On-the-Job-Search, Wage Dispersion and Trade Liberalization

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February 22, 2016

Abstract

This paper builds a dynamic, general equilibrium, open economy model which allows for \textit{ex-ante} homogeneous workers to experience widely different wage growth before and after trade liberalization. Key features of our model include heterogeneous firms, a frictional labor market, on-the-job search, and dynamic wage contracts. We calibrate our model to match salient features of the US economy. Our results indicate four key findings: (1) Our model structure endogenously generates firm growth and export dynamics, (2) Firm heterogeneity and on-the-job search are strong complements in generating wage dispersion, (3) Trade liberalization can induce substantial changes in the shape of the wage distribution, and (4) Trade liberalization will increase long-run residual wage dispersion in equilibrium.

Keywords: exports, wage dispersion, on-the-job search, heterogeneous firms

JEL Classification Numbers: F16, J3, J6

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1 Introduction

Free trade is often supported by the argument that, in the long-run, trade liberalization will induce the reallocation of resources towards more productive uses. This, in turn, has the potential to generate substantial welfare gains both at home and abroad. In contrast, much policy discussion focusses on the impact that trade has on the unequal distribution of returns from trade across workers. This distinction has led to a clear disconnect between policy makers, who stress the impact of trade liberalization on employment and wage dispersion, and trade economists, who generally tend to emphasize the role trade liberalization has on equilibrium prices and resource allocation across countries.

This paper contributes to a growing literature that attempts to fill this gap. We build a dynamic, general equilibrium, open economy model with a frictional labor market. The model in this paper has several important features that distinguish it from earlier studies in the literature. First, we assume that firms offer dynamic wage contracts to attract workers, and the workers direct their search to a particular job offer. Second, the firm’s output depends positively on the worker’s costly effort. Effort, in turn, is induced by the value of the contract.\(^1\) Third, we allow for on-the-job search of employed workers. These features enable us to investigate how trade liberalization affects endogenously-generated heterogeneity among ex-ante homogenous workers and resulting wage dispersion.

The key qualitative implication of this model is that firm heterogeneity, dynamic contracts, and worker mobility through on-the-job search act as complementary processes to generate endogenous heterogeneity among ex-ante homogeneous workers. As in the other models of on-the-job search, the optimal long-term wage contract exhibits an increasing wage-tenure profile. Some will match with productive firms and see wages increase with promotion or by matching with even more productive firms through on-the-job-search. Others will suffer sharp declines in their wages if their job is destroyed and, potentially, persistent unemployment in subsequent periods.

\(^1\)The intuition behind this structure follows that of the large body of efficiency wage models. See Davis and Harrigan (2011) or Amiti and Davis (2012) for examples of the application of efficiency wages to models of international trade. A theoretical basis in a search and matching model is provided by Tsuyuhara (2015).
months. This process endogenously creates wage dispersion; that is, wage variation among ex-ante homogenous workers.

An increasing wage profile also implies that firm output increases with the duration of the match through the endogenous increase in labour services. As in the other trade models with labour market frictions, more productive firms endogenously choose to export, offer higher wages and are characterized by lower turnover. The dynamic contracting environment in combination with endogenous firm dynamics implies that trade liberalization will affect both the set of exporters and nature of export entry and growth across heterogenous firms.

Lastly, related to the above implication, the dynamic contract and the implied firm dynamics also endogenously generate an exporter wage premium among the firms with identical productivity levels. In this sense, the model inherently implies positive correlation between firm output, exporting status and wages.

To illustrate these qualitative implications and to quantitatively evaluate the effects of trade liberalization, we calibrate our model to match salient features of the US economy. In particular, we fit the production side of our model to match key features of firm-heterogeneity and wage-setting behavior among US manufacturers. The model’s frictional labor market is likewise set to replicate observed features of the US labor market. The calibrated model shows that firm-heterogeneity, dynamic contracts and on-the-job-search are strong complements in generating larger wage dispersion among ex-ante homogenous workers. For instance, Hornstein et al. (2011) uses a “mean-min ratio,” the ratio of the average wage to the minimum wage, to evaluate the model’s capability to capture frictional wage dispersion in search models. Our model can generate a mean-min ratio of 2.63, which is significantly larger than that of previous studies and closer to its well-known empirical counterpart (1.98 in US Census data and 2.6 in our dataset).

Finally, using our quantitative model we investigate the impact of trade liberalization on equilibrium wage dispersion among US manufacturing workers. Our counterfactual exercise suggests that trade liberalization will lead to an increase in wage dispersion, though neither the sign or magnitude of the change in wage dispersion is directly predetermined by the model’s structure. Trade liberalization increases the profitability of entering export markets for any
given firm. While this induces previous non-exporters to start exporting, it also encourages firms to offer new higher paying jobs to prospective workers. As workers move through the wage distribution they achieve higher wages through promotion or on-the-job-search than what was previously possible, even if the underlying productivity of their firm does not change. In contrast, with more workers transitioning into high value wage contracts, we might expect to observe a concentration of workers into high wage contracts in equilibrium. While both effects are present in our framework, the calibrated model suggests that the former effect dominates the latter. Across the distribution of ex-post heterogenous workers we find that this effect causes the dispersion of wages to increase by at least 3.2 percent.

1.1 Related Literature

There is a rich and rapidly expanding literature studying the interaction of international trade and labor markets. Early, static models of Davidson et al. (1988) and Hosios (1990) examine the robustness of conventional trade theories in two-sector, small open economy models with search frictions. More recently, Helpman and Itskhole (2010) demonstrate that labor market flexibility can be a source of comparative advantage and that policy differences across countries can affect the pattern of trade. Using a directed search framework, King and Strähler (2014) similarly characterize how differences in endowments and technologies across countries affect equilibrium unemployment after trade liberalization.

A large macro-labour search literature studies wage dispersion and employment dynamics in a frictional labour market. Similar to work by Burdett and Coles (2003, 2010) and Shi (2009), our model endogenously generates wage dispersion among ex-ante homogeneous workers. Here, we augment these frameworks with two additional features in our context: firm-heterogeneity and costly-effort. First, Helpman et al. (2012) emphasize that firm-specific and firm-worker specific differences account for a large percentage of wage variation across firms and workers in an open economy. Our labour market directly captures this first element of wage variability in the sense that firms with higher productivity levels offer systematically higher wages than less productive firms. Second, conditional on firm-productivity, our model will still generate variation
in wages across workers due to the wage-tenure contracts and on-the-job search. Tsuyuhara (2015) integrates costly worker effort on the job and dynamic incentive contracts into the Menzio and Shi (2010) model of directed search. Incorporating this mechanism enables us to capture how dynamic wage contracts and worker mobility interact with endogenous dispersion in domestic revenues, export revenues and export participation in a environment without stochastic firm-level shocks.

Building on these two large bodies of literature, there is a series of recent papers that study the impact of trade in a frictional labour market. In a closely related paper, Felbermayr et al. (2014) studies a monopolistic competition model of trade with heterogeneous firms and labour search frictions. The labor market in their model is based on Kaas and Kircher (2013). Our framework is similar: monopolistic competition and trade with heterogeneous firms (as in Melitz, 2003) and a labor market characterized by directed search. A key difference in our case is that we allow for on-the-job-search and for the firms to offer dynamic wage contracts, instead of fixed-wage contracts. In addition, in our model, dynamic wage contracts generate endogenous productivity variation among ex-ante identical firms due to varying worker effort.

As documented by Moen (1997), directed search without firm heterogeneity generates a degenerate wage distribution and, as such, any dispersion in the wage distribution in Felbermayr et al. (2014) is entirely driven by the exogenous variation productivity levels across firms. In our model directed on-the-job-search endogenously creates wage dispersion even in the absence of firm heterogeneity. In that sense, we study the degree to which firm-heterogeneity, worker effort, and worker mobility through on-the-job search act as complementary processes to generate wage dispersion.

Similarly, Fajgelbaum (2013) studies a monopolistic competition model of trade with heterogeneous firms and a labor market with on-the-job-search. While our paper uses a directed search model to study the impact of trade liberalization and on-the-job search on wage dispersion, Fajgelbaum (2013), in contrast, uses an random search model to characterize the impact of trade and on-the-job-search on average income growth.

Unlike Felbermayr et al. (2014) and Fajgelbaum (2013), where firm growth is captured by
the number of employees, in our setting firm growth is modelled in terms of the output of a single worker. This feature arises due to the technical difficulty of combining dynamic contracts and multiple workers in our model. To characterize the optimal dynamic contract for a single worker firm, the appropriate state variable for the recursive contracting problem is the value of the contract that the firm promises to its worker. With multiple workers, however, we need to compute the accumulated value of contracts that the firm promises to its workers, which depends on the distribution of workers within each firm. We are not aware of an established method to tractably analyze the distribution of accumulated value in a dynamic wage contract setting. Characterizing the optimal dynamic contract in such a setting is beyond the scope of this study.²

Our paper also contributes to the literature which studies the adjustment to trade liberalization. Artuç et al. (2010) and Dix-Carneiro (2014) structurally estimate models of interindustry mobility costs for workers in the US and Brazil during trade liberalization. Both models feature competitive labor markets and find that across industry-labor mobility is extremely low. Kam-bourov (2009) and Ritter (2014) develop models of a frictional labor market where workers search for jobs in a directed fashion across islands of sectors or occupations. Likewise, Coşar (2013) develops a model where workers search for jobs across industries, in which their attachment to each industry is determined through endogenous industry-specific human capital-accumulation. In our model, we abstract from interindustry reallocation. As noted above, most empirical work suggests that few workers reallocate across industries in response to trade liberalization.³ Instead we focus here on the reallocation of workers across heterogeneous firms within a given industry.⁴ While much of the preceding work focuses on the labor market responses of a small open economy, here we study the impact of trade liberalization in a general equilibrium setting between two large countries.

Our paper is naturally related to work which studies firm dynamics and firm entry into export

²The assumptions of fixed-wage contracts and risk-neutral workers in Kaas and Kircher (2014) and thus Felbermayr et al. (2014) allows these authors to compute an accumulated wage bill, which is a more tractable state variable for the value of the firm.
³See Artuç et al. (2010), Dix-Carneiro (2014) and the references therein for an extended discussion.
⁴In this sense our work is closely related to the models of Helpman et al. (2011) and Coşar et al. (2011).
markets. As highlighted in the empirical work of Lopez (2009) and Lileeva and Trefler (2010), growth among new exporters often occurs prior to entry and in response to trade liberalization. Likewise, although we focus on a two country setting, the endogenous export dynamics are consistent with the slow growth of new exporters highlighted by Albornoz et al (2012), Ruhl and Willis (2015), and Rho and Rodrigue (2016).

The next section presents our model that integrates a well-defined product market structure with a search theoretic labor market model. Section 3 describes equilibrium conditions and characterizes the nature of our frictional labor market, dynamic wage contracts, firm growth, and endogenous export decisions. Section 4 describes our calibration procedure and the characterizes the calibrated model’s quantitative implications in the steady-state. Section 5 concludes.

2 Model Environment

We consider an economy with two identical countries: home and foreign. Time is discrete, continues forever, and is indexed by $t$. In each country, there is a continuum of firms and a continuum of infinitely lived workers. There are two markets: the product market and labor market. In the product market, a continuum of horizontally differentiated varieties of consumption goods are traded. A worker and a firm meet in a frictional labor market and create a job. All goods are internationally tradable, while labor service is not.

2.1 The Product Market

The real consumption index for the product market takes the constant elasticity of substitution form:

$$Q = \left( \int_{\omega \in \Omega} y(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}},$$

(1)

where $\omega$ indexes varieties and $\Omega$ is the set of available varieties. We assume $0 < \rho < 1$ so that these goods are substitutes, and the elasticity of substitution between any two goods is given
by \( \sigma = 1/(1 - \rho) > 0 \). The price index corresponding to \( Q \) is given by

\[
P = \left( \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}},
\]

(2)

where \( p(\omega) \) is the price of variety \( \omega \). We choose the aggregate good (at home) as the numéraire, and we normalize \( P = 1 \). Given this market structure, the demand for variety \( \omega \) is

\[
y(\omega) = A^\sigma p(\omega)^{-\sigma},
\]

(3)

where \( A \) is a demand shifter for the home economy. In equilibrium, after normalizing \( P = 1 \), \( A = E^{1-\rho} \) holds where \( E \) is the total expenditure on the varieties within the home economy.

In this monopolistically competitive product market each operating firm produces a different variety of product \( \omega \). Therefore, given the standard demand function (3), each firm’s equilibrium revenue from producing variety \( \omega \) is

\[
r(\omega) = p(\omega)y(\omega) = A y(\omega)^\rho.
\]

(4)

2.2 The Labour Market

The labor market is modeled in a similar way as in Tsuyuhara (2014). All workers and firms are \textit{ex-ante} homogenous. Workers are risk-averse and have a periodical utility function of consumption \( v(\cdot) \), which is strictly increasing, strictly concave, and twice continuously differentiable. We assume that workers cannot borrow and save, so their consumption equals wage \( w \) if employed or unemployment benefit \( b \) if unemployed. Employed workers exert effort on the job, and the disutility of effort is given by \( c(\cdot) \), which is strictly increasing, strictly convex, and twice continuously differentiable. Each worker maximizes the expected lifetime sum of utilities discounted at rate \( \beta \in (0, 1) \).

There is an unbounded number of potential firms entering into the market. Prior to entry,
all firms are identical. Each firm has an access to common production technology:

\[ y = ze, \]

where \( z \) is idiosyncratic productivity and \( e \) is effort exerted by the worker employed in the firm. Each firm maximizes the expected sum of profits discounted at the rate \( \beta \).

Entering firms create a vacancy and post a job offer at a flow cost \( k > 0 \). A job offer is fully described by the long term wage contract, which specifies wages that the worker receives in each period as a function of tenure on the job, namely the \textit{wage-tenure profile}. The contract also depends on the aggregate state of the economy as described below. We assume the offer is binding and no renegotiation occurs after a firm and a worker agree to create a job.

For a given wage-tenure profile, workers can calculate a discounted lifetime expected utility that the contract would deliver, taking into account the possibility of job destruction and the worker’s job-to-job transition. We call it \textit{the promised value}, or simply the \textit{value} of contract. Workers can calculate the value of contract not only at the beginning of tenure but also at any point of tenure in terms of the truncated section of the wage profile.

Using this notion of values of contract, the labor market is organized in a continuum of submarkets indexed by \( x \in X = [\bar{x}, \bar{x}] \), which denotes the value of contract offered in that submarket. That is, submarket \( x \) consists of all the firms that promise to deliver value \( x \), irrespective of the shapes of each individual wage profile. Then, following the traditional view of efficiency-wage models, we assume that a worker’s effort depends positively on the current value of the contract. That is, we assume an increasing and continuously differentiable function \( e : X \to \mathbb{R}_+ \) that determines the worker’s effort on the job.\(^5\)

Once a firm incurs the cost of entry and chooses which submarket to enter, it draws its idiosyncratic productivity \( z \) from a common distribution \( G(z) \). \( G(z) \) has positive support over \((0, \infty)\) and has a density \( g(z) \). The idiosyncratic productivity is constant throughout the duration of

\(^5\)Worker effort is usually modeled as a function of wages, instead of values of contract, in the standard efficiency wage models, such as Solow (1979) and Summers (1988). The existence of such a function is shown in Tsuyuhara (2014) with a more explicit incentive structure.
operation. Upon entry with a low productivity draw, a firm can choose to exit immediately before engaging in the matching process.

Both employed and unemployed workers get the opportunity to search for a job with probability $\lambda_e$ and $\lambda_u$, respectively. Observing these available vacancies, searching workers direct themselves to a particular submarket. In submarket $x$, the ratio between the number of vacancies and the number of searching workers is denoted by $\theta(x) \geq 0$, and we refer to it as the tightness of submarket $x$.

In each submarket, a worker and a firm meet through a frictional matching process, which is summarized by a job finding probability function, $p(\theta)$, and a vacancy filling probability function, $q(\theta)$. We assume that $p : \mathbb{R}_+ \to [0,1]$ is twice continuously differentiable, strictly increasing, and strictly concave, and satisfies $p(0) = 0$ and $p'(0) < \infty$, and that $q : \mathbb{R}_+ \to [0,1]$ is twice continuously differentiable, strictly decreasing, and strictly convex, and satisfies $\theta^{-1} p(\theta) = q(\theta)$ and $q(0) = 1$. In addition, the matching technology is assumed to satisfy the condition that $p(q^{-1}(\cdot))$ is concave.

In each period, an operating firm faces a constant probability $\delta \in (0,1)$ of a bad shock that destroys the match and forces the worker into unemployment. Finally, let $\Phi(x,z)$ be the distribution of workers who are employed at a job that gives the value of contract $X \leq x$ and that have an idiosyncratic productivity $Z \leq z$. Also, let $u$ be the measure of unemployed workers. The state of the labour market is denoted by $\psi_t \equiv (\Phi_t, u_t)$, and the evolution of the state is generically denoted by an operator $\Psi$ so that $\psi_{t+1} = \Psi(\psi_t)$.

2.3 The Worker’s Job Search Problem

A worker who gets the opportunity to search chooses which submarket to enter, taking into account the value of the job offer and the probability of actually finding a job in each submarket. The optimal job search decision depends on the reservation value, which is the promised value of current contract for employed workers, and the value of unemployment for unemployed workers. Consider a searching worker whose reservation value is $\hat{V}$. If he visits submarket $x$, he finds a vacancy and moves to a new job with probability $p(\theta(x, \hat{V}))$ or fails to find a job and stays
with the current job or stays unemployed with probability $1 - p(\theta(x, \psi))$. Therefore, his optimal job search decision maximizes $p(\theta(x, \psi))x + (1 - p(\theta(x, \psi)))\hat{V}$ with respect to $x$. We denote the maximized net expected value of job search given reservation value $\hat{V}$ by

$$D(\hat{V}, \psi) = \max_{x \in X} p(\theta(x, \psi))(x - \hat{V}).$$

The solution to this maximization problem is denoted by $m(\hat{V}, \psi)$, and we define $\bar{p}(\hat{V}, \psi) \equiv p(\theta(m(\hat{V}, \psi), \psi))$ as the probability that the worker finds a vacancy in the optimally chosen submarket given the current promised value $\hat{V}$. Given this optimal search decision, the searching worker’s gross expected value of search is $\hat{V} + D(\hat{V}, \psi)$, and the worker’s expected value for the next period given $\hat{V}$ is $\lambda_k(\hat{V} + D(\hat{V}, \psi)) + (1 - \lambda_k)\hat{V} = \hat{V} + \lambda_k D(\hat{V}, \psi)$ where $k = e, u$.

To define the value of unemployment, let $U(\psi)$ denote the value of unemployment given the state of the labor market $\psi$. In the current period, unemployed workers receive and consume unemployment benefit $b$. If an unemployed worker gets the opportunity to search for a job with probability $\lambda_u$, he solves the above problem with $U(\hat{\psi})$ as his reservation value, where $\hat{\psi}$ denotes the next period state of the labor market. Then, as described above, the expected value for an unemployed worker entering the next period is $U(\hat{\psi}) + \lambda_u D(U(\hat{\psi}), \hat{\psi})$. Therefore, if the unemployment benefit is constant over time, the value of unemployment $U(\psi)$ satisfies the following recursive equation:

$$U(\psi) = v(b) + \beta E_{\psi}(U(\hat{\psi}) + \lambda_u D(U(\hat{\psi}), \hat{\psi})), \quad (6)$$

where the expectation is taken with respect to the state of the labour market $\psi$.

### 2.4 The Firm’s Problem

The firm’s problem is divided into its entry and exit decision and dynamic contract design, including export decision. At each stage, each firm takes as given the aggregate state of the economy, that is, the state of the labor market $\psi$ and the state of the demand side of the
economy \{A_j\}_{d,x}. We denote the aggregate state variables by \( s \equiv (\psi, \{A_j\}_{d,x}) \), and its evolution is generically denoted by \( \Lambda \) so that \( s_{t+1} = \Lambda(s_t) \).\(^6\) It also takes the fixed trade cost \( c \) and the variable trade cost \( \tau \) parametrically as given.

Before explaining each problem, it is useful to describe how the firm revenue function is determined. Conditional on the amount of output \( y \), a firm chooses whether to serve only the domestic market or to engage in exporting as well. In order to export, a firm has to incur a fixed cost \( c > 0 \) and iceberg-type variable trade costs: \( \tau > 1 \) units of a variety must be exported for one unit to arrive in the foreign market. If the firm chooses to export, it allocates its output between the domestic and export market (\( y_d \) and \( y_f \), respectively) to equate its marginal revenues in the two markets. Following the argument in Helpman et al. (2011), the firm’s revenue as a function of its export decision \( \iota \) and its total production level (\( y = y_d + y_f \)) can be expressed as

\[
R(\iota, y, s) = A_d \left[ 1 + \iota(\Upsilon_x - 1) \right]^{1-\rho} y^\rho,
\]

where \( \iota \) equals 1 if the firm exports and 0 otherwise. The variable \( \Upsilon_x \) captures a firm’s “market access,” which depends on the demand shifters of both domestic and export markets:

\[
\Upsilon_x = 1 + \tau^{-\frac{\rho}{1-\rho}} \left( \frac{A_x}{A_d} \right)^{\frac{1}{1-\rho}} \geq 1.
\]

Exporter revenue is decreasing in variable trade costs, \( \tau \), and increasing in the ratio of the foreign demand shifter to the domestic demand shifter, \( \frac{A_x}{A_d} \).

### 2.4.1 Entry and Exit

We now describe an incumbent firm’s exit decision. Even though incumbent firms will not voluntarily choose to exit in a stationary environment, an aggregate state shock, such as trade liberalization, may decrease the value of operation sufficiently so that the firm wishes to exit. To admit this possibility, we allow the firm to exit after an adverse aggregate shock or a policy change.

\(^6\)By construction, \( \Lambda \) includes \( \Psi \), the evolution of worker distribution.
To describe the firm’s exit decision, let \( J(x, z, s) \) be the value of an operative firm, which is the maximal value of the firm’s dynamic contracting problem described below. Given this value function, an incumbent firm chooses to stay operative or exit depending on the aggregate state of the economy. By the assumption of competitive entry, the firm’s outside option in any period is zero. Therefore, if \( J(x, z, s) \) becomes negative for a given state, the firm optimally chooses to exit. If we denote the value of this decision problem by \( I(x, z, s) \), it is defined by

\[
I(x, z, s) = \max\{0, J(x, z, s)\}.
\]

An entering firm draws its productivity only after entering into a submarket. Therefore, the entering firm may also choose to exit before engaging in the matching process if its realized productivity is too low relative to the value of contract that the firm wishes to offer. Following the above argument, the firm decides to produce if the productivity draw yields a positive firm value of operation. Then, using the above notation, we can define a firm’s expected value from entry into a submarket \( x \) conditional on meeting a worker as

\[
E_z I(x, z, s) = \int_0^\infty I(x, z, s)g(z)dz,
\]

where the expectation is taken with respect to the firm productivity \( z \).

### 2.4.2 Dynamic contracts

The incumbent firm’s problem is to design the dynamic wage contract. Because an incumbent firm’s optimal export decision depends on its output level, which is determined by its wage offer, its dynamic contract design jointly determines the export status as well.

Following Tsuyuhara (2015), we characterize the optimal dynamic contract recursively. The state of the problem is the current promised value \( V \), the firm’s idiosyncratic productivity \( z \), and the aggregate state variable \( s \). Given these state variables, the firm’s problem is to choose the current period wage \( w \), the promised utility \( \hat{V}(\hat{s}) \) that the contract delivers to the worker
at the beginning of the next period, and its export decision $\iota$. Let $\xi = \{w, \hat{V}(\hat{s}), \iota\}$. Then, the firm’s maximized value function $J(V, z, s)$ satisfies the functional equation:

$$J(V, z, s) = \max_{\xi} R(\iota, y(z, V), s) - w - \iota c + \beta(1 - \delta(\hat{s}))E_{\hat{s}}(1 - \lambda e \hat{\rho}(\hat{V}(\hat{s}), \hat{\psi}))I(\hat{V}(\hat{s}), z, \hat{s}),$$  

subject to

$$V = v(w) - c(e(V)) + \beta E_{\hat{s}}(\delta(\hat{s})U(\hat{s}) + (1 - \delta(\hat{s}))(\hat{V}(\hat{s}) + \lambda e D(\hat{V}(\hat{s}), \hat{s}))),$$

$$\delta(\hat{s}) = \{\delta \text{ if } J(\hat{V}(\hat{s}), z, \hat{s}) \geq 0, \ 1 \text{ otherwise}\},$$

$$\hat{V}(\hat{s}) \in X.$$ (11)

It is important to note that the firm’s current output and thus revenue is determined by the current state $V$ and the firm cannot change it concurrently. The firm’s contract design, however, affects its future revenue through the future value of the state $\hat{V}$. The choice of $\hat{V}$ also affects how likely the worker stays in the next period, through $\hat{\rho}$, which in turn affects the firm’s expected continuation value. Given the competitive entry of potential firms into the labor market, the firm’s outside option after separation is zero.

The firm’s choices need to be consistent with its promised value $V$. That is, for a given choice of $\xi$, the worker evaluates its value according to the right hand side of equation (9). Then, the first constraint requires that the choice of $\xi$ indeed delivers the current promised value $V$ to the worker. The second constraint (10) requires that the job destruction probability, aside from separation due to the worker’s on-the-job search, needs to be consistent with the exogenous job destruction probability $\delta$ and the firm’s exit decision. The last constraint (11) is a technical requirement that the promised value is chosen from a compact domain of the entire labour market.
3 Equilibrium

Firms enter the labor market competitively. In submarket $x$, a firm fills a vacancy with probability $q(\theta(x, \psi))$. If $q(\theta(x, \psi)) \mathbb{E}_z I(x, z, s)$ is strictly less than the cost of creating vacancy $k$, no firm enters submarket $x$. If the product $q(\theta(x, \psi)) \mathbb{E}_z I(x, z, s)$ is strictly greater than $k$, infinitely many firms enter submarket $x$, which decreases probability of finding a searching worker in that submarket and thus decreases the expected value. Therefore, in any submarket that is visited by a positive number of workers, the market tightness $\theta(x, \psi)$ is consistent with the firm’s optimal job creation strategy if and only if

$$q(\theta(x, \psi)) \mathbb{E}_z I(x, z, s) - k \leq 0,$$

and $\theta(x, \psi) \geq 0$, with complementary slackness.

In this economy, because of monopolistic competition and the fixed cost of entry into the frictional labor market, each incumbent firm earns positive ex-post profit in equilibrium. We assume that all individuals in the economy hold the same portfolio of shares of firms. That implies that the ex-post profits are aggregated across all firms and are redistributed to individuals. Hence, since individuals cannot save, and home and foreign economies are symmetric, balanced trade implies that aggregate revenue must equal aggregate expenditure in equilibrium. Given the distribution of jobs $\Phi(x, z)$, aggregate revenue is computed by $\int R(\iota, y(z, x), s) d\Phi(x, z)$. Then, because $A = E^{1-\rho}$, the demand shifter for home country is given by

$$A = \left(\int R(\iota, y(z, x), s) d\Phi(x, z)\right)^{1-\rho}.$$  \hspace{1cm} (13)

3.1 Definition and solution algorithm

**Definition 1.** A recursive equilibrium is a set of functions $\{I^*, J^*, \theta^*, D^*, m^*, U^*, \xi^*\}$ and transition operators $\Psi^*$ and $\Lambda^*$ for the aggregate variables such that

1. the value of job search $D^*$ and the optimal search policy $m^*$ satisfy equation (5),
2. the value of unemployment $U^*$ satisfies equation (6),
3. the value of incumbent firm $I^*$ satisfies equation (7),
4. the value of operating firm $J^*$ and an optimal contract policy $\xi^*$ satisfy equation (8),
5. the market tightness $\theta^*$ satisfies condition (12), and
6. the aggregate value of $A$ is implied by the aggregate of the individual decisions (13).
7. $\Psi^*$ and $\Lambda^*$ are derived from the policy functions $\xi^*$ and $m^*$ and the probability distribution for $z$.

In general, search models with aggregate state variables are difficult to compute even at the steady state. For a random search model, the distribution of workers across different employment values is an infinite-dimensional state variable for the firm and worker problems. Therefore, an algorithm for computing the steady state of the model becomes a nested fixed-point problem: the outer fixed-point problem to find a steady state aggregate variable, and the inner problem to find a steady state distribution of workers.

The present model substantially simplifies the process for computing the steady state. In this model, because search is directed, a worker’s optimal search depends on the tightness of each submarket but does not depend on the entire distribution of workers. Therefore, a firm’s optimal contracting problem also does not depend on the distribution of workers. It depends on the aggregate state variable $A$ only through its effect on the revenue function. Hence, we can compute the labor market equilibrium for each value of the aggregate variable without searching for the steady state distribution of workers as a fixed-point. Instead, we compute the transition operators $\Psi$ and $\Lambda$ and the resulting stationary distribution as an equilibrium object using the derived optimal policy functions. We then use the computed distribution to update the initial guess of the aggregate variable.

To be precise, we first solve for the labor market equilibrium, namely the firm’s optimal contracting problem together with its entry and exit decisions. The labor market and the product market are linked only through the aggregation condition (13), which determines the firm’s revenue. Since each firm takes $A$ and $\tau$ as given when making its decisions, we treat them parametrically in the first step. If we treat $A$ and $\tau$ as parameters, the labor market
admits the block recursive structure; that is, equilibrium functions do not (directly) depend on the distribution of workers.\footnote{As our main objective is quantitative, we do not prove the existence in this paper. See Menzio and Shi (2010) for details.} After computing the equilibrium functions for a given $A$ (and $\tau$), we can calculate the stationary distribution of workers across employment states as well as the productivity of each job. We use this distribution to compute the updated $A$ from (13). Then, the second step computes the fixed point of this process to find a consistent stationary equilibrium.

### 3.2 Qualitative properties of the firm’s decisions

In this section, we describe key qualitative properties that are useful for describing the model’s behavior. Following the above discussion about the solution algorithm, we drop $z$ from the value function of the firm, and focus on $J(x, z)$ that is independent of the aggregate state variables at the steady state. The qualitative properties described below rely on some technical regularity conditions that we assume for illustrative purposes.\footnote{Proving these properties, though possible in principle, goes beyond the purpose of our paper. See Menzio and Shi (2010) for technical details that provide grounds for these assumptions. Our quantitative exercise, described in the next section, always satisfies these assumptions.}

**Assumption 1.**

1. For all $x \in X$, $J(x, z)$ is increasing in $z$,
2. For all $z \in Z$, $J(x, z)$ is strictly decreasing and concave in $x$.

Intuitively, the value function $J$ is an outcome of optimal profit sharing between the firm and the worker. As higher productivity generates larger output for a given level of work effort, it enables the firm to achieve a higher firm value. Also, at the optimum increasing the value promised to the worker must lower the value left for the firm.\footnote{When $J$ is not concave, the firm can improve its value by randomizing the contracts as discussed in Hopenhayn and Nicolini (1997), which makes the resulting value function concave. Our numerical solutions always generate globally concave value functions without lotteries. Therefore, for expositional clarity, we avoid using the lottery when expressing the firm’s objective function.}
3.2.1 Zero-profit productivity cutoffs for operation

First, Assumptions 1-2 imply that \( E_z I(x, z) \) is decreasing in \( x \). Therefore, there exists a unique \( \tilde{x} \in \mathbb{R} \) such that \( E_z I(x, z) < k \) for any \( x > \tilde{x} \). In equilibrium, no firm enters a submarket with \( x > \tilde{x} \) from the equilibrium condition (12).

Then, conditional on entering submarket \( x < \tilde{x} \), a new entrant needs to decide whether to engage in the hiring process conditional on its productivity draw. It would immediately exit if its value of operation is negative with certain productivity levels. Because \( J(x, z) \) is increasing in \( z \), there exists a cutoff productivity for each \( x \), \( \tilde{z}(x) \) such that \( J(x, z) < 0 \) for any \( z < \tilde{z}(x) \). In each submarket \( x \), if an entering firm draws productivity below \( \tilde{z}(x) \), it immediately exits without hiring a worker. In addition, because \( J(x, z) \) is strictly decreasing in \( x \), it is immediate to show that \( \tilde{z}(x) \) is strictly increasing in \( x \). Therefore, only high productivity entrants can maintain a vacancy in a submarket with higher \( x \). The shaded area of Figure 1 is a set of value-productivity combinations that yield a positive value \( J(x, z) > 0 \), and the firms are operative in this area.

3.2.2 Dynamic contracts

We denote the optimal policy functions associated with \( J^* \) by \( \xi^* = \{\hat{\iota}^*(V, z), w^*(V, z), \hat{\hat{V}}^*(V, z)\} \) as a function of the current state \( (V, z) \). The optimal dynamic contract has the following property.

**Proposition 1.** Under the optimal contract, for any productivity level of the firm, the promised value weakly increases with tenure of the match, i.e., \( \hat{\hat{V}}^*(V, z) \geq V \) for all \( V \in X \) and for all \( z \in Z \). In addition, the optimal policy \( \hat{\hat{V}}^*(V, z) \) is increasing in \( V \); that is, \( \hat{\hat{V}}^*(V_1, z) \geq \hat{\hat{V}}^*(V_2, z) \) for any \( V_1 \geq V_2 \) and for any \( z \).

The proof is in the appendix. This is a key result of labor search models with dynamic contracting and on-the-job search.\(^{10}\)

An immediate corollary to this result is that a firm backloads wages as follows.

\(^{10}\)Shi (2009) proves this result for a model with directed on-the-job search. Tsuyuhara (2015) provides the proof for a model with moral hazard contracting problem.
Proposition 2. Under the optimal contract, for any productivity level of the firm, the wage weakly increases with tenure of the match, i.e., \( w^*(\hat{V}(V,z), z) \geq w^*(V,z) \) for all \( V \in X \) and for all \( z \in z \).

See Tsuyuhara (2015) for the proof. In each period the firm knows that the worker may have the opportunity to search for a new job. If the worker successfully finds a new job, the likelihood of losing the worker depends entirely on whether the future value of that same wage contract is greater at the current firm or the new job. As such, firms will optimally dissuade workers from leaving by offering future wage increases. In addition, by promising increasing value and wages, the firm can induce increasing effort by the worker. These two reasons induce the firm to an offer with increasing values.

Proposition 1 also implies an important property for firm dynamics. A worker’s effort is induced by the current promised value of contract. Therefore, the firm’s choice of promised value implicitly determines its output for the next period. Since the firm optimally chooses a higher promised value next period, its next period output must also be higher.

Proposition 3. Under the optimal contract, a worker’s effort, and thus a firm’s output weakly increases with the tenure of the match.

Hence, the firm optimal contract implicitly determines how its output grows over time conditional on the match continuing. The primary drivers of firm dynamics in this context are the workers’ incentives and the corresponding dynamic contract. This is a novel mechanism for generating heterogeneous productivity among firms, and it contrasts strongly to the standard mechanism that is characterized by any \textit{ex-ante} differences in worker skill as in Helpman et al. (2011).
3.2.3 Export decision

Finally, in each period, for a given value-productivity pair \((x, z)\), the firm chooses \(\iota^*(x, z) = 1\) if and only if the following condition holds:

\[
R(1, y(z, x), z) - c \geq R(0, y(z, x), z) \\
\iff R(1, y(z, x), z) - R(0, y(z, x), z) \geq c \\
\iff \left[ \Upsilon_x^{1-p} - 1 \right] A_d(ze(x))^p \geq c. \tag{14}
\]

It is clear that the left hand side monotonically increases in both \(z\) and \(x\). Therefore, we can characterize the exporting productivity cutoffs \(z_{ex}(x)\), such that, for each \(x \in X\), firms with productivity \(z\) greater than \(z_{ex}(x)\) export. Moreover, it is immediate to show that \(z_{ex}(x)\) is a decreasing function in \(x\). The shaded area of Figure 2 is a set of value-productivity pairs where the firm optimally chooses to export, conditional on operation.

3.2.4 Firm dynamics

The characterization of the firm’s dynamic contracting and productivity cutoffs generates novel implications for firm dynamics in our setting. First, we give an alternative characterization of zero-profit cutoffs; namely, zero-profit value cutoffs. For a given \(z\), \(J(x, z)\) is strictly decreasing in \(x\). Therefore, there exists \(\bar{x}(z)\) such that \(J(x, z) \leq 0\) for any \(x \geq \bar{x}(z)\). The optimal promised value of contract increases with tenure of the match, but it can be shown that the optimal contract in this model does not prescribe a value profile that yields a negative value for the firm.\(^{11}\) Then, we redefine \(\bar{x}(z) = \min\{\{x : J(x, z) = 0\}, \bar{x}\}\) as the upper bound of the value that a firm promises to deliver for a given productivity level \(z\). A firm with productivity \(z\) increases its promised value of contract up to \(\bar{x}(z)\), and once the value reaches to it, the firm keeps the constant wage until the match breaks up.

Similar to the zero-profit cutoffs, we can alternatively define the exporting value cutoffs for each level of productivity. For exporting to be profitable, either \(x\) and/or \(z\) need to be sufficiently

\(^{11}\)There is no present-period gain by committing to a wage profile that yields negative value in the future.
large. Even though $z$ is fixed once a firm starts producing, the optimal dynamic contract implies that the firm raises $x$ over time, and the resulting production level increases. Therefore, for each $z$, there is a threshold $x_{ex}(z)$ such that if the firm’s contract delivers a value higher than this level, the firm starts exporting. Figure 3 and 4 respectively depict the zero-profit value cutoffs and exporting value cutoffs as a function of $z$.

Based on these cutoffs and increasing contract values, firms entering the same submarket will exhibit different dynamics depending on the realized productivity level at the point of entry. Figure 5 illustrates some examples. Suppose a firm enters submarket $x$. if the realized productivity is $z_0$, the value-productivity pair does not meet the zero-profit cutoff, and it immediately exits. If the realized productivity is $z_1$, the firm will start producing once it finds a worker. However, $z_1$ is still too small to make exporting profitable even if it raises the value of contract to the highest possible level ($\bar{x}(z_1) < x_{ex}(z_1)$). If the realized productivity is $z_2$, then the firm initially serves only domestic market. As its value increases and exceeds $x_{ex}(z_2)$, it starts exporting. Finally, if the realized productivity is $z_3$, the firm exports as well as serves the domestic market from its inception.

4 Quantitative Application: Trade Liberalization

This section conducts a quantitative exercise to illustrate the impact of trade liberalization on wage dispersion in a general equilibrium setting. A rich literature explores the impact of trade of wage differences across skilled and unskilled workers, rather than across homogeneous workers. For example, Helpman et al (2011), Helpman et al (2012), Coşar (2013), and Ritter (2014, 2015), among others, characterize the impact of trade on the difference in wages across workers with differing skills, human capital or occupational expertise. A smaller literature studies the impact of trade liberalization on residual wage dispersion. Felbermayr et al. (2014) studies a framework similar to ours; however, in that case there is no role for on-the-job-search. Our work highlights the interplay of firm heterogeneity and on-the-job-search on wage dispersion and firm dynamics. In this sense our work is also closely related to Fajgelbaum (2013) which studies
a model of trade with heterogeneous firms allowing for on-the-job-search. However, our paper uses a directed search model to study the impact of trade on wage dispersion, while Fajgelbaum (2013) uses an undirected search model to characterize the impact of trade and on-the-job-search on income growth.

4.1 Model Calibration

For our quantitative exercise, we will treat the global economy as if it is composed of two symmetric countries: the US and the rest of the world (ROW). We restrict attention to the manufacturing sector and continue to assume that workers do not search for work outside of this industry. While this is an admittedly strong assumption it is consistent with the empirical regularity that workers change industries with very low frequency in response to trade liberalization.12

We first calibrate our model to match US employment, wage, production and trade data. Given the calibrated model we describe the steady state distribution of wages, employment, production and exporting across ex-post heterogeneous firms and workers. We consider the implications of an unexpected trade liberalization, parameterized by a reduction in iceberg transport costs, on wage dispersion and characterize the role of on-the-job-search and it’s interaction with firm-heterogeneity.

4.1.1 Functional Forms

There are several equations for which we will need to impose functional forms to compute the model’s steady-state. The functional forms we choose are collected in Table 1, while our empirical targets and parameter values are documented in Tables 2 and 3, respectively.

We begin by defining the flow utility that a consumer receives, \( v(w) - c(e) \). We assume that we can describe the utility a consumer receives from current consumption by a standard, CRRA utility function \( v(w) = \frac{w^{1-\eta}}{1-\eta} \), where \( \eta \) is the CRRA coefficient. Since effort is an increasing and continuously differentiable function of the current value of the wage contract, \( V \), we write the

effort function as \( e(V) = V^\nu \) for \( \nu > 0 \). The effort exerted by the worker is translated into a utility flow through the worker’s cost of effort function which is modeled as an increasing convex function \( c(e) = \gamma e^2 \) where \( \gamma > 0 \).

<table>
<thead>
<tr>
<th>Description</th>
<th>Functional Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow utility</td>
<td>( v(w) = \frac{w^{1-\eta}}{1-\eta} )</td>
</tr>
<tr>
<td>Effort function</td>
<td>( e(V) = V^\nu )</td>
</tr>
<tr>
<td>Cost of effort</td>
<td>( c(e) = \gamma e^2 )</td>
</tr>
<tr>
<td>Productivity distribution</td>
<td>( G(z) = 1 - (z/\bar{z})^\xi )</td>
</tr>
<tr>
<td>Production function</td>
<td>( y = ze )</td>
</tr>
<tr>
<td>Matching technology</td>
<td>( p(\theta) = \theta (1 + \theta^\phi)^{-1/\phi} )</td>
</tr>
</tbody>
</table>

Notes: Table 1 documents the specific functional forms of the quantitative model equations.

Total firm production is determined by firm productivity, the amount effort induced in the current period from the worker’s contract, and the shape of the production function. We assume that productivity is drawn from a Pareto distribution with shape parameter \( \xi \), \( G(z) = 1 - (z/\bar{z})^\xi \), where \( \bar{z} \) is the lowest possible productivity draw. The production function is specified, as in the theoretical model, as a multiplicative combination of firm productivity and worker effort, \( y = ze \).

Last, we need to specify the matching technology to characterize labor market search. We again chose a standard functional form for the matching technology, \( p(\theta) = \theta (1 + \theta^\phi)^{-1/\phi} \), where \( \theta \) captures market tightness. Given these functional form choices we next describe the empirical values used to pin down model parameters.

### 4.1.2 Parameter Calibration

We appeal to two separate data sources to help pin down the 13 parameters needed to compute the model’s equilibrium. We first turn to the Consumer Population Survey (CPS) to characterize the degree of residual wage distribution among US manufacturing workers. Following Lemieux (2006) we retrieve data from the May CPS and compute measures residual wage dispersion.\(^{13}\)

---

\(^{13}\) We choose to focus on 2002 since this is the same year for which there is well documented production and export statistics. We also consider a longer time period, 2001-2004, but this had virtually no impact on the target
A key difference for our purposes is that we only study the degree of residual wage dispersion among manufacturing workers. Specifically, we consider a regression of a manufacturing worker’s log hourly wage on a host of observable worker characteristics, including age, education, sex and race.\textsuperscript{14} Our measures of residual wage dispersion are based on the residuals obtained from these regressions.\textsuperscript{15}

Similarly, we also use the CPS to compute manufacturing employment transitions in the same year. We assume the length of a period is a quarter and, following the process described in Shimer (2012), we compute the fraction of manufacturing workers who transition from employment to unemployment, from unemployment to employment and from employment at one job to employment at a new job.

To characterize the production side of the economy we appeal to a series of well-established empirical benchmarks from the 2002 Annual Survey of US manufacturers as documented in Bernard et al., (2007) and Bernard, Redding and Schott (2007). In particular, we use moments which characterize the dispersion of manufacturing revenues, the prevalence and intensity of exporting, and empirical wage differences across workers at exporting and non-exporting firms.

Two model parameters can be set to exactly fit their counterpart in the data. First, the iceberg transport cost, $\tau$, is set to 1.12 to match the finding that 14 percent of total sales originate from exports among US exporters (Bernard et al., 2007). Second, we compute that the quarterly transition rate from employment to unemployment among manufacturing workers is 3 percent and set exogenous separation rate, $\delta$, to fit this target moment.

Four more parameters not identified by our empirical targets and are fixed by assuming that they take on the same value as those commonly chosen in the literature. In particular, we assume\textsuperscript{16} that the CRRA coefficient ($\eta$) takes a value of 2, the matching technology parameter moments. We follow Lemieux (2006) in our manipulation of the data with the exception that we only focus on manufacturing workers. See the Appendix for details.

\textsuperscript{14} We have alternatively considered specifications which only use white male workers, but this had little effect on our measures of residual wage dispersion.

\textsuperscript{15} Details are provided in the Supplemental Appendix.

\textsuperscript{16} The elasticity of substitution is set to 3.8 as in Bernard, Redding and Schott (2007) and empirically consistent with the estimates in Simonovska and Waugh (2014). The matching technology parameter implies that the elasticity of substitution between vacancies and applicants is 2/3, and the discount factor is chosen so that the annual interest rate is 5 percent.
(η) is set to 0.5, the elasticity of substitution (σ) across varieties is equal to 3.8, and the quarterly discount factor (β) is fixed to 0.988. Three more parameters are set to 1: \( \lambda_u \), the probability of searching for employment when unemployed, \( \gamma \), the cost of effort parameter, and the lowest possible productivity draw, \( z \).

**Table 2: Calibration Targets**

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data Target</th>
<th>OTJS</th>
<th>No OTJS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean-Min Wage Ratio</td>
<td>2.60</td>
<td>2.63</td>
<td>1.85</td>
</tr>
<tr>
<td>Unemployment to Employment Transition Rate</td>
<td>0.87</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>Employment to Employment Transition Rate</td>
<td>0.08</td>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>Unemployment Insurance Replacement Rate</td>
<td>0.40</td>
<td>0.40</td>
<td>0.43</td>
</tr>
<tr>
<td>Fraction of Exporting Firms</td>
<td>0.18</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td>Exporter Wage Premium</td>
<td>0.06</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Variance of Manufacturing Revenues</td>
<td>0.60</td>
<td>0.61</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Notes: Table 2 documents the target data moments for model calibration.

The remaining 6 parameters are chosen to match 7 target moments from US manufacturing. The first three moments capture of key features of firm-level export behavior. Specifically, we target the fraction of manufacturing firms which export abroad (Bernard et al., 2007), the standard deviation of manufacturing revenues (Bernard, Redding and Schott, 2007), and the export wage premium (Bernard et al., 2007). The export wage premium is defined as the difference paid by exporting and non-exporting firms to similar manufacturing workers. This target moment allows us to capture the difference in wages exporters are willing to pay to induce further output in our model. Likewise, the fraction of exporting firms and the standard deviation of manufacturing revenues are inherently to the fixed export cost parameter and the underlying distribution of productivity, respectively.

The final four moments target features of the US labor market for manufacturing workers over the same period. In particular, we aim to match the fraction of unemployed US manufacturing workers who transition to employment, the fraction of employed workers who transition to employment at a new job, and the degree of wage dispersion among manufacturing workers in the CPS as measured by mean-min wage ratio. All three of these values are computed using data taken from the CPS. Last, we also target the average wage replacement rate among US
workers as commonly parameterized the labour-search literature (Menzio and Shi, 2011). The observed employment transitions and replacement rate intuitively discipline the degree of on-the-job-search and the firm-level entry cost. Last, the observed mean-min ratio disciplines the relationship between wages, effort, and firm revenues.

The target moments are collected in Table 2, while Table 3 displays the calibrated parameter values. To characterize the importance of on-the-job search (OTJS), we also calibrate the model to a setting where workers cannot search on-the-job for comparison. There are two striking results. First, the entry cost is relatively low, especially with OTJS. In general, this is a result of the relatively high probability of finding new employment among unemployed workers. However, with OTJS firms are less willing to enter the market since they have to pay higher wages to keep workers from leaving the job and, as such, lower entry costs are needed to match the target unemployment-to-employment transition rate. Second, the shape parameter of the productivity distribution is relatively large with OTJS and small without OTJS. The reason for this is that productivity and effort are multiplicative complements in the production function and, as such, both affect the dispersion of revenues. The effort exerted by the worker is, in turn, a function of the dynamic wage contract and inherently influenced by the opportunity to search for new employment on-the-job.

### 4.2 Steady State Wage Dispersion

Table 4 presents various statistics describing the degree of wage dispersion in the quantitative model. Across all statistics, the benchmark model with OTJS generates substantially more wage dispersion than the calibrated model without OTJS.
Table 4: Impact of Trade Liberalization on Wage Dispersion

<table>
<thead>
<tr>
<th>Model</th>
<th>With OTJS</th>
<th>Without OTJS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage Variance</td>
<td>0.984</td>
<td>0.738</td>
</tr>
<tr>
<td>Mean-Min Ratio</td>
<td>2.630</td>
<td>1.846</td>
</tr>
<tr>
<td>Ninety-Ten Ratio</td>
<td>4.640</td>
<td>4.507</td>
</tr>
</tbody>
</table>

Notes: The wage variance is the variance of the wage distribution. The Mean-Min ratio the ratio of the average wage to the minimum wage. The Ninety-Ten ratio is the ratio of the wage of workers in the ninetieth percentile of the wage distribution to those in the tenth percentile of the wage distribution.

We note that that the direction of these differences are not obvious and speak directly to the importance of dynamic wage contracts and OTJS in this setting. Specifically, in both cases the model produces similar degrees of revenue dispersion, but through very different mechanisms. In the model without OTJS matching the revenue dispersion is accomplished through a relatively disperse firm-level productivity distribution. In fact, the variance of the underlying productivity distribution in the model without OTJS is roughly 8 times larger than that with OTJS.\(^{17}\) Because the model without OTJS has greater underlying productivity variance, one might expect that it would generate higher wage dispersion since highly productive firms have an incentive to offer higher wages. Rather, we observe the opposite.

As we discussed in Section 3, in our model with OTJS, there are two forces which cause wages to increase over job tenure. First, as in other models with OTJS, firms have an incentive to increase wages to deter workers from leaving the firm through OTJS. Second, and specifically in our model, higher future contract values induce greater effort and thus greater output in the future period. Among low value contracts, both of these forces are in effect, and firms will offer greater wage increases over job tenure. This in turn drives up the average wage relative to the minimum age in the model with OTJS. When the value of the wage contracts become sufficiently high, no firm will enter the submarket to poach an employed worker. Therefore, among high value wage contracts, only the second force is in effect, and wage increases slow down. In contrast, the model without OTJS has only the second force to induce the increase in

\(^{17}\)The variance of the Pareto distribution can be computed as \(Var(z) = (z/\xi - 1)^2(\xi/(\xi - 2))\) for \(\xi > 2\).
wages, and wage variation in that model is significantly smaller. In our setting, the model with OTJS delivers an average wage which is 89 percent greater than that in the model without OTJS. Quantitatively, this impact of having workers move through the wage distribution through job-to-job transitions appears particularly important for matching equilibrium wage dispersion.

4.3 Trade Liberalization

Our aim in this section is to characterize the impact of trade liberalization on workers and firms, in general, and wage dispersion, in particular. In the context of our model, we parameterize the long-run impact of trade liberalization by reducing iceberg trade costs to \( \tau \rightarrow 1 \). Because of the elimination of variable trade costs, lower productivity firms may now grow into export markets, which are less costly to service than before. As the expected profitability of exporting improves, even lower productivity firms offer higher wage contracts to increase their output in anticipation of future exporting. This, in turn, drives up current output and wages. Our calibrated example suggests that aggregate revenues and average wages grow by 3 percent each.

The tariff change also has an important effect on wage dispersion. Table 5 documents that the total variance of wages, the mean-min ratio and the ninety-ten ratio all increase by 3-4 percent after trade liberalization.

Table 5: Impact of Trade Liberalization on Wage Dispersion

<table>
<thead>
<tr>
<th>Percentage Change After Liberalization</th>
<th>Wage Variance</th>
<th>Mean-Min Ratio</th>
<th>Ninety-Ten Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3%</td>
<td>3.2%</td>
<td>4.0%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The wage variance is the variance of the wage distribution. The Mean-Min ratio the ratio of the average wage to the minimum wage. The Ninety-Ten ratio is the ratio of the wage of workers in the ninetieth percentile of the wage distribution to those in the tenth percentile of the wage distribution.

Although it is not obvious, endogenous firm dynamics drive the increase in equilibrium wage dispersion. As exporting becomes less costly, high value wage contracts become more profitable and allow less productive firms to survive in high value submarkets. This is illustrated in Panel
(a) of Figure 6 where the firm survival threshold rotates downwards in high value submarkets. The reduction in the minimum productivity required to survive in high value submarkets has two important effects. First, for a given productivity level, workers can be promoted into higher wage jobs within the firm than what was previously possible. Second, OTJS allows workers who are searching on-the-job to direct their search to thicker, higher value submarkets relative to what was possible prior to trade liberalization.

Our quantitative exercise indicates that the endogenous firm dynamics and worker mobility generate not only the shifts the worker distribution to higher wages but also generates larger wage dispersion. First, the change in wage dynamics leads to greater variation in wages since more firms of different productivities are active in each submarket. This mechanism in general shifts the workers to higher wage jobs. Second, unlike the standard Melitz model, trade liberalization in our model does not eliminate lower productivity-lower wage jobs. These jobs will eventually become more profitable, and knowing the underlying firm dynamics, unemployed workers still search in these submarkets. Therefore, the lowest equilibrium wage in our model does not change before and after trade liberalization. As illustrated in Figure 7, the cumulative wage distribution is generally shifted to the right after trade liberalization even though the minimum wage does not change. These two mechanisms together cause larger wage dispersion after trade liberalization in our model.

As noted above, trade liberalization also has an effect on the export participation decision. As displayed in Panel (b) of Figure 6, the export threshold declines and rotates downwards after liberalization. This has a substantial impact on the number of exporting firms, which increases from 17 to 71 percent, and the export wage premium, which increases from 8 percent to 19 percent, in equilibrium. Nonetheless, export status does not directly increase wages or wage variability, per se. Rather, exporting and firm growth induced through trade liberalization only affect wages indirectly by allowing firms to survive in previously unprofitable submarkets.

Lastly, total employment is nearly unaffected as the unemployment rate rises mildly from 3.2 percent to 3.3 percent after trade liberalization. In our steady state comparison, trade liberalization does not strongly affect worker transition probabilities between unemployment
and employment and between jobs. In our model, job separation rate is largely determined by the exogenous destruction rate. In addition, as mentioned above, trade liberalization in our model does not eliminate lower wage jobs. Therefore, unemployed workers’ job finding rate remains high and hence the unemployment rate does not change significantly. This argument suggests that dynamic contracts and resulting firm dynamics also play the key role in causing this result.

5 Conclusion

This paper developed a dynamic, general equilibrium, open economy model with frictional labor markets to study the impact of trade on residual wage dispersion. Key features of our model include heterogeneous firms, directed search, dynamic wage contracts, and on-the-job search. With dynamic wage contracts and on-the-job search, the model naturally generates positive correlation between firm output, exporting status and wages. It also implies widely different labour market histories among ex-ante homogeneous workers. Our quantitative results suggest two key findings. First, firm-heterogeneity and on-the-job-search are strong complements in generating wage dispersion. Using the mean-min ratio as a measure of wage dispersion, the model calibrated with on-the-job-search generates 43 percent more wage dispersion relative to the a model without on-the-job-search. Second, trade liberalization increases equilibrium wage dispersion by 3-4 percent. This effect is due to the ability of firms to offer workers wages which were previously unprofitable after trade liberalization. Our findings indicate that future research which blends the novel firm dynamics emphasized in our model with a framework which allows firms to hire multiple workers may be particularly fruitful for further explaining the nature of the relationship for between trade, wage dispersion and inequality.

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18 The job finding rate slightly increased from 88 percent to 90 percent, the separation rate is unchanged, and job-to-job transition rate slightly increases from 8.3 percent to 8.5 percent.
References


Appendices

A Omitted Proofs

Proof of Proposition 1. We apply the same method as in Tsuyuhara (2015) to show that the value of contract is increasing with tenure by deriving the inverse Euler equation.

We first denote the consistent wage as an implicit function using the first constraint:

\[ w(V, \hat{V}) = v^{-1}(V + c(e(V)) - \beta(\delta U + (1 - \delta)(\hat{V} + \lambda_e D(\hat{V}))). \]  

It is clear that the consistent wage function is increasing in \( V \) and decreasing in \( \hat{V} \). Using this notation, let \( F \) denote the objective function of the maximization operator as a function of \( V \) and \( \xi \):

\[ F(V, \xi) \equiv R(\iota, y(z, V), z) - w(V, \hat{V}) - \iota c + \beta(1 - \delta)(1 - \lambda_e \hat{p}(\hat{V})) I(\hat{V}, z). \]  

Since concave functions are almost everywhere differentiable, \( J \) is almost everywhere differentiable. It implies that the composite function \( \hat{p}(\hat{V}) \) is almost everywhere differentiable, and its derivative is negative wherever it exists (Menzio and Shi, 2010). Therefore, \( F \) is concave and almost everywhere differentiable with respect to \( V \).

Then, the interior solution to the optimal choice of \( \hat{V}^* \) satisfies the first order condition:\(^{19}\)

\[ \frac{-\beta(1 - \delta)(1 - \lambda_e \hat{p}(\hat{V}^*))}{v'(w(V, \hat{V}^*))} + \beta(1 - \delta)[(1 - \lambda_e \hat{p}(\hat{V}^*)) J'(\hat{V}^*, z) - \lambda_e \hat{p}'(\hat{V}^*) J(\hat{V}^*, z)] = 0. \]  

Dividing through by common terms, it simplifies as

\[ \frac{1}{v'(w(V, \hat{V}^*))} + J'(\hat{V}^*, z) - \frac{\lambda_e \hat{p}'(\hat{V}^*) J(\hat{V}^*, z)}{1 - \lambda_e \hat{p}(\hat{V}^*)} = 0. \]  

\(^{19}\)To complete the argument incorporating the possible nondifferentiable points, we apply the theory of nonsmooth analysis to characterize the solution. See Tsuyuhara (2015) for more details.
By the envelope theorem, we have \( J'(V, z) = \frac{\partial R(\cdot)}{\partial V} - \frac{1}{\nu(w(\hat{V}, V^*))} (1 + c'(e)e'(V)) \), which implies that \( \frac{1}{\nu'(w(\hat{V}, V^*))} = -\frac{1}{\Delta} (J'(V, z) - \frac{\partial R(\cdot)}{\partial V}) \), where \( \Delta \equiv 1 + c'(e)e'(V) \). Substituting this term into the above condition and rearranging terms yield

\[
J'(V, z) - J'(\hat{V}^*, z) \Delta = \frac{\partial R(\cdot)}{\partial V} - \frac{\lambda_e \hat{p}'(\hat{V}^*) J(\hat{V}^*, z)}{1 - \lambda_e \hat{p}(\hat{V}^*)} \Delta.
\]

(19)

Because \( \hat{p}'(\hat{V}^*) \) is negative, the right hand side is positive. Moreover, since \( \Delta > 1 \), \( J'(V, z) - J'(\hat{V}^*, z) \Delta > 0 \) implies that \( J'(V, z) - J'(\hat{V}^*, z) > 0 \). Hence, by the concavity of \( J \), this inequality implies that \( \hat{V}^* \geq V \) for all \( V \in X \).

For the second statement, by Theorem 2.8.1 in Topkis (1998), if the objective function \( F \) is supermodular and has increasing differences in \((V, \hat{V})\) on \( X \times X \), then the solution \( \hat{V}(V) \) as a function of \( V \) is increasing.

**Lemma 1.** \( F \) has increasing differences on \( X \times X \).

**Proof:** Since \( F \) is almost everywhere differentiable function in \( V \), it suffices to show that the derivative \( \frac{\partial F}{\partial V} \) is increasing in \( \hat{V} \) wherever it exists. In addition, the derivative \( \frac{\partial F}{\partial V} \) depends on \( \hat{V} \) only through the implicit function \( w(V, \hat{V}) \). Therefore, \( F \) has increasing differences if and only if \( \frac{\partial^2 w(V, \hat{V})}{\partial \hat{V} \partial V} \geq 0 \).

First, using (15)

\[
\frac{\partial w(V, \hat{V})}{\partial V} = -\frac{1 + c'(e)e'(V)}{\nu'(w(V, \hat{V}))},
\]

(20)

by the inverse function theorem. Then,

\[
\frac{\partial^2 w(V, \hat{V})}{\partial \hat{V} \partial V} = -\frac{1 + c'(e)e'(V)}{(\nu'(w(V, \hat{V})))^2} \left\{ -v''(\cdot) \left( \frac{-\beta(1 - \delta)(1 - \lambda_e \hat{p}(\hat{V}))}{v'(\cdot) v'(\cdot)} \right) \right\}
\]

\[
= -\frac{1 + c'(e)e'(V)}{(\nu'(w(V, \hat{V})))^2} \left\{ v''(\cdot) \left( \frac{\beta(1 - \delta)(1 - \lambda_e \hat{p}(\hat{V}))}{v'(\cdot) v'(\cdot)} \right) \right\}.
\]

(21)

To compute the derivative in the large parenthesis, we use the result that the differentiability of \( J \) implies differentiability of \( D(\hat{V}) \) and that the derivative of \( \hat{V} + \lambda_e D(\hat{V}) \) is equal to \( 1 - \lambda_e \hat{p}(\hat{V}) \)
(Menzio and Shi, 2010). Since \( v'' < 0 \) from concavity of \( V \), (21) implies that \( \frac{\partial^2 w(V, \hat{V})}{\partial V \partial \hat{V}} \geq 0. \) □

A real-valued function \( f \) on \( \mathbb{R}^2 \) is supermodular on \( \mathbb{R}^2 \) if and only if \( f \) has increasing differences on \( \mathbb{R}^2 \) (Topkis 1998, Theorem 2.6.1., 2.6.2, and Corollary 2.6.1.). Therefore, the above lemma implies that \( F \) is supermodular. Hence, Theorem 2.8.1 in Topkis (1998) implies that the optimal solution \( \hat{V}(V) \) as a function of \( V \) is increasing. □
Figure 1: Zero-profit productivity cutoffs.

Figure 2: Exporting productivity cutoffs.
Figure 3: Zero-profit value cutoffs.

Figure 4: Exporting value cutoffs.
Figure 5: Firm dynamics.

Figure 6: Impact of Trade Liberalization on Zero Profit and Export Cutoffs

(a) Zero Profit Cutoffs

(b) Export Cutoffs
Figure 7: Trade Liberalization and the Distribution of Wages
A Manufacturing Worker Flows

To construct manufacturing worker flows we follow the process outlined in Shimer (2012) as closely as possible. We first download the basic monthly CPS files available at the NBER website http://www.nber.org/data/cps_basic.html. Using Stata (version 13.0) we matched the data using the Stata files provided at http://home.uchicago.edu/shimer/data/flows/. We then modify the files provided by Shimer (2012) to (a) focus on manufacturing workers alone and (b) capture transitions between employers following Nagypál (2007). Note that if manufacturing workers leave the manufacturing industry we exclude this record entirely. Following Elsby, Michaels, and Solon (2009), we adjust the measured unemployment rate by a factor of 1.15 to account for CPS discrepancies in measured unemployment. We focus on the year 2002 for the model parameterization. Considering a wider window of years (2001-2004) had little effect on the measured transition rates.

B Manufacturing Wage Dispersion

Following Lemieux (2006) we consider measures of wage dispersion as reported in the May/ORG CPS. To capture a measure of wage dispersion we run a Mincer regression of the log hourly wage, $w_{it}$, on a host of observable characteristics, $x_{it}$, for individual $i$ in year $t$. Observable characteristics include gender, race, education, and age. We follow Lemieux (2006) as closely as possible in the construction of each variable and with regards to interpreting the log hourly wage data. A single, but important exception, is that we restrict attention to manufacturing
workers older than 16 and younger than 64. Our Mincer regression is written as

\[ w_{it} = x_{it}'a_t + \varepsilon_{it} \]

where \( a_t \) is the return to observed characteristics \( x_{it} \) and \( \varepsilon_{it} \) captures residual wage dispersion.

To compute our measure of residual wage dispersion, i.e., the mean-min ratio, we first calculate the average residual wage, \( \bar{w}_t \), as

\[ \bar{w}_t = \frac{1}{N_{it}} \sum_i e^{\hat{\alpha}_0 + \hat{\varepsilon}_{it}} \]

where \( N_{it} \) is the number of individuals in the sample in year \( t \), \( \hat{\alpha}_0 \) is the estimated constant from the above Mincer regression and \( \hat{\varepsilon}_{it} \) is the residual for individual \( i \) in year \( t \). We then compare this value to minimum residual wage in the same year computed in the same manner.

We consider a number of years over the 2001-2004 period but find very similar results regardless of the sample period chosen. As such, the figures reported in the manuscript reflect the 2002 May/ORG CPS since this corresponds to the same year we select other moments to calibrate the model.

References

