AN ANALYSIS OF THE IMPACT OF GOVERNMENT DEBT AND TAXATION ON GROWTH: DOES IT MATTER IF THE DEBT IS HELD ABROAD?

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Abstract

An analysis is conducted into the effect that government debt and taxation have on growth, and whether these results depend upon whether the debt is held domestically or abroad. It is shown that the effect of the size of government debt on growth depends critically on the intertemporal elasticity of substitution of consumption. Higher government debt may reduce growth if it is held internally, but may *increase* growth if the debt is held abroad. The effect of a tax cut on growth can similarly depend on whether the debt is held internally, or held abroad. It is possible that higher initial levels of government debt may increase the resulting growth rate temporarily, but reduce it in the long term. In contrast with existing models, it is shown that a temporary tax cut can have a larger impact on the growth rate than would a permanent tax cut of the same magnitude. A "Debt Trigger" is introduced which prohibits the government from expanding its debt beyond some specific share of GDP. For some parameter values the model exhibits *Unpleasant Fiscal Arithmetic*: reducing the capital tax rate can initiate a drastic growth implosion. Labor-leisure choice is then incorporated into the model, and it is then shown that the growth rate may be increasing in the rate at which the future labor tax is increasing. The results presented here show that there is a complex relationship between the size of the government debt, and the resulting growth and interest rates.

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1 Introduction

In light of the fact that many governments have accumulated high levels of debt, it seems natural to assess the impact that this debt will have on subsequent economic growth. A high level of government debt has implications for the feasibility of subsequent government policies through the government's budget constraint. Nevertheless, this constraint does not pin down a unique path for policy variables, such as tax rates, and so the influence that the government debt can have on future growth is not clear. The analysis conducted below will show that the impact of government debt on growth depends on the nature of the government budget constraint, whether the debt is held domestically or abroad, and also on the consumer's intertemporal elasticity of substitution of consumption. It is shown that, ceteris paribus, higher levels of government debt do not necessarily reduce growth rates contemporaneously. In fact, depending on who holds the government debt, a higher level of debt may even increase the short-term growth rate. Additionally, it is shown that the size of the government debt also influences the response to changes in the tax rate.

One of the earliest and foremost analyses of how the size of the government debt would influence aggregate income is that of Diamond [3]. In his paper higher government debt is shown to reduce output by lowering saving as well as the capital stock.[1] It is natural to think, as Diamond's work suggests, that a higher level of government debt could reduce the growth rate. This also might happen for *different* reasons than suggested by Diamond, such as the fact that the debt must eventually be paid off through higher distortional taxes, which would then reduce the growth rate. However, the model studied here is used to show that this relationship can be much more complex than it first appears. This is because the higher future tax rates (or, the expectations thereof) can have a multitude of different effects on current consumption and investment decisions, which will then influence the current growth rate.[2]

In addition to studying the debt-growth relationship, this paper also seeks to answer other related, important questions, such as: What is the effect on the growth and interest rate of an economy, of a particular change in the tax rate? How does the size of the government debt affect the answer to this last question? Does it make any difference whether this government debt is held domestically, or is held by foreign citizens? To ensure that these questions are coherent, or well defined, it is necessary to make further assumptions about future government policies. This is because changes in current taxes can have many different implications for the future budget constraints of individuals and the government. That is, one cannot change just the tax revenue for a few periods in isolation from other government policy variables, because the government budget constraint will then imply that other things must also change: some combination of future taxes and or spending.

[1] Diamond uses an overlapping generations model. Despite the title of Diamond's paper, one drawback to his analysis is that there is no long-term endogenous growth in his model, and so this model cannot be used to determine how the government debt would influence the growth or interest rate.

[2] Besides the work of Diamond, there are other papers that study the interaction of growth and fiscal policy, but these papers focus on issues that are quite different from the central issues studied here. Saint-Paul [17] studies the effect of government debt in a growth model, but he uses economic models with externalities, and his focus is on dynamic efficiency. Greiner [5], and Greiner and Fincke [6] also studies how the public debt can influence the dynamics of the growth path. However, the focus of both of these papers is quite different from that of the present paper.

Therefore, to investigate how a change in the tax rate will influence the growth rate, one must also make some assumptions about how other *future* government policies are influenced as well.[3] In this paper it will be shown that the impact that a change in the capital tax rate can have on the growth rate will depend critically on various policies and parameters of the economy. Of particular importance is the parameter determining the intertemporal elasticity of substitution of consumption.[4]

In much of the existing literature it is frequently assumed (or shown) that a permanent change in capital tax rates will have a larger impact than a temporary change because the former will have larger incentive effects. However, it is shown below that one notable implication of the model is that this standard result may not necessarily be the case: it is certainly possible for a temporary change in capital taxes to have a *larger* impact on the growth rate than a permanent change.

An important economic feature that is introduced in this paper is that of a "Debt Trigger." This concept is borrowed from Sargent and Wallace [18]. In the model it is assumed that there is some legal or constitutional impediment that prohibits the ratio of government debt to GDP from rising above some pre-specified upper bound, and the government is not permitted to default. Once the debt reaches this upper bound, it is assumed that tax rate must immediately and permanently adjust so that the debt to GDP ratio does not exceed this upper bound.[5] With this mechanism in place, a reduction in the current tax rate may then imply that in the future the tax rate must be increased to prevent the relative size of the government debt from escalating. This mechanism will then imply that for some paths for government taxes, this debt trigger will eventually (and predictably) kick-in, and then government taxes must be immediately imposed at some higher level to keep the debt to GDP ratio constant.

Further support for the existence of a non-linear growth-debt relationship is obtained in the work of Kumar and Woo [12], Panizza and Presbitero [14] and also by Reinhart and Rogoff [15], [16]. These papers suggest that the economic growth rate tends to fall as the government debt to GDP ratio reaches a relatively high level. This would provide support for why one might see a form of a debt trigger in an economy. This model will then provide an economic foundation to show *why* GDP growth may fall once a specific high value of the debt to GDP ratio is reached.

When studying how economies react to a change in taxes, it is usually assumed that any change in tax revenue will be directed into a corresponding change in future government spending or transfers.[6] This assumption appears to be one of convenience, since there does not seem to be any compelling reason why such an offsetting change in future government spending must necessarily be assumed. It is certainly does not appear to be a necessary

[3] To be more precise, what is important is what agents *expect* to be the effect that a change in the current tax rate will have on future government policies.

[4] This parameter has been known to play an important role in many discrete-time dynamic models, particularly when studying issues related to business cycles. But until now this feature does not seem to have been identified as playing an important role in understanding how debt or taxes can influence growth.

[5] This is not entirely an ad-hoc assumption because if there were no such upper bound, then eventually the government budget constraint would be violated because the present value of tax receipts would be less than the current value of the government debt. Therefore, the government budget constraint implies that there must be some upper bound for the debt to GDP ratio.

[6] For example, Trabandt and Uhlig [21] consider both such possibilities in their study of the Laffer Curve.

characterization of the intent of actual policymakers. In this paper an alternative, but no less plausible policy, is assumed. Here a reduction in the tax rate in one period, which reduces government revenue, will necessarily imply an *increase* in future tax rates to make up for the lost revenue. In other words, rather than letting government spending or transfers fall because of the lost revenue, it will be assumed that government spending is unchanged, and so total tax revenue must be held fixed. Therefore, future taxes (rather than spending) must adjust in this case. This analysis is made slightly more complicated because the initial change in taxes may also affect the interest rate, and this feature must also be taken into consideration when calculating how much the discounted government revenue must adjust.

Another important issue addressed here is the different effects that are generated by having foreign-held government debt, as opposed to domestically-held debt. The existing literature seems reticent on the different impacts that might result from this distinction. People frequently talk of "government debt" as if it does not matter who holds it. This is an important issue since, as will be documented below, the last few decades have witnessed a dramatic increase in the fraction of US government publicly-held debt that is held abroad. The model studied below is ideally suited to analyze this issue, and it will be shown that the ownership of the debt can have dramatic implications for fiscal policy, and the growth rate in particular.

It should be stated that the goal of this paper is not normative in nature, in that the objective is *not* to characterize an optimal set of taxes. Instead, the analysis is more positive: the aim is to characterize the implications that alternative fiscal policies may have on an economy. Additionally, another objective is to understand the importance of identifying how current policy changes can influence the expectations of future policy changes. This paper is also a contribution to the literature explaining why capital tax rates seem to have such a negligible impact on the growth rate (see, for example, Stokey and Rebelo [20]). It is also a response to the call by Sims [19] to develop better models which help aid our understanding of how government debt and deficits influence private sector behavior.

The remainder of the paper is organized as follows. In the next section some data is presented on the magnitude of US tax rates and debt levels. Section 3 will present a primitive version of a model without government. Next, Section 4 will describe the structure of the model when government is introduced. Section 5 will then derive the solution to the consumer's intertemporal optimization problem, where government debt is held domestically only. Section 6 will illustrate the model in which the government debt is held abroad. Section 7 will describe the Debt Trigger mechanism, while Section 8 will show how the parameters of the economy are selected. Section 9 analyzes the case of domestically-held government debt. This analysis will focus on how the growth rate is affected by the tax cuts, the size of government debt, and the intertemporal elasticity of substitution. It is also shown that the model may exhibit feature denoted "Some Unpleasant Fiscal Arithmetic": a reduction in the capital tax rate may result in an immediate and continued fall in the growth rate. Section 10 conducts this analysis for the case in which the government debt is held abroad. Section 11 shows how a labor-choice could be incorporated into the model. It is shown that the labor tax can influence the growth rate only if it is changing over time. Final remarks are listed in the final section.

2 Some Data on Debt and Taxes

For illustrative purposes, it may be informative to review some data. In Figure 1, several measures of tax rates that are illustrated. The line labeled "IT" is the ratio of federal income tax revenue to GDP. The line labeled "IT+SS" is the ratio of federal income tax plus social security tax revenue, to GDP. These are very rough measures of the tax rate over many decades. Obviously there has been some growth in this ratio over the past 80 years, but no trend over the past 40 years. The lower dashed line of this figure is the growth rate of per-capita GDP. As has been pointed out by Stokey and Rebelo [20], there is very little relationship between the tax rate, measured in this manner, and the growth rate.[7] One might normally expect a negative relationship between the growth rate and the tax rate. However, the correlation between the growth rate and the income tax revenue to GDP ratio is 0.11, while the correlation between the growth rate and the total tax revenue to GDP ratio is 0.03.

Now of course the top panel in the figure does not really represent a measure of statutory or effective marginal tax rates. It is certainly possible to obtain the highest marginal income tax rates over this period. This is represented by the line labeled "Highest Tax Rate". Once more, there is not a close relationship between this tax rate and the growth rate. The correlation between these two variables is .21, which obviously does not support the notion that higher taxes have a harnessing effect on growth. There are many reasons why there might not be a strong negative relationship between these two variables, and this is explored by Stokey and Rebelo.

One reason why there is not a tight relationship between the growth rate to the tax rate may be that other circumstances (or important variables) have been changing over this period. In particular, Figure 2 shows that the size of the federal government debt to GDP ratio, has certainly changed over this time frame.[8] This raises an important question: Might it be the case that the response of the growth rate, to a change in the tax rate would depend on the size of government debt? In other words, would a reduction in the tax rate affect the growth rate to a degree that depends on the size of government debt? The model studied below will be capable of suggesting some answers to these questions.

Finally, Figure 3 shows the timeline of the foreign and domestically-held private shares of US government debt.[9] Since 1994 there has been a tremendous growth in the share of this debt that is held by foreign entities. From 1994 until 2013 the annualized growth rate in domestic privately-held nominal debt was 2.9%, while for this same period the growth rate for foreign privately-held nominal debt was 12%. In light of these facts, it would then seem important to understand how this domestic-foreign distribution of the debt might affect future growth rates. The model presented below is capable of doing just that.

The fraction of a country's government debt that is held abroad varies a great deal. Table 1 gives a measure of the fraction of the gross debt that is held abroad for a few select

[7] There are many other papers that have arrived at similar findings, such as Easterly and Rebelo [4], and Mendoza, Razin and Tesar [13].

[8] Of course this is only one very rudimentary measure of the indebtedness of the federal government. This does not capture the important obligations (or implicit promises) to individuals, such as through Social Security or Medicare, which are projected to balloon in the coming decades.

[9] Source: *Economic Report of the President*, various years.

economies.[10] With this in mind, it would seem paramount to understand the potential policy differences that might arise from having government debt held abroad, as to being held by domestic consumers.

Table 1

Country	% of Debt That is Held Abroad
Australia	1.8%
China	3.3
Israel	15.2
Brazil	17.5
Canada	29.1
Pakistan	30.3
Argentina	30.5
United Kingdom	32.5
Russia	33.2
Italy	39.4
Turkey	39.8
USA	43.7
Spain	44.4
Sweden	53.9
Poland	61.1
Indonesia	61.8
Mexico	63.7
Portugal	74.1
Bangladesh	75.0

3 The Primitive Structure of the Model

There are many alternative models in the Endogenous Growth literature that could be considered for this analysis. The model studied here must be sufficiently tractable that it is possible to calculate the exact response of the decision rules to a change in the tax rate. The objective here is to accomplish this without resorting to any approximation techniques which may not give precise answers. Precision is important here because the policy changes will impact the interest rate and future government revenue, and it is vital that these effects be calculated accurately, and not be loosely determined by some approximation methods. In a model with endogenous growth, any errors in the calculation of decision rules can be compounded over time and lead to misleading characterizations of present values.[11]

[10] This data is derived from the World Bank Quarterly Public Sector Debt Series (http://datatopics.worldbank.org/debt/qpsd). For each country the data is from either 2014(Q4) or 2015(Q1), whichever is available most recently. Data is available for a very limited number of countries. For each country, total debt is calculated as debt securities plus loans. This excludes SDR's, insurance and pension liabilities. The denominator here is central government gross debt, which would include debt held by central banks. This data is not directly comparable with that in Figure 3, which is measuring debt held by the public in the US.

[11] This issue is complicated even further in this paper because agents will be making decisions that generate a growth path based partly on the *expectation of policy changes that will be forthcoming in the future.*

Therefore it is paramount that the model can be used to precisely characterize the reaction of agent's decision rules to a tax change, as there is a the transition from one growth path to another. Some models are not well-suited for this task.[12] Therefore the model employed here will only be sufficiently complex to exhibit all the necessary features - but no more complex than necessary.

The first model studied will be relatively primitive, and does not rely on potentially confusing complications such as externalities, imperfect competition, or multiple sectors. Additionally, the general equilibrium effects that (perceived or actual) government policies can have on outcomes such as investment, or the interest rate, tend to be very transparent. Other complications, such as endogenous labor, for example, will be introduced and studied later.

The basic structure of the economic environment presented here before proceeding to talk about government policies. The economy will be a one in which there are identical infinitely-lived representative agents. Time will be assumed to be continuous, and indexed by t. Each individual will have preferences of the following CRRA type:

$$\int_0^\infty e^{-\rho t}\left[\frac{c_t^{1-\sigma}}{1-\sigma}\right]dt, \tag{1}$$

where c_t is the flow of consumption. Here ρ is the rate of time preference, $(\frac{1}{\sigma})$ is the intertemporal elasticity of substitution of consumption, and it is assumed that $\sigma > 0$, and $\sigma \neq 1$. There is no uncertainty in the model, and so there is no reason that the parameter σ should be related to a measure of risk-aversion.

The technology is very simple, and for now there will be no labor. At any moment there is a stock of capital (k_t), and this capital depreciates at the geometric rate of δ. Therefore, the constraints of the economy are then written as follows:

$$c_t + i_t = \lambda\left(k_t\right) \tag{2}$$

and

$$\dot{k}_t = i_t - \delta k_t \tag{3}$$

The parameter λ is the fixed productivity parameter, and i_t is investment in new capital.

4 The Model With Government Taxation and Debt

As is typical in such environments, there are multiple ways to describe the competitive equilibrium. The approach here, which may be the simplest, is to have a competitive equilibrium in which households own all the capital, and undertake investment.

In this competitive equilibrium there will be firms that use the capital to produce the consumption good and investment good. The instantaneous before-tax cost of capital to

[12] For example, the two-sector model of Lucas [11] is very useful for some purposes, but not for the issues studied here. The reason is that in such a model a permanent change in the tax rate would generate a non-trivial transition from one steady-state growth path to another, and it would be difficult to accurately characterize the decision rules along this transition path. This is the reason that in the models of Lucas [11] the transitions from one steady-state to another are not calculated, even though he acknowledges that these transitions may be of vital importance in calculating things like welfare costs.

the firm is denoted by r_t. The profits received from the firm for this activity, measured in units of the consumption/capital good, are denoted by

$$\pi = \lambda k_t - (r_t + \delta) k_t.$$

In an equilibrium it must then be the case that

$$r_t = \lambda - \delta. \tag{4}$$

The firm then pays the tax on the net return to capital ($\tau_t r_t k_t$), before paying the remainder after-tax income to the consumer ($(1-\tau)_t r_t k_t$). It is assumed that interest on government debt is taxable. Since there is no uncertainty, both capital and government bonds pay the same return. The consumer's instantaneous budget constraint can then be written as follows:

$$c_t + \left(\dot{k}_t + \dot{b}_t\right) = (k_t + b_t) r_t (1 - \tau_t),$$

where b_t is he per-capita amount of government debt that the consumer holds at date t.

Henceforth, lower case letters will denote variables chosen by the consumer or firm, while upper case letters will be economy-wide averages, which will then determine the government's budget constraint. In equilibrium these will equal each other.

4.1 Government

It will now be assumed that initially there is a fixed stock of real government debt, per-capita, that is outstanding, and this will be denoted as B_0. For convenience, it will be assumed that there is no future government spending to add to this existing stock of debt.[13] This debt must be financed through factor taxation, and since capital is the only factor of production, the government debt must be financed through capital taxation. There can be no default by the government, and this puts considerable discipline on what tax policies are feasible.

The government budget constraint specifies that the excess of interest paid on the debt ($B_t r_t$) over the amount of tax revenue ($(k_t + B_t) r_t \tau_t$), must be financed by increased government borrowing. The per-capita government budget constraint is written as follows

$$\dot{B}_t = B_t r_t - (k_t + B_t) r_t \tau_t. \tag{5}$$

Here K and B represent the per-capita quantities of capital and government bonds. Of course, in equilibrium $K = k$, and $B = b$. Alternatively, this states that the value of government debt at date t must equal the value of subsequent government revenue:

$$\begin{aligned} B_t &= \int_t^\infty \exp\left(-\int_t^s r_z dz\right) (K_s + B_s)(r_s \tau_s) ds && (6) \\ &= \int_t^\infty \exp\left(-\int_t^s r_z (1 - \tau_z) dz\right) (K_s)(r_s \tau_s) ds. && (7) \end{aligned}$$

[13] This makes the analysis of the model more manageable, and also makes it very apparent that changes in the tax rate cannot be absorbed by changes in government spending. This approach also removes the possibility that a modification of the tax rate alone may change the present value of government spending through a change in the interest rate.

Adding government spending would permit the analysis of a whole range of additional experiments. For example, one could study how an expected future increase in government spending could affect current growth through the effect this would have on taxes.

Given an initial value for government debt (B_0), a feasible path for taxes is then a function $\{\tau_s\}_{s=0}^{s=\infty}$ that satisfies equation (6). The important feature here is that it is possible to evaluate the impact of alternative paths for taxes beginning at some initial date.

Note that this government budget constraint shows that a change in the tax rate does not just alter the amount of tax revenue, but it also changes the after-tax interest rate. Furthermore, the relationship between the tax rate and discounted tax revenue is complicated by the fact that the future path of the tax rate obviously influences the path for the capital stock, and so this feature must be taken into consideration when considering the feasible path for the tax rate.

The analysis below will assume an extreme degree of government commitment, in the sense that the government budget constraint must be obeyed. However, this will not be a normative exercise in that these tax rates will not be chosen to solve any particular optimization problem. Instead, the impact of various alternative paths for the tax rate, which satisfy equation (6) will be considered. Obviously not all paths for the tax rate are feasible, in the sense that equation (6) is satisfied.

5 The Consumer's Problem in a Competitive Equilibrium

Since the government only has to finance its existing debt obligation, and government consumption is zero, tax revenue is actually rebated to individual's through the interest payments to holders of government debt. It is then of central importance to characterize the consumer's optimization problem. This problem is then one of maximizing the utility function

$$\int_0^\infty e^{-\rho t}\left[\frac{c_t^{1-\sigma}}{1-\sigma}\right] dt, \tag{8}$$

subject to the constraint that

$$c+\left(\dot{b}+\dot{k}\right)=(b+k)\,r\,(1-\tau_t)\,. \tag{9}$$

5.1 Characterization of the Consumer's Problem

It is straightforward to see that the present value Hamiltonian for this problem is then written as

$$H=e^{-\rho t}\left[\frac{c_t^{1-\sigma}}{1-\sigma}\right]+e^{-\rho t}\mu\left[(b+k)\,r\,(1-\tau_t)-c\right],$$

where μ is the multiplier on the constraint, and has the interpretation of being the utility cost of investment. The optimization conditions for consumption for this problem is then

$$c^{-\sigma}=\mu. \tag{10}$$

Next, there is the following optimization condition for capital or bond holdings, which must hold at each date

$$\frac{\dot{\mu}}{\mu}=\rho-r\,(1-\tau_t)\,. \tag{11}$$

Since any government revenue is used to pay the interest and perhaps principal of the government debt, it follows that all of the output that is produced is consumed by consumers, or invested in capital, in equilibrium (i.e. from equation (2)). For convenience, let ϕ_t denote the fraction of output that is used for consumption in each period while the remainder $(1-\phi_t)$ is used to produce the investment good (i.e. $c = \phi\lambda k$). Next, equation (10) can be used to show that

$$\begin{aligned}\frac{\dot{\mu}}{\mu} &= -\sigma\left(\frac{\dot{c}}{c}\right) \\ &= -\sigma\left(\frac{\dot{\phi}}{\phi}+\frac{\dot{k}}{k}\right)\end{aligned} \tag{12}$$

since $\dot{\lambda}=0$. But equation (11) then implies that

$$-\sigma\left(\frac{\dot{\phi}}{\phi}+\frac{\dot{k}}{k}\right) = \rho - r\left(1-\tau_t\right). \tag{13}$$

Now the before-tax net return to capital is $r = \lambda - \delta$, and the growth rate of capital is as follows:

$$\frac{\dot{k}}{k} = \lambda\left(1-\phi_t\right) - \delta. \tag{14}$$

Then, using equations (13) and (14) implies that

$$-\sigma\left(\frac{\dot{\phi}}{\phi}+\lambda\left(1-\phi_t\right)-\delta\right) = \rho - \left(\lambda-\delta\right)\left(1-\tau_t\right). \tag{15}$$

This can be re-written as

$$\dot{\phi} = \frac{\left[\left(\lambda-\delta\right)\left(1-\tau_t\right)-\sigma\left(\lambda-\delta\right)-\rho\right]}{\sigma}\phi + \lambda\phi^2. \tag{16}$$

This is a non-linear differential equation. Fortunately, it is a Bernoulli differential equation, that has as solution shown in the following proposition.[14]

Proposition 1 *Equation (16) is characterized by the following solution*

$$\phi_t = \frac{\exp\left(\int_t^\infty \left[\frac{[\rho-(\lambda-\delta)(1-\tau_s-\sigma)]}{\sigma}\right]ds\right)}{\int_t^\infty \lambda \exp\left(\int_z^\infty \left[\frac{[\rho-(\lambda-\delta)(1-\tau_s-\sigma)]}{\sigma}\right]ds\right)dz}. \tag{17}$$

Proof. See Appendix B ■

It is useful to check that this is a sensible solution to the problem. First note that if the tax rate is constant ($\tau_t = \tau$), then equation (17) converges to [15]

$$\phi = \frac{\left[\rho-\left(\lambda-\delta\right)\left(1-\tau-\sigma\right)\right]}{\sigma\lambda}. \tag{18}$$

[14] The discrete-time counterpart to this expression is a bit more cumbersonme to characterize. Nevertheless, this is derived in Appendix A. An alternative way to verify that equation (17) is the optimal decision rule is to substitute this equation into both sides of the intertemporal euler equation, and verify that this solution works.

[15] It is noteworthy that equation (16) has other solutions, in addition to that given by equation (17). However, these are either partially or fully "backward-solutions" (or backwardy-looking), and additionally two of the solutions deliver negative values for ϕ, which is impermissible. Since it is not sensible to have investment decisions depend on past taxes, these other solutions are ignored.

It is the working hypothesis that the solutions in equations (17) and (19) are less than unity. Of course, for values of $(\lambda-\delta)$ sufficiently small relative to ρ, this does not have to be the case.

Next, the following proposition shows that this decision rule exhibits sensible economic properties.

Proposition 2 *i) The solution given by equation (17) has the property that ϕ_t is a function of all future taxes, and that $\frac{\partial \phi_t}{\partial \tau_s} > 0$, for $s > t$. ii) Furthermore, $\frac{\partial^2 \phi_t}{\partial s \partial \tau_s} < 0$, for $s > t$, and $\frac{\partial \phi_t}{\partial \tau_s} \rightarrow 0$ as $s \rightarrow \infty$. iii) Along a path where the tax rate is constant ($\tau_s = \tau$), $\frac{\partial \phi_t}{\partial \tau_s}$ is decreasing in σ.*

Proof. See Appendix B ■

Corollary 3 *Since the growth rate of capital is related to $(1 - \phi_t)$, this means that the current growth rate of capital is influenced negatively by the entire path of future tax rates (τ_s).*

The second proposition says that the effect of taxes on growth becomes moderated, the further out into the future the taxes are changed.

A novelty of equation (17) is that this expression shows explicitly how *future* government policies affect the *current* savings rate, and therefore the growth rate. The reader will note that the size of the government debt does not appear explicitly in equation (17). However, it does appear implicitly. The size of the government debt determines the feasible paths for future taxes, through equation (6). And these tax rates influence the growth rate through equation (17). Needless to say, there is not a one-to-one mapping of the size of government debt (B_t) to the contemporaneous growth rate $(1 - \phi_t)$ because there is not a unique set of taxes that finance any amount of initial government debt (B_0).[16]

The sensitivity of the growth rate $(1 - \phi_t)$ to changes in taxes depends on the value of σ. For higher values of σ, changes in the tax rate have a smaller impact on the growth rate. This is an important issue that will be explored below.

It is well known that many growth models exhibit non-linear decision rules or transition paths (see, for example, Turnovsky and Fisher [22]). Equation (17) also shows that, despite the apparent "linear nature" of the present model, that the optimal decision rules can still display a non-linear feature as well.

It is important to note the relationship between the growth rate and the interest rate in the analysis below. The growth rate of capital, is given by equation (14), and depends on ϕ_t. However, the growth rate of consumption is $\frac{\dot{c}}{c} = \frac{\dot{\phi}}{\phi} + \frac{\dot{k}}{k}$, and which depends on $\dot{\phi}$ and ϕ. Although both consumption and capital will grow at the same rate in the steady state (i.e. when $\dot{\phi} = 0$), this will not be the case in some of the experiments below. Equations (17) and the right side of equation (16) then demonstrate *explicitly* how the growth rate would be a function of all *future* capital tax rates. Furthermore, in this model the before tax interest rate is constant (given by equation (4)). In contrast, the after-tax interest rate is given as $r (1 - \tau_t) = (\lambda - \delta) (1 - \tau_t)$, and so the path for the capital tax rate will influence the after tax interest rate. As is typical of a model in which preferences are given by a utility function like equation (8), the growth rate of consumption is determined by the after-tax

[16] Of course, one could consider just the set of policies for which the tax rate is held constant to finance a given feasible level of debt. In this case, a higher level of debt, that is feasibly financed, would entail a higher level of taxes, which in turn imply a lower growth rate.

interest rate, which here is influenced by the capital tax rate. Nevertheless, to make the subsequent analysis more concise, the focus will be on the growth rate of capital.

As will be seen below, given a predetermined amount of government debt, any temporary change in the capital tax rate will influence the long-run after-tax interest rate.

5.2 Equilibrium

To this end, the following definition of a competitive equilibrium will be employed below.

Definition 4 *Given the initial stocks k_0, B_0, for this economy, a* Perfect Foresight Competitive Equilibrium *will be a collection of functions $\{k_t, B_t, c_t, \phi_t, \tau_t,\}_{t=0}^{t=\infty}$, such that for all $t \geq 0$,*

1. *Consumers maximize utility (8) subject to their budget constraint (9). This implies that their consumption/saving decisions are then governed equation (17). Additionally, the transversality condition must hold for government debt and capital.*

2. *Factor prices are given by equation (4).*

3. *The government's budget constraint (6) is satisfied.*

As has been mentioned, the government budget constraint restricts the set of feasible paths of government debt and taxes. In practice, when computing an equilibrium a path for taxes $\{\tau_t\}_{t=0}^{t=T}$ is determined up till some date T. Then some solution (τ^*) is sought for the tax rate thereafter $(t \geq T)$ that will satisfy the government's budget constraint (6).

6 The Model With External Debt

It is of interest to compare the model from the previous section, with an identical economy but where the government debt is held externally, by some foreign entities. In particular, imagine that there is some external economy, which is left un-modeled, in which the consumers and technology is identical to that of the domestic economy. Both economies face the same gross interest rate, or technology parameters. Furthermore, suppose that these foreign consumers initially hold all of the domestic government's debt. In economic terms the difference between this case and the previously specified economy will be that now when the domestic government imposes a capital income tax, there will be a negative income effect, because this tax revenue will be transferred abroad. To facilitate the comparison of this framework with that of the previous model (with domestically-held debt) it will be useful to let ϕ again represent the fraction of gross output that is devoted to consumption.[17]

[17] As indicated above, the objective of this analysis is not to characterize the optimal set of taxes, since there is a considerable literature on this topic. Nevertheless, a natural question to ask is how the set of optimal taxes of an economy might change if the government debt was held abroad, rather than domestically. To the extent that the optimal capital tax might involve various forms of default on the domestically-held government debt, the incentives for such default would seem to be even more pronounced if the debt is held abroad. Such a policy would essentially involve "taxing foreigners", which would usually seem to be a good policy.

Of course, it is assumed here that the foreign holders of these government bonds do not pay taxes back to the issuing government. It may be that these foreign consumers pay taxes on this interest to their own government, but this would not influence the budget constraint of either the domestic consumers or government.

Then the government budget constraints (5) is now given by the following:

$$\dot{B}_t = B_t r_t - (k_t)\, r_t \tau_t,$$

which means that equation (7) must also be altered as well. Here the government's range of feasible tax policies is altered because the tax base is changed: the domestic government does not get to tax the return on government bonds, which are now held abroad.

In contrast, the consumer's budget constraint is no longer equation (9) since they will not own the government debt. In this case the analysis of Section (5.1) still holds, as does equation (13), which captures the distortion from the capital tax. However, now equation (14) must be modified to take into account the fact that there are taxes levied on the return to capital $(\lambda - \delta)$, and that revenue is shipped abroad. Solving for the solution to this model then yields the counterpart to equation (16), which is written as follows:

$$\dot{\phi} = \frac{\left[(\lambda - \delta)\,(1 - \tau_t)\,(1 - \sigma) - \rho\right]}{\sigma}\phi + \lambda \phi^2.$$

Then the solution to this equation, the counterpart to equation (17) is then

$$\phi_t = \frac{\exp\left(\int_t^\infty \left[\frac{[\rho - (\lambda-\delta)(1-\tau_s)(1-\sigma)]}{\sigma}\right] ds\right)}{\int_t^\infty \lambda \exp\left(\int_z^\infty \left[\frac{[\rho - (\lambda-\delta)(1-\tau_s)(1-\sigma)]}{\sigma}\right] ds\right) dz}. \tag{19}$$

Let the solution to equation (17), the model with internally-held debt, be denoted as ϕ_t^i, and let the solution to equation (19), the model with externally-held debt, be denoted as ϕ_t^e. To facilitate a comparison between these two economies, suppose that the parameter values of these economies are such that the resulting values of ϕ are identical. It is straightforward to use these formulae and verify the following result.

Proposition 5 *The responses of a change in ϕ_t^i and ϕ_t^e, in reaction to a change in future taxes, can be characterized as follows:*

$$\frac{\partial \phi_t^e}{\partial \tau_s} = \frac{\partial \phi_t^i}{\partial \tau_s}\,(1 - \sigma)\,, \tag{20}$$

for $s \geq t$. This implies that if $\sigma \in (0,1)$, $0 < \frac{\partial \phi_t^e}{\partial \tau_s} < \frac{\partial \phi_t^i}{\partial \tau_s}$, and that holding everything else constant, a change in future taxes will have a larger (negative) impact on the growth rate if the debt is held internally. However, if $\sigma > 1$, then $\frac{\partial \phi_t^e}{\partial \tau_s} < 0 < \frac{\partial \phi_t^i}{\partial \tau_s}$, and an increase in future taxes will lower the current growth rate if the debt is held internally, but may raise the growth rate if the debt is held abroad. This latter effect is more pronounced the larger is σ.

The reason for the different direction of the changes if $\sigma > 1$ is as follows. If $\sigma > 1$ then there is as pronounced wealth effect from a future change in the capital tax rate, relative to the substitution effect. If the debt is held internally, an increase in a future tax rate (τ_s) causes ϕ_t^i to rise, and growth to fall, because the price of current (as opposed to future) consumption has fallen. This lowers the current growth rate. However, if the government debt is held abroad, then a rise in a future tax rate (τ_s) causes ϕ_t^e to *fall*, in spite of the fact that the relative price of current consumption has fallen. This is because there is a large negative wealth effect: when the higher taxes are imposed the resulting revenue is shipped abroad. Because the intertemporal elasticity of substitution is so small (when $\sigma >> 1$) and the negative wealth effect is so large, the individual responds to the increase in future taxes by *raising saving*, which lowers ϕ_t^e, and this raises the growth rate.

Alternatively, when σ is close to zero, the preferences display a very small wealth effect relative to the substitution effect and so, as equation (20) shows, there is little difference arising from a change in the tax rate, when the debt is held internally or externally.

An alternative way to think of this is that when $\sigma > 1$, the shadow value of government debit is quite different in the two cases: when the debt is held abroad as opposed to being held domestically. The welfare cost of a marginal increase in foreign-held government debt is different from the cost of an increase in domestically-held debt.

It is straightforward to verify that when the government debt is foreign-held, the results from Proposition 2 also hold in the sense that the effect of changes in future taxes is dissipated, the further into the future that these changes take place.

7 The Debt Trigger

The model studied here will exhibit behavior pertinent to many current economies in that the taxes are set in such a way that the government is growing on a path that may be perceived to be unsustainable. Countries seem reluctant to let their government debt exceed twice their annual GDP. Therefore, because of this empirical observation, and in the spirit of Sargent and Wallace [18], for some experiments it will be assumed that there is some upper bound beyond which government debt-to-GDP ratio is not permitted to exceed. Once this level of government debt is reached, it is assumed that the tax rate is then permanently increased to a constant level that will be sufficient to generate enough revenue to finance the current debt.[18] This mechanism will be referred to as a *"debt trigger"*. As a benchmark, it will be assumed this constraint becomes binding when the debt to GDP ratio hits 100%. The exact nature of why this mechanism exists, or why it kicks-in at a specific level, is not specified.[19] That is, it will be convenient to assume that this debt trigger is embedded in

[18] Of course, this does not totally capture the budget difficulties faced by many countries in recent years. In many economies the debt has grown so rapidly not because the interest rate has exceeded the growth rate, but instead because government primary budget deficits have been so large that the government debt has grown much faster than income. Nevertheless, the key feature here is that individuals have to make consumption decisions while knowing that the government budget constraint dictates that at some future *known date*, the tax rate must be raised.

[19] Presumably a better economic foundation for this idea would be to utilize some politico-economic considerations to justify the equilibrium resulting debt to GDP ratio. That is, it might be interesting to have individual agents to vote on *feasible* (or consistent) policies that give rise to a debt to GDP ratio. While interesting, such an ambitious approach is well beyond the scope of the analysis of this paper.

a fixed statute, or in a constitution, and is therefore unchanging, as is the fact that the government cannot default on the debt. The path of the government debt, after the debt trigger is imposed, is characterized in the following proposition.

Proposition 6 *If at some date T, a constant tax is imposed to pay off the current debt, then the debt-to-output ratio will be constant thereafter.*

Proof. Equation (7) can be used to show that the government budget constraint at date T can be written as

$$B_T = \frac{(r\tau)\, k_T}{r\,(1-\tau) - g}, \tag{21}$$

where g is the growth rate of capital ($\left(\dot{k}/k\right) = g$). Then for $t \geq T$, equation (5) then can be written as

$$\frac{\dot{B}_t}{B_t} = r\,(1-\tau) - \left(\frac{k_t}{B_t}\right) r\tau. \tag{22}$$

Substituting equation (21) into (22) reveals that the growth rate of debt is g, which is the same rate as capital or output. ■

Since this is a perfect foresight economy, individuals will know, with certainty, the future path for government debt and tax rates, given the initial level of debt. Again, there is an extreme degree of commitment that is embedded here, in the sense that government is not permitted to renege (or default) on its stated policies. The point here is to study how the level of government debt and taxes in an economy will affect the decisions of agents, and thereby influence the present and future growth rates.

Even without the introduction of the debt trigger, equations (6) and (17) illustrate the non-linear relationship between taxes and growth. But this non-linearity is only exacerbated by the introduction of the debt trigger.[20]

The imposition of a debt trigger in this environment is not completely without justification. After all, it is assumed that the government budget constraint must be satisfied. For the parameter values considered here, if government debt is permitted to grow without bound, then eventually the government budget constraint cannot hold because there is no way that the present value of future taxes will be sufficient to finance the government debt.[21]

The use of such a debt trigger stands in contrast to much of the existing literature, which usually assumes that any changes in tax rates will be absorbed by a corresponding

[20] Jaimovich and Rebelo [10] also focus on the non-linear relationship between taxation and growth, but this is a non-linearity of an entirely different variety.

[21] There are other papers that employ a limit on the size of government debt, or the size of the debt that consumers may hold (and therefore how much the government may issue). For example, Aiyagari, Marcet, Sargent, and Seppälä [1] study a model in which there are assumed limits on the size of consumer and government indebtedness. Their paper is an analysis of optimal taxation, and so the limit on government debt has implications for the dynamic pattern for optimal tax rates. However, their analysis has almost no similarities with the present paper. They do not study many of the issues addressed here, such as the effect the size of government debt may have on growth, or how a change in the tax rate may influence growth path. They also do not have capital accumulation in their model, and so cannot study issues relating to growth rates, which are central to the current paper.

Greiner and Fincke [6] also show that there are limitations on the amount of government debt that can be financed through taxation, and that this has implications for the growth dynamics of an economy.

adjustment in future government spending or transfers.[22] This is often a very convenient assumption, as the changes in government transfers ultimately are reflected in the consumer's budget constraint. For example, Trabandt and Uhlig [21] do just this in their study of the Laffer Curve. These authors are hardly alone, as this assumption seems to be a rather standard operating procedure. However, it seems just as plausible to believe that when government taxes are reduced, that consumers will believe that this will necessitate a future increase in tax revenue, rather than a fall in government spending or transfers. Of course, modeling when the future increase in taxes will be implemented is problematic, since it is not evident when taxes *must* rise, or by what magnitude, because there will be a multitude of different paths for the tax rate that will satisfy the government's budget constraint. Nevertheless, the approach adopted here will be to assume that there will be a known future date at which the tax rate must adjust to a constant level, which will then satisfy the government's budget constraint.

8 Parameter Values

The model studied here will not be designed to mimic any particular economy, and instead the goal will be to illustrate the behavior of the model. In the model a period will correspond to a year. The other parameters value will then be set with this in mind. The discount factor ρ is chosen to be .05. Although this parameter does not have a significant effect on results of experiments conducted below, it affects the level of the growth rate (as shown by equation (17)). The annual depreciation rate is chosen to be $\delta = 10\%$. Given this, the parameter λ is chosen to deliver the desired steady-state growth rate or a particular interest rate. In the benchmark model, a growth rate of 3% will be assumed. Also in the benchmark model, a tax rate of $\tau = 20\%$ will be employed.[23]

Various values will be assumed for the preference parameter σ. As will be shown below this parameter will be important because it is instrumental in determining the response of consumption and saving (or investment) to the change in current and future tax rates. As is well established, for higher value of σ, agents are less willing to substitute consumption intertemporally, and so in this case consumption and saving decisions are likely to be relatively unresponsive to changes in future tax rates. The opposite is true for lower values of σ. Following the work of Hall [8], it is generally assumed that values of σ well above 1.0 are appropriate.[24] Therefore, several different values of this parameter will be utilized to

[22] Bertola and Drazen [2] also employ a notion of a "trigger". However, theirs is a trigger in the path of government spending, and they show that it is important that consumers must take this potential behavior into account when solving their optimization problem. Nevertheless, the "trigger" employed by Bertola and Drazen is very different than that employed here, and they are also not concerned with many of the issues studied here, such as capital accumulation and growth.

[23] This would seem to be somewhat lower than the typical US tax rate on capital. One reason for using this value is that with higher values, and for some versions of the model studied below, it is not possible to analyze the impact of a substantial tax cut (e.g. a cut of 10% for 10 years) because the resulting loss in government tax revenue is too large to be recovered by any feasible future increase in taxes. Alternatively, consideration must be given smaller tax changes, or changes over shorter periods of time. Therefore, for expository reasons a value of 20% is employed.

[24] There are many other studies that come to a similar conclusion. Also, Guvenen [7] uses elasticities as low as 0.1, which correspond to $\sigma = 10$.

illustrate the importance of the intertemporal elasticity of substitution. In the subsequent analysis of the benchmark economy values of σ in excess of unity will be used, although it will also be shown that the model exhibits some distinctive features when σ is less than unity.[25]

The numerical characterization of the model presented below, will be carried out in discrete time, as this is computationally simpler.

9 Analyzing the Model With Domestic Debt

It will be useful to study the behavior of the model with domestically-held government debt first, and then to study the same model with foreign-held debt.

9.1 The Role of Intertemporal Substitution

At the outset it is useful to investigate the impact that intertemporal substitution parameter $\left(\frac{1}{\sigma}\right)$ has on the growth path, for a fixed level of initial debt, with the debt trigger. Therefore, let us consider fixing the initial debt to GDP ratio at 25%. As a benchmark, the initial capital tax rate will be zero, so initially the government debt is being entirely rolled over each period, until the debt trigger kicks in. The debt trigger is then implemented when the debt to GDP ratio hits 100%. For each separate value of σ, the parameter λ will be set to generate a growth rate of 3% in the initial benchmark economy. Of course, this means that for alternative values of σ there will be different values for the equilibrium interest rate. In particular, economies with higher values of σ will have higher interest rates, for a given growth rate of consumption.

Figure 4 shows the growth rate of capital for two different values of this parameter (σ), beginning with a fixed amount of government debt. The relationship between σ and the growth rate is non-monotonic. Initially (i.e. for the first few periods), the economy with higher values of σ has a *higher* growth rate. But the economies with higher values of σ reach the debt trigger sooner, and so their growth rate then falls quicker as a result, than those economies with lower values of σ. The long run impact (after the debt trigger is hit in all economies), is that the growth rate is increasing in the level of σ.

One notable feature in this figure is that the economy with a higher value of σ reaches the debt trigger earlier than would a similar economy with a lower value of σ. The reason for this is that, for a given growth rate, a higher value of σ means a higher interest rate, which in turn implies that the government debt will grow faster, prompting the economy to hit the debt trigger earlier.[26] This also means that for economies with higher interest rates (and therefore higher values of σ), tax revenue in the immediate future is more important than that in the distant future. Therefore, a tax cut of a given size will then have a more dramatic impact on the government budget when the value of σ is higher.

An alternative way to look at the role of the intertemporal elasticity of substitution of consumption is to consider viewing economies with different values of σ, and identical

[25] Havranek, Horvath, Irsovab and Rusnaka [9] show that there seems to be a great deal of variation in estimates of this parameter across countries.

[26] This feature is only partially offset by the fact that a higher interest rate will mean a higher amount of capital income tax revenue.

growth rates when taxes are zero, but which hit the debt trigger at the same date. This means that these economies will have different levels of initial government debt, and different interest rates. Figure 5 illustrates this experiment for three different economies, which hit the debt trigger after 10 periods. The figure shows that the lower is σ, the lower is the initial growth rate, and the final growth rate. This is due to the fact, given a benchmark growth rate, a lower value of σ implies a lower return to capital.

As is well known, with these preferences, for higher values σ, of the income effect dominates any intertemporal substitution effect, as a reaction to the change in the tax rate. Then, for higher values of σ, in reaction to the anticipated future *increase* in taxes, the agent attempts to shift consumption into the future, when taxes are to rise. This raises the current savings rate, which increases the growth rate. Conversely, for very low values of σ, in reaction to the anticipated increase in taxes, the agent attempts to shift consumption into the present from the future, when taxes are to rise. This *lowers* the current savings or investment rate, which then reduces the growth rate. This latter effect is important, because the lower savings rate then reduces the discounted value of future tax revenue, necessitating a sharply higher future tax rate to balance the government's budget constraint.

9.2 The Impact of Debt on the Growth Rate.

It is now possible to address the issue of how the size of government debt can influence the equilibrium balanced growth path. This is illustrated in Figure 6, where the initial levels of debt (B/Y) for three otherwise identical economies are illustrated. Again in this experiment, the benchmark level of growth is 3%, and $\sigma = 5.0$. For two otherwise identical economies, which are different only in their initial levels of government debt relative to GDP, the economy with the higher government debt will then grow at a lower rate. The reason is quite clear: the economy with the higher level of debt will hit the debt trigger earlier, and so higher government taxes will be imposed sooner. This reduces growth more in prior periods, for the economy with higher government debt.

9.3 The Role of the Debt Trigger

It is also interesting to quantify the role of the debt trigger. For example, consider two otherwise identical economies which have $\sigma = 5.0$, and an initial benchmark growth rate of 3% when the tax rate is zero. These economies both begin with levels of government debt relative to GDP of 16.7%. However, one economy has a debt trigger of 100% relative to GDP, while the other has a trigger of 150%. Figure 7 then shows the behavior for the growth paths. The economy with the higher debt trigger initially grows faster because taxes are not going to be raised for 12 periods. But in the long term, after the debt triggers have been hit in both economies, the growth rate is higher for the economy with the lower debt trigger, because the tax rate is eventually lower in this instance.

One lesson from this experiment is that economies with governments that are more tolerant of government debt, may growth faster for a short period, but the long-term growth consequence of this may be lower growth.

9.4 The Impact of a Temporary Tax Cut

This model can also be used to evaluate the impact of a temporary tax rate reduction.[27] To do this, consider an economy in which the initial capital tax rate is initially 20%, and in which the growth rate is then 3%. Suppose that the 20% tax rate is just sufficient to retire the existing stock of government debt. Beginning from this equilibrium, the tax rate will then be cut to 10%, until period 10. At this time, the capital tax rate will then be raised to an amount that will raise the revenue necessary make up for the revenue lost from the tax cut.

Figure 8 shows the response of the growth rate of capital for this experiment, with a value of $\sigma = 5.0$. Here the dashed line indicates the initial growth rate, while the '$\diamondsuit$' symbols indicate the growth rate that would prevail if the tax rate were reduced to 10% forever. For values of $\sigma > 1$, there is a modest short-lived increase in growth immediately after the tax cut, but then the growth rate plummets as higher future taxes are impending. These taxes must be sufficiently high to make up for the lost revenue from the tax cut. For $\sigma = 5.0$, the subsequent tax rate must rise to 38.7%. This experiment illustrates that a tax cut for 9 periods, does not result in substantially increased growth over this entire period, because agents know that the tax cut has repercussions for future tax rates.

Figure 9 shows this same experiment for different values of σ. As can be seen, the initial increases in growth are larger, the larger is σ. However, the long-run growth rate is inevitably lower for all values of σ.

Additionally, this experiment has implications for the behavior of interest rates, and the term structure of interest rates. Consider beginning from a steady-state in which there is a constant tax rate financing a stream of discounted revenue to equal the size of government debt. A temporary tax cut, will then imply a higher future tax rate. This in turn implies that such a tax cut will then raise the current after-tax interest rate, but also imply that the after-tax interest rate will fall in the future. This implies that the current tax cut will cause the term structure of interest rate to become more downward sloping.

9.5 A Digression on Welfare Costs

A natural question that arises when conducting these alternative experiments is: What are the welfare consequences of these tax policies? In a model where "consumption-smoothing" is important, it might seem that such a policy change would reduce welfare. Since the temporary reduction in the distortional tax must be offset by a subsequent increase in the tax rate so as to finance the same amount of government bonds, it is natural to inquire about the welfare costs of these policies. To obtain an answer to this question, let us proceed with measuring the welfare cost in the following manner. Moving from the *benchmark* path in which the tax rate is 20% forever, to the new path in which the tax rate is 10% for 9 periods, and then 38.7%, induces a change in welfare. The welfare cost of this policy will be calculated as the percentage change in capital that would be necessary to make the benchmark path yield the same discounted welfare as under the new tax path.

[27] Strictly speaking, it is not necessary for there to be a debt trigger here, as long as there is *some* mechanism for generating a tax increase at some future date that is sufficiently large to capture all the revenue lost due to the tax cut.

For $\sigma = 5.0$, in Figure 9, the welfare cost of the temporary tax cut is 0.21% of initial capital (or output). Obviously then the benefits of the temporary tax cut are overwhelmed by the subsequent costs of increasing taxes. For $\sigma = 2.0$, in the same figure, the welfare cost of the temporary tax cut is 0.30% of initial output.

9.6 Some Unpleasant Fiscal Arithmetic?

In an influential paper, Sargent and Wallace [18] used a technique that is similar to the debt trigger to derive a striking result. Their finding was that a government that *neglects* to sufficiently increase the money supply to finance some government spending may actually cause *higher* inflation today, because the failure to increase the money supply today signals an even higher increase in the money supply in the future. This latter effect would cause inflation today, provided agent's decisions are influenced by expectations of future prices.

A similar result can be shown to hold in the present model. Here, it is shown that a reduction in the capital tax rate, rather than increasing the growth rate, can immediately result in a permanently *lower* growth rate. The reason for this result is analogous to that of Sargent and Wallace.[28] A cut in the capital tax rate, through the government budget constraint, must imply a large increase in future tax rates. Under certain circumstances, the impact on the growth rate of future tax rates dominates the effect of current taxes. It turns out that the circumstances necessary for this to take place are that the parameter (σ) cannot be too big, or the intertemporal elasticity of substitution $\left(\frac{1}{\sigma}\right)$ cannot be too small.

To illustrate this, consider an experiment identical to those in section 9.4, wherein the tax rate is cut from 20% to 10% in the first period, but raised again after 9 periods to a level necessary to make up for all of the previous reduced revenue.[29] Figure 9 illustrates what happens for a value of $\sigma = .90$. Immediately after the tax cut, the growth rate falls and continues to fall, until period 10. In other words, rather than a tax cut raising the growth rate, it *reduces* growth. For low values of σ, in reaction to the anticipated increase in taxes, the agent attempts to shift consumption into the present from the future, when taxes are to rise. This sharply lowers the savings or investment rate, which reduces the growth rate. Because the lower level of investment reduces the growth rate of capital, this reduces the discounted value of future tax revenue, resulting in a much higher future tax rate to balance the government's budget constraint. In this experiment the tax rate rises from 10% to 27.1% after period 9.

The limits of this experiment need to be explained. This last experiment cannot be studied for values of σ that are too low. The reason is that for sufficiently low values of σ, a tax cut from 20% to 10% for 9 years reduces government revenue so much that there is no feasible way to recover the lost revenue. Similarly, even for higher values of σ, it is possible to cut the tax rate for 9 years and then make up the loss in revenue, but it might not be possible to conduct this experiment for longer periods of time.

[28] There is one notable difference between the framework of Sargent and Wallace, and the model studied here. In their model, there is no compelling reason given as to why the government or central bank *must* increase the money supply (and cause inflation) in the future. There is a variety of feasible policies. However, in the setup of the present paper, there *is* a reason why the government will raise taxes in the future: it must do so for otherwise the government budget constraint will not hold.

[29] Once again, suppose that the existing stock of government debt outstanding is equal to the present value of government revenue that would be yielded by a constant 20% tax rate.

10 Foreign-Held Government Debt

Having illustrated some of the properties of this economy in which there is government debt held by domestic consumers, it is important to investigate how these results change if foreign consumers hold the debt.[30] This section will then illustrate an analogous set of experiments to those just conducted.

First of all, let us study an experiment comparable to that shown in Figure 4. Suppose that initially the economy begins with $\sigma = 5.0$, and the initial government debt to GDP ratio is 16.7%. Suppose that the tax rate is zero until period 10, at which time the tax rate rises keep the debt to GDP ratio constant. Figure 10 then shows how the growth rate responds, in the case of both domestically and foreign-held debt. The case of domestically-held debt is the same as in Figure 4, while with foreign-held debt the behavior is quite different. Why are the responses so dissimilar? With domestically held debt, if taxes are going to rise in the future, the savings rate will fall slightly as the economy approaches the date for the tax change. This fall in the savings rate lowers the growth rate. However, the wealth effect of this is negligible, since the increase in taxes is used to pay interest on the debt, which is held by domestic consumers.

In contrast, if foreign consumers hold the debt, there is a *large negative wealth effect* when taxes are raised in the future, because the interest payments are shipped abroad. Since the intertemporal elasticity of consumption is small in this example, consumers must *raise* their savings rate in anticipation of the future tax increase. In other words, with foreign-held debt domestic consumers will attempt to shift consumption into the future (or increase saving) in order to partially offset the potential fall in consumption produced by the rise in taxes. In the end, the results from these experiments are very different: with foreign-held debt the growth rate rises, while it falls if the debt is held domestically.

This experiment is really just an illustration of the different effects inherent in equation (20). This equation shows that, when $\sigma > 1$, the effect on growth of a change in the tax rate can can be of opposite sign, depending upon whether the government debt is foreign-held, or held domestically. Equation (20) would also seem to imply that the higher is σ, the larger will be the difference between the curves in Figure 10, and so the greater will be the difference in the results between when the government debt is foreign, as opposed to domestically-held.

10.1 Response to a Temporary Tax Cut

Now consider an experiment analogous to that of Figure 8. Suppose that initially the economy begins with $\sigma = 5.0$, and the tax rate is set at 20%, which is just sufficient to pay the interest on the debt and keep the ratio of government debt to GDP ratio constant. Now assume that in period 1 the tax rate is cut to 15% until period 10, at which time the tax rate rises to recover all of the revenue lost from the temporary tax cut. Figure 11 then

[30] Of course, this simple environment does not contain some practical features that may be consequential. For example, foreign borrowing may change the incentives for various forms of default. Secondly, it may be that economies that borrow domestically will ultimately face a different debt trigger than if they borrowed abroad. Lastly, often when governments undertake foreign borrowing it is conducted in units of a foreign currency, and then this raises the issue of exchange rate risk. To the extent that this risk changes the saving behavior of agents, this could certainly change the quantitative nature of the results.

shows the behavior of the growth rates, with domestic and foreign-held government debt. The responses are quite different, despite the fact that the parameters of the economy are identical. With domestically-held debt, a tax cut leads to an immediate modest increase in growth that is temporary. However, with foreign-held debt, the growth rate increases significantly, over the entire period of the tax cut. This experiment might best illustrate the dramatic differences in changes in fiscal policy that result from who holds the government debt.[31]

It is impossible to hold "everything equal" across these two sample economies, even with the same growth rate, and rate of time preference. The reason is that, depending on whether the government debt is held domestically or abroad, the consumers will have different levels of discounted wealth. Obviously if the debt is held domestically the consumers will be wealthier than if the debt is held abroad, and therefore they will likely consume more in the former case, ceteris paribus. Therefore, for the cases illustrated in Figure 11, although the two economies initially have the same after-tax return, if the levels of government debt and capital are identical for these two economies, then the *initial* level of consumption will be higher for the case where the debt is held domestically. This fact is further captured by the different formulae in equations (17) and (19), which encapsulates the fraction of output that is consumed would be different in the two cases. Again, these values for ϕ will differ more, the greater is the value of σ.

In Figure 11 the path labeled '$\diamondsuit$' shows how the growth rate would react if the tax rate were reduced to 10% forever.[32] This experiment indicates that the short run impact on growth of this tax cut is *much larger* than would be the impact of a *permanent* tax reduction. This result contrasts with much of the existing literature where it is generally shown that a permanent change in taxes will have a much larger impact than a temporary impact.

The fact that temporary tax cuts may have larger growth effects than permanent tax cuts, may be a rather surprising result. This shows that in conducting such an experiment it is important to know exactly what is being held fixed, or allowed to change, in the government's budget constraint. Traditionally in the literature, a tax cut translates into a fall in government spending or transfers, and typically this would mean that a temporary tax cut would have a smaller impact than a permanent one. However, here with government spending and debt held fixed, and therefore total discounted government revenue held fixed, a temporary tax cut may have a larger short-term impact on growth than would a permanent tax cut of the same magnitude.

It must be noted that this result is *not* a necessary result of the presence of the debt trigger alone. It also requires a low intertemporal elasticity of substitution of consumption. In the next section it is shown that even with the debt trigger in place, the opposite result might hold.

It should be understood that the higher growth that results when the debt is held externally, is not in itself a good thing. The fact that the debt is held abroad means that

[31] Of course, the initial levels of debt for these two economies are not identical, even if the two economies share the same parameter values. The reason is that the government is paying a different after tax interest rate, depending on whether the debt is held domestically, or abroad.

[32] That is, this would be the path if there were no debt trigger, and it would be feasible to cut the tax rate to 10% forever.

in the future there will be a stream of interest payments that the domestic consumers will have to pay, and this is not a good thing. The temporary increase in growth results from shifting consumption in such a way as to partially offset the welfare loss from these interest payments. Agents would much rather hold the debt domestically. Therefore, the fact that the foreign-held debt leads to higher growth certainly does not imply that agents would prefer that foreigners held their government debt.

10.2 The Role of the Intertemporal Elasticity of Substitution With Foreign Debt

Figures 4 and 5 have shown that the presence of government debt that must be financed in the future can reduce growth prior to the change in taxes. Furthermore, the larger the initial level of government debt, the higher or sooner must be the subsequent rise in taxes. Those experiments were conducted with domestically-held debt, and so it seems important to conduct similar experiments with foreign-held government debt.

Therefore, let us consider fixing the initial debt to GDP ratio at 37%. As a benchmark, the initial capital tax rate will be zero, so initially the government debt is being entirely rolled over each period, until the debt trigger kicks in. The debt trigger is then implemented when the debt to GDP ratio hits 100%. For each separate value of σ, the parameter λ will be set to generate a growth rate of 3% in the initial benchmark economy. Of course, this means that for alternative values of σ will have different values for the equilibrium interest rate. In particular, economies with higher values of σ will have higher interest rates, for a given growth rate of consumption.

Figure 12 shows the growth rate of capital for three different values of this parameter (σ), beginning with a fixed amount of government debt. Once again, the paths are non-monotonic. Initially (i.e. for the first few periods), the economy with higher values of σ has a *higher* growth path, and so it might be said that this parameter has a positive "short run" impact on the growth rate. But the economies with higher values of σ reach the debt trigger sooner, and so their growth rate then falls quicker as a result, than those economies with lower values of σ. Therefore the intermediate term impact is much harder to describe, and is certainly not *increasing* in σ. However, the long run impact (after the debt trigger is hit in all economies), is that the growth rate is increasing in the level of σ. This result is also suggested by noting that the growth rate is related to the size of the term $1-\phi$, and equation (17) implies that

$$\frac{\partial^2 (1-\phi)}{\partial\sigma\partial\tau} = \frac{(\lambda-\delta)}{\lambda\sigma^2} > 0.$$

That is, for a given growth rate, a change in the capital tax rate will produce a smaller reduction in the growth rate if the value of σ is larger.

So what is going on here? Once again, with these preferences, for higher values σ, of the income effect dominates any intertemporal substitution effect, as a reaction to the change in the tax rate. Then, for higher values of σ, in reaction to the anticipated future *increase* in taxes, the agent attempts to shift consumption into the future, when taxes are to rise. This raises the current savings rate, which increases the growth rate. On the other hand, for very low values of σ, because of the future increase in taxes the consumer attempts to raise current consumption (relative to future) in reaction to the anticipated increase in taxes. This *lowers* the current savings or investment rate, which then reduces the growth rate. This lowers the savings rate and reduces the discounted value of future tax revenue, necessitating a sharply higher future tax rate to balance the government's budget constraint.

Once again, it should be noted that the economy with a higher value of σ hits the debt trigger earlier than those with a lower value of σ. A higher value of σ implies a higher interest rate, which implies that the government debt will grow faster, prompting the economy to hit the debt trigger earlier.[33]

A casual view of Figure 12 might lead one to conclude that the different reactions to the reduction in the capital tax rate are due to the fact that the savings or investment rate is more sensitive to the change in the tax when σ is higher. But this is not true, as is shown in Proposition 2, part iii. The curve labeled '$\sigma = 5$' displays a larger increase in growth not because this value generates a larger contemporaneous response to a change in current taxes. In fact for the '$\sigma = .70$' economy, the growth rate is generally more responsive to a contemporaneous change in the tax rate, than for the economies with higher values of σ. Instead, in the case of lower values of σ, although the current growth rate is more sensitive to a change in current taxes, *it is also sensitive to changes in future taxes.* In the figure, for low values of σ, the effect of the low current taxes is simply overwhelmed by the future increase in taxes, and this causes a very muted response of the current growth rate. In contrast, in the case in which $\sigma = 5$, the large increase in future taxes causes the current investment rate to rise, so as to partially offset the future increase in the tax rate.

Note also that although the grow rate is at first increasing for $\sigma = 5$ and $\sigma = 2$, it is non-increasing for $\sigma = .70$. This would imply that the savings or investment rate is falling as the debt trigger is approached.

10.3 The Role of the Size of the Foreign-Held Debt

It is now possible to address the issue of how the size of the foreign-held government debt can influence the equilibrium balanced growth path. To do so, again it will be assumed that there is a "Debt Trigger" in place, so that the debt to GDP ratio cannot exceed some level. The parameter values are chosen so that the economy will be growing, but that the government debt will be rising even faster, which will mean that the debt to GDP ratio will also be growing. Such an economy, if left unchanged, will not satisfy the government budget constraint, and so if the Debt Trigger were not introduced, then some other policy would need to be implemented to make the resulting long-run allocations feasible.

In this experiment the parameters are chosen as described above, and as a benchmark, the preference parameter will be set at $\sigma = 5.0$. In the benchmark economy, with zero debt, the parameter λ will be set so that the growth rate will be 3% when there is no government debt. Then, for this economy, three different scenarios will be considered. The initial government debt to GDP ratio will then set at 37%, 16.7%, or 7.5%.[34]

The resulting growth rate for this experiment is illustrated in Figure 13. These three economies are identical in every respect with the exception of their initial debt to GDP

[33] This feature is only partially offset by the fact that a higher interest rate will mean a higher amount of capital income tax revenue.

[34] These levels are chosen because it is easy to calculate that they will then hit the debt trigger in periods 6, 10, and 14, respectively. In each of these instances the capital tax rate rises to 55% once the debt trigger is hit.

ratios. As the figure shows, the higher is the initial debt level, the higher is the initial growth rate initially. Here, the higher value of the debt to GDP ratio, the closer is the economy to hitting the debt trigger, and so the nearer it is to having to raise taxes. This causes the agent to shift more consumption into the future, which means that the investment rate must rise (and ϕ must fall). Hence, the higher is the debt to GDP ratio, the higher is the growth rate. The punch line would seem to be that, holding other things constant, a higher level of foreign-held government debt leads to higher initial (or short term) growth, but lower growth in the intermediate term because taxes will have to be raised to keep the government debt from escalating. The long-term growth rate is the same for all scenarios because eventually the growth and tax rates assume their long-run levels, with the debt to GDP at 100%, and tax rates that will be sufficient to pay the interest on the debt.

This example illustrates why in the data it may be difficult to discover a tight relationship between the level of government debt and the corresponding growth rate. Suppose one were to observe a number of (otherwise identical) economies, of the sort studied here, that differed *only* in their level of government indebtedness. If one were to look at the corresponding relationship between the level of this debt and the resulting growth rate, this example shows why there might not be any close association, because the relationship can be quite nonlinear.

In some other contexts, the fact that the growth rate is higher prior to the tax increase may be taken to mean that individuals are deliberately realizing capital gains income prior to the tax increase. However, this is clearly not happening here, since there are no capital gains in this model. Instead, in this model the individual can only shift consumption and investment intertemporally in response to the tax changes. Therefore, in this instance the expectation of higher future taxes is causing agents to shift their consumption/saving, and therefore income, across periods. For higher values of σ, the individual will raise saving prior to the rise in taxes, which raises growth. For very low values of σ, the individual will lower saving prior to the rise in taxes, which reduces growth.

Again, this experiment also has implications for the behavior of after-tax interest rates as well. Since the higher level of debt implies that the capital tax will rise sooner, this implies that the higher level of debt will imply that after-tax interest rate will fall sooner. Hence, there is an inverse relationship between the level of government debt, and future after-tax interest rates.

10.3.1 Implications for Government Spending Multipliers

This last set of results also has implications for the evaluation of the size of government spending multipliers, even though there is no explicit government spending in the model. This is a topic of some importance since there has been renewed interest in recent years for evaluating the size of these multipliers.

To proceed with this analysis, consider the following hypothetical experiment. Consider two economies that are identical in every respect, and have the technology and preference parameters specified for Figure 13. Now, assume these economies are growing along identical paths until some specific date. Suppose that at that date, one government engages in some wasteful government spending (or just a pure transfer to consumers) that is financed entirely by a foreign debt issue. Because the spending itself is specious, it has no impact on the future growth path. Now the two economies are no longer identical, but instead one has a higher level of debt. The results from Figure 13 show that immediately after this event, the economy with the higher debt will grow faster than the one with lower debt. An observer, who witnessed this experiment, might naturally conclude that the effect of the government spending was to generate higher economic growth, and may conclude that the spending had a relatively high multiplier. However, as the model shows, this is highly misleading. It is actually *not* the government spending that causes growth to be higher. Instead, it is the higher expected future taxes entailed by the higher debt that causes the individual to invest in capital, which causes growth to be higher. It would be wrong to conclude that the government spending was "productive", and just as wrong to conclude that it was good use of resources. Such a policy must reduce welfare.

This experiment further illustrates that it is not possible to conclusively analyze the impact of a change in government spending without specifying how this will affect future government policies through the government's budget constraint.

10.4 Response to Tax Cuts with Foreign-Held Debt

With foreign-held government debt, the response to a temporary tax cut depends critically on the intertemporal elasticity of substitution. To see this, again suppose that the benchmark growth rate is 3% when the tax rate is 20%. The debt is such that this tax rate is just sufficient to generate enough revenue to finance this level of (foreign-held) debt. Now suppose that in period 1 the tax rate is cut to 15% for 9 periods, and then permanently raised to make up for the lost revenue. Figure 14 shows these same experiments for different values of σ simultaneously. It is apparent the higher is σ, the larger is the increase in the growth rate generated from this *temporary tax cut*. Once again, in contrast with traditional economic thinking, in this environment a temporary tax cut can lead to a larger response in the growth rate than one would find with a permanent tax cut. Furthermore, the magnitude of this increase in the growth rate is related to the size of σ. The higher is σ, the lower is the intertemporal elasticity of substitution, and so the future tax increase prompts the agent to shift consumption into the future (by lowering ϕ, or increasing saving). This has the effect of raising the growth rate.

10.4.1 More Unpleasant Fiscal Arithmetic

It was shown in Section 9.6 (and Figure 9) that if the government debt is held domestically, and the value of σ is sufficiently low, it is possible that a temporary tax cut can immediately lead to *lower growth*. A natural question then is to ask whether this result will hold if the government debt is held abroad as well. It turns out that indeed this is the case. The reason for this is almost identical to the prior case with domestic government debt. Individuals know that the taxes will be raised in the future, and so they raise current consumption relative to future consumption. This essentially lowers the savings rate, and lowers the growth rate.

11 The Model with Labor and Domestically-Held Government Debt

An evident question at this point would be to ask how the results would change if labor were added to the model. One needs to be careful in how this is done, because such a complication can easily make the model unmanageable, or at least, difficult to characterize. Additionally, there is a variety of ways to introduce labor into the model, and the results of the analysis can differ across these different approaches. One way to do this would be to introduce labor/leisure into the preferences, given by equation (8), and also into the production technology. Although this approach is feasible, it introduces a complication into the analysis in that marginal rate of substitution between leisure and consumption turns out to be a very important influence on the results. While this feature is interesting, it also adds a degree of complexity to the analysis and results that would seem rather burdensome at this point.

An alternative way to avoid this issue, and perhaps to simplify the analysis, would be to have a separate group of workers, and owners of capital. The former group will simply work and consume, while not holding capital and government bonds. That is, all government debt will be held domestically by these agents. The holders of capital will then also hold government bonds. In other words, one group will pay the labor tax, while the other will pay the capital tax. This approach is a comparatively simple extension of the model studied earlier, and it will be straightforward to explain the main features of this model with labor. This is the approach adopted below.

11.1 Owners of Capital

There will be a group of agents who hold capital, and they have preferences given again by equation (8). These agents will again maximize this utility function, subject to the same budget constraint, given by equation (9). They will earn a share (α) of aggregate output, and their decisions will directly influence the growth rate. The only slight change to the optimization problem will be the change in the technology, which is now described.

11.2 Technology

The technology for this economy will now have labor. Instead of having production be a function of capital alone, as in the right side of equation (2), the technology will be given as follows:

$$c_t + i_t = z_t \left(k_t^{\alpha}\right) n_t^{1-\alpha}.$$

where n_t is the amount of labor employed. Again, equation (3) still holds. To permit the model to generate balanced growth, it will be assumed that there is an externality in production, so that the technology parameter will be a function of the average amount of capital held across these capital-holders: $z_t = \lambda \left(K_t^{1-\alpha}\right)$.[35] This also implies that the *endogenous, before-tax rate of return on capital* at any date t is given as

$$r_t = \lambda \alpha n_t^{1-\alpha} - \delta.$$

[35] Of course, because of this externality the equilibrium growth rate will not be optimal, but this will not be of direct concern to the analysis below.

Henceforth, the tax rate in the problem of the capital-holders will be denoted by τ_k, which will be the capital tax rate, in order to distinguish it from the labor tax rate τ_n.

11.3 Workers

It will be assumed that there is also a population of workers, and the size of this workforce will be equal to the number of holders of capital. Workers do not save or invest. Each period the supply labor and then consume the proceeds of their after-tax wage income. The one-period preferences of these workers will be given as: $\ln(c_t) + \gamma (1 - n_t)$.

To produce a labor decision-rule that depends non-trivially on the labor tax rate, it will be necessary to assume that some fraction (θ) of the labor income paid to workers is not subject to the labor tax. This is done so as to make the labor decision rule a non-trivial, decreasing function of the labor tax rate. The budget constraint for the workers is then given as follows: $c_t = w_t (1 - \tau_{nt}) n_t + \zeta_t$. Here w_t is the wage, and ζ_t is the income received by workers that is not subject to tax.[36]

$$n = \frac{(1-\tau_n)(1-\alpha-\theta)}{\gamma\left[(1-\tau_n)(1-\alpha-\theta)+\theta\right]}. \tag{23}$$

11.4 The Government Budget Constraint

The government budget constraint is now changed because both labor and capital are taxed. Let τ_{kt} and τ_{nt} denote the date-t capital and labor taxes at date t. Also, let w_t denote the date t wage measured in units of capital. Then the per-capita government budget constraint can be written as

$$B_t = \int_t^\infty \exp\left(-\int_t^s r_z (1-\tau_{kz})\, dz\right)\left[K_s (r_s \tau_{ks}) + N_s (w_s \tau_{ns})\right] ds,$$

with N being the per-capita employment level. Here the equilibrium wage for the individual, measured in units of the consumption good, will be

$$w_t = z_t (1-\alpha-\theta)\left(\frac{k_t}{n_t}\right)^\alpha.$$

In equilibrium the labor tax revenue at date t will be

$$n_t (w_t \tau_{nt}) = \lambda (1-\alpha)(k_t)\left(n_t^{1-\alpha}\right)\tau_{nt}.$$

11.5 Analysis of the Model

The analysis of the decisions of the capital-holders is presented in Appendix C. The growth rate of the model is easily seen to be a function of the investment/saving behavior of the capital-holders, and the employment decisions of the workers. And both of these decisions are influenced by the capital and labor tax rates. Let ϕ_t denote the fraction of capital income that is consumed by the capital holders at date t, and so $(1-\phi_t)$ is analogous to

[36] It is apparent from this expression that if $\theta = 0$, then employment must be constant, or independent of the labor tax rate.

a saving/investment rate. In Appendix C it is then that (ϕ) must satisfy the following differential equation

$$\sigma\dot{\phi} = \phi\left[\left[\alpha\lambda\left(n_t^{1-\alpha}\right) - \delta\right]\left[1 - \tau_{kt} - \sigma\right] - \sigma\left(1 - \alpha\right)\left(\frac{\dot{n}}{n}\right) - \rho\right] + \phi^2\left[\lambda\left(n_t^{1-\alpha}\right)\right]\sigma\alpha$$

Then finally, the non-linear differential equation (16) can be written as

$$\dot{\phi} = p_t\phi + \lambda\phi^2,$$

where

$$p_t = \frac{\left[\alpha\lambda\left(n_t^{1-\alpha}\right) - \delta\right]\left[1 - \tau_{kt} - \sigma\right] - \sigma\left(1 - \alpha\right)\left(\frac{\dot{n}}{n}\right) - \rho}{\sigma}, \tag{24}$$

and

$$q_t = \left[\lambda\left(n_t^{1-\alpha}\right)\right]\alpha$$

The solution to this Bernoulli differential equation is then

$$\phi_t = \frac{\exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty q_z \exp\left(-\int_z^\infty p_s ds\right) dz}. \tag{25}$$

Of course, the labor terms can be eliminated from this expression, by substituting equations (23) and (31). This would make it more apparent how the behavior of the labor tax rate would influence the growth rate.

11.6 Quantitative Effects of a Labor Tax

In such a stylized economy it is not straightforward to assign parameter values for preferences and technology. In the interest of brevity, a detailed analysis of the effect of alternative parameters will not be presented. Instead, a few experiments will be conducted to illustrate the effect of changing the policy parameters (i.e. the tax rates). As shown above, $\left(\frac{1}{\sigma}\right)$ the intertemporal elasticity of substitution for consumption of the capital holders, will again be assumed to be relatively low. Therefore a value of $\sigma = 5.0$ will be employed below.

Of course, α is capital's share of output, and it will be assumed that $\alpha = .30$. It is interesting to note that changing this number has a quantitative impact on the results, but not too much of a qualitative impact. A value of $\theta = .30$ will also be employed.

In the following experiments, the initial tax rate for capital and labor will again be set at 20% initially, $\delta = 10\%$, and the parameter λ will be chosen to produce a benchmark growth rate of 3%. Again, the discount rate will be $\rho = .05$. In period $t = 1$, both tax rates will be reduced from 20% to 10% for precisely 10 periods, and then both tax rates will rise thereafter to such a level that will balance the government's budget constraint. That is, the amount of discounted revenue will be the same under this experiment as if the tax rate had been held at 20% forever.

Figures 15 and 16 show the behavior of the growth rate, for output and capital respectively, to a change in the income tax rate for 10 periods. The final income tax rate is set to 37.6%. The growth rates rise on impact, and continue to rise until the tax rate is increased, whereupon there is a drastic permanent fall in the growth rate. Again, in these figures the

'$\diamondsuit$' symbol shows the behavior of the growth rates if the tax rate were kept at 10% forever. The sharp fall in the growth rate of output is due to both the fall in employment in period 10, due to the increase in the labor tax, and the fall in the saving rate due to the tax increase as well. The magnitude of the peaks and troughs in these figures is an increasing function of σ. For small values of σ (i.e. less than one), these are very small changes in the growth rate.

Since much of the preceding analysis has focused on analyzing the impact of a change in the capital tax rate, it will be of interest to study here a change in the labor tax rate alone. Therefore, consider an experiment very similar to the prior one. The initial tax rate for capital and labor will again be set at 20% initially, which will be just high enough to retire the existing debt. Then, in period 1, the labor tax rate alone is reduced to 10% for 10 years. At that point, it is necessary to raise it to 38.69% in order to recover all the lost revenue. The capital tax rate is 20% forever. Figure 17 shows the resulting growth path for capital. There is a modest increase in the growth rate on impact, but the growth rate increases substantially until period 11. This figure shows that the behavior shown in figures and experiments, which were due to the behavior of the capital tax, can also be produced by a change in the labor tax as well. The reason for this is that changes in the labor tax rate alter the amount of employment, which in turn change the rate of return to capital. Since owners of capital know this, they will then change their investment decisions, which will then affect the growth rate. In particular, the future rise in the labor tax will reduce employment, which in turn will lower the rate of return to capital, making saving less attractive.

It is also of interest to see how the time horizon of the tax cut affects the results. Consider the same economy as that in Figures 15 and 16, where the income tax rate (i.e. both capital and labor), is cut from 20% to 10% for specific number of periods. However, consider doing this for three different cases: let the tax cut be for either 4, 8, or 12 periods. The resulting path of the growth rate is illustrated in Figure 18. As is evident, the largest increase in the growth rate, in the short run, is from cutting the income tax rate for a shorter period of time. While a more long-lasting cut in tax rate results in a smaller increase in the growth rate in the short term, a larger increase in the growth rate in the intermediate term, and the lowest resulting growth rate in the long run. The reason the more prolonged tax cut results in a lower growth rate in the long term is that the tax rate must ultimately rise the most in this case. The table below shows the resulting long-run tax rates for each of these experiments.

Number of Periods of Tax Cut	Final Tax rate
4	25.1%
8	33.1
12	42.3

This is yet another example of an economy in which a short-term change in the income tax rate can have a bigger impact on the immediate growth rate, than would a more long-lasting change in the tax rate of the same magnitude.

Another obvious question is whether the phenomenon of "Unpleasant Fiscal Arithmetic" will exist in this economy in which there is a labor tax. Consider again the economy as that in Figures 15 and 16, where the income tax rate (i.e. both capital and labor), is cut from 20% to 10% for specific number of periods, and then raised after 10 periods to make up for the lost revenue. However, in this case suppose that $\sigma = .60$, so that there is a low elasticity of intertemporal substitution. The resulting growth rate is shown in Figure 19. There is an immediate increase in the growth rate, due to the reduction in the labor tax. However, thereafter the growth rate fall for the same reason described in Section 9.6.

This analysis of the model with labor has been conducted under the assumption that the government debt is held domestically. However, it was shown in Sections 9 and 10 that the effect of policy changes can differ substantially, both quantitatively and qualitatively, depending on who holds this debt. This suggests that the experiments of the present section may also change considerably, if the government debt is held abroad.

12 Final Remarks

Several generations of research into dynamic models have taught us how vital it is to integrate expectations into the decision-making of agents. This is especially evident when studying issues related to tax reform. The model studied here makes it quite apparent that one cannot hope to capture the effects that government policies will have without also knowing what these also imply about future policies. This is not some fictional consideration of only theoretical consequence: because changes in policies at one date *must* have an influence on future policies.

This paper can be viewed as an answer to Sims [19] call to "more accurately model ... the wealth effects on private sector behavior of debt and expected future fiscal policy."[37] In this model it has been shown that analyzing the effect of a change in the tax rate, or in the size of the government debt, can be problematic because of the complicated dependence that future government policies will have on current policies. The government budget constraint presumably imposes discipline on what policies are feasible. Agent's expectations must take into account how this constraint will affect future decisions. If agents believe that a reduction in current taxes necessarily implies a future reduction in spending, then the impact will be vastly different than if agents believe that spending is likely to be unchanged, and instead future taxes must necessarily be raised.

The model studied here assumed that a fixed amount of predetermined government debt had to be financed through distortional taxation. It was shown that the impact that the size of this debt had on the growth rate depends on some important parameters of the economy. In one extreme case, a *reduction* in the capital tax rate causes a growth *implosion* because of the expectation that the government budget constraint must imply that taxes will rise even higher in the future.

Although the analysis makes it very apparent that characterizing the impact of a policy change critically depends on understanding the expectations of agents, it is far from clear how to do this in practice. There are other important related issues that also arise: It is also evident that the credibility of announced government policies is also of vital importance.

The model studied here is necessarily highly stylized, as there is only one technology in which individuals can invest. In practice, there is generally a multitude of vehicles

[37] See Sims [19], page 1202.

that individuals can employ to shift consumption intertemporally. This may be another important feature that would affect the quantitative nature of the results.

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13 Appendix A

In this Appendix the discrete-time counterpart to model section 5 is presented. The model is then re-written in the following manner. The preferences of the individuals are written as follows

$$\sum_{t=1}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} \right]$$

Let the technology be

$$c_t + i_t = \lambda \left(\phi_t k_t \right)$$

and

$$k_{t+1} = (1-\delta) k_t + i_t.$$

The budget constraint faced by the individual consumers is written as

$$c_t + (b_{t+1} + k_{t+1}) = (b_t + k_t) \left[1 + r_t (1 - \tau_t) \right].$$

The euler equation for a consumer in this economy is of the usual form:

$$\left(c_t^{-\sigma} \right) = \beta \left(c_{t+1}^{-\sigma} \right) \left[1 + r_{t+1} (1 - \tau_{t+1}) \right]$$

Also, the interest rate can be seen to be $r = \lambda - \delta$. But since $c_t = \lambda(\phi_t k_t)$, and $k_{t+1} = [(1-\delta) + \lambda(1-\phi_t)] k_t$, this equation can be written as

$$(A(\phi_t k_t))^{-\sigma} = \beta\left(\lambda\left(\phi_{t+1}k_{t+1}\right)\right)^{-\sigma}\left[1 + r_{t+1}(1-\tau_{t+1})\right]$$

or

$$(\phi_t)^{-\sigma} = \beta\left(\phi_{t+1}\right)^{-\sigma}\left[(1-\delta) + \lambda(1-\phi_t)\right]^{-\sigma}\left[1 + (\lambda-\delta)(1-\tau_{t+1})\right],$$

which can be re-written as

$$\phi_t = \phi_{t+1}\left[(1-\delta) + \lambda(1-\phi_t)\right]\left\{\beta\left[1 + (\lambda-\delta)(1-\tau_{t+1})\right]\right\}^{-1/\sigma}.$$

This equation is a non-linear difference equation, and is the discrete-time counterpart to the non-linear differential equation (16). In principle this difference equation could be "solved forward", but it would be a nasty mess. Nevertheless, this equation illustrates how the current value of ϕ_t is then going to be a function of all present and future values of the tax rate τ_{t+s}, for $s \geq 0$.

There is another approach to this problem that is illuminating. Imagine a fictitious planner who is maximizing the welfare of consumers in such an environment, while facing a series of intertemporal returns, denoted $1 + r_{t+1}(1-\tau_{t+1})$. Let the state variable for such a planner be k_t. It is straightforward to verify that the value function for the dynamic programming problem, that corresponds to the planning problem that corresponds to the competitive equilibrium, is of the form

$$v(k_t) = \Pi_t (k_t)^{1-\sigma}.$$

This value function is written as a function of the capital stock, and not the capital stock and the stock of bonds, because in this environment all output is consumed by consumers, and so the capital stock is a sufficient state variable to characterize the amount of consumption. It can be shown that the coefficient Π_t is a non-linear function of the parameters of the economy, as well as all the future capital tax rates $(\tau_s, s \geq t)$.

14 Appendix B

14.1 Proof of Proposition 1:

Equation (16) is of the following form:

$$\dot{\phi} = p_t\phi + \lambda\phi^2.$$

Writing the candidate solution as

$$\phi_t = \frac{\exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz}. \tag{26}$$

Taking the derivative of this expression with respect to t yields

$$\begin{aligned}
\dot{\phi} &= \frac{p_t \exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz} - \frac{(-\lambda)\left[\exp\left(-\int_t^\infty p_s ds\right)\right]^2}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]^2} \\
&= p_t\phi + \lambda\phi^2.
\end{aligned}$$

14.2 Proof of Proposition 2:

Using equation (26) it is possible to show that, for $j > t$,

$$\begin{aligned}
\frac{\partial \phi_t}{\partial \tau_j} &= \frac{\partial \phi_t}{\partial p_j} \cdot \frac{\partial p_j}{\partial \tau_j} \\
&= \left[\frac{\exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz} - \frac{\exp\left(-\int_t^\infty p_s ds\right) \int_t^j \lambda \exp\left(-\int_z^\infty p_s ds\right) dz}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]^2} \right] \left[\frac{\lambda - \delta}{\sigma} \right] \\
&= \left[1 - \frac{\int_t^j \lambda \exp\left(-\int_z^\infty p_s ds\right) dz}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]} \right] \phi_t \left[\frac{\lambda - \delta}{\sigma} \right] > 0. \qquad (27)
\end{aligned}$$

Note that the first term in square brackets is unity when $j = t$, and goes to zero as $j \to \infty$, so that very distant taxes have a negligible effect. Then it is straightforward to show that

$$\frac{\partial^2 \phi_t}{\partial j \partial \tau_j} = \left[\frac{-\exp\left(-\int_j^\infty p_s ds\right)}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]} \right] \phi_t \left[\frac{\lambda - \delta}{\sigma} \right] < 0.$$

Equation (27) gives the change in ϕ_t, for a change in τ_j per unit of time. To obtain the effect of of changing the future tax rate over some interval if time (Ω) in the future ($> t$), it is necessary to use the expression in equation (27) to calculate

$$\int_{j \in \Omega} \left(\frac{\partial \phi_t}{\partial \tau_j} \right) dj.$$

It is then possible to verify that a permanent change in the tax rate has the impact that would be calculated from equation (18).

Lastly, if the tax rate constant ($\tau_s = \tau$), then equation (27) shows that $\frac{\partial \phi_t}{\partial \tau_j}$ is decreasing in σ.

15 Appendix C

The optimization problem for the holders of capital is similar to that in Section (5.1). The Hamiltonian for this problem is then written as follows

$$H = e^{-\rho t} \left[\frac{c_t^{1-\sigma}}{1-\sigma} \right] + e^{-\rho t} \mu \left[(b + k) r (1 - \tau_{kt}) - c \right],$$

The optimization condition for consumption is then

$$c_t^{-\sigma} = \mu.$$

The euler equation for this problem is then written as follows:

$$\begin{aligned}
-\sigma \left(\frac{\dot{c}}{c} \right) &= \left(\frac{\dot{\mu}}{\mu} \right) \\
&= \rho - r (1 - \tau_{kt}).
\end{aligned}$$

Now, in equilibrium the before tax rate of return to capital is easily seen to be

$$r_t = \alpha z_t \left(k_t^{\alpha-1}\right)\left(n_t^{1-\alpha}\right) - \delta.$$

Using the definition of z_t, this can be written as

$$r_t = \alpha\lambda\left(n_t^{1-\alpha}\right) - \delta.$$

The euler equation can then be written as

$$\sigma\left(\frac{\dot{c}}{c}\right) = \left[\alpha\lambda\left(n_t^{1-\alpha}\right) - \delta\right](1-\tau_{kt}) - \rho.$$

It must be remembered that the holders of capital only get a share of income that is α. Now with this in mind, to solve this problem, let us use the following decision rules:

$$\begin{aligned} c &= \phi\alpha z_t\left(k_t^{\alpha}\right)\left(n_t^{1-\alpha}\right) \\ &= \phi\alpha\lambda\left(k_t\right)\left(n_t^{1-\alpha}\right), \end{aligned} \tag{28}$$

and

$$\dot{k} = (1-\phi)\,\alpha\lambda\left(k_t\right)\left(n_t^{1-\alpha}\right) - \delta k. \tag{29}$$

Equation (28) then implies that

$$\frac{\dot{c}}{c} = \frac{\dot{\phi}}{\phi} + \frac{\dot{k}}{k} + (1-\alpha)\left(\frac{\dot{n}}{n}\right), \tag{30}$$

but using equation (29) this can be rewritten as

$$\frac{\dot{c}}{c} = \frac{\dot{\phi}}{\phi} + \left[(1-\phi)\,\alpha\lambda\left(n_t^{1-\alpha}\right) - \delta\right] + (1-\alpha)\left(\frac{\dot{n}}{n}\right).$$

Now substituting equation (30) into this expression yields

$$\sigma\left(\frac{\dot{\phi}}{\phi} + \left[(1-\phi)\,\alpha\lambda\left(n_t^{1-\alpha}\right)\right] + (1-\alpha)\left(\frac{\dot{n}}{n}\right) - \delta\right) = \left[\alpha\lambda\left(n_t^{1-\alpha}\right) - \delta\right](1-\tau_{kt}) - \rho,$$

or

$$\sigma\left(\dot{\phi}\right) = \phi\left[\left[\alpha\lambda\left(n_t^{1-\alpha}\right) - \delta\right]\left[1-\tau_{kt}-\sigma\right] - \sigma(1-\alpha)\left(\frac{\dot{n}}{n}\right) - \rho\right] + \phi^2\left[\lambda\left(n_t^{1-\alpha}\right)\right]\sigma\alpha.$$

Using equation (23), it is also possible to show that

$$\frac{\dot{n}}{n} = \frac{-\theta\dot{\tau}_n}{(1-\tau_n)\left[(1-\tau_n)(1-\alpha-\theta)+\theta\right]}. \tag{31}$$

Figure 1

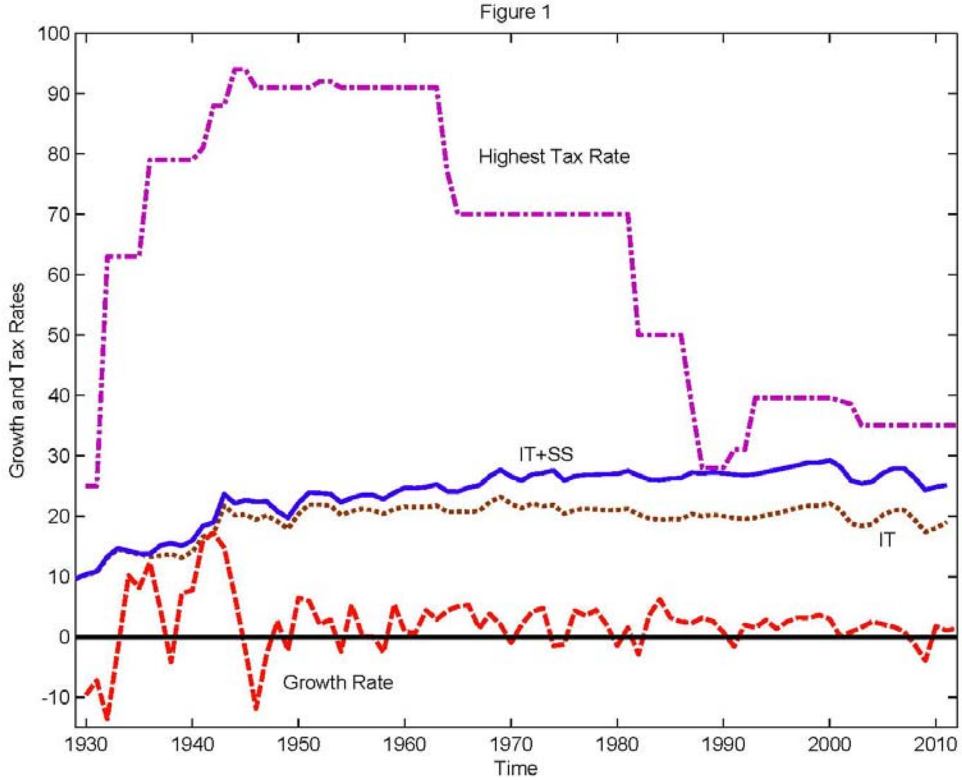
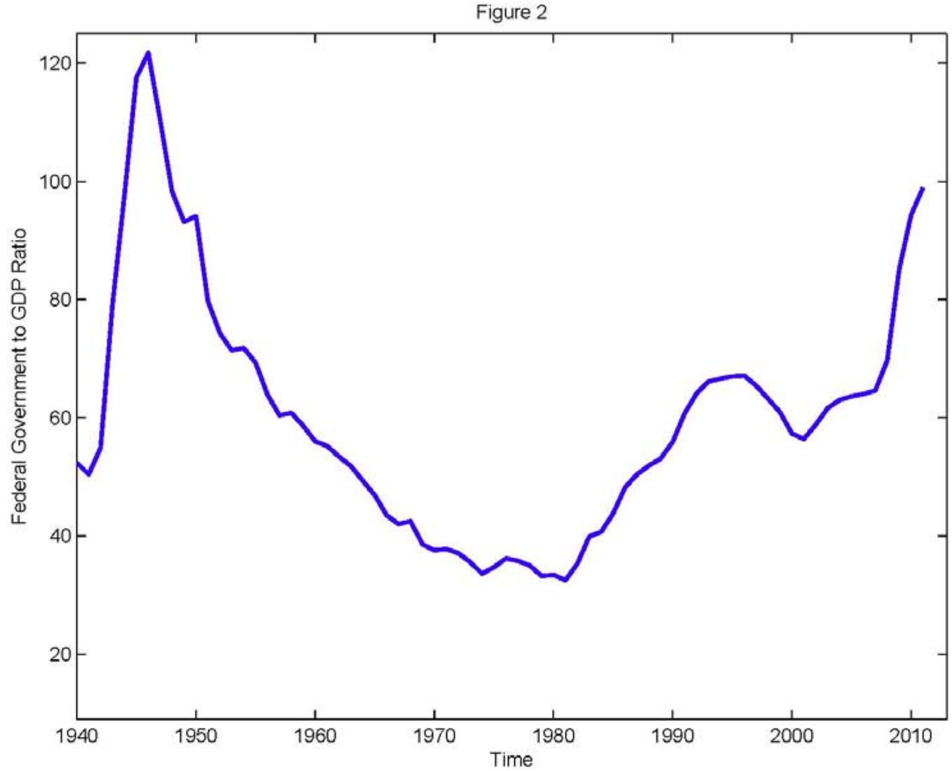


Figure 2



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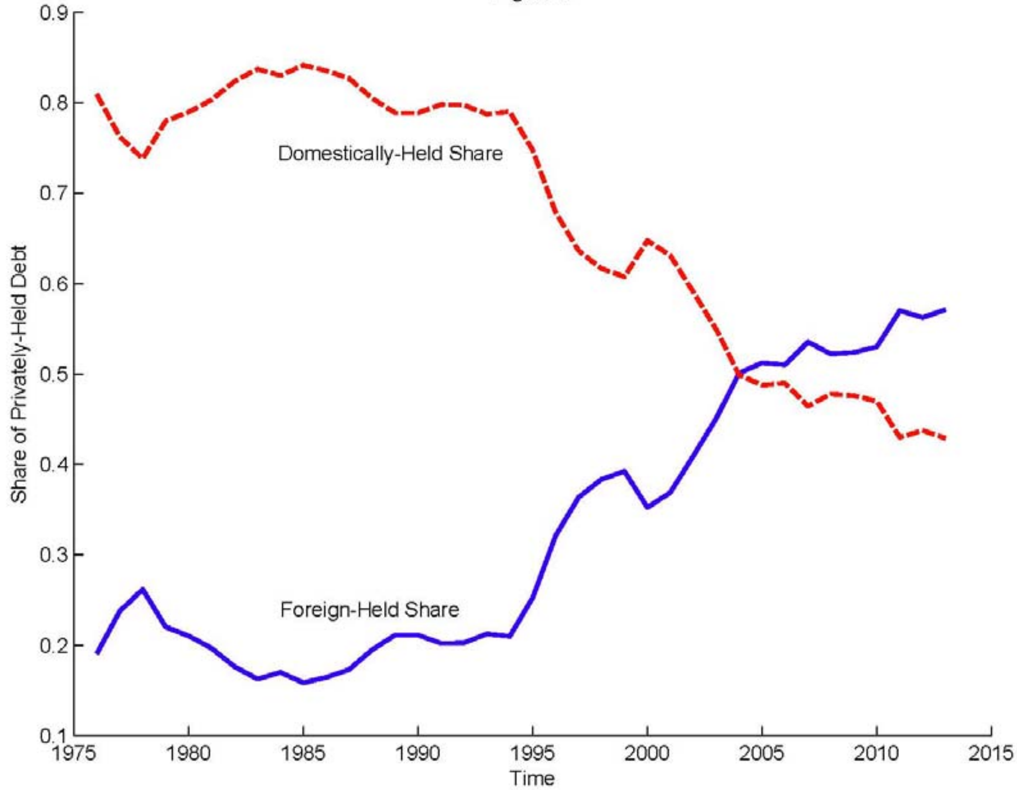

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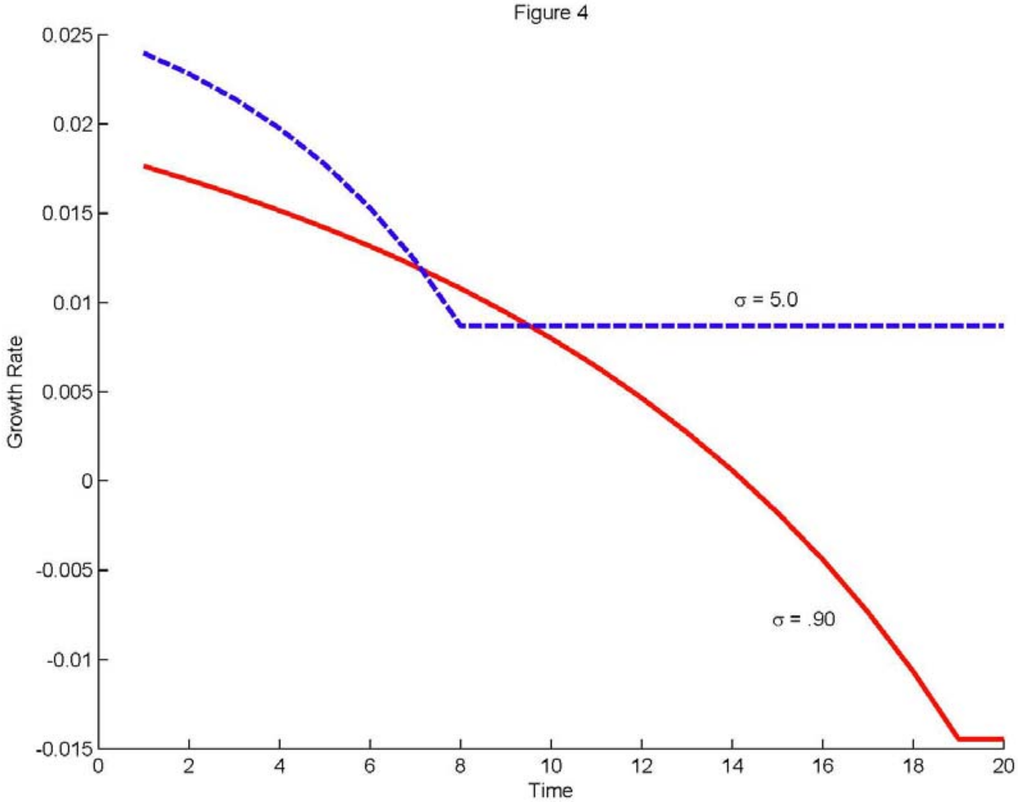
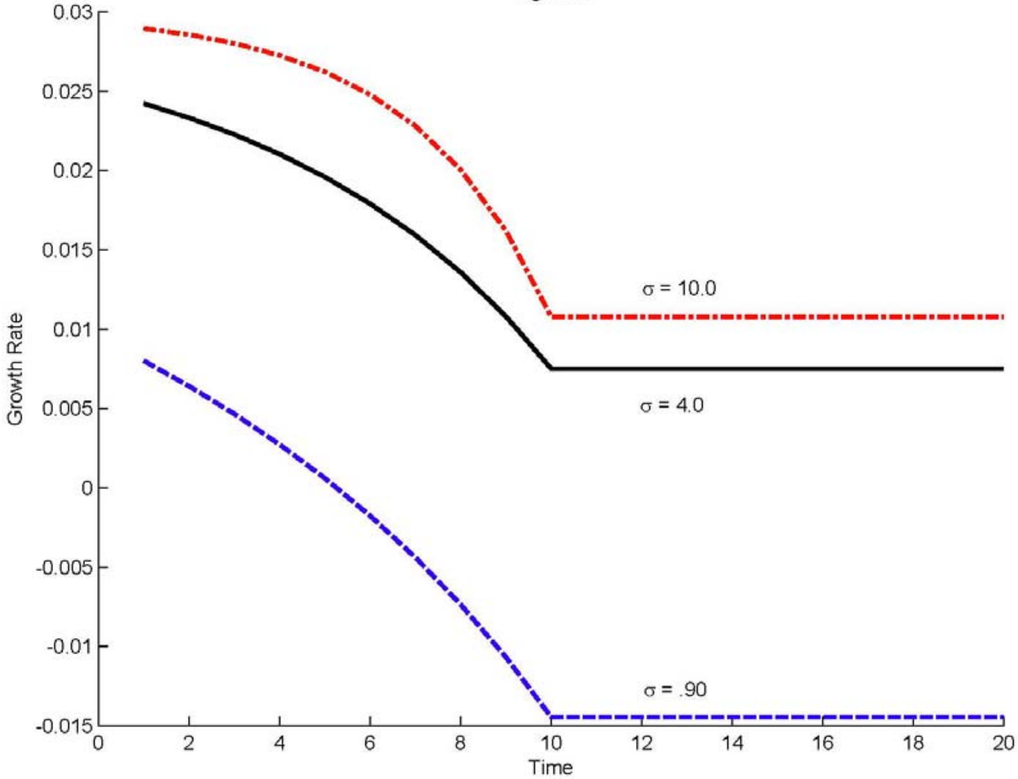


Figure 5



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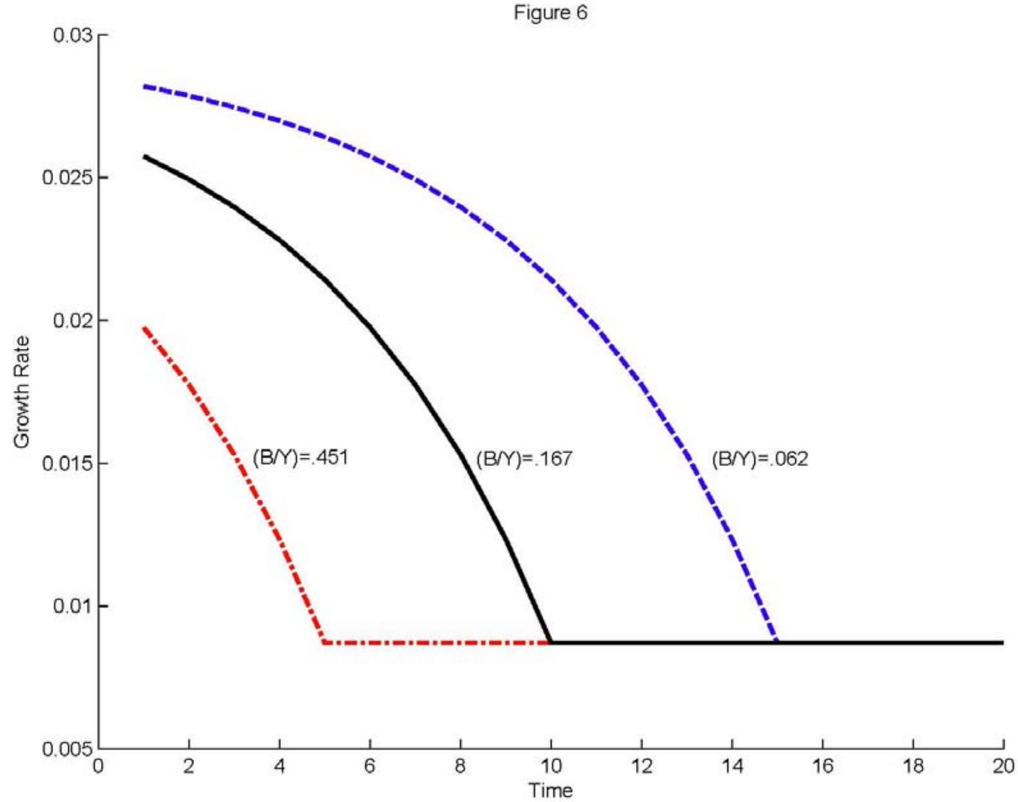

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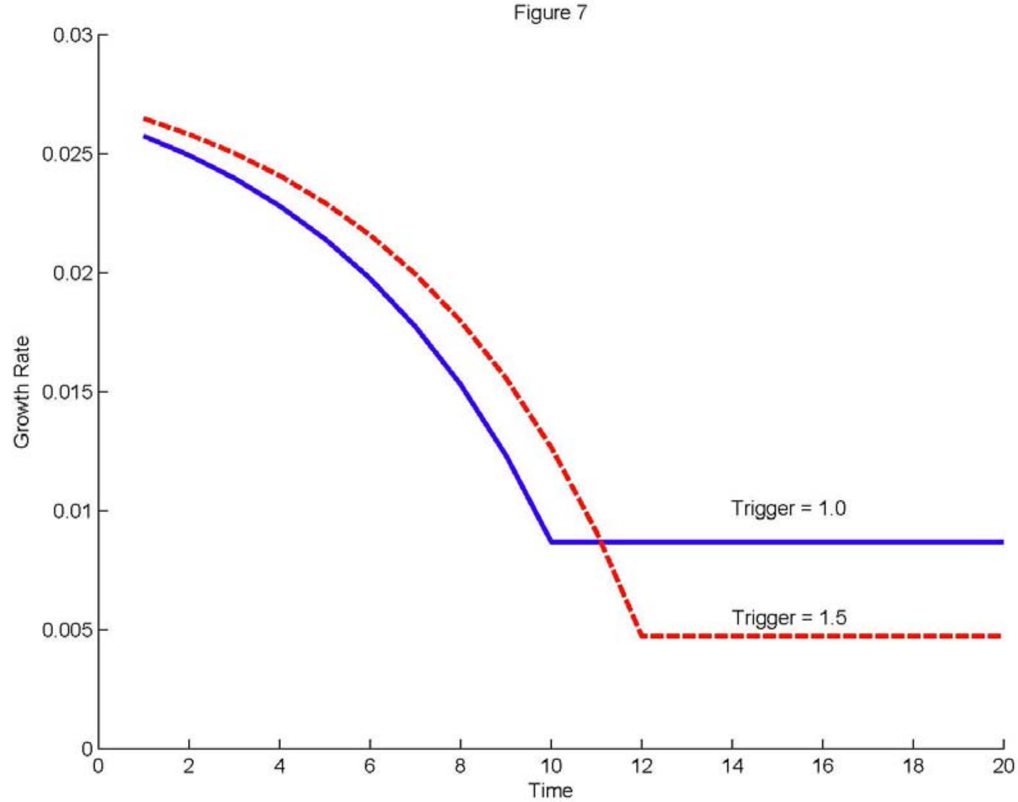

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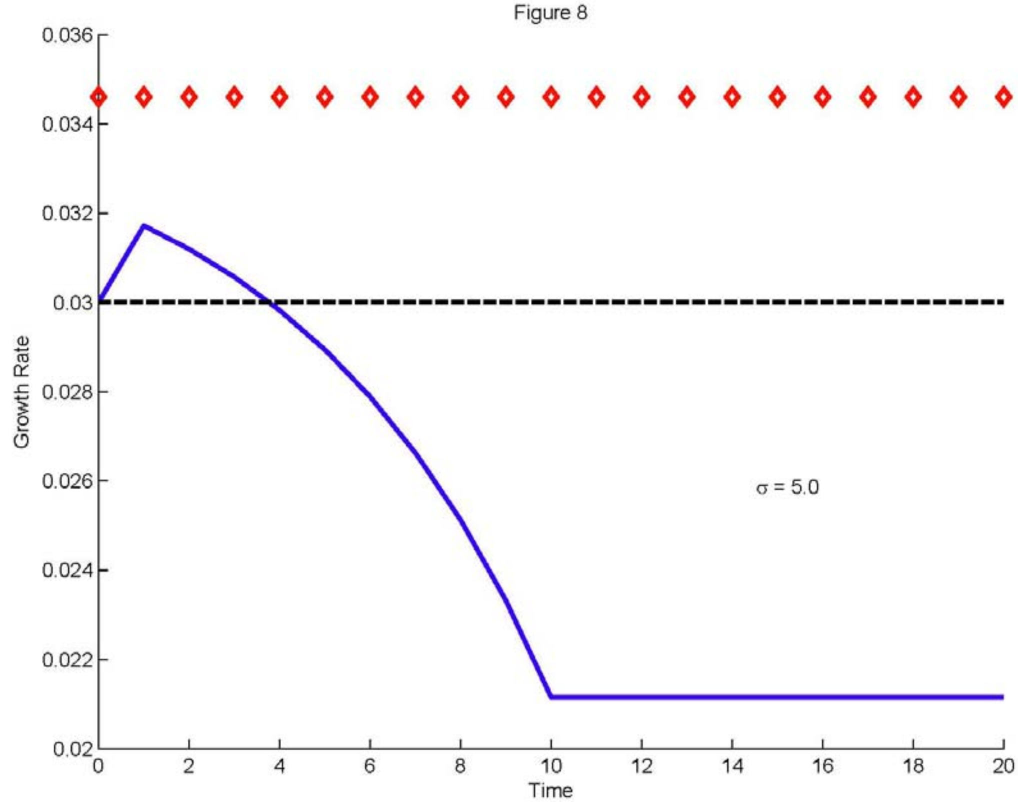


Figure 9

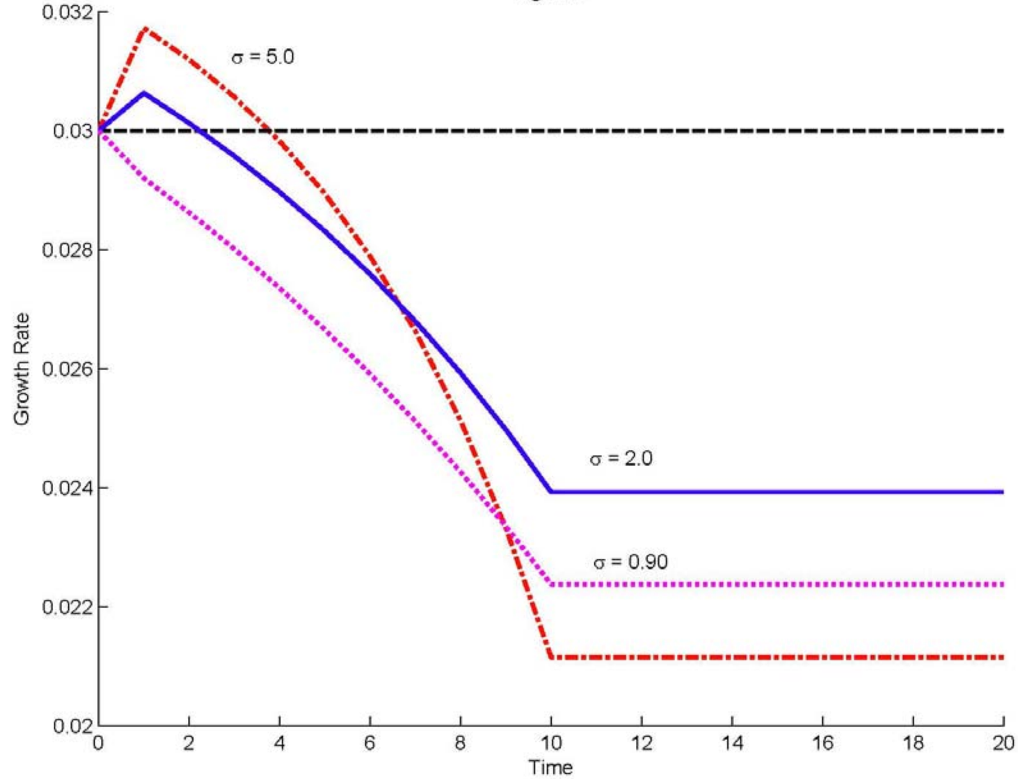
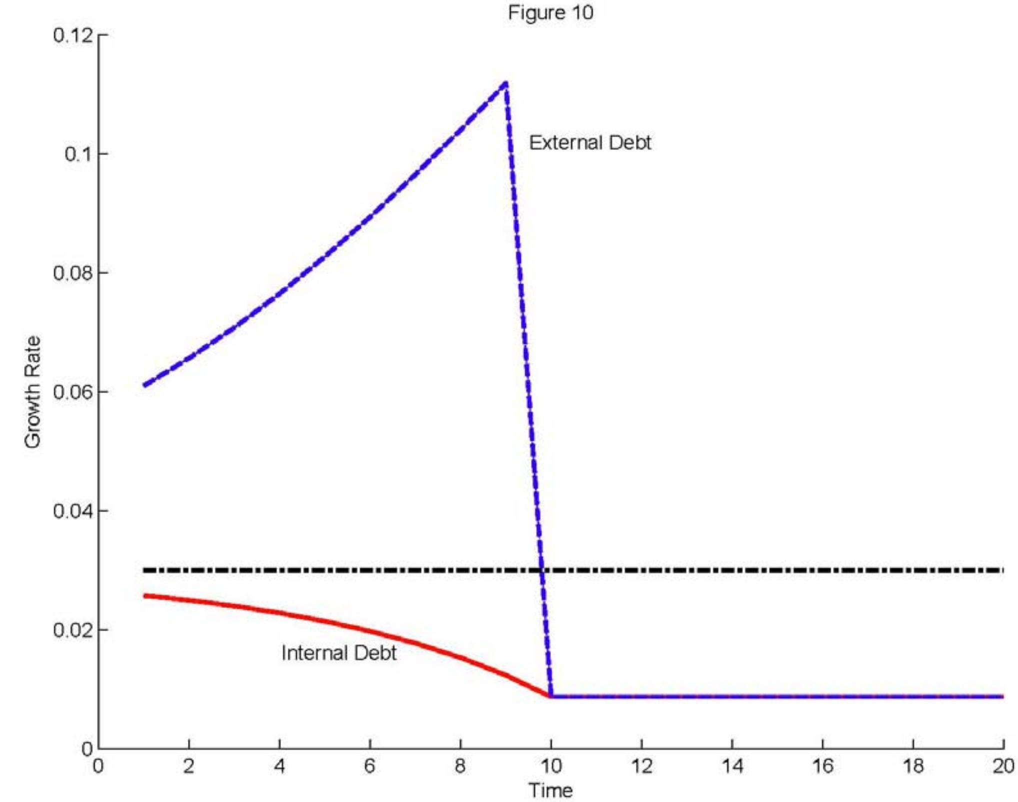


Figure 10



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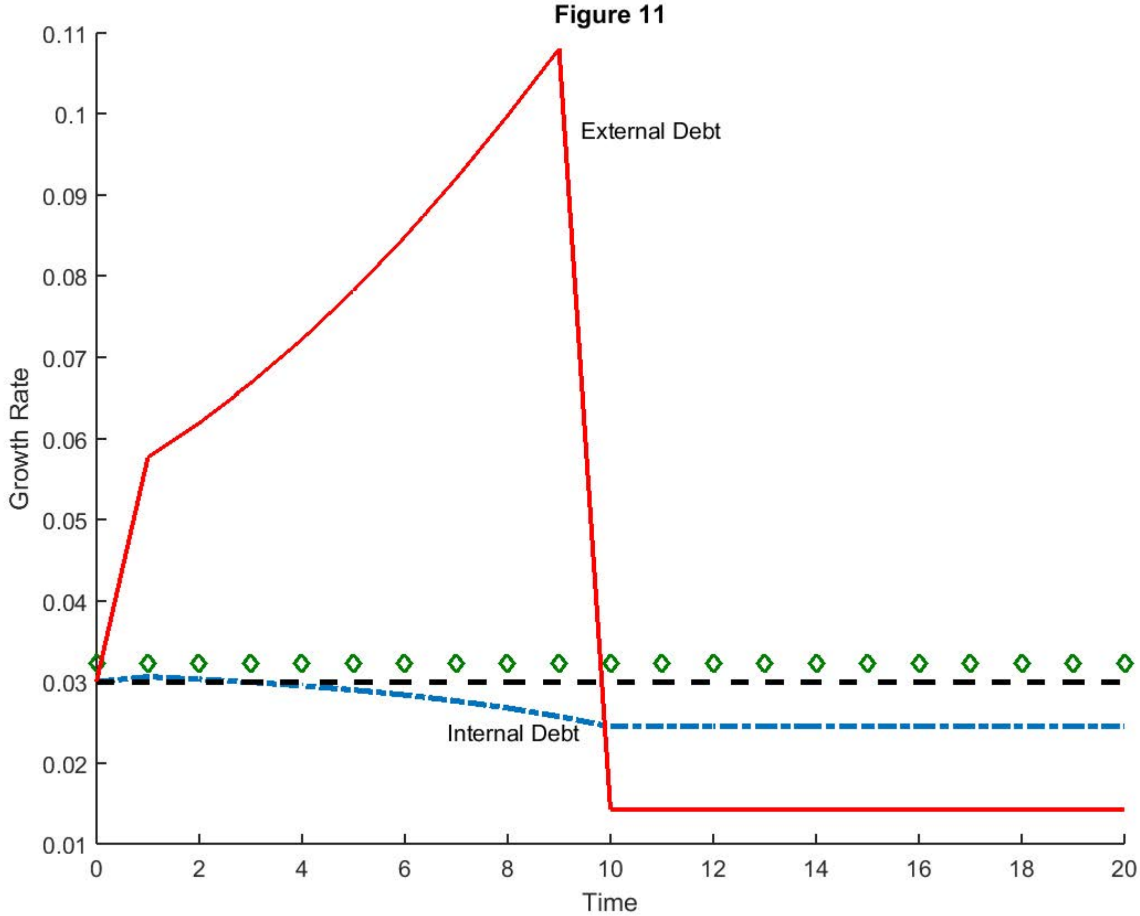

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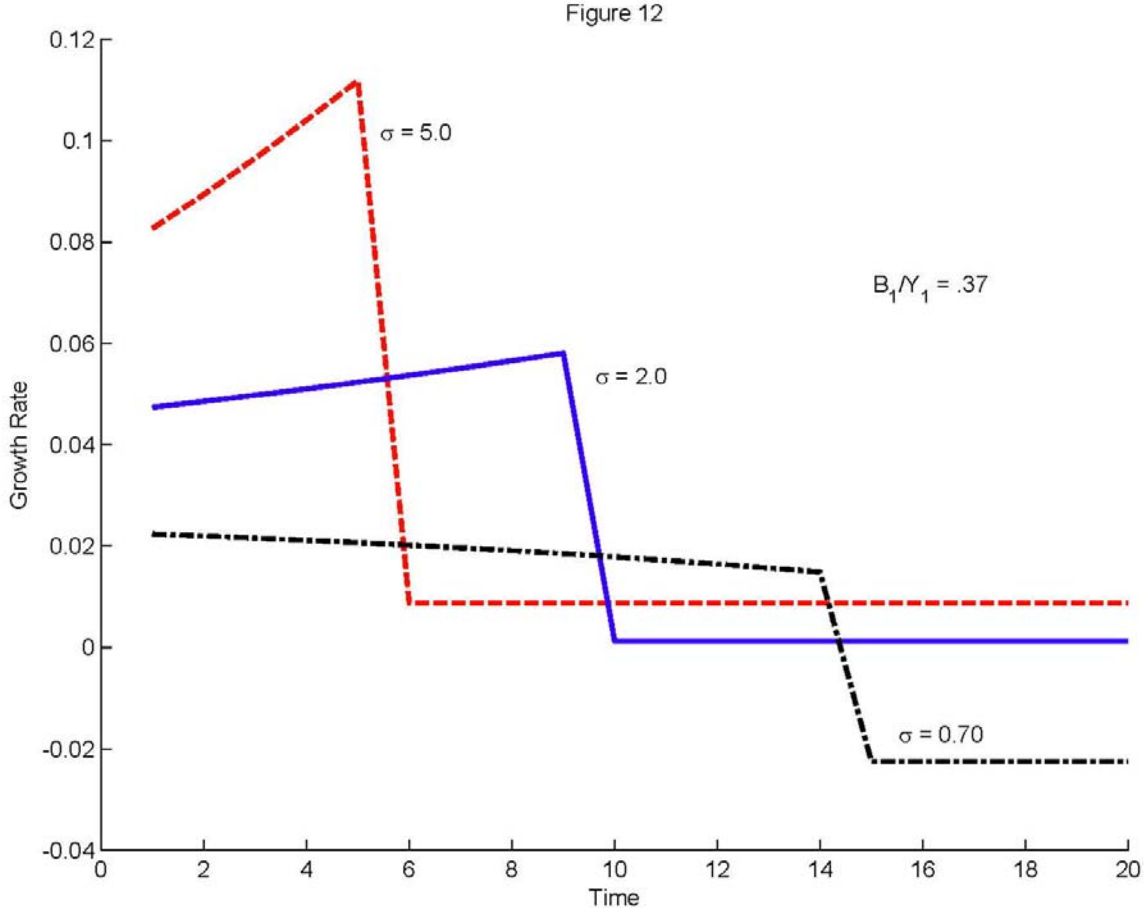

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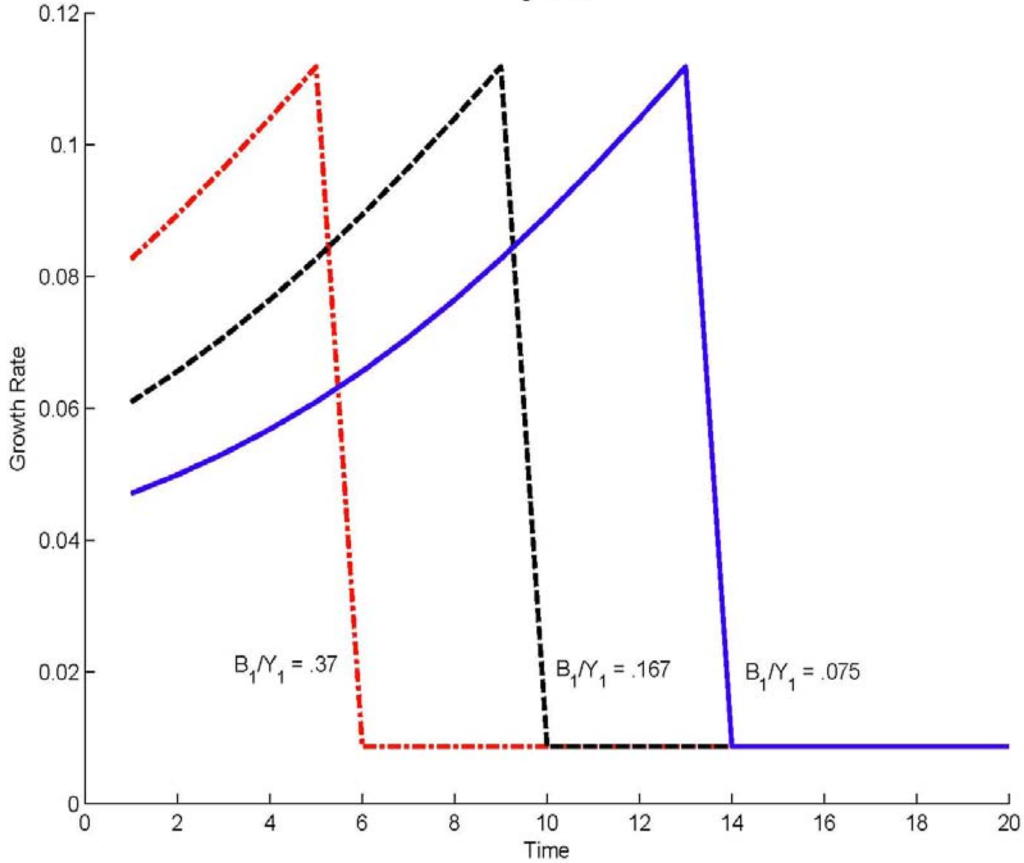


Figure 14

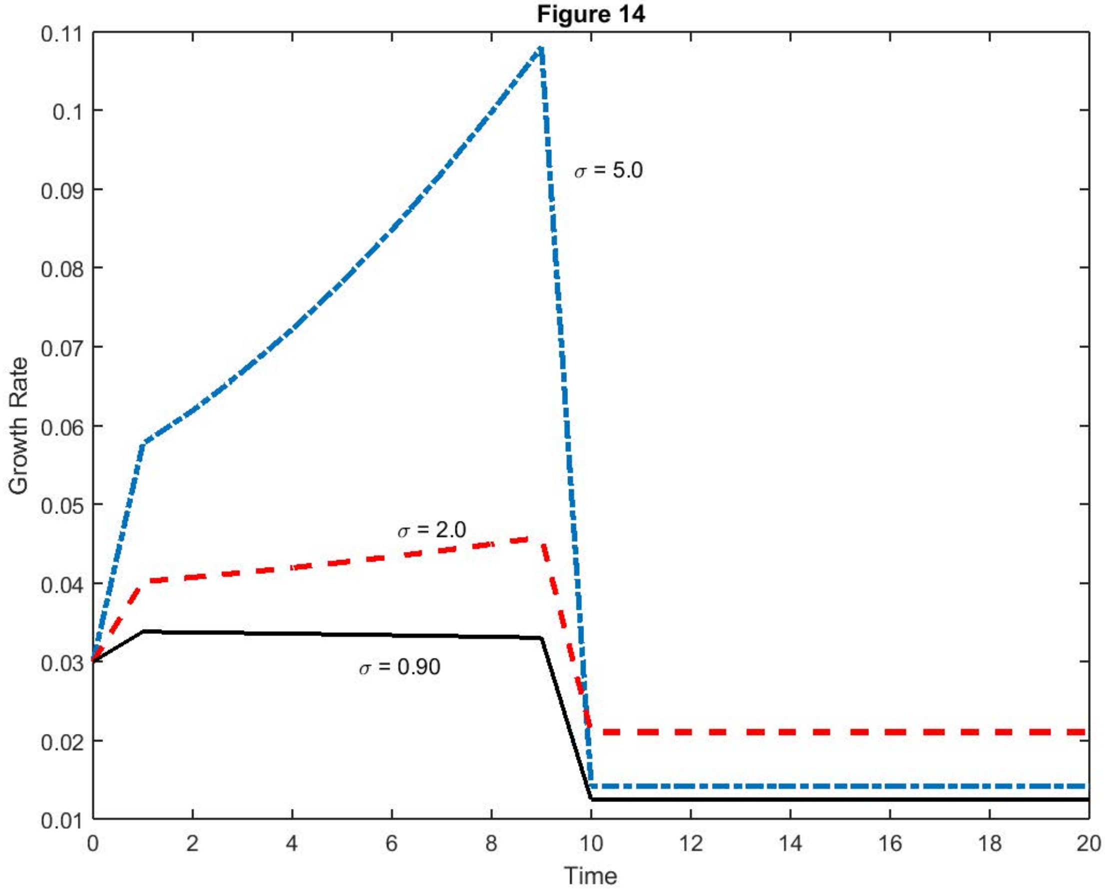


Figure 15

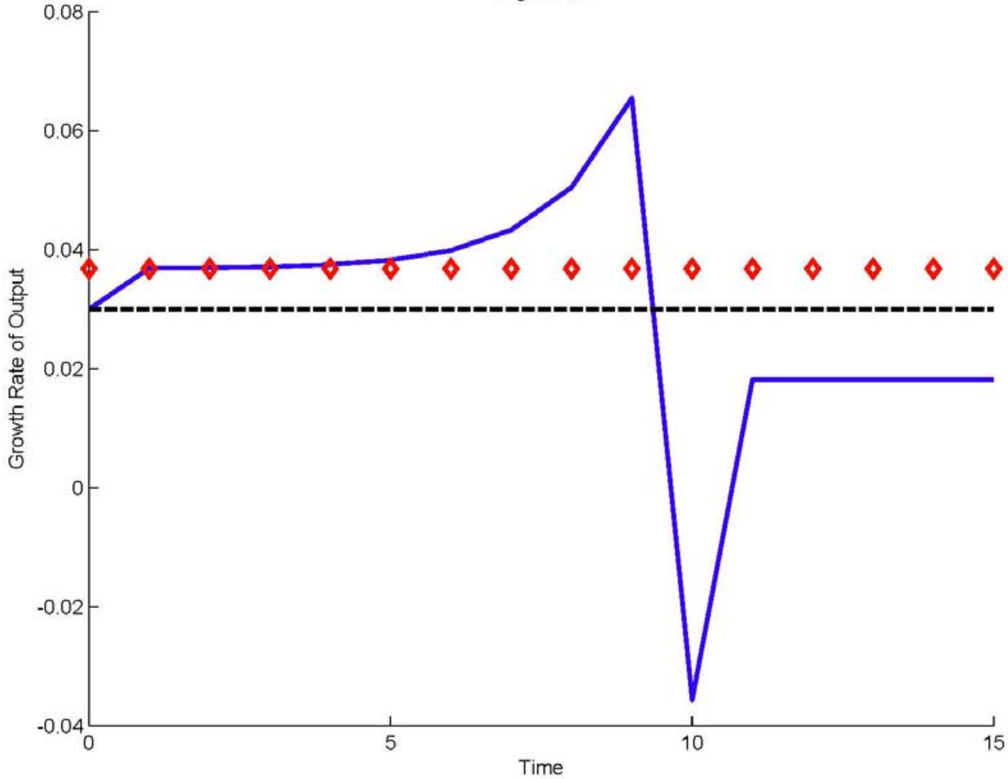
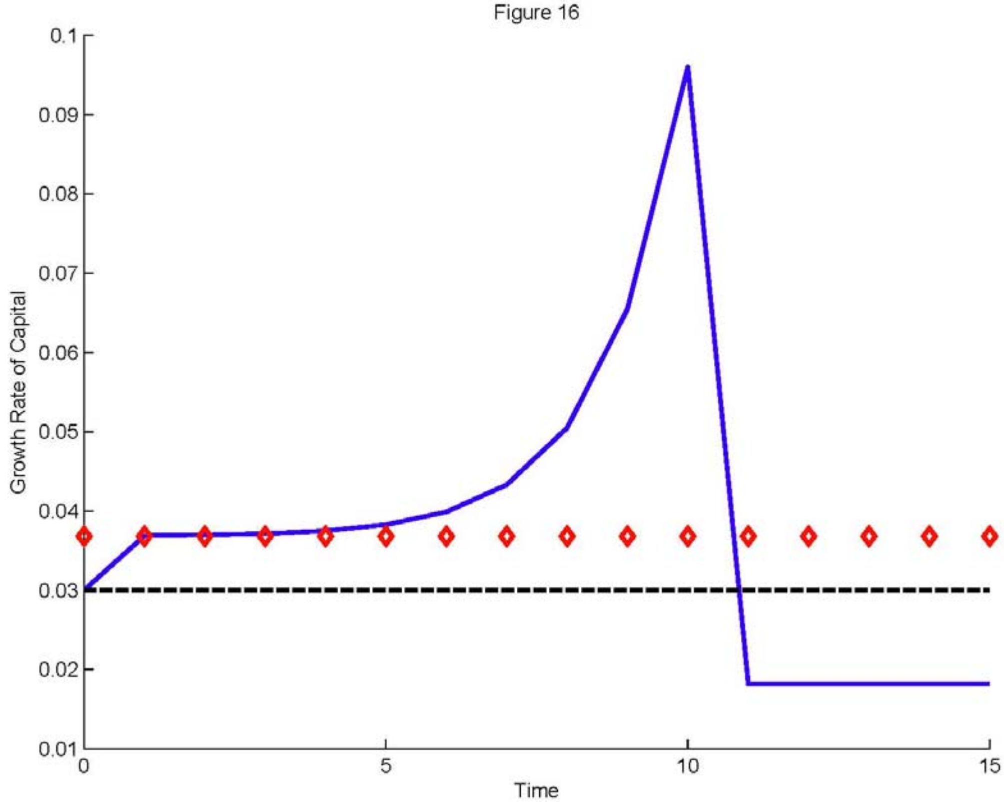


Figure 16



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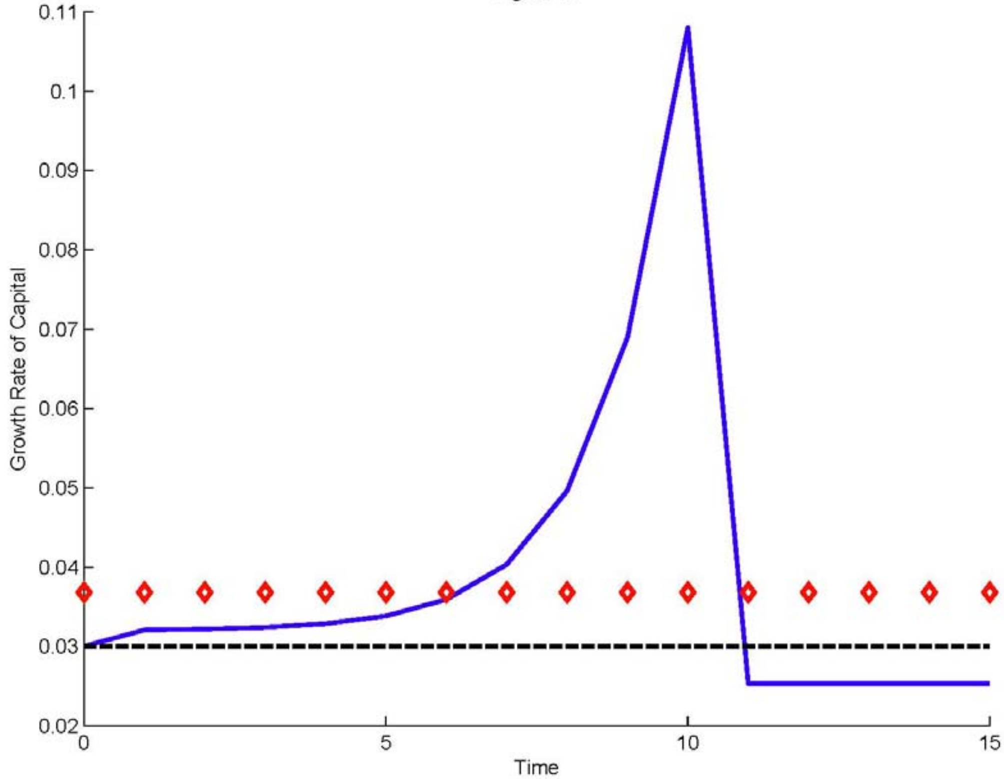

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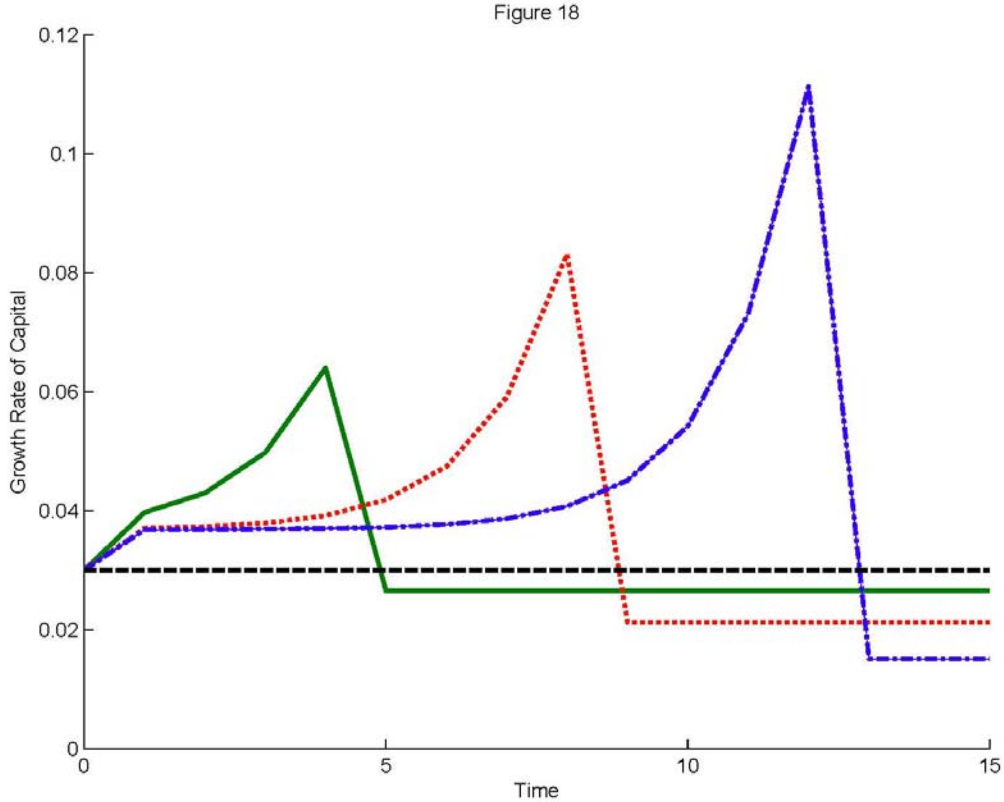

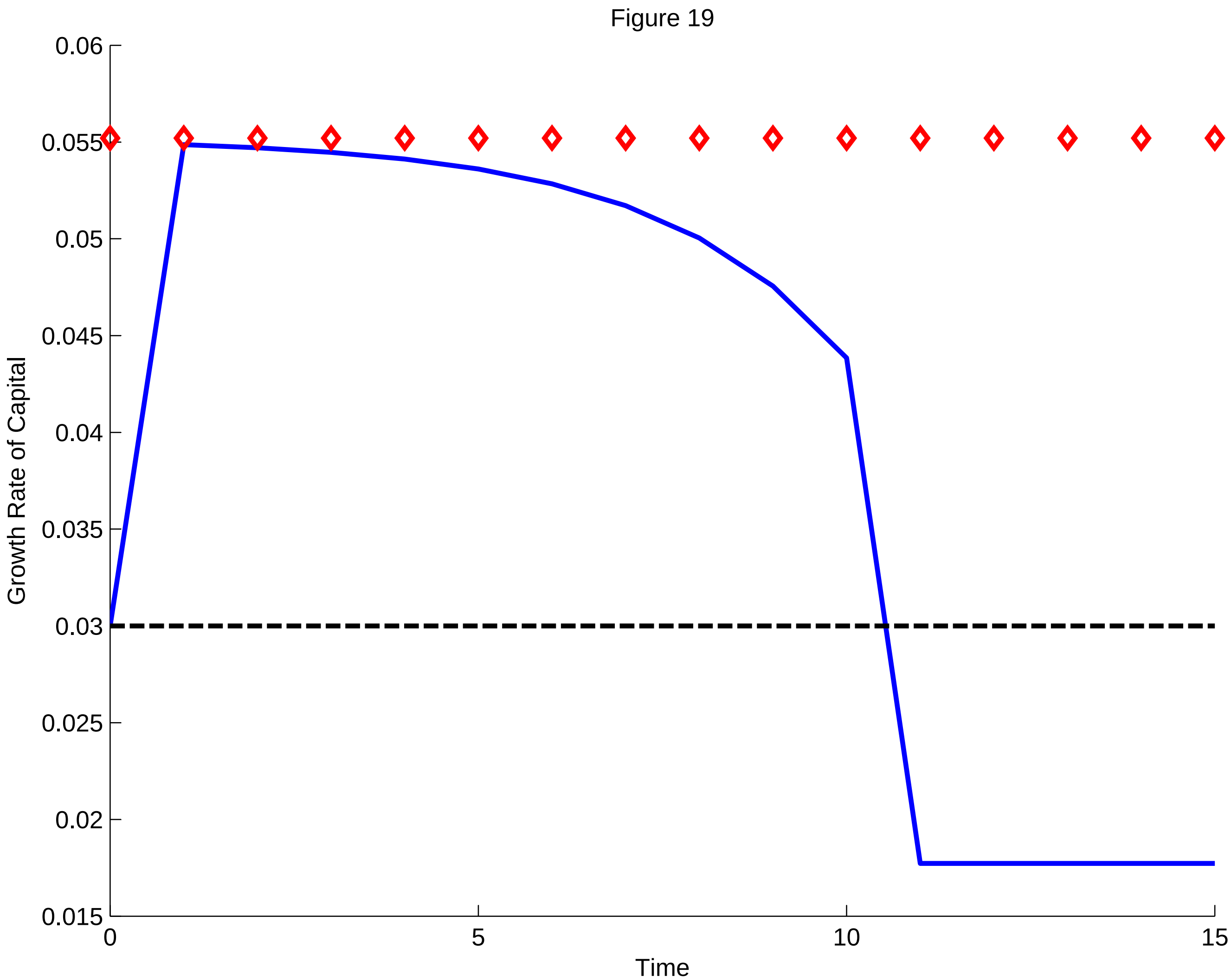
Figure 19
Growth Rate of Capital
Time
0.06
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0
5
10
15