Probabilistic versus continuum descriptions of rarefied sediment particle motions and transport

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Abstract. Sediment transport in many hillslope and riverine settings involves rarefied conditions in which moving particles are at low concentrations. Such conditions are at odds with conventional continuum formulations of transport. In this first companion paper we describe rarefied sediment particle motions and transport from the probabilistic perspective of statistical mechanics. Quantities that are considered deterministic in continuum systems — notably particle fluxes — are stochastic quantities with decided uncertainty in sediment systems. This uncertainty inextricably depends on the spatial resolution and the averaging time, which set the rate of convergence to ensemble expected conditions. Moreover, particle trajectories matter: owing to the strongly unidirectional motions of bed load particles, flux components measured at large angles from the mean motion may be poorly resolved, requiring unreasonably large averaging times for small spatial scales. In contrast, flux components associated with the radial symmetry of rain splash trajectories are insensitive to coordinate orientation. The Exner equation represents noise — a Lévy process with drift — at small space and time scales. Convergence to the expected rate of change in the surface elevation depends on the magnitude of this expected rate and therefore the intensity of forcing, the spatial resolution and the interval of time averaging. A spatial resolution that is necessarily smaller than the changing topographic feature (e.g. bedform) might require a large averaging time to achieve convergence to expected conditions, so the feature must be sufficiently large or change sufficiently slowly to achieve quasi-steady conditions nominally represented by the Exner equation. Characterizing the fluctuations in the flux and its divergence therefore is key to interpreting the noisiness and uncertainty of the responses of topographic features to both steady and unsteady conditions of particle excitation and transport. In our data-rich companion paper we present details for rain splash and bed load transport with the contrasting example of continuum conditions of air.

1 Introduction

Sediment transport in many natural and experimental settings involves rarefied conditions in which moving particles are at low concentrations. Rarefied conditions occur with transport of soil particles by rain splash (Furbish et al., 2007, 2009, 2017a; Dunne et al., 2010), the skittering of rockfall material over scree slopes (Gerber and Scheidegger, 1974; Kirkby and Statham, 1975; Statham, 1976; Dorren, 2003; Tesson et al., 2020; Furbish et al., 2021; Williams and Furbish, 2021a) and the raveling of particles on hillslopes following disturbances or release from storage behind vegetation (Roering and Gerber, 2005; Lamb et al., 2011, 2013; DiBiase and Lamb, 2013; DiBiase et al., 2017; Doane, 2018; Doane et al., 2018, 2019; Roth et al., 2020). Rarefied conditions occur with transport of bed load particles near threshold conditions (Ancey et al., 2008; Ancey, 2010; Furbish et al., 2012; Roseberry et al., 2012; Chartrand et al., 2022; Pierce et al. 2022) and transport of sand by wind. In these situations the interactions between particles and the surface over which they move are far more important in determining the dynamical behavior of the particles than are their interactions with other moving particles (Furbish et al., 2021a), akin to granular shear flows at high Knudsen number (Risso and Cordero, 2002; Kumaran, 2005, 2006). And, when particle motions are coupled with surrounding fluid motions, particle-surface interactions combined with alternating states of motion and rest hold a central role in particle behavior (Einstein, 1937; Furbish et al., 2012a; Roseberry et al., 2012; Fathel et al., 2015; Pierce and Hassan, 2020; Pierce et al., 2022). Rarefied particle motions are therefore distinct from granular flows and suspensions. In addition, when viewed with respect to their small number concentrations, the motions and transport of sediment tracer particles involve rarefied conditions. This includes natural tracer particles in soils, for example, particles identified by their cosmogenic nuclide concentrations or luminescence ages or intensities (Furbish et al., 2018a, 2018b; Gray et al., 2020), and the movement of artificial tracer particles in natural (e.g. Ferguson and Wathen, 1998) and experimental (e.g. Hill et al, 2020) rivers.

Descriptions of sediment transport, including rarefied conditions, conventionally adopt a continuum-like framework. Yet rarefied conditions are at odds with continuum formulations of transport (Furbish et al., 2012; 2017a, 2017b, 2018c, 2021a; Ancey, 2020a, 2020b; Furbish and Doane, 2021; Williams and Furbish, 2021; Pierce, 2021; Hassan et al., 2022). As Furbish and Doane (2021) note:

> The continuum hypothesis... stands as a triumph of the physical sciences... [allowing] us to envision many solid and fluid materials at our ordinary macroscopic scale of observation as being continuous things whose properties and behavior can be described using that part of the calculus given to continuously differentiable functions — even though when we focus our attention on the scale of the elements of a "continuous" material, that is, at the particle scale, we discover that it is decidedly discontinuous. That said, this lovely continuum siren is to be avoided as a de facto starting point in descriptions of sediment motions and transport.

Furbish and Doane (2021) use the example of rarefied particle motions on hillslopes to outline philosophical and technical aspects of pursuing a statistical mechanics description of rarefied transport conditions without assuming a continuum behavior at the outset. This is within the context of a broader effort. For example, Furbish and Doane (2021) offer a sample of 81 papers representing recent work on probabilistic elements of sediment motions and transport in five topical areas: bed load particle motions and transport; bed load tracer particle motions, including effects of particle-bed exchanges; nonlocal sediment transport on hillslopes; particle motions in soils, including tracer particles; and rain splash transport. Among these, Schumer et al. (2009) provide a valuable primer on using advection-dispersion equations to describe tracer particle transport and Ancey (2020a, 2020b) provides a timely review of the state of research efforts focused on bed load transport. Importantly, the probabilistic nature of this body of work is not conditioned by a continuum framework, and it in part reflects insights gained from increasing access to advanced computations and high-speed imaging techniques. Much of this work is focused on the kinematics of particle motions and transport, but with increasing efforts to explicitly incorporate mechanics. For example, this includes efforts to couple bed load particle motions with fluid motions using a Langevin or Langevin-like equation (Ancey and Heyman, 2014; Fan et al., 2016; Pierce et al., 2022) and consideration of the energetics of particle motions on hillslopes (Furbish et al., 2021a, 2021b, 2021c).

Herein we step back just a bit, stay at the kinematic level, and address a key element of the problem. Namely, we examine the consequences of the relatively small numbers of particles in motion during rarefied transport, paying close attention to the geometry of particle motions, in relation to what we envision for continuum conditions. We elaborate how definitions and measurements of the particle flux can involve substantial, unavoidable uncertainty that inextricably depends on the spatial resolution and time averaging involved (e.g. Ancey and Pascal, 2020; Furbish and Doane, 2021; Pierce et al., 2022). We show that this uncertainty is closely connected with the orientation of the coordinate system relative to the mean particle motion as a consequence of the directional anisotropy of particle trajectories. And we show how the uncertainty associated with definitions and measurements of the particle flux varies with position and time in relation to waxing and waning particle excitation. Our analysis clarifies the meaning of the particle flux, distinguishing between ensemble conditions versus what occurs in any realization, and it shows why the Exner equation is illusory and represents noise — a Lévy process with drift — at small space and time scales. A key outcome of the analysis is a description of the scales at which the particle flux may be viewed as varying smoothly.

Questions concerning the use of a continuum framework to describe moving granular materials are not new. In his pioneering work on granular flows as a fluid mechanical phenomenon, Haff (1983) cautions for the need to assess whether such flows actually satisfy the continuum hypothesis, and indeed Haff's cooling law is limited to the homogeneous cooling state (Brito and Ernst, 1998; Nie et al., 2002; Brilliantov and Pöschel, 2004; Dominguez and Zenit, 2007; Brilliantov et al., 2018; Yu et al., 2020). Owing to limitations of continuum-like descriptions, studies of granular gases have largely proceeded using numerical simulations of particle behavior wherein inelastic collisions have a central role in clustering, collapse and aggregation behaviors (Brilliantov and Pöschel, 2004; Brilliantov et al., 2018). Continuing efforts to explore hydrodynamic formulations of granular gases typically focus on restrictive conditions, for example, the onset of clustering as a hydrodynamic instability (Mitrano et al., 2014; Louge, 2014) and the appearance of hydrodynamic behavior at certain scales (Duffy and Brey, 2003). Similarly there are ongoing questions concerning the rheological behavior of dense granular flows, for example, the significance of nonlocal effects (Henann and Kamrin, 2013) and the importance of dynamic compressibility in continuum models relating the effective viscosity to the inertial number (Heyman et al., 2022). Thus, the analysis described in this paper centered on rarefied sediment transport is part of a larger effort to clarify consequences of the specifics of particle behavior during transport rather than assuming these specifics intrinsically admit a conventional continuum-like description.

In this paper and its companion we focus on two transport processes: rain splash transport and bed load transport. We choose these specifically because the geometry of particle trajectories are in each case reasonably well-constrained, yet distinctive, as revealed by high-speed imaging, and because the mechanisms of particle excitation offer a key contrast for points we want to make. Our analysis therefore is based on well-known attributes of sediment particle motions combined with principles from statistical mechanics and probability adapted to sediment particle systems. The analysis rests on no moot assumptions and contains no surprises. The analysis is supported with theory, simple numerical simulations and experimental data, including time series of bed load transport involving a mixture of particle sizes sampled at 1 Hz (Chartrand, 2017; Hassan et al., 2022).

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