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Local system identification analyses of the dynamic response of soil systems

Mourad Zeghal*, Caglar Oskay

Department of Civil and Environmental Engineering, Rensselaer Polytechnic Institute Troy, 4028 Jonsson Engineering Center, Troy, NY 12180, USA

Abstract

The seismic behavior of massive geotechnical systems exhibits complex response patterns and mechanisms under severe loading conditions. Some of these mechanisms are localized in space, but nevertheless impact significantly the entire system response and ultimately its stability. A thorough monitoring and identification of the whole response of a distributed soil system commonly constitutes a significant challenge, and would generally be prohibitively expensive. This study presents a point-wise identification technique of soil-systems using the acceleration records provided by local instrument arrays. The newly developed identification algorithm consists of: (1) evaluation of strain tensor time histories using the motions recorded by a cluster of instruments (arranged in an appropriate multi-dimensional configuration), (2) estimation of the stress tensors corresponding to the evaluated strains utilizing a pre-selected class of constitutive models of soil response, (3) computation of accelerations associated with estimated stresses using the equations of motion, and (4) calibration and evaluation of an optimal model of soil response by minimizing discrepancies between recorded and computed accelerations. Computer simulations and analysis of centrifuge tests of a soil–quay wall system showed that the proposed technique provides an effective tool to identify local soil characteristics and properties.

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1. Introduction

Massive and distributed geotechnical systems exhibit a broad range of response patterns when subjected to seismic excitations. The response of sites, embankment and other massive deposits depends on soil properties, water level and degree of saturation, stratigraphy, topography, and other factors. Under severe loading conditions, soils also exhibit local response mechanisms which affect significantly the whole system behavior. Some of these mechanisms reflect the particulate nature of soils, and others are associated with abrupt changes in properties. For instance, liquefaction of a thin soil layer at a certain depth may lead to excessive site lateral spreading and failure [1].

Identifying and calibrating models of the dynamic response of a full soil system is a complex endeavor, especially when the behavior of this system is marked by the development of localized response mechanisms (e.g. formation a shear band or liquefaction of a thin soil stratum). Commonly, the measured or recorded global response of a massive system does not provide enough

information to uniquely and accurately identify local mechanisms. Global identification approaches based on boundary value problem formulations smear these mechanisms over the entire system, and possibly lead to erroneous idealizations. To reduce the problem indeterminacy, a significant number of sensors would generally be needed to monitor the system local and global response as well as the boundary conditions. Furthermore, such a thorough monitoring may be prohibitively expensive and technically challenging.

This study presents an effective alternative to global system analyses. The multi-dimensional constitutive response of massive soil-systems is assessed locally using point-wise identification analyses. These identifications are performed using the accelerations recorded by local multi-dimensional arrays of closely spaced instruments. The developed identification algorithm does not use or require the availability of recordings or measurements of boundary conditions, nor the solution of boundary value problems. Computer simulations and centrifuge model tests of a soil system behind a quay wall were used to demonstrate the capabilities of the proposed identification technique.

In Section 2, a general formulation of the identification technique is presented. Thereafter, results of the conducted

* Corresponding author.

E-mail address: zeghal@rpi.edu (M. Zeghal).

analyses using numerical simulations and centrifuge model tests are described.

2. Identification algorithm

As mentioned earlier, local soil properties and mechanisms may have considerable impact on the behavior of geotechnical systems. Accurate identification of these local mechanisms based on modeling of a whole system requires a significant number of sensors. The problem is even more complex for semi-infinite systems, such as sites and slopes. Adequate monitoring of the boundary conditions of such systems when subjected to multi-dimensional excitations remains an unresolved issue. Generally, the information provided by a limited number of sensors would suffice only when evaluating the main global features of a system response, and may lead to erroneous identifications of local mechanisms or properties.

A new system identification technique is developed in this paper to analyze soil system local response mechanisms using the motion recorded by a cluster of closely spaced accelerometers appropriately located and arranged in 2D or 3D configurations (depending on the dimensionality of the system response). The developed algorithm (Fig. 1) consists of the following: (1) evaluation of strain tensor time histories using the recorded accelerations, (2) estimation of the corresponding stress tensor utilizing a pre-selected class of constitutive models of soil response, (3) computation of the accelerations associated with the estimated stress tensors using the equilibrium equations, and (4) evaluation and calibration of an optimal model of soil response. Model calibration is based on a minimization of the discrepancies between recorded and computed accelerations using optimization techniques. The approach focuses on the evaluation of actual soil dynamic response characteristics and properties without interfering with the mechanisms of seismic wave propagation. Fig. 2 shows schematically a range of local conditions of a quay wall–soil system which may advantageously be analyzed

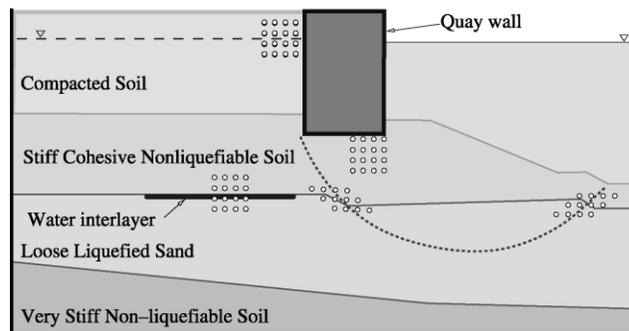


Fig. 2. Schematic of a quay wall–soil system, showing a range of local conditions which may advantageously be analyzed using the developed local identification technique.

using local instrument arrays. The proposed algorithm requires specific configurations for local instrument arrays, as described below. Such configurations are expected to become widely used in centrifuge and 1g shake table model tests, and eventually in full-scale system instrumentation.

2.1. Evaluation of the strain field

For small deformations, the strain tensor, ϵ , is given by:

$$\epsilon = \frac{1}{2}(\mathbf{u}\nabla + \nabla\mathbf{u}) \quad (1)$$

where $\mathbf{u} = \{u_1, u_2, u_3\}^T$ is displacement vector, $\nabla = \{\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3\}^T$ is Laplace differentiation operator, and x_i ($i = 1, 2, 3$) refers to an orthogonal Cartesian coordinate system. The displacements may be obtained through double time integration of the corresponding acceleration records. When a soil system is monitored by an array consisting of a cluster of accelerometers (Fig. 2), estimates of the strain field may be evaluated within the instrumented zone using interpolation or finite differentiation techniques [2]. The accuracy of these strain estimates is a function of spacing and number of instruments used in interpolations. For a 2D analysis, second order accurate estimates may be obtained at instrument locations if a cluster of five appropriately distributed sensors is used. Fig. 3 exhibits the required instrument configuration to achieve such an accuracy for interior, edge and corner cluster locations. Second order accurate strains may also be evaluated at the center of a group of four sensors within a rectangular or a parallelogram configuration (Fig. 3), as discussed below.

Evaluation of the components of a 2D strain tensor also requires recordings of soil response in two orthogonal or skew rectilinear directions (i.e. x_1 and x_2 in Fig. 4). A larger number of sensors are needed to achieve a higher order accuracy (nine are required for fourth order accuracy). Similarly, seven instruments distributed along three orthogonal or skew rectilinear directions are necessary to estimate second order accurate strains in 3D analyses.

For 2D plane-strain conditions, the discrete counterpart of strains at the (p, q) instrument location in Fig. 3 (using

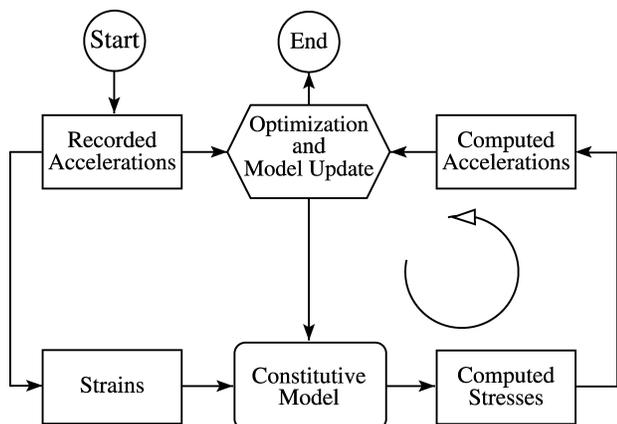


Fig. 1. Algorithm of the developed local system identification approach.

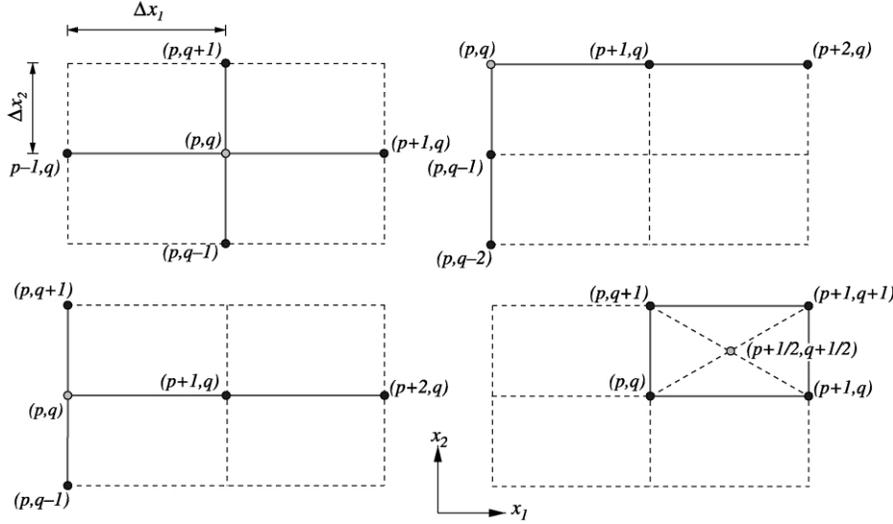


Fig. 3. Instrumentation configurations for evaluation of second order accurate 2D strain estimates (at locations (p, q) and $(p + 1/2, q + 1/2)$).

a two-index numbering) is given by:

$$\boldsymbol{\epsilon}^{(p,q)} = \frac{1}{2}(\mathbf{u}\nabla^{(p,q)} + \nabla^{(p,q)}\mathbf{u}) \quad (2)$$

in which $\nabla^{(p,q)}$ is a discrete counterpart of the Laplace differentiation operator ∇ :

$$\nabla^{(p,q)}(\cdot) = \left\{ \begin{array}{l} \frac{(\cdot)^{(p+1,q)} - (\cdot)^{(p-1,q)}}{\Delta x_1} \\ \frac{(\cdot)^{(p,q+1)} - (\cdot)^{(p,q-1)}}{\Delta x_2} \end{array} \right\} \quad (3)$$

Eq. (3) requires constant instrument spacing, Δx_1 and Δx_2 , in the x_1 and x_2 directions, respectively. Alternative equations should be employed with nonuniform spacing, as described by Zeghal et al. [3] for a 1D case.

In this study, second order accurate strains were therefore evaluated using clusters of 9 (3-by-3 instruments within parallelogram 2D configurations, Fig. 4) at locations of the intermediate nodes $(p - 1/2, q - 1/2)$, $(p + 1/2, q - 1/2)$,

$(p - 1/2, q + 1/2)$, and $(p + 1/2, q + 1/2)$, and expressed in terms of the skew rectilinear coordinates ξ_i , ($i = 1, 2, 3$). The corresponding stresses obtained at these same locations (Section 2.2) are then used to evaluate second order accurate accelerations (Section 2.3) at the central location of the cluster, as described below (Fig. 4). At these intermediate nodes, the strain tensor is given by Ref. [4]:

$$\boldsymbol{\epsilon} = \boldsymbol{\Phi}\hat{\boldsymbol{\epsilon}}\boldsymbol{\Phi}^{-1} \quad (4)$$

in which $\hat{\boldsymbol{\epsilon}}$ is the stress tensor in the skew rectilinear coordinate system ξ_i ($i = 1, 2, 3$) (Fig. 4):

$$\hat{\boldsymbol{\epsilon}} = \frac{1}{2}(\hat{\mathbf{u}}\hat{\nabla} + \hat{\nabla}\hat{\mathbf{u}}) \quad (5)$$

In Eqs. (4) and (5), $\hat{\mathbf{u}}$ and $\hat{\nabla}$ (with $\hat{\nabla}_{ij}(\cdot) = \partial(\cdot)/\partial\xi_j$) are displacement and differentiation vectors expressed in the skew coordinate system, and $\boldsymbol{\Phi}$ is transformation tensor

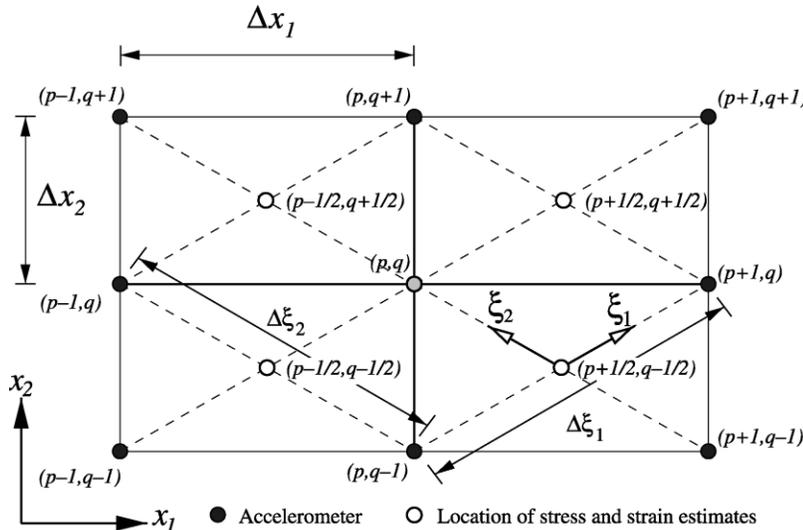


Fig. 4. Instruments, strain and stress point locations for a 2D local (point-wise) identification analysis.

from the skew to the Cartesian coordinate system [4]:

$$\Phi_{ij} = \frac{\partial x_j}{\partial \xi_i}, \quad i, j = 1, 2, 3. \quad (6)$$

2.2. Constitutive modeling and estimation of the stress field

Under conditions of multi-dimensional wave propagation, nonparametric estimates of stresses could not be evaluated directly from recorded accelerations. Such nonparametric identification is possible for level sites under conditions of vertical propagation [3]. Any parametric identification therefore requires the use of a pre-selected class of constitutive models to evaluate stresses from the estimated strains. The class of models may then be calibrated using the measured response. To demonstrate the capabilities of the proposed technique, this study focused on analyses of clayey soils under short term loading conditions. A Von-Mises criterion was therefore used to describe the yield condition. A multi-surface plasticity technique was used to idealize soil nonlinear, hysteretic and path dependent stress–strain response and to address the problem of load reversals associated with dynamic excitations [5]. The elasto-plastic constitutive relations for clayey soils are given by:

Yield function:

$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}, k) = \frac{3}{2}(\mathbf{S} - \boldsymbol{\alpha}) : (\mathbf{S} - \boldsymbol{\alpha}) - k^2 = 0, \quad (7)$$

Flow rule:

$$d\boldsymbol{\epsilon}^p = \langle dL \rangle \mathbf{P} \quad (8)$$

Hardening rule:

$$d\boldsymbol{\alpha} = \langle dL \rangle a \boldsymbol{\mu} \quad (9)$$

in which $\boldsymbol{\sigma}$ is stress tensor, \mathbf{S} is deviatoric stress tensor ($\mathbf{S} = \boldsymbol{\sigma} - p\boldsymbol{\delta}$, with $p = 1/3\text{trace}(\boldsymbol{\sigma})$ and $\boldsymbol{\delta}$ is Kroneker delta unit tensor), $\boldsymbol{\alpha}$ and k are plastic internal variables defining location and size of the yield surface, $d\boldsymbol{\epsilon}^p$ is plastic strain tensor, \mathbf{P} is a symmetric second order tensor giving the direction of plastic deformations, $\boldsymbol{\mu}$ is a second order tensor defining the direction of kinematic hardening, a is kinematic hardening parameter, $\langle \cdot \rangle$ are Mac-Cauley brackets ($\langle dL \rangle = dLH(dL)$, with H the Heavyside step function), and dL is plastic loading function increment. The elasto-plastic constitutive equation may then be expressed as [5,6]:

$$d\boldsymbol{\sigma} = \left(\mathbf{E} - \frac{(\mathbf{E} : \mathbf{P})(\mathbf{Q} : \mathbf{E})}{H_p - H_0} \right) : d\boldsymbol{\epsilon} \quad (10)$$

in which ‘:’ denotes the scalar product of two tensors, \mathbf{E} is the elastic constitutive tensor (as a function of the soil shear modulus G and Poisson’s ratio ν), \mathbf{Q} is unit outward normal to the yield surface, $H_0 = \mathbf{Q} : \mathbf{E} : \mathbf{P}$, and H_p is the plastic modulus. Under short term conditions, clayey materials exhibit mostly an associated flow rule ($\mathbf{P} = \mathbf{Q}$). Thus, the

variations of the elastic and plastic moduli, H_0 and H_p , are direct functions of the initial and elasto-plastic shear moduli [5], G_0 and G :

$$H_0 = 2G_0 \quad \text{and} \quad \frac{1}{H_p} = \frac{1}{2G} - \frac{1}{2G_0}. \quad (11)$$

A purely hysteretic material energy dissipation commonly does not account for soil damping observed at low-strains. Therefore, material damping is supplemented by a viscous dissipation mechanism:

$$d\boldsymbol{\sigma}^{\text{vep}} = d\boldsymbol{\sigma} + \eta d\dot{\boldsymbol{\epsilon}} \quad (12)$$

in which $\boldsymbol{\sigma}$ is given by Eq. (10), and η is a viscous damping factor.

2.2.1. Recognition of the most consequential parameters

The number of variables which may be estimated in identification analyses is limited by the observational data, and recognition of the most consequential parameters is therefore essential. The model variables consists of the constitutive equation parameters, which are the material elasticity parameters, G_0 and ν , and plasticity parameters, f , $\boldsymbol{\alpha}$, k , a , and $\boldsymbol{\mu}$. The mass density can be measured accurately in laboratory or in situ.

Analyses of uniaxial constitutive relations suggest that the elastic and plastic moduli are the most phenomenological parameters that control a stress–strain response. The modulus H_p , which may be expressed as:

$$H_p = \frac{\mathbf{Q} : d\boldsymbol{\sigma}}{\mathbf{P} : d\boldsymbol{\epsilon}^p} \quad (13)$$

plays the role of the plastic modulus for multi-dimensional stress conditions. Furthermore, the plastic modulus variations are directly related to the dependence of the yield surface f on the plastic internal variables $\boldsymbol{\alpha}$ and k and on evolution rules of these two internal variables [5]. Therefore, selecting the yield surface and associated internal variables or the plastic modulus as main parameters are basically equivalent, and it is only a matter of convenience. In view of Eq. (11), the identification analyses were limited to evaluation of the initial and elastoplastic shear moduli, G_0 and G . Variations of the Poisson’s ratio of clayey soils were assumed to be not significant under short term dynamic conditions.

2.3. Estimation of accelerations

For a total stress formulation, the system acceleration and stress tensor are related through the equilibrium equations:

$$\ddot{\mathbf{u}} = \frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma} + \mathbf{b} \quad (14)$$

where \mathbf{b} is body force (i.e. gravity). A discrete counterpart of Eq. (14) within the skew rectilinear coordinate system was used to evaluate the acceleration corresponding to the computed stresses $\hat{\boldsymbol{\sigma}} = \Phi^{-1} \boldsymbol{\sigma} \Phi$ (Eq. (12)). For a 2D setting and when the instrument array consists of 9 (3×3) 2D

instruments (as shown in Fig. 4), the accelerations are computed at the central point (p, q) using the computed stresses at the intermediate locations $(p - 1/2, q - 1/2)$, $(p + 1/2, q - 1/2)$, $(p - 1/2, q + 1/2)$, and $(p + 1/2, q + 1/2)$. When a cluster is larger than 3×3 instruments, accelerations may be evaluated at the location of all interior accelerometers (not located on an edge or a corner). The evaluated accelerations are then transformed from the skew rectilinear coordinate system to the Cartesian one:

$$\ddot{\mathbf{u}}(\mathbf{x}) = \Phi \hat{\mathbf{u}}(\xi) \tag{15}$$

2.4. Model calibration

The selected class of models were calibrated using a simple deterministic approach. Stiffness and damping parameters of the models described earlier were estimated using a generalized (weighted) least-squares optimality criterion [7], so as to minimize the difference between observed acceleration time histories, $\ddot{\mathbf{u}}^{(o)}$, and those predicted by the selected models, $\ddot{\mathbf{u}}^{(m)}$. The optimality criterion is given by:

$$O = \int_0^T \|\ddot{\mathbf{u}}^{(o)} - \ddot{\mathbf{u}}^{(m)}\|_{\mathbf{W}} dt + \|G - G^{(a)}\|_{\mathbf{W}_G} + \|\eta - \eta^{(a)}\|_{\mathbf{W}_\eta} \tag{16}$$

in which $\|\ddot{\mathbf{u}}^{(o)} - \ddot{\mathbf{u}}^{(m)}\|_{\mathbf{W}}$ refers to the weighted Euclidean norm of $\ddot{\mathbf{u}}^{(o)} - \ddot{\mathbf{u}}^{(m)}$, \mathbf{W} is a positive definite weighting matrix which takes into account the available information regarding the model performance, G and η refer collectively to the shear stiffness and damping parameters described below, $G^{(a)}$ and $\eta^{(a)}$ are a priori estimates of G and η , and \mathbf{W}_G and \mathbf{W}_η are associated weighting matrices. The term $\|G - G^{(a)}\|_{\mathbf{W}_G} + \|\eta - \eta^{(a)}\|_{\mathbf{W}_\eta}$ reflects the uncertainty and confidence level in the a priori parameter estimates and are used to improve the problem conditioning [8].

The model optimal parameters are then given by:

$$\min_{G>0, \eta>0} O \tag{17}$$

This deterministic approach, which was adopted because of its simplicity, does have an underlying equivalent probabilistic model [9]. A general functional was pre-selected for the shear modulus based on expected variations for soil media [10]:

$$G(\gamma) = \frac{G_0 - G_p}{\left(1 + \left(\frac{\gamma}{\gamma_0}\right)^n\right)^{(n+1)/n}} + G_p \tag{18}$$

in which G_0 and G_p are initial and large-strain moduli, γ_0 and n are parameters defining the variations as a function of shear strain amplitude, γ . Thus, the identification was reduced to the estimation of G_0 , G_p , γ_0 , n , and η . The numerical evaluation of optimal parameters was performed using quasi-Newton nonlinear optimization

algorithms [11]. Nonconstrained identifications were employed using logarithmic change of variables.

3. Computer simulations and convergence properties

Analytical and mathematical analyses of the convergence and accuracy characteristics of the developed identification technique are feasible only for the simple cases of elastic constitutive behaviors, and are not possible for the more realistic nonlinear inelastic soil models. Computer simulations were therefore conducted to assess the capabilities of the developed identification algorithm. The performed analyses were specifically aimed at analyzing the impact of discretization errors on the convergence characteristics of the developed identification algorithm. Finite element models were used to generate simulated acceleration records of a hypothetical quay wall–soil system (Fig. 5). Shear wave velocity profile of soil behind the quay wall was marked by the presence of a weak soil layer. Simulated recorded response of this weak layer in the vicinity of the quay wall (Fig. 5) was used to demonstrate the algorithm capabilities in capturing local response characteristics and properties.

The simulation analyses revealed that when the spacing between accelerometers is equal to the finite element dimensions, the inverse problem solution exhibits sensible errors. These errors decrease when the spacing between accelerometers is two or more times the dimensions of the finite elements and reflect the impact of using two different discretization techniques in the forward problem finite element solution and the inverse problem finite difference algorithm. Such errors have little relevance to the identification of real system. Fig. 6 shows a close agreement between the simulated recorded motion and optimal accelerations identified using instruments separated by distances equal to four times the finite element dimensions (2.0 m \times 2.0 m). Close agreements were also obtained between the estimated

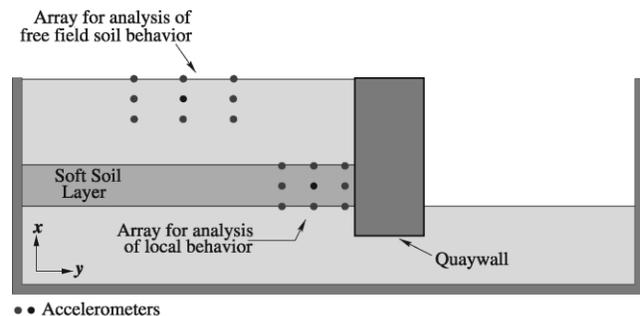


Fig. 5. Model configuration for numerical simulations.

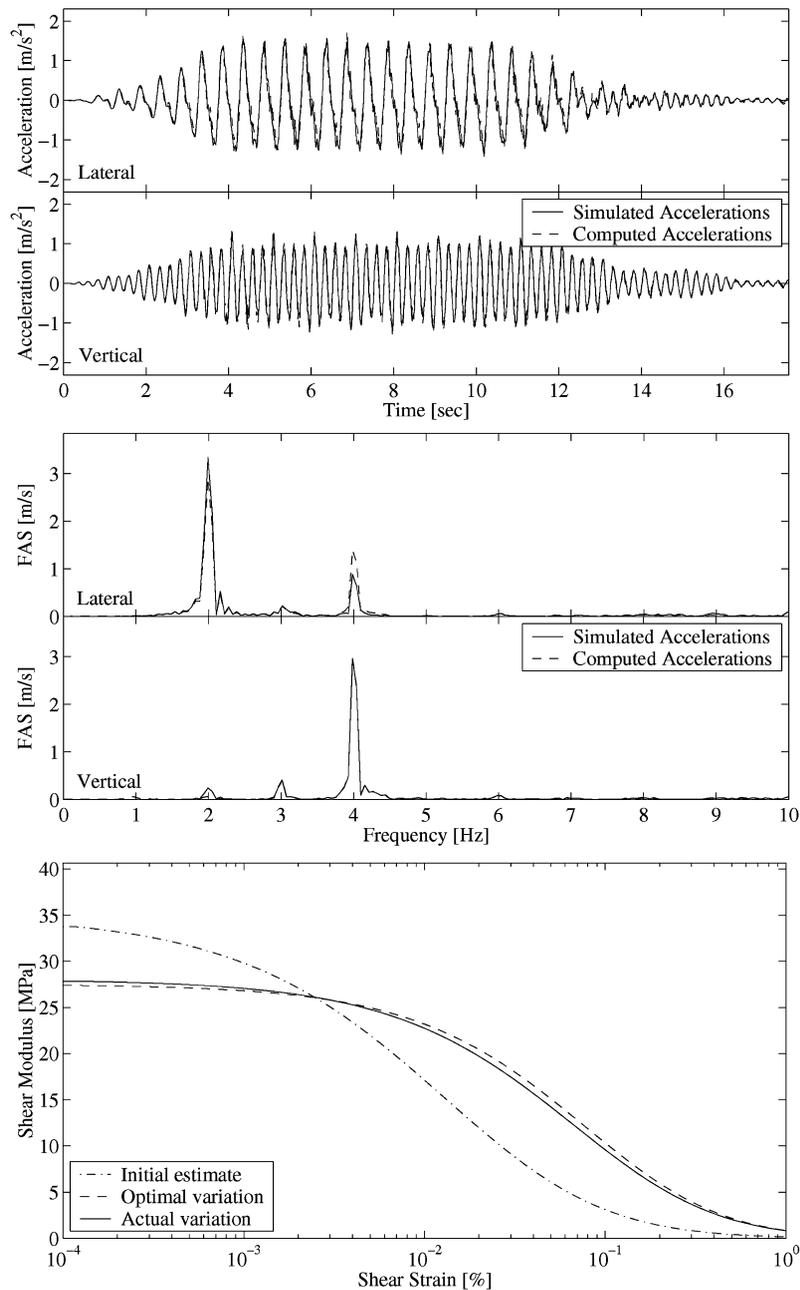


Fig. 6. Local identification of a soil–quay wall finite element simulation model (Fig. 5) subjected to a strong shaking: acceleration time histories, Fourier amplitude spectra, and shear moduli at the central instrument station (next to the wall).

and actual variations of the shear modulus (Fig. 6). Errors were of the order of 2% or less.

When the spacing between accelerometers becomes significant (i.e. higher than 1/5 of involved wave lengths) the optimal solution was characterized by sensible errors in strain estimates leading to unbalanced stresses and a drift in computed accelerations. This drift is reduced significantly if higher order interpolations (e.g. 5 point approximations) are used.

4. Centrifuge model tests

A series of centrifuge tests of a soil–quay wall model was conducted in a rigid box, as shown in Fig. 7. Stiff and soft clays were used for the soil. The models were built and instrumented at 1g (g = acceleration of gravity) with transducers installed to capture the 2D (lateral and vertical) clayey soil response. Because of technical constraints (e.g. number of channels for data acquisition) the accelerations

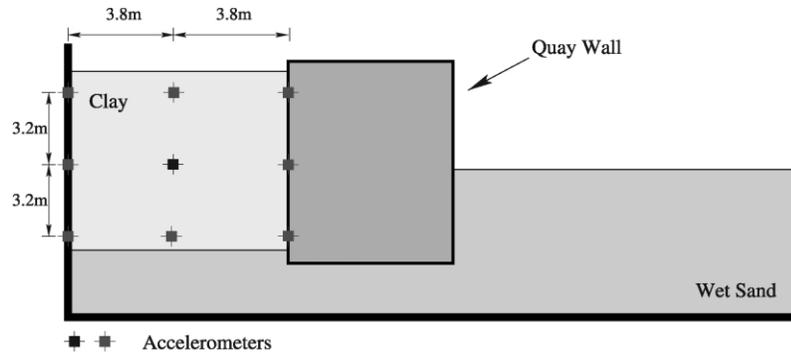


Fig. 7. Configuration of soil–quay wall centrifuge test model.

were measured only at nine locations (Fig. 7), providing experimental data to assess the constitutive stress–strain relationship at one single location. In fact, the conducted analyses were based on the assumption that the soil and the rigid box or the quay wall have identical accelerations at the interface. The discrepancies between these accelerations are thought to have only minor effects on the conducted analyses. 1D shaking was imparted along the model base using an electro-hydraulic shaker. Because of the model geometry and the presence of noise, the recorded vertical accelerations behind the quay wall were sensible.

Fig. 8 show the recorded and computed acceleration time histories and Fourier amplitude spectra at the central recording location within a stiff clay layer for a low amplitude shaking. This shaking induced mostly a linear response, and the estimated low-strain shear modulus ($G_0 = 29$ MPa) was found to be in close agreement with the modulus estimated using 1D nonparametric stress–strain analysis ($G_0 = 25$ MPa) using a vertical array composed of the three central accelerometers [3]. Reasonably good agreement were also obtained between computed and recorded accelerations for a strong shaking

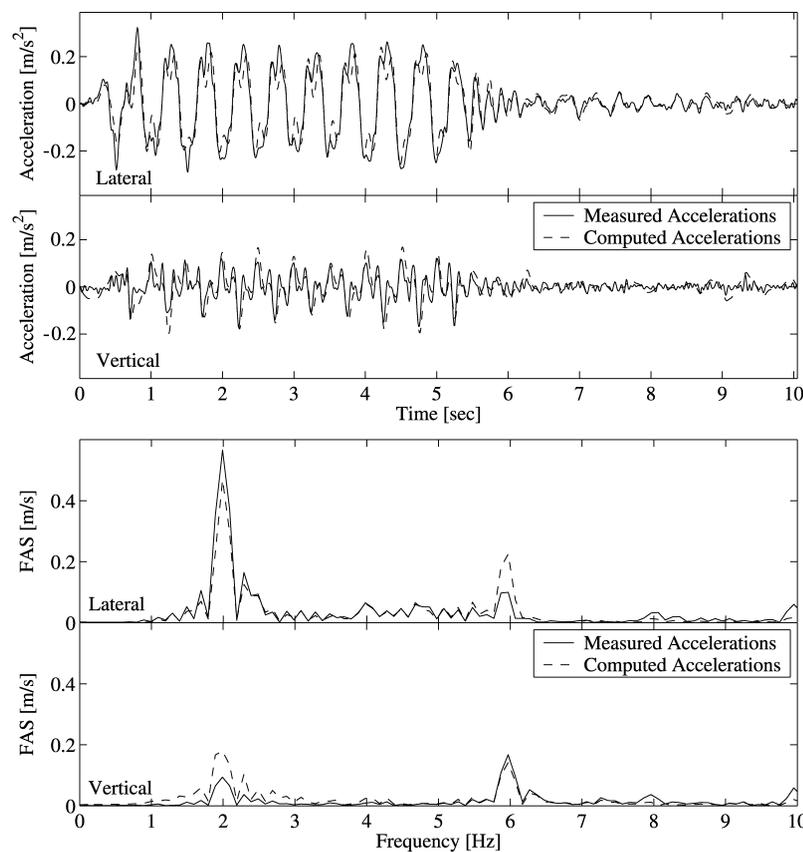


Fig. 8. Local identification of the soil–quay wall centrifuge model of Fig. 7 subjected to a low-amplitude shaking: acceleration time histories and Fourier amplitude spectra at the central instrument station.

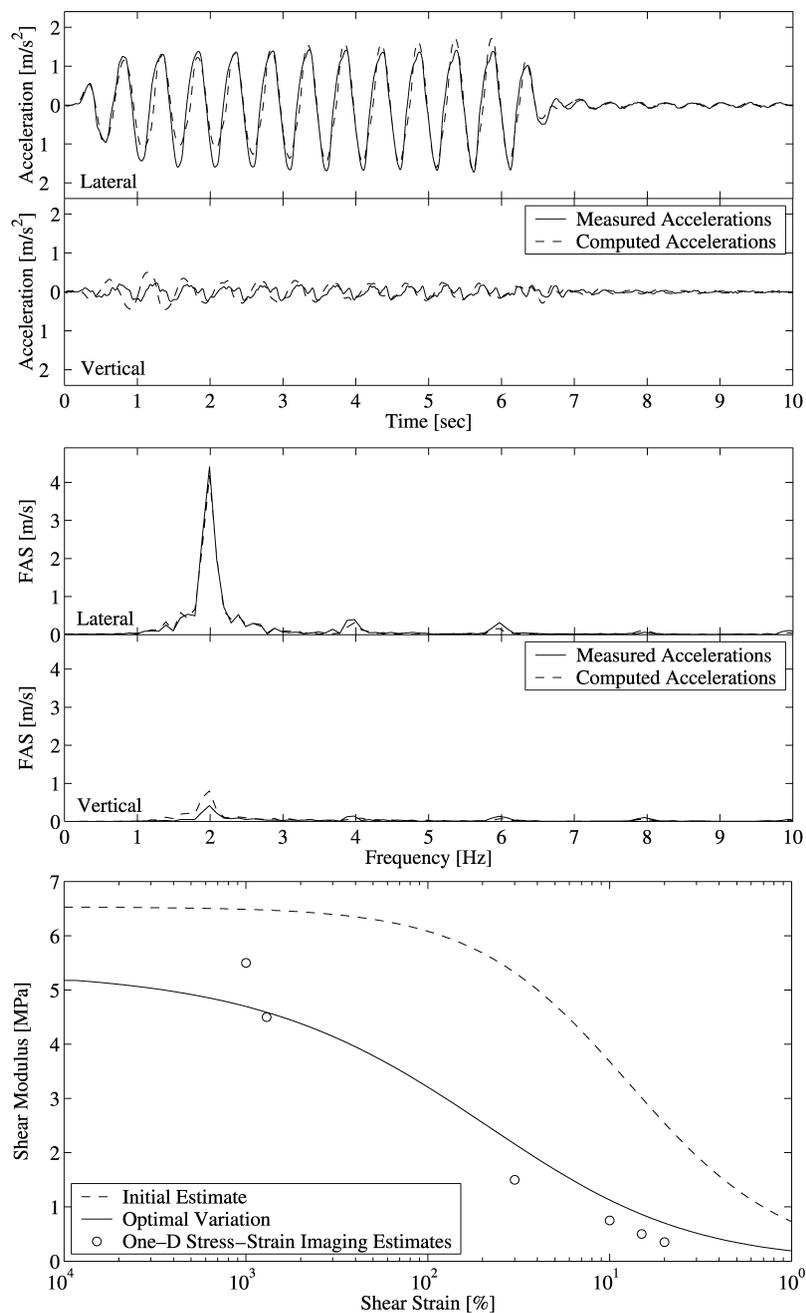


Fig. 9. Local identification of the soil–quay wall centrifuge model in Fig. 7 subjected to a strong shaking: acceleration time histories, Fourier amplitude spectra, and shear moduli at the central instrument station.

of a weaker clay layer as shown in Fig. 9. The associated shear modulus variations are also displayed in this figure. These variations were found to be consistent with the trend obtained from a 1D nonparametric stress–strain analysis [3]. The relatively lower quality of fitness for the vertical accelerations reflects the higher noise-to-signal ratio in this direction and the fact that the above assumption regarding soil accelerations at the interface with the box and the quay wall is more valid for the lateral direction than for the vertical one.

5. Conclusions

This paper presents a multi-dimensional local system identification technique of soil systems. Local mechanisms of soil response are analyzed using accelerations recorded by a cluster (2D array) of closely spaced instruments. This technique does not use or require the availability of recordings or measurements of boundary conditions, nor the solution of the boundary value problem associated with an observed system. Such an approach is particularly

advantageous when a system behavior is affected by local mechanisms such as at the interface of soil and structural elements. Global identification approaches based on boundary value problem formulations will generally fail to capture the effects of local mechanisms. Computer simulations and analysis of centrifuge tests of a soil quay system showed that the proposed technique provides an effective tool to identify local characteristics and properties of soil system response.

Acknowledgments

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