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Article

Olga Nikolajevna Bondareva was born in St Petersburg on 27 April 1937. She joined the Mathematical Faculty of the Leningrad State University in 1954, and completed a Ph.D. in mathematics at the Leningrad State University in 1963, in part under the supervision of Nicolaj Vorobiev. Her thesis was entitled ‘The Theory of the Core in an n-Person Game’. Bondareva rose through the ranks at Leningrad State University to become a senior research fellow in 1972 and a leading research fellow in 1989. Because she sympathized with a student who wished to emigrate to Israel, however, she was prohibited from teaching from 1973 until 1989. With *perestroika* and increased freedom to travel outside the Soviet Union, she became an active and energetic international figure in game theory. She died as a result of a traffic accident on 9 December 1991.

Bondareva published over 70 works on game theory and mathematics, supervised seven Ph.D. students, and was a member of the editorial board of *Games and Economic Behavior*. Her work on the core of a cooperative game plays a central role in game theory, and her insights can be seen underlying recent work on the theory of price-taking equilibrium and the core.

The following is a brief description of Bondareva's celebrated result. To allow us to see the relationship of this result to more recent research on games and economies with many players, it is stated for games with player types and requiring only ‘essential superadditivity’ in the definition of feasible payoffs.

Define a (pre)game with T types of players as a function ψ from vectors of non-negative integers $s \in \mathbb{Z}^T_+,\ s \neq 0$, called *profiles* of coalitions, into the non-negative real numbers $\mathbb{R}_+$. Given a vector $m \in \mathbb{Z}^T_+$, representing the total player set of the game and $s \in \mathbb{Z}^T_+,\ s \leq m,\ \psi(s)$ is interpreted as the total payoff to a coalition of players consisting of s_t identical players of type t, $t = 1, \ldots, T$. Let $(s^\ell : \ell = 1, \ldots, L)$ denote the collection of all profiles $s^\ell \leq m$. A *partition of a profile s* is determined by a collection of non-negative integers $(n_1, \ldots, n_L)$ satisfying the condition that $\Sigma n_\ell s^\ell = s$. With the domain of ψ restricted to profiles $s^\ell \leq m$, the pair (m, ψ) determines a cooperative game. Let $\psi^*(m)$ denote the maximum, over all partitions of m, of $\Sigma n_\ell \psi(s^\ell)$. A payoff vector $\overline{x} \in \mathbb{R}^T$ is in the (equal treatment) *core* if and only it holds that. $\overline{x} \cdot m \leq \psi^*(m)$ ($\overline{x}$ is *feasible*) and for each $\ell, \psi(s^\ell) \leq \overline{x} \cdot s^\ell$.

Now consider the following linear programming (LP) problem:

$$\min_{\overline{x}} \overline{x} \cdot m \text{ subject to } \psi(s^\ell) \leq \overline{x} \cdot s^\ell \text{ for all profiles } s^\ell \leq m.$$

A vector $\overline{x}^*$ is in the core if it is a solution to the above LP problem *and* $\overline{x}^* \cdot m = \psi^*(m)$. The dual LP problem is:

$$\max_{\omega_1, \ldots, \omega_L} \sum_\ell \omega_\ell \psi(s^\ell) \text{ subject to } \sum_\ell \omega_\ell s^\ell = m \text{ and } \omega_\ell \geq 0 \text{for all } \ell$$

From the fundamental duality theorem of linear programming, there is a solution to the first LP problem if and only if there is a solution to the second, and, in this case, it holds that the optimal values of the objective functions in the two LP problems are the same.

For the second LP problem, let (ω^*_ℓ) denote the solution for the ‘balancing weights’ (ω_ℓ). The game is *balanced* if and only if $\sum_\ell \omega^*_\ell \psi(s^\ell) = \psi^*(m)$. It follows that a game is balanced if and only if it has a non-empty core, Bondareva's result. (See also Shapley, 1967.)

Numerous applications of game theory to economics have employed the concept of balancedness. An outstanding contribution is Shapley and Shubik (1969), who show an equivalence between the set of totally balanced games (balanced games with the property that every subgame also has a non-empty core) and market games (cooperative games derived from economies where all players have concave utility functions). Bondareva's result as formulated above is a key ingredient in Wooders (1994), showing that under mild conditions games with many players are market games. Scarf (1967) demonstrates non-emptiness of the core of a balanced game without side payments (where the payoff set for a coalition S is a subset of R^S rather than a real number). Bondareva's result also underlies the approximate balancedness of economies with clubs or relatively small effective or nearly effective coalitions. While this result has been demonstrated in much generality, the key is simple. Since the coefficients of the dual LP problem are integers, when the total player set is replicated (becomes rm, $r = 1, 2, \ldots$) and no new effective coalitions are permitted (that is, if $\psi(s) \geq 0$ then $s \leq m$) then there is an integer k such that all replicated games with total player profiles given by rkm are balanced (Wooders, 1994, and references therein). The integer k clears the denominators of the (rational) extreme points of the convex set of balancing weight vectors of the dual LP problem.

In recent works on the theory of clubs and local public goods, balancedness plays a crucial role; see Demange and Wooders (2005) for several recent examples and additional references.

We refer the reader to Rosenmueller (1992) for some additional details of Olga Bondareva's life. See also Kannai (1992) for an excellent review of research on the core and balancedness.

See Also

- game theory

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