{x}\}$, integrate and simplify to obtain:

$$ETC_2(x_1) - ETC_0 = \exp\{-(x_1 - \underline{x})(1 + K\lambda)/K\}[(1 - \exp\{-2K\lambda\})/2\lambda - K/(1 + K\lambda)].$$

This expression is negative for all $x_1 \in [\underline{x}, \infty)$ because the term in square brackets is negative for $K > 0$ and $\lambda > 0$.

iii) Now consider F(•) to be an arbitrary distribution on $[\underline{x}, \bar{x}]$. A sufficient condition for $ETC_2(x_1) - ETC_0 < 0$ for all $x_1 \in [\underline{x}, \bar{x})$ is that (assuming $\bar{x} - \underline{x} < 2K$) the integrand is point-wise negative:

$$\exp\{-(x_2 - \underline{x})/K\} - \exp\{-(x_1 - \underline{x})/K\}[(x_1 + 2K - x_2)/2K] < 0$$

for all $x_2 \in [x_1, \bar{x}]$ and $x_1 \in [\underline{x}, \bar{x})$. This will be true as long as $\bar{x} - \underline{x} \leq \alpha K$, where α is the solution to the equation $\exp\{-\alpha\} + \alpha/2 = 1$. QED.

Why is the use of an MFN always part of an equilibrium settlement demand made by P_1?

We have claimed that, under conditions such that every P_1 type gains from including an MFN in its settlement demand (and D is indifferent), all P_1 types will include an MFN. A more formal argument is as follows. Suppose that D conjectures that every type of P_1 will include an MFN. Then upon observing an MFN, D continues to entertain the possibility that the associated demand could have come from any type $x_1 \in [\underline{x}, \bar{x}]$ and, in particular, D assigns the type $x_1 = S_1 - k^D$, giving rise to the equilibrium strategies provided in Proposition 3. Thus, if D conjectures that every type of P_1 will include an MFN and behaves accordingly, then it will be optimal for every type of P_1 to include an MFN, and thus D's conjectures are correct. The out-of-equilibrium beliefs that support this are as follows: upon observing a demand S_1 with no MFN, D believes that P_1 would have made this error regardless of type. Thus, D reverts to the no-MFN rejection function but continues to infer that $x_1 = S_1 - k^D$. Given this inference and his anticipated payoff from the second-period bargaining game without an MFN, D is indifferent between acceptance and trial against P_1 and thus is willing to randomize according to the no-MFN rejection function. Moreover, given that D is expected to use the no-MFN rejection function upon observing no MFN, P_1 would continue to (optimally) use the settlement demand function $S_1 = x_1 + k^D$ even if no MFN were included. However, since every type of P_1 receives a lower payoff in the no-MFN equilibrium, no type is tempted to defect to not including an MFN. Thus, these out-of-equilibrium beliefs support the inclusion of an MFN by all types of P_1.

Could there be another equilibrium in which no P_1 type includes an MFN and play proceeds as in Proposition 1? The answer is "No." Suppose, to the contrary, that D conjectures that no P_1 type would include an MFN. Upon observing no MFN, D would play according to the equilibrium in Proposition 1 (the no-MFN case). In order to deter the use of an MFN, D would have to maintain out-of-equilibrium beliefs that punish P_1 for including an MFN. For instance, suppose that upon seeing an MFN, D believes that P_1 is the lowest possible type, $\underline{x}$, and thus rejects any demand in excess of $\underline{x} + k^D$. But then the type $\underline{x}$ would defect to $\underline{S} = \underline{x} + k^D$ with an MFN (from the putative equilibrium in which no P_1 type uses an MFN), since this demand is accepted for sure in both cases and the MFN adds a strictly positive expected future payment for this type. Moreover, there would be a neighborhood of types near $\underline{x}$ who would also defect to $\underline{S} = \underline{x} + k^D$ with an MFN from their no-MFN demands. Thus, even these extremely punishing out-of-equilibrium beliefs cannot support an equilibrium in which no P_1 type includes an MFN.