Abstract. Understanding how shocks to the technology of war fighting affect the probability a conflict devolves to war is central to many aspects of security studies. In the standard crisis bargaining model, changes to war payoffs have no effect on the equilibrium probability that bargaining ends in war. This neutrality result implies that shifts in military capabilities, as well as tools of statecraft such as third-party intervention, alliances, and arming, have no effect on war onset. The empirical record as well as received wisdom seems to contradict this observation. Can we simultaneously accept the existing empirical findings on war onset and maintain the theory that war is the result of bargaining breakdown? We show that a series of individually innocuous assumptions combine in the standard crisis bargaining model to produce this result. While it is true that changes to the war payoffs make one player more aggressive and the other less aggressive, these effects need not balance out. We show that the exact balancing and ensuing neutrality result relies on a form of symmetry in how external shocks influence the tradeoffs between the payoffs each side derives from fighting and reaching a peaceful settlement.
1. Introduction

The crisis bargaining model has become a fixture in theories of conflict.\(^1\) One variant of the model, which has received the majority of attention in the literature, is a standard ultimatum bargaining game with one-sided incomplete information. This version is often used to illustrate how uncertainty about payoffs is a sufficient friction to reach bargaining failure and inefficient war. The model now represents a popular baseline, and variants of it are used to study a host of other important topics ranging from the effects of the balance of power on conflict,\(^2\) to the role of mediation\(^3\) and domestic politics,\(^4\) to the efficacy of third-party intervention\(^5\) as well as alliances and deterrence.\(^6\)

The standard crisis bargaining model has substantial appeal. It is both relatively tractable and has many comparative statics that are consistent with stylized facts generally accepted as important in the field of conflict studies. The standard model also produces some surprising results, probably the most unexpected of which is called “neutrality,” which states that changes in the war payoffs are offset by equilibrium bargaining behavior in such a way as to result in no change in the equilibrium probability of conflict.\(^7\)

The mechanism behind the neutrality result is compelling. If one country is made stronger, it will behave more aggressively while its opponent becomes more conciliatory in the bargaining process. Increases in the strengthened country’s war payoffs lead it to make more aggressive demands, and, with all else equal, there results a higher chance of bargaining failure. However, all else is not equal. As a result of the increase in the probability of victory for the strengthened country, the weakened country is now willing to make more concessions to its opponent. This conciliatory behavior reduces the risk of

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\(^1\)Fearon 1995.
\(^2\)Powell 1996, 1999; Reed 2003.
\(^3\)Kydd 2003, Rauchhaus 2006.
\(^6\)Yuen 2009, Benson 2012.
war. In the standard model, these offsetting incentives lead both countries to take actions that in equilibrium neither increase nor decrease the probability of war.

However, as we show, neutrality is an unintended consequence of individually reasonable simplifying assumptions in the original specification of the model. To demonstrate this, we show why the neutrality result obtains, how it relates to assumptions about players’ utilities, and then provide a generalization of the original formulation that is better suited for studying the effects of changes to the technology of war fighting or the distribution of power in the crisis bargaining framework with asymmetric information. The analysis illustrates how a key comparison of the way that war payoffs change influences whether the risk of war increases or decreases. Our point is not that the modeling choices in Fearon’s original paper or other studies using this model are wrong. We accept that every model is stylized in particular ways. Rather we hope that this note will reveal how particular features of this widely used model impact its applicability to questions about the effect of changes in the probability of victory on the onset of war. This note offers guidance into how the choices made about war and settlement payoffs can influence the model’s ability to provide leverage on certain types of questions.

We are not the first to discuss the source of the neutrality result. Previous scholars have incompletely attributed the result to the assumption of risk neutrality, but as we show it is not risk neutrality but rather the assumption that changes in one player’s payoffs exactly offset changes in the other player’s payoffs at equilibrium that drives the result. Although certain assumptions that relax risk neutrality will also introduce the relevant asymmetry, we caution scholars against equating risk neutrality with the neutrality result.

We think this is an important question because many issues in international relations can be framed in terms of understanding how changes to the war payoffs of countries influence their behavior in crisis bargaining. Such changes may result from exogenous shifts in the distribution of power or intervention by third-party defenders. Related to

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9Powell 1999.
the models of third-party intervention in interstate war are models of third-party intelement in intrastate conflict.\textsuperscript{11} Other factors that may influence war payoffs and crisis bargaining include international organizations,\textsuperscript{12} domestic politics,\textsuperscript{13} and previous actions by the disputants in a dynamic setting such as in arms races.\textsuperscript{14}

Typically modelers incorporate shifts in war payoffs by either considering changes in the distribution of capabilities or opponents’ costs of war. For example, Powell (1999) models a country’s decline in power as a decrease in its capabilities relative to its opponent, or the probability it will win in the war lottery. In Yuen (2009), third-party intervention also affects disputants’ relative capabilities. Werner (2000) models third-party intervention as altering both the distribution of capabilities as well as war costs. In Chapman and Wolford (2010), support from an external international organization might affect a country’s costs of war. In Filson and Werner (2004), becoming more democratic increases war costs. An exception to shifts affecting either capabilities or war costs is Tarar (2006), in which a leader choosing to go to war might be rewarded by a domestic audience by retaining office. In this setup, the leader’s war payoff increases by the amount of the private value of public office.

In our analysis below, we provide a general intuition for how changes in war payoffs influence the aggressiveness of states in bargaining and affect the probability of war. We show that exact neutrality is a fragile condition, though the cushioning effect of bargaining dynamics is robust. If shocks change the payoffs to war fighting relative to the marginal value of settlements in ways that do not exactly balance out, then the neutrality result will fail. We further show that increases and decreases in the probability of war result from predictable asymmetries in changes in war payoffs. Establishing general conditions for violating neutrality may create opportunities for further theorizing about how policies create asymmetries in payoffs that affect war onset. We provide several applications of

\textsuperscript{11}Cetinyan 2002, Kydd and Straus 2013.
\textsuperscript{12}Chapman and Wolford 2010.
\textsuperscript{13}Filson and Werner 2004, Tarar 2006.
\textsuperscript{14}Kydd 2000.
the result to published models studying the relationship the distribution of power and war onset.

2. Crisis Bargaining and the Neutrality Result

Crisis bargaining considers situations where players, negotiating over an issue or territory, have an outside option in the form of war. In a setting with two countries, say $A$ and $B$, we may conceive of the negotiations as relating to the division of some disputed territory, and possible solutions are characterized by the countries’ relative shares. The share to player $A$ is denoted $x$ and the share to player $B$ is $1-x$. It is standard to assume that the payoffs from such a split are linear in the shares and thus given an agreed split of $(x, 1-x)$ the players’ preferences can be represented by an expected utility function $EU_A(x) = x$ and $EU_B(1-x) = 1-x$.\footnote{Fearon (1995) appears to be more general as the model is formulated to allow players to have non-linear utility over the shares. But all of the analysis of the relevant model (claim 2 in Fearon’s appendix) takes the utility over shares to be linear. The literature seems to have followed this simplification. We begin with this assumption and then relax it in the next section.} If an agreement is not reached, then the players obtain payoffs from fighting a war. With normalized payoffs for owning the entire “pie” the standard formulation is

$$EU_A(\text{war}) = p - c_A,$$

and

$$EU_B(\text{war}) = 1 - p - c_B.$$

The term $p$ then is interpreted as the probability that $A$ wins the full prize while $B$ wins it with probability $1-p$. The costs, $c_A$ and $c_B$, capture losses from war fighting. Natural interpretations of war costs in the literature include destruction of property, loss of life, or time costs associated with delay in resolving the issue. The bargaining protocol is then the ultimatum game. Player $A$ makes a proposal, $(x, 1-x)$. Player $B$ then decides to accept, resulting in the payoffs $x$ and $1-x$, or to reject, resulting in the war payoffs.

With no uncertainty the unique subgame perfect equilibrium involves $A$ offering exactly
$1 - x = 1 - p - c_B$ so that the dispute is peacefully resolved with the share $x = p + c_B$ going to $A$.

However, a rationalist explanation for war is provided if $A$ is assumed to face uncertainty about the term $c_B$. For simplicity, we follow the most common formulation and assume that while $B$ knows her war cost, $A$ treats it as a random variable drawn from a distribution $H(\cdot)$ with density $h(\cdot)$ on the interval $[0, 1]$. The characterization of a Perfect Bayesian Equilibrium in which there is a risk of war is mechanical and well known. $B$ will accept an offer in which $1 - x \geq 1 - p - c_B$. This means that from the perspective of $A$ deciding what offer to make, the probability of war is a function of her offer, $\pi(x) = H(x - p)$. Accordingly, $A$ selects $x$ to maximize\(^\text{16}\)

\[H(x - p)(p - c_A) + (1 - H(x - p))x.\]

The first order condition is

\[h(x - p)(p - c_A - x) - H(x - p) + 1 = 0.\]

Throughout we follow the literature and assume that the parameters support an equilibrium in which $A$’s offer involves risk. This is the assumption that is central to using uncertainty as a rationalist explanation for war.\(^\text{17}\) In this case, let $x^*$ denote the solution to the first order condition. In this context the neutrality result speaks to a comparative static on how changes in $p$ influence the equilibrium probability of war, $\pi(x^*)$. Expanding our notation to capture the dependencies, we may write $x^*(p)$ to denote the solution to the first order condition and $\pi^*(p) = \pi(x^*(p))$ as the equilibrium probability of war.

**Proposition 1.** (neutrality): *In the canonical model the probability of war is constant in $p$, i.e., $\frac{d\pi^*}{dp} = 0$.*

\(^\text{16}\)Note that the above derivation parallels that given in Fearon's proof of claim 2.
\(^\text{17}\)See Fey, Meirowitz, and Ramsay (2013) for necessary and sufficient conditions.
Proof. The proof is an application of the implicit function theorem and the chain rule. The equilibrium effect of \( p \) on \( x^* \) is found by way of the implicit function theorem, \( \frac{dx^*(p)}{dp} = 1 \).

Recalling that \( \pi(x^*(p)) = H(x^*(p) - p) \), the chain rule yields

\[
\frac{d\pi^*}{dp} = h(x^*(p) - p)(\frac{dx^*(p)}{dp} - 1).
\]

Substituting \( \frac{dx^*(p)}{dp} = 1 \) into the above yields the conclusion, \( \frac{d\pi^*}{dp} = 0 \). \( \square \)

3. Rethinking payoffs

It is not uncommon for authors to investigate changes to war onset and bargaining outcomes stemming from external shocks to the probability of victory, \( p \), in the standard expected utility set up. But clearly Proposition 1 says we should not bother, because there is no effect. To gain some clarity regarding the effects of policy levers that change the payoff to war, consider a more general form of the war payoff.

Let the war payoffs to \( A \) be denoted by a function

\[
EU_A(\text{war}) = p(a),
\]

and let the war payoffs to \( B \) be denoted

\[
EU_B(\text{war}) = w(a) - c_B.
\]

For convenience we include the private cost, \( c_A \), in the term \( p(a) \). Because private information about \( B \)'s cost is the friction that drives inefficiency (risk of war), we keep this term separate for country \( B \).

The parameter \( a \) is included to allow for comparative statics. Here we can think of \( a \) as an input to the war payoffs, such as a level of arms, a state of military technology, the amount of military assistance from an ally, etc. To facilitate comparative statics analysis we assume that the functions \( p(\cdot) \) and \( w(\cdot) \) are twice differentiable. The assumption that
\( p'(a) \geq 0 \) and \( w'(a) \leq 0 \) allows us to interpret an increase in \( a \) as a change that favors player A’s war payoff.

With these changes we may repeat the analysis above with very minor changes. Country B will accept any offer \( x \) satisfying the inequality

\[
c_B \geq w(a) + x - 1.
\]

Thus the probability of war given an offer \( x \) is

\[
\pi(x; a) = H(w(a) + x - 1).
\]

Given this rule, A’s offer maximizes

\[
H(w(a) + x - 1)p(a) + (1 - H(w(a) + x - 1))x.
\]

The first order condition is now

\[
h(w(a) + x - 1)(p(a) - x) - H(w(a) + x - 1) + 1 = 0.
\]

The effect of a change in \( a \) on the equilibrium offer \( x^*(a) \) is found by using the chain rule and the implicit function theorem,

\[
\frac{dx^*(a)}{da} = \frac{-h(w(a) + x^*(a) - 1)\left[p'(a) - w'(a)\right] - h'(w(a) + x^*(a) - 1)\left[p(a) - x^*(a)\right]w'(a)}{-h(w(a) + x - 1)[2 + h'(w(a) + x - 1)[p(a) - x]].
\]

Note that in the canonical model \( p(a) = a \) and \( w(a) = 1 - a \), in which case the above simplifies to 1. Using the chain rule again we can express the change in the equilibrium probability of war as

\[
\frac{d\pi^*(a)}{da} = h(w(a) + x^*(a) - 1)[w'(a) + \frac{dx^*(a)}{da}].
\]

Recall that \( w'(a) \) is negative and so whether this term is positive, zero, or negative depends on whether the magnitude of \( \frac{dx^*(a)}{da} \) is larger, the same, or smaller than the magnitude of \( w'(a) \). To make this comparison it is convenient to factor \(-w'(a)\) out of the expression.
for \( \frac{dx^*(a)}{da} \). Doing this we obtain

\[
-w'(a) \frac{-h(w(a) + x - 1)[1 - w'(a)] + h'(w(a) + x - 1)[p(a) - x]}{-h(w(a) + x - 1)[2] + h'(w(a) + x - 1)[p(a) - x]}. 
\]

The fraction is greater than or equal to one if \( \frac{-w'(a)}{w'(a)} \) is greater than one and it is less than or equal to one if the reverse is true. Accordingly, substituting this fraction into the probability of war, we see that the sign of \( \frac{dx^*(a)}{da} \) coincides with the sign of \( p'(a) + w'(a) \). Thus if changes in \( a \) produce larger changes in \( p(a) \) than they do in \( w(a) \), the increase in \( a \) will increase the equilibrium probability of war. We have thus established the following result.

Proposition 2. (generalized war payoff model) If changes in the term, \( a \), have a larger effect on the war payoff to \( A \) than they do on the war payoff to \( B \), then an increase in \( a \) will increase the equilibrium probability of war. If the effect on \( B \)’s payoff is larger, then an increase in \( a \) will decrease the equilibrium probability of war.

The foregoing analysis demonstrates how changes in war payoffs affect probabilities of war onset. Proposition 2 establishes asymmetry conditions in war payoffs that lead neutrality to fail. Because neutrality depends on changes in war payoffs relative to the marginal value of settlements exactly balancing out, we now relax the restriction that players have linear payoffs over settlement shares.

Fearon motivated his study with a model in which the players’ payoffs over shares were arbitrarily increasing functions. So \( A \) preferred larger values of \( x \), and \( B \) preferred lower values of \( x \). In the class of models covered by Proposition 2 it is assumed that these functions are linear and so settlement payoffs are \( x \) and \( 1 - x \) respectively. One might, however, believe that players obtain diminishing returns from the settlement. One plausible specification characterizes the split \((x, 1-x)\) as yielding payoffs of \( x^{\frac{1}{2}} \) and \((1-x)^{\frac{1}{2}}\) respectively. With this specification and the assumption that war payoffs are just \( p - c_A \) and \( 1 - p - c_B \) as in the canonical specification, if \( c_B \) is uniform on the unit interval.
then the equilibrium probability of war for a parameterization which induces war with non-degenerate probability is given by the expression

\[ 1 - p - \frac{1}{4}2^{\frac{3}{2}}[c_A^2 - c_A(c_A^2 + 8)^{\frac{1}{2}} + 4]^{\frac{1}{2}}, \]

which is linearly decreasing in \( p \). Thus, as \( A \) is made stronger the likelihood of war decreases.\(^{18}\)

To extend Proposition 2 to capture this flexibility, we assume that \( A \) obtains payoff \( s_A(x) \) and \( B \) obtains payoff \( s_B(1 - x) \) from a settlement of \((x, 1 - x)\) with both \( s_A(\cdot) \) and \( s_B(\cdot) \) being increasing twice differentiable functions that have a non-vanishing second derivative on the interval. Define the ratio \( \gamma(x) = \frac{s_A(x)}{s_B(1-x)} \) and the equilibrium cutpoint, \( C^*(a) = w(a) - s_B(1 - x^*(a)) \). In this notation, we obtain the following characterization.

**Proposition 3.** *(generalized war and settlement payoff model)* For a fixed model with parameter \( a \) and equilibrium offer \( x^*(a) \), the sign of \( \pi'(a) \) is the same as the sign of

\[
\left(1 - H[C^*(a)]\right)\gamma'(x) - h[C^*(a)]s_B'(1 - x)(\gamma(x) + \frac{p'(a)}{w'(a)}) \\
\frac{h'[C^*(a)]}{h[C^*(a)]}(p(a) - s_A(x))s_B'(1 - x) - h[C^*(a)](2\gamma(x)s_B'(1 - x)) + (1 - H[C^*(a)])\gamma'(x). 
\]

The proof of this result follows the same steps as before, so we leave the details of the analysis in the appendix. Looking at Proposition 3, the result implies that neutrality requires that the numerator of the expression is 0. Formally, then, we need

\[
(1 - H[C^*(a)])\gamma'(x) = h[C^*(a)]s_B'(1 - x)(\gamma(x) + \frac{p'(a)}{w'(a)}).
\]

This expression can be written as

\[
\frac{(1 - H[C^*(a)])}{h[C^*(a)]} = \frac{s_B'(1 - x)(\gamma(x) + \frac{p'(a)}{w'(a)})}{\gamma'(x)}.
\]

The numerator of the right hand side simplifies to

\[\text{In this specification, the equilibrium offer solves a second order polynomial. The solution is thus found in closed form, but is not worth reproducing here. Once the appropriate root is substituted into the probability of war function and simplified the above expression obtains.}\]
\[ s'_A(x) + s'_B(1 - x) \left( \frac{p'(a)}{w'(a)} \right), \]

which leads to the following corollary.

**Corollary 1.** In the more general model, neutrality occurs at an equilibrium \( x^*(a) \) if and only if

\[ \frac{s'_A(x^*(a))}{s'_B(1 - x^*(a))} + \frac{p'(a)}{w'(a)} = 0. \]

So when the ratio of the rate of change of utilities for the settlement at equilibrium are the negative of the ratio of the rates of change of the war payoff, neutrality holds. For example, in the standard model, \( \frac{s'_A(x^*(a))}{s'_B(1 - x^*(a))} = 1 \) because the utilities were simply the size of the share, and when the probabilities of winning are \( p \) and \( 1 - p \), then \( \frac{p'(a)}{w'(a)} = -1 \) and we have neutrality. But of course with relaxed functional forms this balancing need not occur.

### 4. Applications

In the more general presentation of the bargaining game in Fearon (1995) each player has a utility \( u(x) \) that was increasing and concave on the unit interval and normalized so that \( u(1) = 1 \) and \( u(0) = 0 \). Thus \( p(a) = p - c_A \), \( w(a) = 1 - p \) and \( \frac{p'(a)}{w'(a)} = -1 \). Although this general version is not solved in Fearon, it is easy to apply our result to see that when both players have the same strictly concave \( u(x) \), as long as \( x^* \neq 1/2 \) and \( u'(x) \neq u'(1 - x) \), then by Corollary 1 neutrality does not hold. This is easily seen in our example above where the utilities for a settlement are \( \sqrt{x} \). This leads to an equilibrium probability of war described in Equation 1, which is decreasing in \( p \).

In another example, if we follow Stam (1996), the payoffs to war for countries \( A \) and \( B \) are given by

\[ p(a) = \pi(a) + q(a)\delta V_A - c_A \]
\[ w(a) = q(a)\delta V_B + (1 - \pi(a) - q(a)), \]
where \( \pi(a) \) is the probability that war ends this period with a victory for \( A \), \( q(a) \) is the probability of a stalemate where the game will move to the next period, which is discounted and valued at \( V_i \), and \( (1 - \pi(a) - q(a)) \) is the probability that country \( B \) wins this period. With standard linear payoffs to settlement, by Corollary 1, there can only be neutrality in this model if \( q'(a) \) is zero at the equilibrium offer or \( \delta V_A = \delta V_B - 1 \).

A natural application of the results here is to reconsider work on the balance of power. A longstanding question for theoretical and empirical scholars of security has been whether a balance of power results in greater stability\(^{19}\) or more conflict.\(^{20}\) Powell (1996) develops a formal model to help answer this question. In Powell’s treatment changes to the balance of power influence the probability that the proposer makes an offer with risk, but the balance of power does not have a direct effect on the probability of conflict conditional on an offer with risk being made. In the Powell model \( u_i(t) = t \), \( w(a) = p \), and \( p(a) = 1 - p - c_2 \). If we differentiate the relevant functions and evaluate them as required, then Corollary 1’s neutrality condition is satisfied. In fact, substituting the solution in Powell (1996, p. 266) for the optimal offer into the probability of war function, then whenever \( x^* \) is greater than the status quo the probability of war is \( (1 - c_2)/2 \), which is not a function of \( p \).

Stepping outside of Powell’s model we observe that the empirical findings on the balance of power are mixed. More precisely, it is not that no significant results have been found, but rather different papers find different results. This suggests that perhaps the comparative static depends on other features of the landscape and different empirical exercises are picking up different patterns. To see how the logic behind Propositions 2 and 3 might support further work connecting theory and empirics, consider a very stylized example with no additional non-linearities but one additional form of heterogeneity. Suppose that country \( A \) derives value \( \gamma \) from winning the prize while country \( B \) derives value 1 from winning the prize. Importantly then the gain from winning over losing to \( A \) is higher than it is to \( B \) if \( \gamma > 1 \) and the opposite is true if \( \gamma < 1 \). In this case we can just let \( a \) denote

\(^{19}\)Claude 1962, Morgenthau 2006, Mearsheimer 1990.

the probability that $A$ wins and we have the functions

\[ p(a) = \gamma a - c_A \]
\[ w(a) = 1 - a - c_B. \]

In this case we have

\[ p'(a) = \gamma \]
\[ w'(a) = -1. \]

Thus by Proposition 2, when $\gamma > 1$ increases in the capacity of $A$ will increase the probability of war, and when $\gamma < 1$ increases in the capacity of $A$ will decrease the probability of war. These conclusions do not depend on whether $a$, here interpreted as the probability that $A$ wins, is close to or far from $\frac{1}{2}$. So here we find a monotone relationship between strength and stability where the direction of changes depends on whether we are considering increases in the strength of the side for whom the war fighting stakes are higher or lower. Our point is not to forward this example as a strong model of balance of power, but rather to illustrate that even in models very close to that in Fearon (1995), failure to account for asymmetries such as this could help explain why the empirical record is so mixed.

These examples show that many of the models we regularly study will not satisfy the neutrality condition. The fact that the result of the comparative statics exercises depends on how one changes the payoffs should not be taken as an indictment of these game-theoretic models, but rather a confirmation of the relevance of formal modeling. The fact that the results depend on some nuances of how states value the tradeoff between war fighting and settlements points to a subtlety which is otherwise unknown to the literature on the consequence of the distribution of power. Wittman (1979) foresaw an indeterminacy when thinking about balance of power and war risk when he wrote:
The analysis in this article suggests that this debate [balance of power] is irrelevant. There is no relationship between the probability of winning and the probability of war and therefore we will find no consistent empirical relationship between the two.... If one side is more likely to win at war, its peaceful demands increase; but at the same time the other side’s peaceful demands decrease. Thus we do not know whether an overlap is more or less likely. Furthermore, if one side’s increase in subjective probability of winning is equal to the other side’s decrease in subjective probability of winning, there will be no change in the probability of war (p. 751).

But Wittman was writing prior to the application of bargaining models with asymmetric information in international relations and thus he does not actually conduct equilibrium analysis on a model which can sustain the existence of war. Both Fearon and Powell worked with models in which the risk of war emerged endogenously but they specify payoffs which lack the flexibility of Wittman’s formulation. Although Wittman’s conjecture that offsetting changes in war payoffs will have no affect on the probability of war is correct, his overall conclusion is nevertheless too strong. Our analysis illustrates exactly what additional information is needed to understand the relationship between balance and stability, and the two specifications that we applied the result to illustrate how one can incorporate richer models of war fighting into the canonical bargaining setting.

5. Conclusion

Although the crisis bargaining game developed in Fearon (1995) has been quite influential, it has an important limitation when used as a benchmark model for the analysis of changes to war payoffs. We show that the neutrality result is not a limitation of the underlying framework but rather a consequence of the interaction of some individually reasonable simplifying assumptions that add up to this result. The basic structure and logic of the Fearon model is quite amenable to natural extensions that render the model more appropriate when the theory is focused on changing the military balance. In other
words, close cousins of the standard model can be well suited to answering questions regarding the underlying effect of changes in the probability of victory. Specifically, the equilibrium probability of war depends on only whether the effect on A’s war payoff is larger or smaller on the affect of B’s war payoff. When the effect on the privileged disputant dominates, then the risk of war increases. Of course, the choice of the appropriate variant of this model for a given question is where the art lies.

A final point is in order. It is of conceptual value to recognize that the neutrality result is not an artifact of risk aversion as others have speculated (Fearon 1992, Kydd 2010). In fact in a model with only two possible outcomes, the idea of risk aversion is not particularly relevant. Perhaps a more fitting description of what previous scholars have meant is linearity of utility over the shares of the prize. Under this interpretation too we find that previous assessments of what drives the neutrality result miss the mark. Our analysis leading to Proposition 2 maintains linearity of utility over the shares and relaxes the requirement that shocks result in offsetting changes to the payoffs from fighting in a model with two possible war outcomes. Although we conclude that departures from risk neutrality or linearity are not essential to violations of the neutrality result, our Proposition 3 does illustrate some role for non-linearities in the payoffs from settlement shares to fuel violations of the neutrality result. Taken together, these theoretical findings indicate that the choice of how to model war fighting is important, surely merits more thought in subsequent theoretical work, and can likely be accommodated within the confines of canonical bargaining models.
Proof of Proposition 3.

Proof. Player A selects $x$ to maximize

$$H[w(a) - s_B(1 - x)]p(a) + (1 - H[w(a) - s_B(1 - x)])s_A(x).$$

The first order condition for an interior $x$ is

$$h[w(a) - s_B(1 - x)][p(a) - s_A(x)] + (1 - H[w(a) - s_B(1 - x)])\gamma(x) = 0.$$

For ease of exposition, we suppress the arguments when the meaning is clear, writing $H[w(a) - s_B(1 - x)]$ as $H[\cdot]$, with similar choices for $h$ and the derivative of the density $h'$. Applying the implicit function theorem to the first order condition we obtain,

$$\frac{dx^*(a)}{da} = -\frac{h'[\cdot](p(a) - s_A(x)) + h[\cdot](\frac{w'(a)}{w'(a)} - \gamma(x))}{h'[\cdot](p(a) - s_A(x))s'_B(1 - x) - h'[\cdot](2\gamma(x)s'_B(1 - x)) + (1 - H[\cdot])\gamma'(x)}.$$

Applying the chain rule and fact that $\pi^*(a) = H[w(a) - s_B(1 - x^*(a))]$, we obtain

$$\frac{d\pi^*(a)}{da} = h[\cdot](w'(a) + s'_B(1 - x)\frac{dx^*(a)}{da}).$$

Since $h[\cdot]$ is strictly positive, the sign of the above derivative coincides with the sign of $w'(a) + s'_B(1 - x)\frac{dx^*(a)}{da}$. Substituting we see that this expression reduces to

$$1 - s'_B(1 - x)[\frac{h'[\cdot](p(a) - s_A(x)) + h[\cdot](\frac{w'(a)}{w'(a)} - \gamma(x))}{h'[\cdot](p(a) - s_A(x))s'_B(1 - x) - h[\cdot](2\gamma(x)s'_B(1 - x)) + (1 - H[\cdot])\gamma'(x)}].$$

Finding a common denominator, we can simplify this to

$$\frac{(1 - H[\cdot])\gamma'(x) - h[\cdot]s'_B(1 - x)(\gamma(x) + \frac{w'(a)}{w'(a)})}{h'[\cdot](p(a) - s_A(x))s'_B(1 - x) - h[\cdot](2\gamma(x)s'_B(1 - x)) + (1 - H[\cdot])\gamma'(x)}.$$
References


