

Project Outcomes Report

Project Title: Collaborative Research: Leveraging Comparison and Explanation of Multiple Strategies (CEMS) to Improve Algebra Learning

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When it comes to learning strategies for solving math problems, students often memorize the steps without comprehending the underlying mathematical principles or attending to when it is most appropriate to use each strategy. Asking students to compare and discuss multiple strategies for solving the same problem encourages higher order thinking and develops students' knowledge of why and when to use different strategies. In addition, teachers need materials and routines to support their use of these practices in their classrooms. We revised and evaluated instructional materials and routines to *Compare and Discuss Multiple Strategies (CDMS)* for Algebra I topics, beginning with materials developed in a prior grant ([DRL0814571](#)).

Our materials and instructional routines ensure that multiple strategies for solving the same problem are clear and visible simultaneously to students and use well understood terminology and visual cues to promote comparison (see example). These materials include reflection questions to encourage student explanation of key points.

The major advances in this grant were instructional routines and expanded teacher professional development. Because many high school math teachers continue to structure their teaching around lecture and independent practice, we developed and scaffolded an instructional routine for supporting small group work and whole class discussion. A key discussion question is posed (e.g., "Is there a situation where substitution would be better than elimination, or vice versa?"), and then students think for a minute on their own, pair with another student to discuss their answers, and then share ideas in a whole class discussion (e.g., Think-Pair-Share). At the end of the discussion, the teacher summarizes the main points of the discussion and asks students to write a summary in their own words. This routine is supported by a graphic organizer that prompts students to engage in and take notes on each phase. Teachers are provided with sample questions to help draw out multiple student responses to the same question and to ask students to build on one another's ideas. Although we had encouraged these practices in the past, developing and supporting a clear instructional routine with a graphic organizer led to much more consistent and high-quality implementation.

Professional development (PD) occurred in the summer and throughout the school year. In the summer, teachers were introduced to our curriculum during a one-week, 35-hour professional development institute that we designed and administered. The PD included videotaped exemplars of other teachers using the curriculum, and experiences planning and teaching sample lessons to peers. During the school year, teachers received brief one-on-one support to help plan when to use the materials and feedback that highlighted what they were doing well and specific ways to improve implementation.

In the first 2 years of the grant, 9 teachers integrated our CDMS lessons into their Algebra I instruction, as we refined our approach. We also tested whether the depiction of strategies as being used by particular fictitious students in our materials was effective and found that it was.

This grant culminated with a yearlong evaluation of our approach in Algebra I classrooms in Massachusetts and New Hampshire. There were 16 treatment teachers leading 25 sections with 550 students and 13 control teachers leading 21 sections with 498 students. Control teachers used their typical algebra instruction while treatment teachers received professional development and access to our CDMS lessons. Treatment teachers typically used 28 of our 40 CDMS lessons during the school year, spending an average of 17 minutes per lesson. Treatment teachers dedicated extended time to comparison of multiple strategies, use of small group work, and use of class discussion in most observed lessons. In contrast, control teachers did these things infrequently. When using our materials, treatment teachers often asked conceptual and open-ended questions, elicited extended responses from students, and supported procedural flexibility. Students in the treatment condition had greater algebra knowledge than students in the control condition for the unit on linear equations, although not for three other units after we accounted for pre-existing differences between the groups. Thus, our CDMS approach shows promise for improving classroom instructional quality, but has not yet led to robust improvements in student knowledge.

In summary, the intellectual merit of the project includes an expanded theory of algebra learning that incorporates how students develop robust knowledge for working with symbolic strategies, including how CDMS supports procedural flexibility, conceptual knowledge and procedural knowledge.

In addition to the broader impact of this research on the quality of instruction that participating students received, dissemination efforts have made our CDMS instructional approach available to algebra teachers throughout the country at <https://www.compareanddiscuss.com/>. The research project has also provided valuable research opportunities for undergraduates, including two students at a local Historically Black College/University who went on to become mathematics teachers.

Gloria and Tim were asked to solve $5(x + 3) = 20$.

Gloria's "distribute first" way

Tim's "divide first" way

First I distributed.

Then I subtracted on both sides.

I divided by 5.

Here is my answer.



$$5(x + 3) = 20$$

$$5x + 15 = 20$$



$$\begin{array}{r} 5x + 15 = 20 \\ -15 \quad -15 \\ \hline \end{array}$$



$$\frac{5x}{5} = \frac{5}{5}$$



$$x = 1$$



$$5(x + 3) = 20$$

$$\frac{5(x + 3)}{5} = \frac{20}{5}$$



$$\begin{array}{r} x + 3 = 4 \\ -3 \quad -3 \\ \hline \end{array}$$



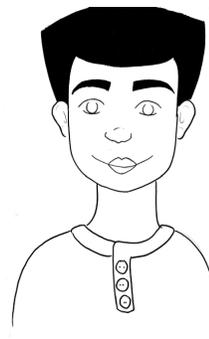
$$x = 1$$



First I divided by 5.

Then I subtracted from both sides.

Here is my answer.



How did Gloria and Tim find the solution to the equation?



Which method is better? What are some important differences between Gloria's "distribute first" method and Tim's "divide first" method?

Discuss Connections

If solving $5(x+2) + 7 = 12$, which method is better? why?



Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair



Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?



Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words: