Shanks Workshop on Combinatorics and Graph Theory

April 13 – 14, 2024

Vanderbilt University

Nashville, United States

Venue

Room 1307
Stevenson Center Building 1
Vanderbilt University
Nashville, TN 37240

Organizing Committee

Mark Ellingham (Vanderbilt University), mark.ellingham@vanderbilt.edu
Xiaonan Liu (Vanderbilt University), xiaonan.liu@vanderbilt.edu

Schedule

Venue: Stevenson Center 1307

Saturday, April 13th		
Time	**Speaker**	**Title**
9:00-9:50	Zi-Xia Song	Hadwiger's Conjecture
Coffee Break		
10:20-10:45	Vaidy Sivaraman	Chromatic number: Problems, puzzles, and paradoxes
10:55-11:20	Tom Kelly	Hypergraph embeddings and decompositions: robustness via spreadness
Lunch Break		
14:00-14:25	Anton Bernshteyn	Correspondence Coloring from Random Covers
14:35-15:00	Zilin Jiang	Beyond the classification theorem of Cameron, Goethals, Seidel, and Shult
15:10-15:35	Fan Wei	Limit points of the top eigenvalues of d-regular graphs
Coffee Break		
16:00-17:40	Open Problems Session	

Sunday, April 14th		
Time	**Speaker**	**Title**
9:00-9:50	Xingxing Yu	Linkages in graphs
Coffee Break		
10:20-10:45	Tung Nguyen	A few steps towards the Erdős–Hajnal conjecture
10:55-11:20	Songling Shan	An Ore-type condition for hamiltonicity in tough graphs and the extremal examples
11:30-11:55	Rose McCarty	Colorings of the plane cannot avoid prime distances

Title & Abstracts

Hadwiger's Conjecture

Zi-Xia Song
University of Central Florida

Hadwiger's Conjecture from 1943 states that for every integer $t \geq 1$, every graph either can be t-colored or has a subgraph that can be contracted to the complete graph on $t+1$ vertices. This is a far-reaching generalization of the Four-Color Theorem and perhaps the most famous conjecture in graph theory. Until recently the best known upper bound on the chromatic number graphs with no K_t minor is $O(t(\log t)^{1/2})$, obtained independently by Kostochka and Thomason in 1984, while joint with Norin and Postle, we improved the frightening $(\log t)^{1/2}$ term to $(\log t)^{1/4}$ in 2019. The current record is $O(t \log \log t)$ due to Delcourt and Postle. In this talk we will survey the history of Hadwiger's Conjecture and discuss some aspects of the proofs.

Chromatic number: Problems, puzzles, and paradoxes

Vaidy Sivaraman
Mississippi State University

The chromatic number of a graph is an invariant of fundamental importance in structural and algorithmic graph theory. How does a graph look like if it has large chromatic number? What structures can we find in it? This talk will feature several open problems relating the chromatic number to induced subgraphs, minors, and graph complementation.

Hypergraph embeddings and decompositions: robustness via spreadness

Tom Kelly
Gerogia Institute of Technology

A graph H embeds in a graph G if G contains a subgraph isomorphic to H, and it decomposes G if the edges of G can be partitioned into subgraphs isomorphic to H. Questions about when a graph embeds in or decomposes another are central in combinatorics. "Dirac-type" embedding results address minimum-degree conditions to ensure an embedding of some graph. Block designs, a fundamental object of Design Theory, are decompositions of complete graphs.

In this talk, we will discuss robustness of embeddings and decompositions. For example, given a hypergraph of large minimum degree, we will discuss the threshold for a random subhypergraph to have a perfect matching or Hamilton cycle. We will also discuss the threshold for constructing block designs using only a random selection of blocks. All of these results utilize the recent Park–Pham Theorem or one of its variants. A crucial notion for this is that of the spreadness of a certain type of probability distribution.

Correspondence Coloring from Random Covers

Anton Bernshteyn

Georgia Institute of Technology

DP-coloring (also called correspondence coloring) is a generalization of list coloring introduced by Dvořák and Postle, in which the lists of available colors vary from vertex to vertex and the identifications between the lists vary from edge to edge. In this talk I will focus on the case where the correspondences between the lists are chosen uniformly at random and present some results, problems, and conjectures that arise in this setting. Joint work with Daniel Dominik, Hemanshu Kaul, and Jeffrey Mudrock.

Beyond the classification theorem of Cameron, Goethals, Seidel, and Shult

Zilin Jiang

Arizona State University

The classification of graphs with smallest eigenvalues at least -2 culminated in a beautiful theorem of Cameron, Goethals, Seidel and Shult, who related such graphs to root systems from the representation theory of semisimple Lie algebras. In this talk, I will explore graphs with smallest eigenvalues between -2 and $-\lambda$, where λ is about 2.0198, and I will explain why the mysterious number λ is a barrier for classification. Joint work with Alexander Polyanskii and Hricha Acharya.

Limit points of the top eigenvalues of d-regular graphs

Fan Wei

Duke University

For each $d \geq 3$, what is the set of all limit points of the second largest eigenvalue of growing sequences of d-regular graphs? This question can be viewed as a variation of the inverse spectral problem, whose analogue for all hyperbolic surfaces (with genus tending to infinity) is a consequence of the recent breakthrough work of Hide and Magee. We also discuss a more general question of identifying all vectors which are limit points of the vectors of the top k eigenvalues of sequences of d-regular graphs. Our technique provides a method to construct d-regular almost Ramanujan graphs with large girth and localized eigenvectors corresponding to eigenvalues larger than $2\sqrt{d-1}$, extending in some aspect a result of Alon, Ganguly, and Srivastava. This is a joint work with Noga Alon.

Linkages in graphs

Xingxing Yu
Georgia Institute of Technology

A fundamental result in structural graph theory is the characterization of graphs that are not 2-linked. Those are graphs in which there exist distinct vertices a_1, a_2, b_1, b_2 with no disjoint paths from a_1, b_1 to a_2, b_2, respectively; and such graphs are essentially planar with a_1, b_1, a_2, b_2 on the boundary of a face in cyclic order. We consider a more general question: For two sets of vertices S_1, S_2 in a graph G, when does G contain disjoint connected subgraphs G_1, G_2 such that $S_1 \subset G_1$ and $S_2 \subset G_2$? This is difficult to answer even for the case when $|S_1| = 2$ and $|S_2| = 3$. In this talk, I will present recent results on this question, based on joint work with Xiying Du, Yanjia Li, and Shijie Xie.

A few steps towards the Erdős–Hajnal conjecture

Tung Nguyen
Princeton University

A cornerstone of Ramsey theory says that every graph contains a clique or independent set of logarithmic size, which is asymptotically optimal for almost all graphs. The Erdős–Hajnal conjecture from 1977 predicts a very different situation in graphs with forbidden induced subgraphs; more precisely, the conjecture asserts that for every graph H, there exists $c = c(H) > 0$ such that every n-vertex graph with no induced copy of H has a clique or independent set of size at least n^c. This conjecture remains wide open, and we will discuss recent progress on it in the talk.

An Ore-type condition for hamiltonicity in tough graphs and the extremal examples

Songling Shan
Auburn University

Let G be a t-tough graph on $n \geq 3$ vertices for some $t > 0$. It was shown by Bauer et al. in 1995 that if the minimum degree of G is greater than $\frac{n}{t+1} - 1$, then G is hamiltonian. In terms of Ore-type hamiltonicity conditions, the problem was only studied when t is between 1 and 2, and recently the second author proved a general result. The result states that if the degree sum of any two nonadjacent vertices of G is greater than $\frac{2n}{t+1} + t - 2$, then G is hamiltonian. It was conjectured in the same paper that the "$+t$" in the bound $\frac{2n}{t+1} + t - 2$ can be removed. Here we confirm the conjecture. The result generalizes the result by Bauer, Broersma, van den Heuvel, and Veldman. Furthermore, we characterize all t-tough graphs G on $n \geq 3$ vertices for which $\sigma_2(G) = \frac{2n}{t+1} - 2$ but G is non-hamiltonian. This is joint work with Masahiro Sanka.

Colorings of the plane cannot avoid prime distances

Rose McCarty
Georgia Institute of Technology

We prove that in any coloring of the plane with finitely many colors, there exist two points which receive the same color and whose (Euclidean) distance is a prime number. Our approach is based on a breakthrough result of Davies which solved the odd distance problem. This talk aims to give a high-level overview of the approach. The key idea is to apply a spectral bound for infinite Cayley graphs, and estimate the eigenvalues by approximating sums with integrals.This is joint work with James Davies and Michał Pilipczuk.
