Multidimensional Vector Model of Stimulus–Response Compatibility

Motonori Yamaguchi
Vanderbilt University

Robert W. Proctor
Purdue University

The present study proposes and examines the multidimensional vector (MDV) model framework as a modeling schema for choice response times. MDV extends the Thurstonian model, as well as signal detection theory, to classification tasks by taking into account the influence of response properties on stimulus discrimination. It is capable of accounting for stimulus–response compatibility, which is known to be an influential factor in determining choice-reaction performance but has not been considered in previous mathematical modeling efforts. Specific MDV models were developed for 5 experiments using the Simon task, for which stimulus location is task irrelevant, to examine the validity of model assumptions and illustrate characteristic behaviors of model parameters. The MDV models accounted for the experimental data to a remarkable degree, demonstrating the adequacy of the framework as a general schema for modeling the latency of choice performance. Some modeling issues involved in the MDV model framework are discussed.

Keywords: Simon effect, psychological scaling, sequential sampling model, Thurstonian model, vector model

Task performance is better when stimuli are compatible with the responses than when they are not. For instance, when people are asked to press left and right keys in response to lights that occur on the left and right sides, they typically respond faster when the light locations correspond to the positions of the keys. This stimulus–response compatibility (SRC) effect is robust and obtained in a wide range of task conditions, including those of applied settings such as driving a car and piloting an aircraft (Yamaguchi & Proctor, 2006, 2010, 2011b). Thus, SRC has been one of the major principles in designing tools and systems (Proctor & Van Zandt, 2008; Proctor & Vu, 2006; Wickens, Lee, Liu, & Gordon-Becker, 2004). Also, the SRC effect and its variant phenomena (e.g., the flanker compatibility and Stroop effects) are thought to reflect the abilities to control actions and processes underlying them. These effects have been taken as indices of cognitive capabilities, especially in comparing across subject populations of different ages or with distinct pathological conditions (e.g., Cagigas, Filoteo, Stricker, Rilling, & Friedrich, 2007). At the same time, the SRC effect has also served as an experimental tool to investigate cognitive processes and test cognitive theories in a variety of research fields (e.g., Inhoff, Rosenbaum, Gordon, & Campbell, 1984; Leonhard, Ruiz Fernández, Ulrich, & Miller, 2011; McCann & Johnston, 1992). Additionally, the SRC effect itself has been the subject of active research programs. In particular, researchers in the studies of action and perception have taken it as a central construct in their theorization of cognitive performances (see, e.g., Hommel & Prinz, 1997). Nevertheless, previous theories are concerned mostly with qualitative characteristics of the phenomena, and only limited attempts have been made to approach SRC from a quantitative perspective.

Generally speaking, a modeling framework serves three purposes. First, it serves as a metaphorical description of hypothetical cognitive entities and their relations. Examples of metaphorical descriptions in cognitive psychology include theories of attention (e.g., resources, spotlight, etc.), modes and stages of cognitive processing (e.g., automatic/controlled modes, Donders/Sternberg processing stage models), and kinds of memory pools, among numerous others. These metaphorical terms are typically borrowed from the information-processing framework (Newell, Shaw, & Simon, 1958) and physiology (i.e., neurochemical mechanics) but are not limited to them. A metaphor is an effective way to communicate abstract ideas in a tangible manner and helps bridge between different fields of research.

The second purpose of a modeling framework is to demonstrate what a theory of cognition is able to do or not to do by simulating hypothesized processes and systems. It is a means to confirm or disconfirm the workability of a theory, and unknown behaviors of the hypothesized systems are often discovered by means of simulations that can also be tested against data. Prominent examples of this nature include architectural models such as the ACT-R theory (Anderson et al., 2004), EPIC (Meyer & Kieras, 1997), and Soar (Newell, 1990), which are computational extensions of information-processing metaphors, and connectionist models such as the PARP framework (Rumelhart, McClelland, and the PDP Research Group, 1986), which are physiological metaphors of cognition.

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Correspondence concerning this article should be addressed to Motonori Yamaguchi, Department of Psychology, Vanderbilt University, Nashville, TN 37203. E-mail: motonori.yamaguchi@vanderbilt.edu
Finally, the third purpose of a modeling framework stems from the Fechnerian tradition (Fechner, 1860/1966). A mission of the Fechnerian approach is to discover functional relationships between observable and unobservable quantities. Operationally, this approach is reduced to the process of mapping between physical measurements and psychological concepts via mathematical principles. From this perspective, the model ultimately defines concepts mathematically, and data are interpreted in light of these concepts. In effect, a model serves as a measure of psychological concepts that provides an efficient means to summarize characteristics of complex data and the behaviors that generate the data sets. Such an approach is sometimes called cognitive psychometric (e.g., Wagenmakers, van der Maas, & Grasman, 2007). A most successful example of this sort may be signal detection theory (Green & Swets, 1966), which has become a standard measurement model in studies of sensory and perceptual processes. More recently, sequential sampling models, such as random walk and accumulator models (e.g., Brown & Heathcote, 2008; Ratcliff, 1978), are becoming a standard means to summarize response time (RT) data in various domains. The practical utility of these modeling frameworks depends on the representability of the data (Dzhafarov, 1993); they become theoretical languages when they are capable of representing a range of data adequately.

The SRC studies have been dominated by qualitative, metaphorical models (e.g., Kornblum, Hasbroucq, & Osman, 1990; Nicoletti & Umiltà, 1985; Proctor & Reeve, 1985), and the trend still continues at present (e.g., Hommel, Müsseler, Aschersleben, & Prinz, 2001; Proctor & Cho, 2006). Some of the metaphorical models have been elaborated into computational ones and have successfully accounted for basic findings (e.g., Tagliabue, Zorzi, Umiltà, & Bassignani, 2000; Zhang, Zhang, & Kornblum, 1999; Zorzi & Umiltà, 1995). However, there has not been any attempt to advance the field from the third, cognitive psychometric perspective, nor have modelers who work with those representational frameworks been concerned with SRC. The present research departs from previous studies in that its goal is to take a step toward a quantitative framework that satisfies the third purpose of cognitive modeling. The framework we develop is called the multidimensional vector (MDV) model. In the following sections, we first introduce recent developments in the studies of SRC. Then, we describe theoretical foundations and characteristic properties of MDV. A general account of the SRC effect (or the Simon effect; see a later section for the definition) is considered based on the framework, and MDV models are developed for five experiments that utilize the SRC tasks and test how well the MDV models can account for the experimental data. Finally, we discuss some limitations and possible extensions of the present approach, and some representative multidimensional scaling approaches are compared to the present framework.

Stimulus–Response Compatibility

The term stimulus–response compatibility was coined by Paul Fitts (Fitts & Seeger, 1953; also see Small, 1990) to refer to the phenomenon in which responses are faster and more accurate when stimuli are paired to responses that are physically or conceptually similar to, or have dimensional overlap with, the stimuli (Kornblum et al., 1990). Indicating its robustness, the SRC effect can be observed with various types of stimulus and response modes and based on task-irrelevant stimulus and response features (see Lu & Proctor, 1995; Proctor & Vu, 2006, for reviews). The effect based on task-irrelevant stimulus features is specifically called the Simon effect (after its discoverer, J. R. Simon; Simon, 1990), a variant of which is the main focus of the present study. An explanation of the Simon effect is based on a dual-route concept, according to which response activation occurs via automatic and controlled response-selection routes (e.g., De Jong, Liang, & Lauber, 1994; Kornblum et al., 1990). The accounts state that if a stimulus is compatible with the assigned response, automatic activation of that response occurs, facilitating responding. On the other hand, if a stimulus is incompatible with the assigned response, automatic activation of a wrong response occurs, interfering with making the correct response. Previous computational models (Zhang et al., 1999; Zorzi & Umiltà, 1995) are instantiations of the dual-route accounts and have shown that the basic pattern of the SRC effect can be accounted for by the models.

Although multiple information-processing routes are apparently required to model the Simon effect, the mechanisms underlying the effect appear to be more flexible than those assumed in the previous dual-route models. For instance, the dual-route models adopt the traditional distinction between automatic and controlled processes that states that these processing modes are fundamentally different (e.g., Posner & Snyder, 1975; Schneider & Shiffrin, 1977), but recent developments in the SRC literature provide a number of observations that contradict this strict automaticity assumption. In particular, studies have shown that the Simon effect does not emerge unconditionally on any stimulus features that overlap with responses (Hommel, 2000). For instance, Eimer and Schlaghecken (1998) showed that the spatial compatibility between subliminally primed arrows and responses influenced performance when the target stimuli were also arrows but not when they were letters, suggesting that processing irrelevant subliminal primes depends on what kind of stimuli the observer intends to process. Similarly, Miles, Yamaguchi, and Proctor (2009) demonstrated that the Simon effect can be diluted when a task-irrelevant neutral stimulus is presented, but the dilution occurs only when the neutral stimulus is categorically similar to a task-irrelevant stimulus that produces the Simon effect. This result implies that processing of task-irrelevant stimuli is capacity limited and context dependent. Moreover, Ansorge and Wühr (2004) showed that a task-irrelevant stimulus dimension produces the Simon effect only if response alternatives used in the task are discriminated in that psychological dimension, and Yamaguchi and Proctor’s (2011b) study suggested that the Simon effect depends on how much attention is allocated toward specific response components. These observations imply that influences of task-irrelevant stimulus dimensions are contingent on task goals. Thus, in contrast to the dual-route accounts, recent studies suggest that the Simon effect is sensitive to the task context (e.g., task goal, response mode) that sets the actor’s anticipation or intention of making specific actions. That is, to a large extent, which of the task-irrelevant stimulus dimensions produces the Simon effect is determined by what actions are required in that task context.

A purpose of the MDV framework is to address this issue. As can be seen in the following sections, the context of stimulus representations is defined in MDV based on feature dimensions that are necessary to represent both stimuli and responses. Consequently, when there is overlap between stimulus and response
dimensions, they produce the SRC effect, consistent with earlier accounts of SRC (e.g., Kornblum et al., 1990; Lu, 1997), but without committing to the traditional idea of automatic versus controlled processes, in that selectivity across feature dimensions is central in the MDV concept. Also, recent studies suggest that stimulus features are encoded according to their relevance to the type of actions required to make their responses (e.g., Fagiolini, Hommel, & Schubotz, 2007; Wykowska, Schubö, & Hommel, 2009). These findings indicate that one is more likely to pay attention to aspects of stimuli that are related to features of the required actions. That is, the SRC effect is a consequence of attentional control, rather than the lack of control thereof, as the automaticity accounts may imply. MDV is one way to instantiate such a process.

Because SRC describes the fact that performance is better when stimuli and responses are similar than when they are not, a theory must offer an explanation as to why similarity between stimuli and responses leads to better performance. A fundamental issue rests on the question of how stimuli and responses are represented psychologically (e.g., Hommel et al., 2001; Proctor & Cho, 2006; Wallace, 1971). The SRC effect should naturally arise in the response-selection process based on such representations. Nevertheless, the existing computational models of SRC are not concerned with this issue, assuming that similar stimuli and responses are somewhat connected to or associated with each other and producing automatic response activation.

A traditional discipline of psychological investigations that is most directly related to the issues of mental representations is that of psychological scaling. Since the dawn of Fechnerian psychophysics, researchers have developed elaborate techniques to recover the psychological representations of physical stimuli or the task context from human performance data. Unfortunately, however, these techniques are not sufficiently integrated into contemporary studies of human cognition, so no attempt has been made in the SRC literature to approach the representational problems from the scaling perspective. To achieve the present purpose, it is necessary to develop such an integrated framework for modeling RT data.

**Psychological Scaling Models**

Psychological scaling is a process of translating outcomes of psychological tasks into hypothetical constructs, called *utility*, which stands for the psychological value of a physical object or event. Psychological scaling models are generally classified into two families known, respectively, as fixed- and random-utility models. Fechner’s theory of just noticeable difference is based on a fixed-utility model, which assumes that physical stimuli are translated into mental representations that are constants along a continuum. The model did not take into account the probabilistic nature of human performance (but see Dzhafarov & Colonius, 2001, 2011, for a more contemporary reformulation of Fechnerian scaling). The fixed-utility family is currently known as the *Bradley-Terry-Luce model* (Bradley & Terry, 1952; Luce, 1959), which also assumes constant stimulus representations with a probabilistic selection rule to account for the probabilistic outcomes of choice behaviors. On the other hand, the random-utility family is generally known as a *Thurstonian model*, after its inventor L.L. Thurstone (Thurstone, 1927). Thurstone generalized Fechnerian scaling by assuming that psychological representations are mapped onto random variables rather than onto constants. Although the two families differ in their fundamental assumptions, researchers have found that these approaches are formally related and often equivalent under certain conditions (e.g., see Luce, Bush, & Galanter, 1963). This means that models from both traditions often account for the same data equally well, and choice between them is not necessarily consequential.

Subsequently, researchers have developed hybrid models, one of which is named the *wandering vector* (WV) model (Carroll, 1980; De Soete & Carroll, 1983). The model starts with a fixed-utility assumption and ends with a random-utility model. The original vector model (Slater, 1960; Tucker, 1960; Tucker & Messick, 1963) was developed to account for individual differences in preference-ranking behavior, which allows decomposing the outcomes of preferential judgment (i.e., a dominance table) into three matrices that represent three aspects of the task behavior. The first matrix represents the characteristics of choice objects (stimuli), summarized in a multidimensional space; the second matrix represents psychological dimensions that contributed to the task outcomes, which determine the orthogonal axes of the psychological space; and the third matrix represents the general preferences of individual subjects along the psychological dimensions, in which each individual subject is represented as a vector (*subject vector*). That is, the results of preference judgments of a particular individual on given objects are obtained as an inner product of the subject vector and the object matrix. In geometrical terms, this operation corresponds to orthogonal projection of stimulus coordinates onto the subject vector (DeSarbo & Cho, 1989). And the resulting quantities are the *subjective utilities* of the stimuli, the utilities that are unique to the given individual and task context.

WV is a probabilistic generalization of the vector model. In the original vector model, stimulus coordinates and subject vectors are deterministic, so it is a fixed-utility model. WV also defines deterministic stimulus coordinates in a multidimensional space, but it introduces a probabilistic factor by assuming that the subject vector is a random vector. That is, in WV, the subject vector fluctuates randomly across trials, and when orthogonally projected onto the subject vector, stimuli are also randomly distributed along the subject vector. The stimulus distribution along the subject vector reflects the subjective utility of the choice object for the particular individual. The subjective utility is represented by the extension of the projected vector in the direction of the subject vector in this schema. In Figure 1A, for instance, two stimuli X and Y are represented in a two-dimensional space according to their attributes. They are projected onto the subject vector, and the extents of these projections in the direction of the subject vector determine their subjective utilities. In the example depicted in the figure, the subjective utility of Y is larger than that for X, so the observer chooses Y over X.

Thurstonian models have been generalized to multidimensional stimuli in various ways (see, e.g., Ashby, 1992, for various Thurstonian models, including WV), and WV may not be the most straightforward extension of the framework. However, the machinery assumed in WV simplifies the formal expression of the model to a greater degree as compared to more straightforward extensions of the Thurstonian approach to multidimensional cases (e.g., Ashby & Townsend, 1986; Tanner, 1956). Therefore, we chose WV to be the starting point of our formal approach to SRC. We do
not present detailed formulations of WV, but readers who are interested in the mathematical treatments should consult De Soete and Carroll (1983, 1992).

**Multidimensional Vector Model Framework**

Psychological scaling models have been studied extensively, but there have been only a few attempts to integrate these models into contemporary cognitive psychology. A possible reason for this is that psychological scaling models are not concerned with the main dependent variable used most frequently by cognitive researchers: that is, whereas scaling models have traditionally been developed to account for choice probability, cognitive theories are built based on RT in many cases. Thus, to integrate the psychological scaling approach into contemporary studies of cognition is to extend the domain of the models to RT. At the same time, although similar, the tasks used in these fields of research are different, so some reformulations of scaling models are necessary to bridge between the fields. In the present study, the MDV framework extends the domain of psychological scaling (WV in particular) to binary classification that is the standard form in the studies of SRC and many other areas of cognitive research. We provide formal treatments of MDV in the Appendix and keep mathematical expressions minimal in the following discussions.

It should be acknowledged that the general context model (GCM; Nosofsky, 1986) and the general recognition theory (GRT; Ashby & Townsend, 1986) are multidimensional approaches that have also been extended to account for RT data (see, e.g., Nosofsky & Palmeri, 1997; Nosofsky & Stanton, 2005, for GCM; Ashby, 2000; Ashby & Maddox, 1994; Maddox & Ashby, 1996, for GRT; also see Fific, Little, & Nosofsky, 2010). We discuss formal and theoretical relations of these previous works to the current framework in the General Discussion.

**Multidimensional Vector Model for Classification**

In a classification task, a single target stimulus is presented on each trial. Participants judge whether the stimulus belongs to one response category or the other based on a prespecified attribute of the stimulus. MDV represents the classification process in terms of comparing the stimulus utility against a decision criterion that is established according to the task definition. The process starts with mapping the given stimulus onto a point in the multidimensional space according to its attributes (see Figure 1B). The stimulus point (denoted by $x_i$ in the figure) corresponds to the intrinsic characteristics of the stimulus that are assumedly stable across experimental conditions and individuals. Thus, the intrinsic utility of the stimulus is fixed points in the space. Subsequently, the stimulus utility is translated into the subjective utility that varies according to the task context. The subjective utility of a given stimulus feature can be high in one context but low in another context. Such fluctuations in subjective utility happen all the time in reality, depending on one’s current needs. MDV assumes that the fluctuations reflect the subjectivity involved in the decision processes that are determined by the actor’s goal and task requirements.

In MDV, the subjective utility of a stimulus is computed by translating the intrinsic stimulus utility in the multidimensional space into the decision space. This translation is an orthogonal projection of the stimulus onto a random vector, the process that defines a vector model. In Figure 1B, the process is schematically depicted as a shift from the multidimensional space to a unidimensional axis. The stimulus is classified by comparing its subjective utility against an arbitrary decision criterion (denoted by $c$ in the figure) that partitions the decision space into alternative response categories (A and B in Figure 1B). We call the random vector the *decision axis*, the term borrowed from signal detection theory (Green & Swets, 1966). The decision axis represents an internalized task context, which is influenced by the task goal and prior experiences. Due to the fact that the decision axis in MDV is a random vector, the same stimulus is projected to slightly different points on the decision axis at different points in time during a trial. Hence, the subjective utility of a stimulus is a random variable distributed along the decision axis. We assume that the decision axis is normally distributed in the multidimensional space, and this assumption leads to normally distributed stimulus representations along the decision axis (see the Appendix). Thus, when the stimulus distributions along the decision axis are concerned, the model is formally equivalent to signal detection theory applied to binary classification (see Figure 1B, right side). Response selection is performed by sampling from the stimulus distribution and observing on which side of the decision criterion the sample falls. To generate RT distributions, we assume that this sampling process is repeated during a trial until the amount of evidence reaches a threshold (see the next subsection).
In formal terms (see the Appendix for details), the stimulus $S_i$ is represented in the $n$-dimensional psychological space with fixed coordinates $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$. The decision axis is also expressed as a random vector $v = (V_1, V_2, \ldots, V_n)$, of which each element is a normal random variable. Then, with orthogonal projection, the subjective utility of $S_i$ is summarized as a random variable $X_i = \mathbf{v}^T \mathbf{x}_i$, which has mean $\mu_i = \mathbf{x}_i^T \mathbf{\mu}$ and variance $\delta_i^2 = b \mathbf{x}_i^T \mathbf{x}_i$, where $\mathbf{\mu}$ is the mean vector of $\mathbf{v}$ and $b$ is a constant that represents the variability of $\mathbf{v}$. In the presentation of the stimulus $S_i$, the probability that a sample provides evidence for the response category A is

$$P[A|S_i] = \Phi\left(\frac{c - \mu_i}{\delta_i}\right).$$

with $\Phi$ being the cumulative distribution function (CDF) of standard normal variable and $c$ being the decision criterion that is an arbitrary point on the decision axis. Suppose that the decision criterion is unbiased; then it is readily seen that $(c - \mu_i)/\delta_i$ is equivalent to the sensitivity measure ($d'$), or discriminability, as defined in signal detection theory. Consequently, MDV defines the discriminability of stimuli in terms of the distribution of subjective utility on the decision axis. As described in the Appendix, $c$ can be set at zero, so Equation 1 is rewritten as

$$P[A|S_i] = 1 - \Phi\left(\frac{\sum_{r=1}^{n} x_{ir} w_r}{b \sum_{r=1}^{n} x_{ir}^2}\right),$$

where $w_r = \cos \theta_r$ being the weight associated with the stimulus feature in the $r$th dimension with the angle $\theta_r$ between the decision axis and the $r$th dimension axis. The weight determines the extent to which the $r$th stimulus dimension contributes to the task outcome (i.e., the larger it is, the greater the contribution of the corresponding dimension to the outcome is). The weights are geometrically constrained by the identity $\sum_r w_r^2 = 1$ and range between 1 and $-1$.

**Counter Model Extension**

At this point, the model only predicts choice probability. To extend the framework to RT, we adopted a type of sequential sampling model known as a counter model (Audley & Pike, 1965; see Figure 2). In a sense, this choice is arbitrary because other sequential sampling models are also available (e.g., Brown & Heathcote, 2008; Ratcliff, 1978). Nevertheless, there are three desirable properties of the counter model in the present context. First, the counter model is mathematically compatible with the MDV concept (see the Appendix). Second, the counter model can be expressed in a closed form. This allows use of maximum likelihood estimation, which is technically easy to implement and probably most reliable (e.g., Myung, 2003; Ratcliff & Tuerlinckx, 2002; Van Zandt, 2000), and allows use of information criteria to select from alternative models (see the present model applications below). Finally, the counter model can easily be extended to a general case where more than two alternative choices are available. The present study is not concerned with conditions that involve more than two alternative choices, but this characteristic opens a future possibility to extend the framework to multiple-choice cases.

In the counter model, distinct response categories are represented by buffers called counters that accumulate evidence favoring their corresponding response categories. Evidence arrives at the counters with a variable time interval, and this interarrival interval is distributed exponentially with rate parameter dependent on the given stimulus. At each sampling, a counter is incremented by a fixed amount (e.g., 1). The sampling process repeats until either of the counters reaches a response threshold. Therefore, the counter model is a race between independent Poisson counters toward a response threshold. The latency of this evidence accumulation process is considered to be decision time (DT), and RT is the sum of DT and nondecision time ($t_0$):

$$RT = DT + t_0.$$  

The nondecision time is often assumed to be a constant but can be another random variable (e.g., Ratcliff & Smith, 2004). In the MDV version, evidence is sampled from the stimulus distribution along the decision axis, and the probability of the sample providing evidence for the response category A is given by Equation 2. Also, the response threshold is a uniformly distributed random variable. As a consequence, the counter model is formally defined by

$$f_{DT}(t;A|S_i) = f(t;A|S_i)[1 - F(t;B|S_i)],$$

which is the probability density function (PDF) of DT for the response category A when the stimulus $S_i$ is presented. The first term of the right side of Equation 4 is the PDF of processing time of the response category A, which is given by

$$f(t;A|S_i) = \frac{1}{K_{\max} - K_{\min} + 1} \sum_{j=K_{\min}}^{K_{\max}} (\lambda_{d(|j-1)])[j-1]! \lambda_{d(|j)\exp}(-\lambda_{d(|j)\lambda_{d(|j)}}).$$

The second term is one minus the CDF of processing time of the competing response category B, which is

$$F(t;B|S_i) = 1 - \frac{1}{K_{\max} - K_{\min} + 1} \sum_{j=K_{\min}}^{K_{\max}} \sum_{j=0}^{j-1} (\lambda_{d(|j)\lambda_{d(|j)}})^j j! \exp(-\lambda_{d(|j)\lambda_{d(|j)}}).$$
In effect, Equation 4 expresses the PDF of the finishing time for which the counter A reaches the response threshold before the counter B.

The parameters \( \lambda \) and \( K_{\text{min}} \) and \( K_{\text{max}} \) are, respectively, the rate parameter and the minimum and maximum values of the response threshold \( K \). As shown in the Appendix, the rate parameter for the response category A is obtained by

\[
\lambda_{a,0} = \lambda_i P(A|S_i),
\]

where \( \lambda_i \) is called the base accumulation rate given the stimulus \( S_i \) (we assume that \( \lambda_i \) is identical for all \( i \)). The base accumulation rate is equal to the sum of accumulation rates for all alternative responses: \( \lambda_i = \sum_{k \in R} \lambda_i^{k|0} \), where \( R \) is a set of all competing counters that represent the response alternatives in the task. The conditional probability in Equation 7 is given by Equation 2.

To summarize, MDV assumes that stimuli are first encoded into a multidimensional psychological space according to their intrinsic utilities. Depending on the task goal and requirements, the actor adjusts his or her internal state (decision axis) and translates the intrinsic stimulus utility into the subjective utility. The subjective utility is a random variable due to the fluctuations in the internal states, expressed in terms of a probability distribution. Decision making or response selection is performed by sampling the subjective utility from the distribution until evidence for a response alternative reaches the threshold. These assumptions are formally defined and result in a set of unique properties. We discuss some of the main characteristic properties of the MDV model below.

**Some Mathematical Properties of MDV**

**RT–distance hypothesis.** The latency of the sequential sampling process is determined by two factors. The first is the distance from the starting point of the accumulation process to the termination point of the process (i.e., the response threshold). The greater the distance between these two points is, the longer it takes to reach the response threshold. The response threshold is assumed to be subject to cognitive control and independent of stimuli. Speed–accuracy tradeoffs are attributed to the adjustment of the threshold value (Vickers & Lee, 1998). The second factor, which is of more importance in the present context, is the accumulation rate; the larger the accumulation rate is, the quicker the sampling process terminates. As shown above, the accumulation rate for a particular response in the MDV counter model is determined by the probability of a stimulus sample being drawn from the corresponding region on the decision axis (i.e., hit rate in signal detection theory; see Equation 2), which depends on the discriminability of that stimulus. If the variance of a distribution is held constant, discriminability is a function of the decision criterion and the location, or mean, of the distribution of subjective utility. The discriminability increases as the distribution mean moves away from the decision criterion and, subsequently, increases the accumulation rate. As a result, RT (or DT) becomes shorter as the distance from the decision criterion to the mean of subjective utility increases. This relationship between RT and the decision criterion is known as the RT–distance hypothesis (e.g., Ashby & Maddox, 1994; Murdock, 1985; Nosofsky & Palmeri, 1997).

As noted, the subjective utility of a stimulus is given by a dot-product of the stimulus coordinates and the decision axis, \( X_i = \mathbf{v}^T \mathbf{x}_i \), which is normally distributed with mean \( \mu_i = \mathbf{v}^T \mathbf{\mu} \) and variance \( \delta_i^2 = b \mathbf{v}^T \mathbf{x}_i \). Importantly, the mean can be expressed as a weighted sum of the stimulus coordinates \( \mu_i = \sum w_i \mathbf{x}_i \). Thus, in MDV, the subjective utility of the stimulus is equal to a weighted sum of the subjective utilities of stimulus features in the respective dimensions. The weight \( w_i \) is called dimensional saliency because it is determined by the salience, or pertinence, of that dimension in a given task context. This parameter is geometrically represented by the cosine of the angle between the decision axis and the dimensional axis, \( \cos \theta_i \), and can be interpreted as corresponding to selective attention to that psychological dimension. Note that the decision process of MDV is a limited capacity model because the weights are constrained by the identity \( \sum_i w_i^2 = 1 \).

**Saturation.** The variance of subjective utility is another factor that influences the accumulation rate. As Equation 2 shows, the subjective utility (discriminability) decreases as the variance increases, so the accumulation rate becomes smaller and RT increases with the variance. The variance depends on the decision axis parameter \( \Sigma \) (the covariance matrix), which represents the stability of the (internalized) task context. As suggested by De Soete and Carroll (1983), the covariance matrix is set \( \Sigma = b \mathbf{I} \) (the identity matrix), so the variance of subjective utility is given by \( \delta_i^2 = b \mathbf{v}^T \mathbf{x}_i \). Thus, the scalar \( b \) controls the size of the variability in subjective utility. If this value is large, the decision axis fluctuates more, so the variance of subjective utility increases. More importantly, it shows that the variance is proportional to the norm (or length) of the stimulus coordinate vector \( \mathbf{x}_i \). In other words, the variability of the subjective utility depends on the distance between the origin and the point at which the stimulus is represented in the multidimensional space. This is an interesting characteristic of MDV: Given that the mean is held constant, the subjective utility saturates as the stimulus representation gets removed away from the origin of the space, resulting in a smaller accumulation rate for that stimulus.

In scaling approaches similar to the present one, such as GCM (Nosofsky, 1986) and GRT (Ashby & Townsend, 1986), the origin of the similarity space can be chosen arbitrarily because similarity is measured in terms of interpoint distances between stimuli, which are unique up to an additive constant (i.e., model predictions are unaffected if a constant is added to all stimulus utilities). This is not the case in MDV. Because the variance of subjective utility depends on the norm of the stimulus coordinates in the multidimensional space, model predictions are affected if the origin of the space changes. In an extreme case where a stimulus is located at the origin, the variance is zero, so no variability should be involved in the decision process for that stimulus. At the opposite extreme where a stimulus is too far away from the origin, the variance is infinitely large, so performance is always at a chance level. Therefore, the origin of the multidimensional space is fixed at the decision criterion. This definition ensures that the origin of the space not only is systematically determined but also is consistent with the presupposition that the decision axis is a vector emanating from the origin because the decision criterion should always be on the decision axis by definition. This is also consistent with the fact that stimuli and responses are referentially coded according to a salient reference point, rather than being fixed by physical stimulus or response properties, the observations prevalent in the SRC literature (e.g., Cho & Proctor, 2003; Guiard, 1983; Hommel, 1993; Vu & Proctor, 2002).
Multidimensional Vector Model of the Simon Task

Although the Simon effect is primarily measured in terms of RT, error rates often show that responses are more accurate for compatible trials than for incompatible ones. That is, RT and percentage error (PE) are positively correlated. Therefore, we propose that the Simon effect can be modeled most parsimoniously in terms of accumulation rate. In the MDV counter model, the accumulation rate is determined by the probability of a sample being taken as evidence for a particular response, which depends on the discriminability of the stimulus. The Simon effect is attributed to discriminability of stimuli that changes according to the compatibility between stimulus and response.

Suppose, for example, that red and green stimuli are assigned to left and right responses. The Simon effect implies that the discriminability of a red stimulus is greater when it occurs on the left than when it occurs on the right. Such an outcome is typically interpreted as indicating that response is biased toward stimuli that appear on the left rather than on the right. However, the Simon effect also implies that the discriminability of a green stimulus is greater when it occurs on the right than on the left, which then implies that the response bias produced by the stimulus location is reversed according to the color of stimuli (i.e., the responses mapped to the color). Thus, the influences of stimulus location on stimulus discriminability are inconsistent between the two color conditions, so a purely stimulus-based account of discriminability cannot explain the Simon effect. Instead, the Simon effect indicates that the discriminability of stimuli should be considered to be a joint product of stimulus and response properties, and a joint representation of stimuli and responses is required to model the processes that underlie the Simon task (Hommel et al., 2001). The present adoption of a vector model is due to its ability to construct such a representation.

MDV Account of the Simon Effect

An MDV model of the standard Simon task is schematically depicted in Figure 3. In a typical Simon task, there are four stimulus conditions (e.g., red and green stimuli that occur in the left and right locations), and one pair of stimuli is mapped to one response and the other pair is mapped to the other response (e.g., red stimuli are mapped to the left response, and green stimuli are mapped to the right response). In MDV, the four stimuli are represented by four points in a two-dimensional space (see the figure).
upper panel of Figure 3). Note that these four points are fixed in the space, and here they are assumed to be equidistant from the origin of the space (i.e., the variances of the stimulus distributions along the decision axis are equal). Response selection is performed by projecting these multidimensional representations onto the decision axis.

A critical assumption in accounting for the Simon effect is that the parameters of the decision axis are dependent on the response properties. That is, if the left and right responses are assigned to red and green stimuli, as in the example, then the response representations involve both the spatial and color features. In similarity-based categorization models (e.g., Logan, 2002; Nosofsky, 1986), responses are considered to represent assigned stimulus categories. That is, if stimuli are to be categorized based on their colors, response categories are the colors used in the task, even though the actual responses are pressing of keys, for example. Such models disregard intrinsic properties of responses, but studies suggest that certain intrinsic features are involved in response representations if the alternative responses can be discriminated based on those features (Ansorge & Wühr, 2004). By contrast, previous models of SRC assume that responses are represented based on general response codes that represent their intrinsic properties (e.g., Zhang et al., 1999; Zorzi & Umilta, 1995), ignoring the stimulus features that are assigned to these responses. Yet De Houwer (2004) demonstrated that representations of nonspatial vocal responses involve spatial features that are assigned to the responses, so that they produce a Simon-like effect (termed the extrinsic Simon effect) when the responses are made to a nonspatial attribute of spatial stimuli. Therefore, response representations should involve both intrinsic (response-discriminating) features and extrinsic features (i.e., the task-relevant features of stimuli) of responses.

In the current example, MDV assumes that the left response is represented in terms of the lefthess of the physical response feature as well as the redness of a task-relevant stimulus feature that is assigned to that response. Likewise, the right response involves the rightness of the physical response feature as well as the greenness of a task-relevant stimulus feature assigned to that response. MDV postulates that the decision axis is the vector that connects between these response representations. The lower panel of Figure 3 depicts the stimulus distribution along the decision axis. Because all stimuli are equidistant from the origin of the multidimensional space, their variances are identical, so the accumulation rates for the four stimulus conditions are determined solely by the locations of the distributions (i.e., means). For red stimuli, the distribution is located farther away from the decision criterion in the red/left stimulus condition than in the red/right stimulus condition; for green stimuli, the distribution is located farther away from the decision criterion in the green/right stimulus condition than in the green/left stimulus condition. From the RT–distance hypothesis, responses are expected to be faster (and more accurate) for the red/left and green/right stimuli (i.e., compatible trials) than for the red/right and green/left stimuli (i.e., incompatible trials).

**Formulation and Implementation of the MDV Model**

As an initial demonstration, we fit the MDV model for the standard Simon task to experimental data collected for our prior research (Proctor, Yamaguchi, Dutt, & Gonzalez, 2011). In the experiment, 24 participants performed a Simon task for which they pressed a left or right key on the standard keyboard (the z or l key, respectively) in response to the color (red or green) of circles that appeared on the left or right side of the screen. They were instructed to ignore the stimulus location as much as possible and focus on the stimulus color. One group of participants pressed the left key to red circles and the right key to green circles; the mapping was reversed for the other group. Each participant performed four separate blocks of 80 trials in a 30-min session, in which the four combinations of the stimulus color (red, green) and locations (left, right) were intermixed. The MDV model for this experiment was instantiated as follows.

**Model instantiation.** The four stimulus conditions are represented by the stimulus vector \( \mathbf{x}_i = (x_{i1}, x_{i2}) \), \( i = 1, 2, 3, 4 \), with the first dimension \( (x_{i1}) \) corresponding to the task-relevant color dimension and the second dimension \( (x_{i2}) \) to the task-irrelevant spatial dimension. The alternative responses are represented by the two points in the same multidimensional space, and the line connecting the two representations establishes the decision axis, expressed by the normal random vector \( \mathbf{v} \). The parameters of the decision axis are the mean vector \( \mathbf{\mu} = (\mu_1, \mu_2) \) and covariance matrix \( \Sigma = \mathbf{I} \).

Given \( \mathbf{x}_i \), the probability that a stimulus sample indicates the left response is

\[
P_{\text{L|X}}(x_i) = \phi\left( \frac{x_{i1} \mu_1 + x_{i2} \mu_2}{\sqrt{\mu_1^2 + \mu_2^2}} \right)
\]

The probability for the right response is, thus, \( P_{\text{R|X}} = 1 - P_{\text{L|X}} \), which is the probability of sampling errors. If the base accumulation rate is \( \lambda \), the accumulation rate is \( \lambda_{\text{L|X}} = P_{\text{L|X}} \lambda \) for the left response and \( \lambda_{\text{R|X}} = P_{\text{R|X}} \lambda \) for the right response.

In the Simon paradigm, however, data are typically summarized as a function of compatibility between stimulus and response locations rather than that of the factorial combination of stimulus and response locations. Consequently, the model may be expressed more economically when the task-irrelevant stimulus dimensions are defined based on compatibility; that is, \( x_{i2} = a_2 \) if the stimulus is compatible and \( x_{i2} = -a_2 \) if it is incompatible. In this case, the task-relevant dimension can always be fixed to be \( x_{i1} = a_1 \). Here we assumed that the two values of each stimulus dimension are complementary, meaning that there is no perceptual bias for green or red and for left or right, or that the bias is averaged out. From Equation 2, the probability of a stimulus sample indicating the correct response is

\[
P_{\text{Comp}} = \phi\left( \frac{a_1 w_1 + a_2 w_2}{\sqrt{a_1^2 + a_2^2}} \right)
\]

for the compatible trials and

\[
P_{\text{Incomp}} = \phi\left( \frac{-a_1 w_1 + a_2 w_2}{\sqrt{a_1^2 + a_2^2}} \right)
\]

for the incompatible trials. The corresponding probabilities for error responses are \( q_{\text{Comp}} = 1 - P_{\text{Comp}} \) and \( q_{\text{Incomp}} = 1 - P_{\text{Incomp}} \). Therefore, the rate parameters for the correct and error responses on the compatible and incompatible trials are given, respectively, by \( \lambda_{\text{Comp}} = P_{\text{Comp}} \lambda \), \( \lambda_{\text{Incomp}} = P_{\text{Incomp}} \lambda \), and \( \lambda_{\text{E|Comp}} = q_{\text{Comp}} \lambda \) (the subscripts C and E are for the correct and error responses, respectively). Substituting into Equations...
4–6, the MDV counter model for the standard Simon task is given by

\[ f_{DF}(t; C[\text{Comp}]) = f_{C[\text{Comp}]}(t)\left[1 - F_{E[\text{Comp}]}(t)\right], \tag{11} \]

where

\[ f_{C[\text{Comp}]}(t) = \frac{\lambda_{C[\text{Comp}]} \exp(-\lambda_{C[\text{Comp}]})}{K_{\max} - K_{\min} + 1} \sum_{i=K_{\min}}^{K_{\max}} \frac{\left(i - 1\right)!}{(i + 1)!}, \tag{12} \]

\[ F_{E[\text{Comp}]}(t) = 1 - \frac{\exp(-\lambda_{E[\text{Comp}]})}{K_{\max} - K_{\min} + 1} \sum_{i=K_{\min}}^{K_{\max}} \sum_{j=0}^{i-1} \frac{\lambda_{E[\text{Comp}]}^j}{j!}. \tag{13} \]

The model for error responses is obtained if the subscripts \(C\) and \(E\) are switched, and that for the incompatible trials is obtained if the subscripts \(\text{Comp}\) and \(\text{Incomp}\) are switched.

**Model implementation.** For the present application, the coordinate parameters \(a_1\) and \(a_2\) were set at 1, so the number of free parameters was six in the present case (\(\lambda, b, w_1, K_{\min}, K_{\max}\), and \(t_0\)). They were estimated by using the maximum likelihood method, with the likelihood function \(L(\Theta|D) = \prod f_{DF}(D; \Theta)\), where \(\Theta\) and \(D\) are the set of parameters and the RT data, respectively, and \(f_{DF}\) is the PDF of predicted RT, with \(f_{DF}(t) = f_{DF}(t - t_0)\) and \(f_{DF}\) being as specified by Equation 11. To estimate the threshold boundaries \(K_{\min}\) and \(K_{\max}\), a grid search was used (see Van Zandt, 2000). Both parameters assumed a positive integer between 2 and 15, based on the preliminary examinations of the model. For each pair of the boundaries, a genetic algorithm (Haupt & Haupt, 2004) was used to search for an approximate position of the global maxima first, which was followed by a hill-climbing search with the simplex algorithm (Nelder & Mead, 1965) that used the result of the genetic algorithm as the initial position. Then, the result was further fed back to the genetic algorithm to shuffle out of a possible local minimum (like simulated annealing), which was followed by another simplex routine. This procedure was repeated five times for each pair of \(K_{\min}\) and \(K_{\max}\). Note that parameter estimation was performed on the data pooled across participants. It has been shown that estimation based on pooled data can result in more accurate parameter recovery than the average of parameters estimated based on individual data (e.g., Cohen, Sanborn, & Shiffrin, 2008), especially when the number of replications per condition is small for each participant. Because sample sizes for individual participants were not sufficiently large for the present experiment (and typical Simon experiments in general), the pooled method is more appropriate.

The fitting procedure suggested the following parameters as the best values; \(\lambda = 18.582 \times 10^{-3}\), \(b = 0.610\), \(w_1 = 0.997\) \((w_2 = 0.077)\), \(K_{\min} = 3\), \(K_{\max} = 4\), and \(t_0 = 235\). Figure 4 shows the theoretical PDF predicted by the MDV model (dashed lines) superimposed on the observed RT distributions (i.e., histograms normalized to yield the area under the curve equal the unity) for the compatible and incompatible trials. As apparent, fits to the correct trials were highly successful: The predicted PDF almost entirely overlaps with the empirically estimated density function.

![Figure 4](image-url)  
**Figure 4.** Normalized histograms of the observed response times (RTs) and predicted density functions for the standard Simon task.
However, fits to error data were poorer: The predicted error RT distributions shifted to the right, indicating that predicted RTs tended to be longer than the observed values. This misprediction of error RTs by the counter model has been well documented in previous studies, especially when error RTs are faster than correct RTs (Ratcliff & Smith, 2004; Townsend & Ashby, 1983; Van Zandt, 2000). We attempted to remedy this problem by introducing the variability of response thresholds (see the Appendix), but the modification was not sufficient to resolve the problem. Nevertheless, considering that more than 95% of trials were correct trials in the present experiment, it is possible that the majority of errors were due to processes that are outside the scope of the present model (e.g., loss of vigilance). Also, the contribution of the error data to model fits is proportional to the frequency of error trials, so the model successfully accounted for approximately 95% of the present data. Predicted mean RTs were 422 ms for the compatible trials and 428 ms for the incompatible trials, which compare to the observed values of 420 ms and 435 ms, respectively. The predicted error rates were 3.09% and 5.26% for the compatible and incompatible trials, which compare to the observed values of 3.52% and 4.59%, respectively. Although the result is generally encouraging, the present data set may not be rich enough to test the MDV model framework because the experiment involved only two trial conditions. Hence, the next section tests the model in more complex task conditions.

### Model Applications

To further examine the robustness of the MDV framework, the MDV counter model was tested in five additional experiments that involved far more complexity than the standard Simon task of the previous section. The first two experiments are reported in Yamaguchi and Proctor (2011a), whereas the subsequent three experiments are reported in Yamaguchi and Proctor (2011b), so readers should consult these articles for further details of the experiments. The MDV models were derived for the five experiments based on the standard model developed in the previous section. The fitting procedure was the same as that used earlier.

For those experiments, the goodness of fit was examined in two ways. First, theoretical mean RTs were derived from the MDV counter model for each experimental condition. If the model predictions are sufficiently accurate, these values should be linearly correlated with the observed mean RTs. This linear relationship can be shown graphically by plotting the theoretical mean RTs against the observed mean RTs. In the graph, the plotted points should lie close to the diagonal line (i.e., the linear function, \( \text{theoretical RT} = \text{observed RT} \)), and errors in predictions are represented by the vertical distance from this diagonal line to the plotted points. These observations can be examined statistically in terms of the linear correlation coefficient \( r \) between the theoretical and observed mean RTs and root-mean-square error (RMSE) of the plot.

Second, several model instances were developed for each experiment and compared in terms of their goodness of fit to the data. For this purpose, we computed Bayesian information criterion (BIC) and Akaike’s information criterion (AIC), which are defined, respectively, as

\[
\text{BIC} = -2 \ln(L) + m \ln(N),
\]

\[
\text{AIC} = -2 \ln(L) + 2m,
\]

where \( L \) is the maximized likelihood, \( m \) is the number of free parameters, and \( N \) is the sample size. The smaller these values are, the better the model is. For each measure, the first term represents goodness of model fit, and the second term represents the penalty as a function of the number of free parameters in the model. The penalty is needed because a model may fit better than another just because it involves a greater number of free parameters and is more flexible. Because the information criteria provide a principled manner to penalize according to the flexibility of models, they are usually preferred for the purpose of model selection. However, these measures do not speak to the quality of model fits to the data. Thus, to examine the similarity between model predictions and the actual data, \( r \) and RMSE are more preferred indices than the information criteria. Hence, these two measures complement each other.

### Experiment 1

In Experiment 1, participants responded to the shape of stimuli (circle or square) that could appear at one of eight stimulus locations (designated by outline boxes) arrayed horizontally on the computer screen (see Figure 5, left side). Responses were made by pressing the left or right key on a computer keyboard. This experiment is a simple extension of the Simon task to multiple stimulus locations, and the data provided a good test bed for a unique property of the MDV model that leads to a counterintuitive prediction that RT becomes longer for more compatible stimuli than for less compatible ones.

An MDV representation of the present experimental conditions is shown in Figure 6, which depicts only the conditions where the stimulus shape is a square that is assigned to the right keypress. Let the eight stimulus locations be labeled as L1, L2, . . . , and L8, from the left to right, and squares that appear at these locations be denoted by \( S_1, S_2, \ldots, S_8 \). As described earlier, these stimuli are normally distributed with mean \( \mu_s = x_t^s \mathbf{\mu} = w_x x_{t1} + w_y x_{t2} \) and variance \( \sigma_s^2 = \sum_{i=1}^2 x_t^s x_{t_i} = bx_t^s x_{t_i} = b(x_{t1}^2 + x_{t2}^2) \), where \( x_{t_j} \) is the coordinate in the task-relevant dimension (shape) and \( x_{t_2} \) is that in the task-irrelevant dimension (location). Because \( x_{t_1} \) is the same for all eight stimuli, the distance between stimulus and the decision criterion \( (\mu_s - c) \) increases with \( x_{t_2} \), as can be seen in the lower panel of Figure 6, implying that the degree of compatibility (or incompatibility) increases as the distance from the decision criterion increases. This would suggest that RT decreases as the stimulus locations shifts outward. However, because the variance is proportional to the squared distance of the stimulus from the origin, it increases as the distance between the stimulus and the decision criterion increases. Consequently, the discriminability, as defined in MDV—that is, \((\mu_s - c)\sigma_s^2\) can be smaller for those stimuli that are more compatible with the assigned responses than for those that are less compatible. This property of the MDV model implies that RT can be longer for those stimuli that are more compatible with the assigned response but are more distant from a reference point.

It is possible that the distance effect occurs just because visual stimuli at the peripheral positions have lower resolution, so their perceptual representations are noisy and produce larger variances. Alternatively, it may be that because attention needs to travel
toward the stimulus location over time, the stimulus representation becomes more variable due to changes of the representation caused by the attention shift. Such variability can be larger at more distant locations for which attention needs to travel longer. Regardless of the interpretation of the effect, this property of the MDV model implies that RT can be longer for those stimuli that are more compatible with the assigned response but are more distant from a reference point. Moreover, MDV predicts that the distance effect interacts with the Simon effect, such that the effect is larger for the incompatible trials than for the compatible trials. This is because the probability in Equation 2 is affected more when the distribution is located closer to the decision criterion (i.e., incompatible trials) than when it is located farther away (compatible trials). These predictions of the MDV model are rather counterintuitive and cannot be predicted by the existing models of the Simon effect. In fact, a contrary result can be predicted from previous models that assume passive decay of the Simon effect with prolonged perceptual processing due to the eccentricity of stimuli (i.e., a smaller Simon effect at outer locations than for inner locations; e.g., De Jong et al., 1994; Hommel, 1994; Zorzi & Umilta, 1995). Therefore, should these predictions be experimentally confirmed, MDV receives strong support for its validity as a model of the Simon effect.

Figure 5. The displays used in Experiment 1 (left) and Experiment 2 (right).

Figure 6. A multidimensional vector representation of Experiment 1 (only experimental conditions with square stimuli are shown).
The results are summarized in Figure 7 (left side). As predicted, responses for the compatible trials were slower for the outer stimulus locations than for the inner stimulus locations, despite the fact that the former locations were more compatible with the corresponding responses. The same pattern appears for the incompatible trials, but the distance effect for these trials was apparently greater than that for the compatible trials. As a result, the Simon effect increased from the inner to outer locations. At the individual-subject level, the majority of the participants showed a pattern consistent with the averaged data. The results are generally consistent with the MDV predictions.

Although consistent, the results can also be interpreted differently. For instance, the slowing of responding may be due to a peripheral or anatomical factor; that is, given that the fixation cross was presented on each trial, it took longer to saccade to outer locations than to inner locations. Such a model can be constructed in the MDV framework by assuming an effect of stimulus eccentricity on the nondecision time \( t_0 \), which increases with the distance from the fixation cross. The model is a viable alternative and produces two predictions that are different from the earlier model. First, the model predicts no distance effect on PE because the variability in PE emerges from the decision process. Second, the model should predict that the effect of eccentricity is additive with the Simon effect; that is, responses on the compatible and incompatible trials should be influenced by the eccentricity equally. Both predictions are inconsistent with the data; PE increased with the eccentricity, which is especially clear in the incompatible trials, and the eccentricity effect was larger for the incompatible trials than for the compatible trials.

At the same time, it is also possible that the response slowing occurred due to change in the response threshold with stimulus eccentricity. Because the response threshold is typically assumed to be preset before stimulus presentation, it is not common to assume dynamic changes of the parameter according to trial conditions that are intermixed. Yet it is still possible that participants could reactively heighten the threshold when stimuli occur at outer positions. Thus, the model is also a viable alternative. A problem of the model rests on its prediction; change of the response threshold with stimulus eccentricity \( x_1 \) is set equal to 1 for all trials, and this parameter determines the scale of the psychological space. The coordinate in the task-relevant dimension (location; \( x_2 \)) varies according to the compatibility between the stimulus location and the response location. If the stimulus location in the 4th condition is compatible with the correct response, \( x_2 \) is positive; if it is incompatible, \( x_2 \) is negative.

For the coordinate model, the magnitude of \( x_2 \) varies according to how far away it is located from the screen center. From the near to far locations, we define \( x_2 = a_1, a_2, a_3, \) or \( a_4 \), where \( 0 < a_1 < a_2 < a_3 < a_4 \). Two model variations were tested for these parameters. In the first model (free-coordinate model), the stimulus coordinates in the task-relevant dimension (\( a_1 \)) were unconstrained, so they were free parameters to be estimated. In the second model (fixed-coordinate model), the stimulus coordinates were fixed at arbitrary values; \( (a_1, a_2, a_3, a_4) = (0.25, 0.5, 0.75, 1) \). For both variations, the nondecision time \( t_0 \) was set equal for all trials. There were six free parameters for the fixed-coordinate model and 10 for the free-coordinate model.

The nondecision-time and response-threshold models were basically the same as the MDV counter model for the standard Simon task described in the previous section (Equations 9–13). For the nondecision-time model, we assumed that \( t_0 \) was a linear function of the stimulus eccentricity; that is, \( t_0 = \alpha + \gamma e_i \), where \( e_i \) is the eccentricity \( e_i = 1, 2, 3, 4 \) (from inner to outer locations) and \( \alpha \) and \( \gamma \) are constants. For the response-threshold model, the response threshold was allowed to freely across the stimulus locations. To avoid including too many free parameters, \( K_{\text{min}} \) and \( K_{\text{max}} \) were constrained to be equal at each stimulus location for this model. Neither model involved the variability in the spatial coordinates; that is, \( a_i = 1 \) for all \( i \).

**Model-fitting results.** The fitting results are summarized in Table 1. The best parameters estimated for the two coordinate models are similar. The free-coordinate model suggested that the stimulus coordinates for the two inner positions were nearly identical with those of the fixed-coordinate model but that the two outer positions were less spread than them. The free-coordinate model had a greater penalty for the number of free parameters, but it still yielded smaller BIC and AIC. On the other hand, the fixed-coordinate model showed slightly smaller RMSE and larger r, suggesting that the predictions of the model were more similar to the data than the predictions of the free-coordinate model at the level of mean RTs. The inconsistency between the information criteria and the similarity measure is not necessarily surprising because they measure different aspects of the goodness of fit. In any case, both models provided good accounts of the data. On the other hand, the nondecision-time and response-threshold models

\[
\lambda_{c(t_0)} = \lambda \Phi \left( \frac{w_1 x_1 + w_2 x_2}{\sqrt{b(x_1^2 + x_2^2)}} \right) \tag{14}
\]

for correct responses and

\[
\lambda_{e(t_0)} = \lambda \left[ 1 - \Phi \left( \frac{w_1 x_1 + w_2 x_2}{\sqrt{b(x_1^2 + x_2^2)}} \right) \right] \tag{15}
\]
were less successful as compared to the coordinate models. The nondecision-time model predicted the increasing RTs with stimulus eccentricity, and the correlation for mean RTs was as high as those of the preceding two models. However, the correlation for PEs was lower than those obtained for the coordinate models. Also, BIC and AIC were substantially higher than those of the coordinate models. The response-threshold model provided information criteria values that were as small as those of the coordinate models, which was somewhat surprising. Yet the correlation between predictions and observations was remarkably lower. The best parameters show flat response thresholds for the four stimulus eccentricity. This result is due to the fact that change of the response threshold would alter the model predictions too much, given that other parameters were held constant across the conditions.

Overall, the coordinate models provided better accounts of the present data. The predictions of the two coordinate models are similar, so we focus on those of the fixed-coordinate model, which are shown in Figure 8. Figure 8A shows a scatterplot of the predicted and observed mean RTs. The points lie near the diagonal line that represents the perfect fit of the model to the data (i.e., theoretical RT = observed RT). The linear correlation between the predicted and observed RTs was very high ($r = .992$), which implies that more than 98.4% of the variability in the observed mean RTs was accounted for by the model ($r^2 = .984$). To see the pattern of the predicted and observed RTs, they were plotted as a function of trial condition in Figure 8B. As described earlier, observed RTs were generally shorter for the compatible trials than for the incompatible trials, and there was an increasing trend of RT from inner to outer stimulus positions. This trend was more evident for the incompatible trials than for the compatible trials. These patterns correspond to the observations. Finally, Figure 8C depicts the observed RT distributions and the theoretical PDFs predicted by the fixed-coordinate model. Given that only six free parameters were needed to generate the predictions (only five free parameters were actually effective, since the upper and lower boundaries of response threshold were identical), the similarity between the theoretical and observed RT distributions is remarkable.

Experiment 2

Experiment 2 was structurally similar to Experiment 1 but provided a distinct pattern of the data. As in Experiment 1, the present experiment involved eight stimulus locations horizontally arrayed on the screen (see Figure 5, right side). The imperative stimulus was a solid square colored in green or red. Participants were instructed to ignore the stimulus location and respond to the stimulus color by pressing the left or right key. Unlike Experiment 1, the display was clearly partitioned into left and right halves by larger frames, and two underline placeholders appeared to one side or the other of a centered cross in one of the frames, prior to onset of the imperative stimulus, designating both the frame in which the stimulus would appear and whether it would be to the left or right of the cross.

In contrast to Experiment 1, there was an oscillating pattern of RTs across stimulus locations (see Figure 7, right side). Similar results were obtained by Lamberts, Tavernier, and d’Ydewalle (1992) with a similar method, and they suggested that this Simon effect pattern could be explained if the eight spatial positions were represented in terms of three spatial dimensions: hemispace, for which the target stimulus was defined with respect to the left or right side of the screen center; hemifield, for which the target was defined with respect to the left or right side of the cross that appeared in the hemispace; and relative position, for which the target was defined with respect to whether it appeared above the left or right placeholder in the hemifield. We tested this claim by fitting three versions of the MDV counter model. The first version is the fixed-coordinate model of Experiment 1. This model assumed that the spatial representation for the present task was one-dimensional (1-D), as opposed to the three-dimensional (3-D) representation proposed by Lamberts et al. The second and third versions assumed that the spatial representation involved three proposed dimensions, but they differed with respect to specifications of the stimulus coordinates (continuous vs. categorical).

Model instantiation. The accumulation rates for the 1-D model are given by Equations 14 and 15. Those for the 3-D versions are derived as follows. The model assumes that the

Figure 7. Mean response times (RTs; in milliseconds) and percentage errors (PEs) as a function of stimulus location (L1–L8) and response location (left, right) in Experiment 1 (left) and Experiment 2 (right).
underlying psychological space consists of four dimensions: one task-relevant dimension (color) and three task-irrelevant dimensions (hemisphere, hemifield, relative position). Hence, the accumulation rates for the $i$th trial condition are

$$\lambda_{c(i)} = \lambda \Phi \left( \frac{w_1 x_i + w_2 x_{i2} + w_3 x_{i3} + w_4 x_{i4}}{\sqrt{b(x_{i1}^2 + x_{i2}^2 + x_{i3}^2 + x_{i4}^2)}} \right)$$

(16)

for correct responses and

$$\lambda_{e(i)} = \lambda \left[ 1 - \Phi \left( \frac{w_1 x_i + w_2 x_{i2} + w_3 x_{i3} + w_4 x_{i4}}{\sqrt{b(x_{i1}^2 + x_{i2}^2 + x_{i3}^2 + x_{i4}^2)}} \right) \right]$$

(17)

for error responses, where $x_{ij}$ (task-relevant dimension) is set to be 1 for all $i$ and $x_{ij} (j = 2, 3, 4)$ varies according to the relationship between the stimulus location and the correct response location in the $i$th trial condition. The subscript $j = 2$ corresponds to hemisphere, $j = 3$ to hemifield, and $j = 4$ to relative position. The coordinate parameter $x_j$ is set $a_j$ if the stimulus location in the $i$th dimension is compatible with the correct response in the $i$th condition and $-a_j$ if it is incompatible.

The first version of the 3-D model (3-D categorical) assumed that the magnitude of $x_j$ is constant (i.e., $a_j = 1$ for all $i$ and $j$), which is equivalent to assuming that each dimension is categorical or binary (left or right), and the magnitude in the respective dimension is not consequential. On the other hand, the second 3-D model (3-D continuous) assumed that the magnitudes of $x_j$ vary according to the distance from the respective reference point. That is, in the hemispace, the reference point is the center of the screen, so the stimulus locations L1, L2, L3, and L4, for example, have the magnitudes $a_{12} = 1$, $0.75$, $0.5$, and $0.25$, respectively, as specified in the fixed-coordinate model of Experiment 1. In the hemifield, the reference point is the center of a rectangular frame, which is located between L2 and L3, so L1, L2, L3, and L4 have the magnitudes $a_{12} = 0.5$, $0.25$, $0.25$, and $0.5$, respectively. In the relative position, the reference point is at the middle of the two horizontal bars, so all stimulus locations have the magnitude $a_{14} = 0.25$. Note that the numbers of free parameters in the two versions of the 3-D model are the same ($m = 8$) because these coordinates are prespecified as in the fixed-coordinate model of Experiment 1. There were six free parameters for the 1-D model.

**Model-fitting results.** The results of fitting are summarized in Table 2. We expected that the two 3-D models would perform better than the 1-D model would, which was generally confirmed. Most remarkably, as expected, the 1-D model was unable to produce the oscillating pattern of RTs, producing a poor similarity measure ($r = .459$). Although the two 3-D models were similar, the continuous version fit better in terms of the information criteria, so we focus on that model.

The comparisons between the predictions of the 3-D continuous model and the experimental data are shown in Figure 9. Figure 9A shows that the predicted RTs lie close to the diagonal line, suggesting that the model accounted well for the RT pattern observed in the experimental data, except for one data point which represented the trial condition labeled $kE$ in Figure 9B. As mentioned, the model predictions captured the most salient feature of the data, the oscillating pattern of RT across stimulus locations, and the amplitude of the oscillation became smaller for the outer positions than for the inner positions, which is also consistent with the
observed pattern. The model fit was reasonably good ($r = .856$), although not as good as that obtained in Experiment 1, which is possibly because the present experimental conditions were more complex and noisier than those of Experiment 1. At the individual-subject level, there was larger between-subject variability in the present experiment as compared to that of Experiment 1, suggesting that different individuals might have adopted different modes of encoding the stimulus locations. Also, there could have been unknown factors not accounted for by the present model, but we do not try to speculate on such a factor here. Importantly, Figure 9C shows that the theoretical distributions are very similar to the observed ones.

Experiments 3–5

The following three experiments involved further complexity in the task (see Yamaguchi & Proctor, 2011b, for details). The experiments used a version of the Simon task for which there were multiple response components that could establish SRC independently of each other. Whereas several previous studies (including the present Experiment 2) demonstrated that the Simon effect occurs based on multiple stimulus dimensions, the present experiments showed that the Simon effect could be produced based on multiple response components. Also, it was found that the contributions of these components were dependent on the task require-
ments, suggesting that attentional factors played a key role in producing the Simon effect. We propose possible models for the data sets provided by the three experiments.

In the experiments, participants were presented with an aviation display and asked to control the aircraft by using a flight yoke. The imperative stimuli consisted of one task-relevant dimension (tone pitch for Experiment 3, and stimulus color for Experiments 4 and 5) and one task-irrelevant spatial dimension, as in the standard Simon task. Thus, participants were instructed to turn the aircraft to the right for one tone pitch or stimulus color and to the left for the other tone pitch or stimulus color, while ignoring the spatial information (i.e., the side of the ear to which tones were presented or the location of colored stimuli).

The attitude information (i.e., roll) of the aircraft was simulated on the display located in front of the participants. Two groups of participants used different display formats, known as horizon-moving and aircraft-moving displays (see Figure 10). For the aircraft-moving display, the aircraft symbol on the screen moved according to the simulated aircraft’s attitude. Thus, the symbol moved in the same direction as the movement of the simulated aircraft (e.g., a right rotation of the aircraft is shown as a right tilt of the aircraft symbol in the display). This display presented a motion compatible with the actual rotation direction of the simulated aircraft. Therefore, when the aircraft was rotated to the direction compatible with the stimulus location, the display motion was also compatible with the stimulus location. On the other hand, for the horizon-moving display, the aircraft symbol was stationary at the center of the screen, but the horizon symbol moved to represent the attitude of the simulated aircraft. In this format, the horizon symbol moved in an opposite direction to the movement of the aircraft (e.g., a right rotation of the aircraft is shown as a left tilt of the horizon symbol in the display). Thus, this display presented a motion incompatible with the actual rotation direction of the aircraft. Therefore, when the aircraft was rotated to the direction compatible with the stimulus location, the display motion was incompatible with the stimulus location. Consequently, the use of these display formats dissociated the compatibility between stimulus location and the aircraft movement from the compatibility between stimulus location and the display motion.

At the same time, we varied the relationship between the simulated aircraft and the control device. In one trial block, the relation was normal; to turn the aircraft, participants needed to turn the flight yoke to the desired direction, so the condition was compatible. When the aircraft was rotated to the direction compatible with the stimulus location, the flight yoke also rotated to the direction compatible with the stimulus location. In the other block, the relation was reversed; to turn the aircraft, participants needed to turn the flight yoke to an opposite direction to the desired one, so the condition was incompatible. When the aircraft was rotated to the direction compatible with the stimulus location, the flight yoke rotated to the direction incompatible with the stimulus location. The use of these control relations dissociated the compatibility between stimulus location and the aircraft movement from the compatibility between stimulus location and the control direction.

To summarize, these manipulations dissociated between three response components and their compatibility relationships with the target stimuli. The response components included the movement directions of (a) simulated aircraft, (b) display symbol, and (c)
hands that operated the flight yoke. Each response component had two alternative values (left and right), so there was a total of eight trial conditions in which the compatibility relations between the stimulus location and the three response components orthogonally varied (see Table 3). In all conditions, tasks were instructed in terms of the simulated aircraft. Thus, participants intended to move the aircraft, not the display object or the flight yoke, to the left or right according to the pitch of tones or the color of stimuli.

As can be seen in Figure 11, the patterns of the Simon effect differed across the experiments, indicating that the influences of the three response components varied. In Experiment 3, the Simon effect was significant with respect to the three response components, but the influence of display format was eliminated in Experiments 4 and 5. Moreover, Experiment 5 showed that the Simon effect emerged based only on the aircraft movement. We interpreted these results as suggesting that the influences of response components are dependent on selective attention to the respective response components. Also, throughout the three experiments, RT was longer for the reverse-control than for the normal-control, which indicates the effect of compatibility between the simulated aircraft and the operating hand. This type of compatibility effect is known as response–effect (R-E) compatibility effect (Kunde, 2001) or tool–hand correspondence effect (Kunde, Müsseler, & Heuer, 2007). This compatibility effect was independent of the

Figure 9. Comparisons between the predictions of the categorical model and the observed response times (RTs) in Experiment 2. A: Scatterplot of the predicted and observed mean RTs. B: Predicted and observed mean RTs and percentage errors (PEs) as a function of trial condition (Co = compatible, Ic = incompatible, 1–4 = innermost to outermost stimulus locations). C: Normalized histograms of the observed RTs and the predicted density function of the 3-D continuous model.
Simon effect because it remained significant even if the influence of hand movement disappeared in the Simon effect in Experiment 5. Also, the task involved compatibility between the simulated aircraft movement and the movement of the display symbol (aircraft or horizon). However, the effect was not statistically significant in any of the experiments.

**Model instantiation.** Because stimuli were two-dimensional in these experiments, the stimulus conditions can be described as in those for the standard Simon task (see Figure 3). That is, any stimulus condition can be expressed by the vectors \( \mathbf{x}_i = (x_{i1}, x_{i2}) \) with \( i = 1, 2, 3, \) and \( 4 \). To express influences of multiple response components, separate decision axes were assumed for distinct elements, with the subscripts representing aircraft \( 1, \) hand \( 2, \) and display \( 3 \). Hence, the projections of the response components on the respective decision axes are distributed with the mean vectors \( \mathbf{p}_1, \mathbf{p}_2, \) and \( \mathbf{p}_3, \) respectively. This means that the mean vectors represent the amount of attention allocated to the respective stimulus dimensions, which is assumed independent of the response components. This means that the mean vectors are expressed by \( \mathbf{p}_h = (w_{1h}, \pm w_{2h}) \) for all \( h \), where the sign of the second elements is determined by the consistency of the compatibility relations across the response components.\(^2\) Given \( \mathbf{x}_i \), the probability that a stimulus sample indicates the response \( A \) is given by

\[
p_{A_{X_{i}},h} = \Phi \left( \frac{x_{i1}w_{1h} \pm x_{i2}w_{2h}}{\sqrt{b_{1h}(a_{1h}^2 + a_{2h}^2)}} \right) \tag{18}
\]

and that for the response \( B \) as \( p_{B_{X_{i}},h} = 1 - p_{A_{X_{i}},h} \). For simplicity, we assumed \( b_h = b \) for all \( h \).

Furthermore, attention to the three response components was modeled by varying the base accumulation rates \( (\lambda_1, \lambda_2, \) and \( \lambda_3) \) associated with the respective decision axis. This may mean that the actor prepares to react to an incoming signal by activating the relevant set of actions by increasing the base activation of these action representations. The degree to which the response component \( h \) contributes to the eventual response activation is given by the ratio \( \pi_h = \lambda_h / \lambda \) where \( \lambda = \sum_{h=1}^{m} \lambda_h \) where \( m \) is the number of relevant response components (= 3 for the present case). In turn, each of the base accumulation rates can be expressed by \( \lambda_h = \pi_h \lambda \), where \( \sum_{h=1}^{m} \pi_h = 1 \). Consequently, given the stimulus \( \mathbf{x}_i \), the accumulation rate for the response \( A \) on the decision axis \( v_h \) is determined to be \( \lambda_{A_{X_{i}},h} = p_{A_{X_{i}},h} \pi_h \lambda \) and the accumulation rate for the correct response is given by

\[
\lambda_{A_{X_{i}},h} = \sum_{h=1}^{m} \lambda_{A_{X_{i}},h} = \lambda \sum_{h=1}^{m} p_{A_{X_{i}},h} \pi_h \lambda \tag{19}
\]

where \( h = 1 \) for aircraft, \( 2 \) for display, and \( 3 \) for hand. The present model is expressed with reference to the correspondence between the stimulus location and the movement direction of aircraft because the response component was task relevant. Hence, given \( \mathbf{p}_h = (w_{1h}, \pm w_{2h}) \), the probability of a stimulus sample indicating the correct response on the decision axis \( h \) may be derived by

\[
p_{\text{Comp},h} = \Phi \left( \frac{a_{1h}w_{1h} \pm a_{2h}w_{2h}}{\sqrt{b(a_{1h}^2 + a_{2h}^2)}} \right) \tag{20}
\]

for the compatible trials and

\[
p_{\text{Incomp},h} = \Phi \left( \frac{a_{1h}w_{1h} \pm a_{2h}w_{2h}}{\sqrt{b(a_{1h}^2 + a_{2h}^2)}} \right) \tag{21}
\]

for the incompatible trials. The present experimental conditions were even more complex than those of Experiment 2, but the main model parameters were essentially the same. For the present experiments, seven free parameters \( (K_{\text{max}}, K_{\text{min}}, \lambda_1, \lambda_2, \lambda_3, w_1, \) and \( r) \) were needed to predict DT.

\(^2\)To understand this formulation, let us assume that the response \( A \) consists of left aircraft movement, left display movement, and right hand movement, and the response \( B \) consists of right aircraft movement, right display movement, and left hand movement. In such a condition, the movements of aircraft \( (v_r) \) and display \( (v_d) \) are consistent, whereas that of hand \( (v_h) \) is inconsistent with the other two components. Then, if the responses \( A \) and \( B \) are mapped to red and green stimuli, respectively, the decision axes that represent the aircraft movement and the display movement have the mean vector \( \mathbf{p}_1 = (w_1, w_2) \), whereas the decision axis that represents the hand movement has the mean vector \( \mathbf{p}_3 = (w_1, -w_2) \). In other words, if two response components are consistent, the second element of the mean vector, \( w_2 \), has the same sign for the decision axes that represent these components, but if they are inconsistent, \( w_2 \) has the opposite sign.

Figure 10. The aircraft-move (A) and horizon-move (B) formats of the attitude indicator used in Experiments 3–5 (see Yamaguchi & Proctor, 2011b).
As mentioned earlier, there were two additional compatibility relations involved in the task. Although they are theoretically important, we do not attempt to model these additional components but simply added nondecisional constants $t_1$ and $t_2$. For the R-E compatibility, we introduced an indicator variable $I_1$, defined by

$$I_1 = \begin{cases} 
0 & \text{if control is normal} \\
1 & \text{if control is reversed}
\end{cases}$$

with the R-E compatibility effect given by $t_1I_1$. Similarly, for the display compatibility, we introduced an indicator variable,$\]

$$I_2 = \begin{cases} 
0 & \text{if the display is aircraft-moving} \\
1 & \text{if the display is horizon-moving}
\end{cases}$$

with the display compatibility effect being $t_2I_2$. Consequently, RT for this model is given by

$$RT = DT + t_0 + t_1I_1 + t_2I_2. \quad (22)$$

Because the display compatibility effect was not statistically significant, both model variations with and without the display effect were fitted to the three experiments separately in the same manner as in the preceding experiments.

Model-fitting results. The fitting results are summarized in Table 4. Excellent fits were obtained for all experiments in terms of the similarity measures, despite the fact that several simplifications were made in the model. For Experiment 3, the models with and without the display compatibility component fit to the data equally well, but because of the greater penalty for the model with display compatibility, the fit was better without the component in terms of the information criteria, suggesting that the parameter was not effective in accounting for the data variability across conditions. For both models, the base accumulation rate for the aircraft movement ($\lambda_1$) was largest, followed by the rate for the hand movement ($\lambda_2$), and that for the display movement ($\lambda_3$) was smallest, suggesting the least contribution of the last factor. These results indicate that the task context was represented based primarily on the direction of aircraft movement, which was expected because this component was task relevant (i.e., participants were instructed on the task in terms of aircraft movement). Thus, the recovered parameter values are consistent with the observed data. The comparisons between the observed RT and the predictions of the model without assuming the display effect are shown in Figure 12. The correlation between the data and the model predictions was very high ($r = .942$), and the overall errors of the predictions were small ($RMSE = 8.44$ ms).

For Experiments 4 and 5, the model assuming display compatibility outperformed the model without that component. The correlation between the data and the model predictions was evidently higher for the former model, accounting for more than 90% of the variability in the data in both cases. As can be seen in Figures 13A and 14A, the data for these experiments were more scattered than those of Experiment 3. Consequently, the variability between experimental conditions was larger, resulting in larger $RMSE$ as compared to that obtained for Experiment 3. Nevertheless, Figures 13B and 14B suggest that the model predictions capture the observed RT pattern quite well. The RT distributions appear to involve greater noise than those of Experiment 3 (see Figures 13C and 14C). Thus, the theoretical PDFs were somewhat off from the observed distributions in some of the conditions, but they were not far from the observations.

According to the parameters recovered for Experiment 4, there was an increase in the base accumulation rate for aircraft movement ($\lambda_1$) and a decrease for hand movement ($\lambda_2$), as compared to Experiment 3. These results reflect the dependency of the Simon effect on these response components. Although the influence of the

Table 3

<table>
<thead>
<tr>
<th>Compatibility relation</th>
<th>Aircraft-move</th>
<th>Horizon-move</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal-control</td>
<td>Reverse-control</td>
</tr>
<tr>
<td>Stimulus-aircraft ($h = 1$)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Stimulus-display ($h = 2$)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Stimulus-hand ($h = 3$)</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Note. Positive sign = compatible with the stimulus location; negative sign = incompatible with the stimulus location.

Figure 11. The Simon effect as a function of display format (aircraft-moving or horizon-moving) and control relation (normal or reverse) in Experiments 3–5 (see Yamaguchi & Proctor, 2011b).
display movement ($\lambda_2$) was statistically nonsignificant, the model suggests that the influence actually increased in Experiment 4 as compared to Experiment 3. This may explain the fact that the model with display compatibility resulted in a better fit in that experiment. Moreover, for Experiment 5, the accumulation rate for the aircraft movement was very high, and those for the other two components were very small. This is consistent with the results obtained in the data, suggesting that the Simon effect occurred based only on the aircraft movement. The recovered dimensional saliencies for the task-relevant and -irrelevant stimulus dimensions ($w_1$ and $w_2$) are very similar across experiments. In fact, the three experiments show similar magnitudes of the Simon effect in the condition for which the compatibilities of the three components were consistent (i.e., the normal-control condition with the aircraft-moving display). Once again, the recovered parameters agree with the obtained results.

Last, although the model fits were reasonably good for RT, those for PE were less so. Because the current MDV model was developed to model the Simon effect, it does not account for other influential factors (i.e., the display and R-E compatibility effects). Especially, R-E compatibility appeared to have a significant influence on PE because the reverse-control condition produced more errors in general. This explains the lower correlations between the model predictions and the observed PE data throughout the three experiments. Possibly, a second counter process could be added to account for the R-E compatibility effect in RT and PE. Because our main focus in the present research is to model SRC, we leave the possible extensions of the present models to future investigations.

### Summary and Discussion of Model-Fitting Results

Researchers have argued that there are at least two different forms of representation for spatial information. The first form, called coordinate spatial relations (Kosslyn, 1994) or perceptual representations (Logan, 1994), consists of metric information of objects. The second form, called categorical spatial relations (Kosslyn, 1994) or conceptual representations (Logan, 1994), consists of semantic or propositional information of objects (e.g., left, right, above, or below). Most accounts of the Simon effect have assumed the latter form of stimulus and response representations, whereas MDV adopts a metric representation of the task context. The results of Experiments 1 and 2 suggest that metric models performed better. The variable degrees of the Simon effect of Experiment 1 could not be produced by the decision-time and response-threshold models that assumed binary stimulus coordinates (left and right of the fixation), which is equivalent to the categorical-coding notions assumed in previous models. Also, Experiment 2 suggested that the continuous model accounted for the data better, although the categorical model still performed at a satisfactory level. Therefore, the metric framework developed here is a novel contribution to the field.

Similarly, Experiment 1 confirmed the unique prediction of the MDV model that the Simon effect increases with the eccentricity of stimuli, while RT also increased with the eccentricity. It may be worth mentioning that in a recent Simon study (Klein, Dove, Ivanoff, & Eskes, 2006) for which participants responded to the color of stimuli by pressing left/right, top/down, or two diagonal keys to stimuli arranged in two layers of clock-face positions, the

### Table 4: Parameter Estimates of the Multidimensional Vector Counter Models in Experiments 3, 4, and 5

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment and model</td>
<td>$\lambda_1 \times 10^4$, $\lambda_2 \times 10^4$</td>
</tr>
<tr>
<td>Experiment 3</td>
<td>8.493, 1.061</td>
</tr>
<tr>
<td>Without display comp.</td>
<td>6.857, 1.153</td>
</tr>
<tr>
<td>With display comp.</td>
<td>8.304, 1.137</td>
</tr>
<tr>
<td>Experiment 4</td>
<td>11.613, 1.141</td>
</tr>
<tr>
<td>Without display comp.</td>
<td>11.591, 1.137</td>
</tr>
<tr>
<td>With display comp.</td>
<td>11.613, 1.141</td>
</tr>
<tr>
<td>Experiment 5</td>
<td>9.129, 1.124</td>
</tr>
<tr>
<td>Without display comp.</td>
<td>9.032, 1.124</td>
</tr>
<tr>
<td>With display comp.</td>
<td>9.129, 1.124</td>
</tr>
</tbody>
</table>

Note: Values in brackets are not free parameters. $\lambda_1$ and $\lambda_2$ are correlation coefficients for mean response times; $\beta$ and $\gamma$ are correlation coefficients for percentage errors; $\lambda_1$ is the base accumulation rate for aircraft; $\lambda_2$ is the base accumulation rate for hand.
The magnitude of the Simon effect was also found to be a function of the angular distance from the response locations (e.g., left–right responses were faster when stimuli appeared at the horizontal positions, slower when stimuli appeared at the diagonal locations, and slowest when stimuli appeared at the vertical locations; see also Simon & Wolf, 1963, for similar results). These findings are consistent with MDV since the stimulus utility in the spatially overlapping dimension decreases as the angular distance increases.

Klein et al. (2006) also found that responses were generally faster when stimuli occurred at the inner layer (near the fixation) than when they occurred at the outer layer, which is also consistent with the property that the variance of subjective utility increases with the absolute distance from the origin of the space.

Of course, these outcomes could be modeled differently. For instance, one could have attributed the increased RT for more peripheral stimuli to nondecision time and the increased Simon effect to the accumulation rate, which is also a function of the eccentricity. However, we suggest that the MDV models (i.e., the coordinate models) provide more parsimonious accounts because, in these models, both effects arise from the way stimuli are represented in the multidimensional space. One could falsify the present assumption by dissociating the eccentricity effect from the

Figure 12. Comparisons between the predictions of the multiple-response component model and the observed response times (RTs) in Experiment 3. A: Scatterplot of the predicted and observed mean RTs. B: Predicted and observed mean RTs and percentage errors (PEs) as a function of compatibility (Co = compatible, Ic = incompatible), control relation (normal, reverse), and display format (aircraft-move, horizon-move). C: Normalized histograms of the observed RTs and the predicted density function of the multidimensional vector model without display effect (Aircraft = aircraft-move, Horizon = horizon-move).
Simon effect. Thus, whether this property of the MDV model holds in psychological domains other than that of spatial representation will be interesting to see.

We also demonstrated that the MDV model can be used effectively to compare alternative hypotheses. In Experiment 2, for example, the model-fitting results suggested that the data are better accounted for by the 3-D models as speculated by Lamberts et al. (1992). The 1-D model that accounted for Experiment 1 failed to capture the most salient pattern of the data, suggesting the flexibility of stimulus encoding in the Simon task. This observation also contradicts the traditional assumption of strict automaticity in previous models of the Simon effect. More recent studies also showed that the Simon effect can occur based on multiple stimulus dimensions (e.g., Lleras, Moore, & Mordkoff, 2004; Wühr, Biebl, & Ansorge, 2008), suggesting that the multidimensionality should be a default mode of modeling the Simon effect. Finally, Experiments 3–5 showed that the Simon effect emerged based on multiple response components, and we proposed an MDV model to account for the findings. The results were generally encouraging, although there were certain mispredictions of the model for these experiments (e.g., those of PEs), which are most likely due to factors that are outside of the present model (i.e., R-E compatibil-

Figure 13. Comparisons between the predictions of the multiple-response component model and the observed response times (RTs) in Experiment 4. A: Scatterplot of the predicted and observed mean RTs. B: Predicted and observed mean RTs and percentage errors (PEs) as a function of compatibility (Co = compatible, Ic = incompatible), control relation (normal, reverse), and display format (aircraft-move, horizon-move). C: Normalized histograms of the observed RTs and the predicted density function of the multidimensional vector model with display effect (Aircraft = aircraft-move, Horizon = horizon-move).
ity). Thus, further improvements of the model would be beneficial in future studies. Nonetheless, the estimated parameters for these experiments closely matched the statistical analyses of the experimental data, and the resulting values were reasonably interpretable, suggesting that the model successfully recovered the cognitive parameters underlying the task performance.

Some considerations of the parameter estimates are in order. For most of the models tested in the present research, the upper and lower boundaries of response threshold ($K_{\text{min}}$ and $K_{\text{max}}$) resulted in identical values. This implies that the trial-to-trial variability in the response threshold did not help increase the goodness of fit to the present data. The variability in the response threshold was thought to be important to account for fast error responses, the condition in which mean RT for error responses is faster than mean RT for correct responses (e.g., Brown & Heathcote, 2008; Ratcliff & Smith, 2004). Without this assumption, the counter model predicts that mean RT for correct responses will always be faster than that for error responses (Townsend & Ashby, 1983; Van Zandt, 2000). This inability to produce fast error responses has been taken seriously as indicating incorrectness or incompleteness of the model (Brown & Heathcote, 2008; Ratcliff & Smith, 2004). The inclusion of the variable response threshold allows fast error responses to be accounted for.

![Figure 14](image_url)

**Figure 14.** Comparisons between the predictions of the multiple-response component model and the observed response times (RTs) in Experiment 5. A: Scatterplot of the predicted and observed mean RTs. B: Predicted and observed mean RTs and percentage errors (PEs) as a function of compatibility (Co = compatible, Ic = incompatible), control relation (normal, reverse), and display format (aircraft-move, horizon-move). C: Normalized histograms of the observed RTs and the predicted density function of the multidimensional vector model with display effect (Aircraft = aircraft-move, Horizon = horizon-move).
responses because error trials are predominantly those for which the response threshold for the incorrect response happens to be lower than that for the correct response, so that a slow (error) response alternative can win the race. Yet fits of the present counter model to error data were still poor (but their predictions of the error probabilities are reasonably similar to the observed values; see the respective figures). There might be a fundamental difficulty fitting a counter model to error RT distributions (see also Van Zandt, 2000; Van Zandt, Colonius, & Proctor, 2000). This aspect of the present formulation of the MDV model is subject to future scrutiny.

### General Discussion

A main consequence of the MDV model framework is an extension of stochastic RT models to account for SRC by incorporating theoretical constructs developed in psychological scaling models. The other side of this consequence is an extension of the Thurstonian framework to account for choice RT. In principle, a sequential sampling model should be capable of reproducing any RT distribution if the parameters are allowed to vary freely across different conditions (Dzhafarov, 1993). Thus, it is possible to apply a sequential sampling model to the SRC paradigm by allowing parameters to vary freely across compatible and incompatible trials. Such a modeling approach may still be meaningful in that it is a way to summarize experimental data by taking detailed aspects of RT distributions that go beyond conventional analyses of means (e.g., Balota & Yap, 2011; Heathcote, Popiel, & Mewhort, 1991), but the present research went a step farther. The MDV models assumed formal relations between parameters across experimental conditions and provided constraints on the parameter recovery. In this sense, MDV provides a rationale behind modeling the SRC paradigm. The choice of the vector model is, however, rather arbitrary, and there could be other scaling models that might have been adopted for the present purpose. We consider such alternative possibilities in comparison to the present MDV model approach.

### General Recognition Theory

One of the most general forms of the multidimensional Thurstonian model (or signal detection theory) is GRT (Ashby & Perrin, 1988; Ashby & Townsend, 1986). In GRT, stimulus representations are multivariate random variables with arbitrary distribution functions. The multidimensional space is partitioned into response regions with linear or nonlinear boundaries, and categorization is performed according to which response region a stimulus sample falls in. In contrast to GRT, MDV assumes that stimulus representations are constants in a multidimensional space, and not the space but the decision axis is partitioned to represent alternative response categories. Although the two frameworks differ in many aspects, there are also formal relationships between GRT and MDV. For instance, a GRT model with a linear boundary has a corresponding MDV model with the decision axis that is perpendicular to the linear response boundary. This can be depicted schematically as in Figure 15. In the linear-boundary GRT model, all stimulus samples that fall below a linear function \( y = ax + b \) are categorized as one response (in a gray zone). Note that the intercept \( b \) can always be set equal to zero (i.e., the response boundary passes through the origin) because the origin of the space is arbitrary in GRT. Then, the line perpendicular to the response boundary, \( y = (1/a)x \), can be considered to be a decision axis by which all stimulus samples below the response boundary are orthogonally projected onto one side of the origin, indicating that they are all categorized as the same response. In fact, the response boundary does not need to be linear as long as the function can be transformed to a linear function, so a function such as \( y = ae^{bx} \) is also permissible. In such a case, the log-transformation can be applied to the \( y \)-axis, so that the transformed \( y \) values are linearly related to \( x \)—that is, \( \ln(y/a) = bx \)—and, with the distorted psychological space, one may find a decision axis perpendicular to the linear function to translate into an MDV model.

GRT and MDV also agree with respect to RT predictions, at least in the general trend. Ashby and Maddox (1994) proposed that RT for a given stimulus sample is a negative exponential function of the distance between the sample location to the response boundary. Thus, the farther the stimulus sample is located away from the response boundary, the shorter the RT for that stimulus (RT–distance hypothesis). If the same stimulus sample is projected onto a line perpendicular to the linear response boundary, the distance from the projected point to the response boundary is unchanged and equivalent to the subjective utility as defined in MDV, which determines the accumulation rate and thus RT. Of course, GRT has not been applied to the SRC paradigm, so it does not specify how response properties should influence the response boundary. However, these formal relationships between GRT and MDV indicate that MDV has the potential to account for task settings to which GRT has been applied successfully. Also, one may exploit these facts by applying GRT to a psychological task for the purpose of constructing a multidimensional space and then derive an MDV model for another task based on that GRT representation of stimuli. In essence, doing so would provide a set of transformations and parameters that are necessary to relate two different experimental paradigms.

### General Context Model

GRT is Thurstonian in which psychological representations of stimuli are considered to be random variables. An alternative
approach is the fixed-utility branch of psychological scaling. The fixed-utility assumption has also been adopted in cognitive theories (e.g., Bundesen, 1990; Logan, 2002). One of the models that represent contemporary multidimensional approach is GCM (Nosofsky, 1986). GCM was developed based on Shepard’s (1957, 1987) law of stimulus generalization (or identification), but it extended Shepard’s model to categorization tasks by assuming that stimulus categories consist of a set of exemplars in long-term memory, which was proposed by a precedent context model by Medin and Schaffer (1978). In the categorization process, a stimulus is classified into a categorical class if the sum of the psychological distances between the stimulus and the category exemplars is smaller than that between the stimulus and the exemplars in other categories. A major result of the GCM framework is that a unified account of identification and categorization data is possible if the scale of psychological dimensions can be modified according to the task requirements. Nosofsky (1986) attributed this process to selective attention to one or the other psychological dimension and formally modeled it with weight parameters associated to the feature dimensions (see also Carroll & Chang, 1970; Luce, 1959; Shepard, 1957, 1964). This concept is a common mechanism assumed in theories of visual attention (e.g., Logan, 1996; Müller & Krummenacher, 2006; Treisman & Sato, 1990). Geometrically, the weights are used to elongate or shrink the respective psychological axes and alter the psychological distances between exemplars and stimulus according to the task context. This key component of GCM is akin to the rotation of the decision axis in MDV; the difference is that the psychological space is distorted by attention in GCM but not in MDV.

GCM is also linked to the Thurstonian approach via the solution to the discrepancy between Shepard’s (1957, 1958) original formulation of the generalization law and Nosofsky’s (1985, 1986) results. Shepard’s law states that the probability that the stimulus $S_i$ elicits the response $R_j$ that is associated with $S_j$ is a function of the similarity $s_{ij}$ between $S_i$ and $S_j$, which is generally expressed as

$$Pr[R_j|S_i] = f(s_{ij}).$$

In turn, the similarity is a function of the psychological distance between the two stimuli

$$s_{ij} = g(d_{ij}) = g\left(\sqrt{\sum x_{ir} - x_{jr}}^\alpha\right),$$

where $x_{ir}$ is the fixed utility of $S_i$ in the $r$th dimension. The distance function $d_{ij}$ is known as city-block if $\alpha = 1$ and Euclidean if $\alpha = 2$. Shepard suggested that similarity is best described by an exponential decay function of the distance, $s_{ij} = \exp(-d_{ij}^\alpha)$, with the city-block metric for separable dimensions. On the contrary, Nosofsky found that a better fit was obtained when the model assumed the Euclidean metric related to the similarity via a Gaussian function, of the form $s_{ij} = \exp(-d_{ij}^2)$. Shepard (1986) noted that the discrepancy might have arisen from the fact that Nosofsky used more confusable stimuli. That is, the perceptual representations of stimuli are normally distributed and entail the Euclidean metric, but the generalization (classification) process might be characterized by the exponential decay function of the city-block distances. Ennis (1988; Ennis, Palen, & Mullen, 1988) formulated Shepard’s conjecture and confirmed by simulation that a model with the exponential decay function of the city-block distance mimicked the Gaussian decay function of the Euclidean distances when the underlying stimulus representations were multivariate normal. The proposed modification transforms GCM into a model that is formally similar to a multidimensional variant of Coombs’s (1950) unfolding theory, for which both stimulus representations and ideal points are stochastic (Zinnes & Griggs, 1974; see also Ennis & Johnson, 1994, for a more general formulation). Such a model may be utilized to model the SRC paradigm if the ideal points are considered to represent alternative response categories. This possibility will be discussed more fully in the next section, relating to a sister model of WV, named the wandering ideal-point (WIP) model (Böckenholt & Gaul, 1986; De Soete, Carroll, & DeSarbo, 1986).

Wandering Ideal-Point Model

WIP is a Thurstonian model whose theoretical constructs are very similar to WV, so it could have been adopted in the present research to account for SRC as well. WIP is a variant of the unfolding theory (Coombs, 1950) where preferential judgment is performed by comparing stimuli against an ideal of the stimulus category. Stimuli and the ideal are points in a multidimensional space, and the subjective utility of stimuli depends on their distances to the ideal point. As in WV and MDV, WIP assumes that stimulus representations in the multidimensional space are constants but that the ideal point is a random variable. Consequently, the interpoint distance to the ideal point is also a random variable. One way to apply the model to a two-alternative classification task is to assume two ideal points corresponding to two alternative response categories, and a stimulus is classified into a category if it is closer to one corresponding ideal point than to the other. The model can be seen as an instantiation of the prototype theory (Rosch, 1973). The other way is to assume an exemplar-based comparison process as in GCM where there are multiple ideal points to compare. In either case, these models provide a joint representation of stimuli and responses, a precondition to account for the SRC effect.

Nevertheless, a straightforward extension of WIP may not be as simple as the WV model. For simplicity, we focus on the former version with a single ideal point for each response category. Formally, the model can be expressed as follows. Let $x^{i}$ denote the stimulus representation on the $i$th trial, and $v_A$ and $v_B$ be ideal points that represent the response categories A and B, which are multivariate normal variables. If

$$(x^{i} - v_A)^T(x^{i} - v_A) < (x^{i} - v_B)^T(x^{i} - v_B),$$

then the response A is selected. This inequality can be arranged into

$$2x^{iT}(v_B - v_A) + v_A^Tv_A - v_B^Tv_B < 0,$$

where the first term is a univariate normal variable and the remaining two terms are weighted noncentral chi-square variables. Compared to the extension of the WV model, WIP is more complicated and involves a greater number of parameters (at least twice as many parameters for the ideal points as for the decision axis in MDV). A possible simplification may be made by altering the assumptions of WIP, so that stimulus representations are now
multivariate normal variables and the ideal points are constants. Then, on each trial, there is only one stimulus representation, and the model can be expressed by a univariate normal variable (i.e., in the above inequality, \( \mathbf{x}_i \) becomes a random vector, and \( \mathbf{v}_A \) and \( \mathbf{v}_B \) are constant), which is as simple as MDV. Of course, the degrees of freedom in this revised model drastically increase as the number of stimuli increases. Hence, some restrictions (e.g., equal variance assumption) will be needed to make the model feasible, as in GRT.

An interesting property of the revised WIP model is the fact that the subjective utility is determined by the differences between the two ideal point vectors, that is, the first term of the above inequality. \( \mathbf{x}_i^T(\mathbf{v}_B - \mathbf{v}_A) \). This indicates that the dimensional saliencies are implicitly defined in that model as the component-wise differences of the ideal points in the respective dimensions; that is, the subjective utility, \( \mathbf{x}_i^T(\mathbf{v}_B - \mathbf{v}_A) \), indicates that its \( r \)th component is \( w_r x_r = (v_{rB} - v_{rA})x_r \). Consequently, the WIP model implies that if the two response categories did not differ with respect to the \( r \)th dimension, that dimension does not contribute to computations of the subjective utilities of stimuli. As described earlier, this property is consistent with recent findings in the SRC studies (Ansorge & Wühr, 2004; Vu & Proctor, 2002), which have shown that a response-discriminating feature dimension contributes to the SRC effect. Therefore, the WIP model also appears to provide promising characteristics to model the SRC paradigm. In fact, an interpretation of the formal expression of the model (see Equation 26) is that the difference vector between the two ideal points becomes a decision axis, and the stimulus representation is projected onto the axis. The magnitude of the projected utility in the direction of the difference vector corresponds to the subjective utility. Hence, WIP is formally a variant of the vector model family.

Concluding Remarks

There have been considerable developments of multidimensional Thurstonian approaches in the last decades. MDV adopted a special case of the multidimensional solutions, but an alternative approach could have been taken as well. A possibility is the multidimensional unfolding of Zinnes and Griggs (1974; also see Zinnes & MacKay, 1983) or the WIP model mentioned earlier, both of which are models of preferential judgments based on the psychological distances from the ideal point. These models are contained in the model of triadic judgments (i.e., given three stimuli, participants judge which two of the stimuli are more similar to each other than other possible stimulus pairs; Ennis, 1993; Ennis & Johnson, 1994; Mullen & Ennis, 1991; also see Torgerson, 1958). The basic idea that would underlie applications of these models to the SRC contexts is that not only the stimulus but also its relations to alternative responses are the properties of the comparison process, providing a joint representation of the two ends of performance. Consequently, the subjective utility of the given stimulus is determined by its relation to the responses. This is the fundamental principle that motivates the present research and the development of the MDV model framework.

SRC has been known to be one of the most influential task variables that determine the efficiency of response selection. Fitts and Deininger (1954) suggested that the degree of SRC depends on “(a) the selection of congruent stimulus and response sets, and (b) the generation of congruent pairings of these stimuli and response elements” (p. 490). In MDV, the first factor determines the dimensionality of the multidimensional psychological space that describes the task context, and the second factor is realized as a result of the orthogonal projection of stimulus utilities onto the decision axis, which is analogous to the stimulus discriminability as defined in signal detection theory. Therefore, MDV states that stimulus discriminability is determined partly by response properties. This conclusion is important because the influence of response factors on stimulus discrimination has rarely been considered in previous studies. However, such consideration is not as peculiar as it might sound. In fact, recent studies provide evidence supporting that sensitivity to stimulus features depends on the relevance of these features to the type of actions required to make their responses (e.g., Fagiolli et al., 2007; Wykowska et al., 2009), indicating that attention is directed to aspects of stimuli that are pertinent to implementing the required actions. We propose MDV as a way of instantiating such a process.

As discussed in the introduction, MDV was developed as a modeling framework for choice RT. Yet, when details are considered, it also makes certain predictions about experimental results as a consequence of the mathematical formulation assumed in MDV. As considered in the present research, testing these predictions is important, but our main purpose of including the applications of MDV to the five experiments was to demonstrate how the framework could be used in practice. It was shown that empirically testable model instances can be derived from the framework. An important point is that MDV suggests different model instances with unique representational structures to account for different data patterns. As we demonstrated by fitting to the actual experimental data from our previous study, one can decide which of these alternatives provides a better account of the data. It is for this reason that MDV should be considered to be a general modeling framework that can be applied beyond the SRC paradigm. In a sense, MDV is a generalization of SDT that incorporates considerations of response properties into the underlying process. The extent to which this framework can be generalized to other experimental paradigms is the most interesting empirical issue to be addressed in future investigations.

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Appendix

Formulation of the Multidimensional Vector Model

The basic machinery of the multidimensional vector (MDV) model is provided by the wandering vector (WV) model developed by De Soete and Carroll (1983, 1992), so the formal development of MDV follows that of the WV model. Yet the psychological interpretations of the machinery do not necessarily reflect the interpretations of those authors. In this appendix, we first describe a general mechanism assumed in MDV. Then, we describe the counter model based on the MDV concept.

Basic Machinery

In MDV (see Figure 1B in the main text), stimuli $S_i$ are represented as fixed points in an $r$-dimensional space, $x_i = (x_{i1}, x_{i2}, \ldots, x_{ir})$. These stimulus coordinates reflect the intrinsic utilities of stimuli that are invariant across task contexts and individuals. Without a specific task context, the multidimensional space can be regarded as an affine space, since the origin of the space cannot be determined until a specific task context is given. As described in the main text (see the section entitled Multidimensional Vector Model Framework), the origin of the space is given only after the location of a decision criterion is determined, which provides a reference point for stimulus encoding. To determine the location of a decision criterion, the responses (A and B) used in the task must also be mapped onto the multidimensional space. The coordinates of response representations involve response-discriminating, or intrinsic, features (e.g., left and right; Anserte & Würr, 2004) as well as the task-relevant stimulus, or extrinsic, features (e.g., red and green; De Houwer, 2004). Consequently, to represent stimuli and responses used in the task, the feature dimensions in the space involve, at least, task-relevant stimulus dimensions (e.g., color) and response-discriminating feature dimensions (e.g., spatial location).

The alternative response representations are connected to form a decision axis $v$. The decision axis is a random vector that fluctuates over time, and the variability is assumed to be normally distributed with mean $\mu_i$ and covariance matrix $\Sigma$. De Soete and Carroll (1983) suggested that the covariance matrix is fundamentally indeterminate and can be assumed to be $\Sigma = bI$, where $b$ is a scalar and $I$ is the identity matrix. The variability of the decision axis can be considered to reflect fluctuations of dimensional weights (selective attention) as described in the main text. The response-selection process is performed on the decision axis, so a point on the decision axis is arbitrarily chosen as the decision criterion $c$, which then serves as the origin of the multidimensional space and incidentally ensures that the decision axis always passes through the origin of the space as required in the vector model. Once stimuli and responses are mapped onto the multidimensional space and the origin is determined, the resulting spatial configuration provides the task context. Response selection is performed based on this spatial configuration. That is, the decision criterion partitions the decision axis into two regions, each corresponding to one of the alternative responses. The subjective utility of the stimulus is compared against the decision criterion to examine whether it provides evidence for one or the other response.

At any moment during a trial, a decision axis is sampled, and the stimulus presented on that trial is orthogonally projected onto the axis. This projected point on the decision axis represents the subjective utility of the stimulus on that particular moment, which is given by

$$x'_i = v(v^Tv)^{-1}v^Tx_i. \quad (A1)$$

Though the utility of stimulus $x_i$ is fixed, the decision axis $v$ is random, so the subjective utility $x'_i$ becomes a random variable distributed along the decision axis. Thus, this procedure is equivalent to sampling from the distribution of the subjective utility along the decision axis. A sample provides evidence for a response depending on the response region it comes from. This can be determined by comparing the magnitude of $x'_i$ in the direction of $v$ against the magnitude of $c$. In general, the decision criterion can be written as $c = c(v^Tv)^{-0.5}v$, with $c$ being the scalar that expresses the magnitude of $c$ in the direction of the vector $v$. Thus, the probability that the magnitude of $x'_i$ in the direction of $v$ is greater than that of the decision axis is given by

$$Pr\left[\frac{v^Tx_i}{|v|} > c \right]. \quad (A2)$$

The variable $X_i = v^Tx_i$ is normally distributed with mean $\mu_i = x_i^T\mu$ and variance $\delta_i^2 = x_i^T\Sigma x_i = bx_i^Tz_i$. Let $c = 0$, so that the decision criterion is always at the origin. Also, $|v|$ is always positive, so the sign of the left side of the inequality is determined by $v^Tx_i$. That is, the above probability is equivalent to

$$Pr[X_i = v^Tx_i > 0].$$

Thus, the response-selection process may be expressed by the following decision rule:

$$\begin{cases} \text{Selected response A if } X_i \leq 0 \\
\text{Selected response B if } X_i > 0 \end{cases}. \quad (A3)$$

Given the stimulus $x_i$, the probability of a stimulus sample indicating the response A is

$$Pr[A|x_i] = Pr[X_i \leq 0] = 1 - \Phi\left(\frac{\mu_i}{\sigma_i}\right), \quad (A4)$$

where $\Phi$ is the standard normal cumulative distribution function (CDF). The probability of selecting the response B can be obtained as one minus the above probability:

$$Pr[B|x_i] = \Phi\left(\frac{\mu_i}{\sigma_i}\right). \quad (A5)$$

(Appendix continues)
As emphasized in the main text (and also mentioned above), the decision axis can be viewed as a weighting function of the psychological dimensions. Because the mean vector of \( \mathbf{v} \) can be assumed to be a unit vector (since the length of the decision axis is inconsequential), the mean of \( X_i \) can be written as

\[
\mu_i = \mathbf{x}_i \mu = \sum_{k=1}^{n} x_k \cos \theta_{x_k},
\]

where \( x_i = (x_{i1}, \ldots, x_{in}) \) and \( \mu = (\cos \theta_1, \ldots, \cos \theta_n) \), with \( \theta_i \) being the angle between the decision axis and the \( r \)th dimensional axis. That is, the mean of the stimulus distribution along the decision axis is a weighted sum of the component utilities in the respective dimensions, with \( w_r = \cos \theta_r \) being the weight associated with the stimulus feature in the \( r \)th dimension. The weight determines the extent to which the \( r \)th stimulus dimension contributes to task performance, and it is constrained by the identity \( \sum w_r^2 = 1 \).

**MDV Counter Model**

For a two-choice alternative task, the counter model assumes two independent counters, each corresponding to one of the two alternative responses. As detailed above, MDV assumes that, during a trial, the stimulus \( x_i \) is repeatedly projected onto the decision axis \( \mathbf{v} \) and the position of \( x_i \) on the decision axis is observed. This process is referred to as *stimulus sampling*. Each stimulus sampling yields a realization of the random variable \( X_i \) which provides evidence for one of the two responses according to the response-selection rule given in A3. When evidence for one response is obtained, the counter associated with that response is incremented by a fixed amount (= 1 in the present study). The sampling process continues until either counter reaches the response threshold \( K \). The motor command is then transmitted to the next processing stage to execute the selected response.

A formal expression of the counter model can be derived as follows. First, note that the time interval between two sampling events is a random variable \( T \) exponentially distributed according to the CDF

\[
F_T(t) = 1 - e^{-\lambda t}, \quad \lambda > 0,
\]

and the probability density function (PDF)

\[
f_T(t) = \lambda e^{-\lambda t}, \quad \lambda > 0.
\]

This sampling process is known as a Poisson process, which is often used to model the activity of neurons (Townsend & Ashby, 1983). Then, the counter model can be considered to be a race between two parallel, independent Poisson processes. A convenient property of the exponential distributed variables is the fact that, if two exponential variables are independent, the minimum of the two variables is another exponential variable with the rate parameter given by the sum of the rate parameters of the two competing processes. More formally, if \( T_a \) and \( T_b \) are two exponentially distributed time intervals with the rate parameters \( \lambda_a \) and \( \lambda_b \), respectively, then \( T = \min(T_a, T_b) \) is also exponentially distributed interval with the rate parameter \( \lambda = (\lambda_a + \lambda_b) \).

From this property, we obtain the following fact: The time intervals \( T_a \) and \( T_b \) correspond to the time that it takes to increment the counters A and B, respectively. Suppose that the probability that the counter A is incremented at any sampling is \( p_a \), and the probability that the counter B is incremented at any sampling is \( p_b = 1 - p_a \). The former probability is equal to the probability that the counter A is incremented before the counter B; that is,

\[
p_a = \int_0^\infty f_a(t)[1 - F_b(t)]dt.
\]

The functions \( f_a \) and \( F_b \) are, respectively, the PDF of \( T_a \) and the CDF of \( T_b \). Thus,

\[
p_a = \frac{\lambda_a}{\lambda_a + \lambda_b} = \frac{\lambda_a}{\lambda}.
\]

Then, one gets \( \lambda_a = p_a \lambda \) and, in the same manner, \( \lambda_b = p_b \lambda \). As mentioned earlier, \( \lambda = \lambda_a + \lambda_b \) is the rate parameter of the variable \( T = \min(T_a, T_b) \). The parameter \( \lambda \) is called the *base accumulation rate*. In MDV, the probabilities \( p_a \) and \( p_b \) are determined by the distribution of subjective utility along the decision axis (see Equations A4 and A5).

Furthermore, let us define the processing times \( T_A \) and \( T_B \) to be the latencies for the counters A and B to reach their respective response thresholds \( K_A \) and \( K_B \). Although response biases can be modeled by the difference between \( K_A \) and \( K_B \), we assume no response bias (\( K_A = K_B = K \), with \( K \) being a natural number). Because the counters are incremented by one unit at each sample, the counter process is terminated when either of the counters is incremented \( K \) times. That is, one can write

\[
T_A = T_a + T_{a2} + \ldots + T_{aK},
\]

\[
T_B = T_b + T_{b2} + \ldots + T_{bK},
\]

where \( T_a \) and \( T_b \) are the interval between the \( (i - 1) \)th and \( i \)th increments in the respective counter. The processing times \( T_A \) and \( T_B \) are the sums of exponential variables \( T_{ai} \) and \( T_{bi} \), which are the well-known gamma (or Erlang) variables. Consequently, the PDF and CDF of \( T_A \) are, respectively,

\[
f_{T_A}(t) = \frac{(\lambda_d)^K}{(K - 1)!} \lambda_a \exp(-\lambda_d t)
\]

and

\[
F_{T_A}(t) = 1 - \sum_{i=0}^{K-1} \frac{(\lambda_d)^i}{i!} \exp(-\lambda_d t).
\]

For \( T_B \), the respective functions are obtained by replacing \( \lambda_a \) with \( \lambda_b \). Because the decision time (DT) is determined by the latency that it takes for one counter to reach the response threshold before the other counter reaches it, the PDF of DT for the response A is given by

\[
f_{DT}(t;A) = f_a(t)[1 - F_b(t)].
\]

Analogously, the PDF of DT for the response B is obtained by switching the subscripts A and B. Note that the above function is a joint function that expresses the simultaneous occurrence of the

(Appendix continues)
two events: \( E_1 = \{ DT \leq t \} \) and \( E_2 = \{ \text{Response A is selected} \} \). This function does not integrate to 1 but to \( \Pr[E_2] \); hence, it is called defective or incomplete PDF (Brown & Heathcote, 2008; Dzhafarov, 1993). The conditional PDF can be obtained if Equation A13 is divided by \( \Pr[E_2] \), which is a complete PDF.

Although the counter model is typically defined by substituting Equations A11 and A12 into Equation A13 (with appropriate subscripts), an additional refinement can be made. Namely, the response threshold \( K \) is assumed to be a uniformly distributed (discrete) random variable in the interval \([K_{\text{min}}, K_{\text{max}}]\). It is also assumed that \( K \) is identically and independently distributed for all counters. Then, the joint CDF of \( K \) and \( T_A \) is

\[
F_A(t, k) = \frac{1}{K_{\text{max}} - K_{\text{min}}} \left[ 1 - \sum_{i=0}^{k-1} \frac{(\lambda_i t)^i}{i!} \exp(-\lambda_i t) \right], \tag{A14}
\]

and summing the CDF over all values of \( K \) gives the marginal CDF of \( T_A \),

\[
F_A(t) = 1 - \frac{1}{K_{\text{max}} - K_{\text{min}}} \sum_{i=0}^{\text{max}} \sum_{j=K_{\text{min}}}^{K_{\text{max}}} \frac{(\lambda_i t)^i}{i!} \exp(-\lambda_i t), \tag{A15}
\]

and the corresponding PDF

\[
f_A(t) = \frac{1}{K_{\text{max}} - K_{\text{min}}} \sum_{i=0}^{\text{max}} \frac{(\lambda_i t)^i}{i!} \exp(-\lambda_i t). \tag{A16}
\]

Note that, strictly speaking, the parameters \( p_a \) and \( p_b \) (and, thus, \( \lambda_a \) and \( \lambda_b \)) are always conditional upon given stimuli. Hence, we use \( p_a|x_i \) to indicate the conditionality of \( p_a \) on the variable \( X_i \) or \( x_i \) when necessary. In addition, though the base accumulation rate could also be conditional on given stimuli, it is assumed to be equal for all stimuli in the present study. See the main text for more specific instantiations of the MDV counter models.