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Cyclic Games: An Introduction and Some Examples[1]

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We introduce a model of a cyclic game. Designed to take advantage of the recurring nature of certain economic and social situations, a cyclic game differs from an extensive form game in that a cyclic game does not necessarily have an end. The same situations, although with different players, may be repeated infinitely often. We provide an example showing that, even though a cyclic game has, in a sense, perfect information, it may not have an equilibrium in pure strategies. We demonstrate existence of equilibrium and illustrate the application of our model to an oligopolistic industry. *Journal of Economic Literature* Classification Numbers: C72, C73, D43.

1. CYCLIC GAMES: AN INTRODUCTION

In economics and game theory there are many situations where activities occur over time, with the same situation replicated again and again but with

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different players. For example, over time different entrepreneurs may enter any industry, over time their plants may deteriorate, and they may leave the industry. Other entrepreneurs may enter the industry at the same time or at different times. While individual firms may be short-lived, the industry continues indefinitely. Alternatively, a player may take an active role in an activity only at infrequent intervals, where these are so infrequent that he can be assumed to ignore the effects of his current actions on the situation he confronts when he is next in the market. For example, a player may enter the housing market several times during his lifetime, but only infrequently. Each time he purchases a new home, he may view the game and the players as completely separate from those of his previous ventures into the housing market; nevertheless, between the periods when he is active in the market, the activity of the market continues.

To enable us to treat recurring situations, we introduce the framework of a cyclic game. The cyclic game framework is one of perfect information and includes extensive form games with perfect information as a special case. In general, however, unlike an extensive form game, a cyclic game is not necessarily a tree; the same situation may recur infinitely often. Intuitively, we can imagine a player entering into the activity of the game, playing out his role (which can depend on the situation of the game into which he was born), receiving payoffs, and leaving the game. Eventually the game may arrive at a point where a player with the same role again becomes active in the game. In this paper we describe such a cyclic game, show existence of a stationary equilibrium point, and develop two examples.

Because of the nature of a cyclic game, in this context a stationary strategy has a somewhat different interpretation than in an extensive form game. We can interpret a player as an individual who enters and exits the game possibly an indefinite number of times. Alternatively, we can view a player as short-lived, entering the game just once, and then, when he exits after some finite number of moves, never returning to the game. In a cyclic game, the short-lived player, however, may be a member of a dynasty; he is preceded by an individual who faced the same choices when he entered into a situation with the same circumstances, and is followed by another individual who will face the same choices when confronted with the same circumstances. Essentially, the short-lived player is reincarnated, possibly infinitely often.

In this paper we label players according to their roles. This allows us to avoid indexing of players who have the same roles and to avoid keeping track of the number of "rounds" of the cyclic game. With this in mind, in our discussion we refer to the individual player as entering and exiting the game, possibly repeatedly. Consistent with our treatment of the player, only stationary equilibria, which assign the same behavior for a player each time he is in the same situation, will be considered.

A stationary equilibrium is defined by local optimization. The equilibrium is stationary in the sense that the strategy combination at a decision point does not depend on how the point is reached. A global stationary strategy, specifying a local strategy for each decision point, is a stationary equilibrium if no player can increase the expected payoff at any of his decision points by changing his strategy choice at that point, given his choices at his other decision points and given the choices of other players.

Our examples of cyclic games are sufficiently simple so that we can depict the games by graphs. The first example illustrates that a cyclic game might have no stationary equilibria in pure strategies. The second illustrates an oligopolistic industry where profits may be positive, even though the market is contestable (see Baumol *et al.*, 1982, for a discussion of contestable markets).

Cyclic games may be viewed as special cases of stochastic games[3] and are closely related to recursive games, as in Everett (1957). Games with recurring situations and changing players, as in the current paper, were independently introduced in Jackson and Kalai (1995, 1997), where they are called recurring games.[4] There are, however, some significant differences between the Jackson–Kalai model and ours. Our framework is more specialized in that we require complete information, while Jackson and Kalai allow incomplete information. Our framework, however, allows more flexibility in the player set than does Jackson and Kalai. Recall that Jackson and Kalai consider games where the player set of the nth recurrence of the game is a copy of the player set of the initial game and players who are in the nth game leave at the end of that game and do not appear in any subsequent game. While our paper allows this possibility, it also allows overlapping generations, for example. In general, we cannot partition a cyclic game into one recurring game. A player may appear in a cyclic game, a second player may leave, a third player may enter, a player with the same role as the second player may reappear, and then the first player may leave. It is possible also that a player may leave a cyclic game and no one in that role ever reappears in the game. Jackson and Kalai have in mind one game played repeatedly by different players, whereas we have in mind situations where there are possibly several generations of players active at any one time. The results of the Jackson and Kalai papers are also distinct from ours. Jackson and Kalai consider learning and convergence to Bayesian equilibrium while we study existence of stationary equilibrium.

[3]We are grateful to Jean-Francois Mertens for pointing this out to us.

[4]Although drafts of the current paper have been circulated since 1992 and the paper was presented at New York University in 1993 it was not circulated as a published Discussion Paper (Selten and Wooders 1995) until December 1995, a few months after the publication of the Jackson and Kalai (1995) Discussion Paper.

2. CYCLIC GAMES

A *cyclic game* $G = (X, Z, E, P, A, \pi, h)$ has the following constituents:

X a finite set, whose members are called *decision points*. An arbitrary member of X is denoted by x.

Z a finite set, whose members are called *payoff points*. An arbitrary member of Z is denoted by z.

E a subset of Z, whose members are called *exit points*.

P a partition $P = \{X_0, X_1, \ldots, X_n, Z_1, \ldots, Z_n\}$ of $X \cup Z$, where $\cup X_i = X$ and $\cup Z_i = Z$. The set X_0 is called the set of *random points* and, for $i \neq 0$, the set X_i is called the set of *decision points for player i*. The set Z_i is called the set of *payoff points for player i*. We establish the convention that x denotes a member of either X or Z. Each set Z_i contains a nonempty subset E_i, called the set of *exit points for player i* and satisfying $E = \cup E_i$.

A a function $A(\cdot)$ from $X \cup Z$ to subsets of $X \cup Z$. Together with the set $X \cup Z$ the function $A(\cdot)$ describes a directed graph K with the points in $X \cup Z$ as vertices and the connections between points $x \in X \cup Z$ and $y \in A(x)$ as edges directed from x to y. A *chain from* x_1 to x_m is a sequence $x_1, \ldots x_m$ with $x_1 \in X \cup Z$ and $x_{k+1} \in A(x_k)$ for $k = 1, \ldots, m-1$. A chain $x_1, \ldots x_m$ is *terminating* if $A(x) = \emptyset$ and it is *looped* if $x_k \in A(x_m)$ for a point x_k on the chain. The function $A(\cdot)$ is required to satisfy the following properties:

(a) If $x \in Z$ then $|A(x)| \leq 1$, where $|\cdot|$ denotes the number of elements in a set.

(b) For $i = 1, \ldots, n$, the following holds: If $x_1, \ldots x_m$ is a terminating or looped chain, with $x_1 \in X_i \cup Z_i$, then one of the points on the chain is in E_i.

π a function which assigns a probability distribution $\pi(\cdot|x)$ over $A(x)$ to every $x \in X_0$. The notation $\pi(y|x)$ is used for the probability of $y \in A(x)$.

h a system of payoff functions $h_i(\cdot)$ for the players $i = 1, \ldots, n$. Player i's payoff function assigns a real number $h_i(z)$ to every payoff point $z \in Z_i$.

One can think of the cyclic game G as a directed graph K introduced in the formal explanation of A, supplemented by additional specifications. With this interpretation in mind, we call $A(x)$ the *successor of* x and A the *successor function*.

Note that a difference between the constituents of a cyclic game and those of an extensive form game with perfect information is that in a cyclic game payoff points may have successors. Also, since it is convenient, we have only one player receiving a payoff at each payoff point so, for example, there may be several payoff points, one following another. Another difference between a cyclic game and an extensive game is that a cyclic game does not necessarily have an initial node or point. Moreover, although it is not ruled out, we do not require that the graph K of a cyclic game is a tree. In fact, for any two points x and y, it may be the case that $A(x)$ and $A(y)$ intersect; there may be more than one way of getting to a point.

If we cut the graph K at the exit points of one of the players $i = 1, \ldots, n$, we obtain a new directed graph K_i with an associated successor function A_i defined as follows:

$$A_i(x) = \begin{cases} A(x) & \text{if } x \notin E_i \\ \emptyset & \text{otherwise.} \end{cases}$$

A *chain in* K_i is a sequence $x_1, \ldots, x_m$ with $x_{k+1} \in A(x_k)$ for $k = 1, \ldots, m-1$. For the case $x_m = x_1$ the chain is a *cycle in* K_i. We also transfer the notions of *terminating* and *looped* to this new context.

Condition (b) imposed on A in the definition of a cyclic game has the consequence that the directed graphs K_i are *acyclic* in the sense that they have no cycles. A cycle in K_i would also have to be a cycle in K. Therefore an exit point of i would have to be on such a cycle. This is impossible since $A_i(x) = \emptyset$ holds for points $x \in E_i$. Accordingly we call (b) the *acyclicity requirement.*

The acyclicity requirement ensures that a player cannot arrive at the same situation twice without passing through an exit point and that at any decision point a player will face a finite future. It is possible that a player could arrive at the same decision point from two different preceding decision points. For example, we may have the sort of situation depicted in Fig. 1a.

Figure 1 illustrates two situations that may appear within the context of a cyclic game and one situation that is ruled out. In Fig. 1a the two points ∇_1 and ∇_2 are exit points for Player i. Both these exit points lead eventually to the same decision point, labeled $\bullet_3$. Informally, the player can "enter" the game in the situation given by $\bullet_1$ or the situation given by $\bullet_2$. From either point, he may eventually reach the situation given by $\bullet_3$. At $\bullet_3$ the differences in the histories that have brought him to that point are ignored. Figure 1b depicts the possibility that the player may exit at either ∇_1 or ∇_2 but in either case he enters into the same situation given by his decision point $\bullet_1$. The situation depicted in Fig. 1c is ruled out by our definition; no one exit point can lead to two decision points for the same player.

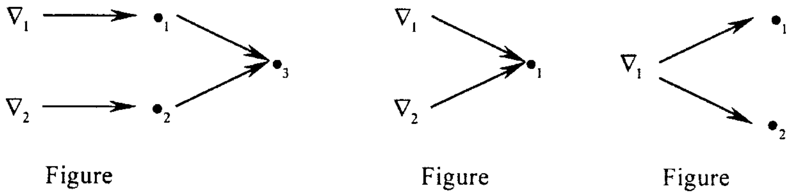

FIGURE 1

3. AN EXAMPLE

Before formally defining strategies and the equilibrium concept, we develop a simple example with two players. The example to follow illustrates a cyclic game and shows that a cyclic game does not necessarily have an equilibrium in pure strategies.

Figure 2 describes the game. In the figure, a decision point x for Player i is depicted by a black point $\bullet^i$ superscripted by i, while an exit point z is depicted by a downward-pointing triangle ∇^i superscripted by i and with the amount of the payoff $h_i(z)$ written $[h_i(z)]$. For this example, all payoff points are exit points.

Suppose a play of the game begins with the decision point $\bullet^1$ for Player 1. He has two choices, either α^1 or β^1. If he chooses α^1, the next node $\triangledown^2$ is a payoff point for Player 2 and at that point, Player 2 gets a payoff of 0. The successor of the payoff point is a decision point $\bullet^2$ for Player 2. If Player 1 chooses β^1, then Player 2 exits with a payoff of 2, Player 1 exits with a payoff of 1, and it is then Player 2's move (for a new Player 2).

Player 2 also has choices labeled α^2 and β^2. If he chooses α^2, then Player 1 gets a payoff of 2 and the next node is the decision point $\bullet^1$ for Player 1. If Player 2 chooses β^2, Player 1 gets a payoff of 0, Player 2 gets a payoff of 1, and the next node is a decision point for Player 1.

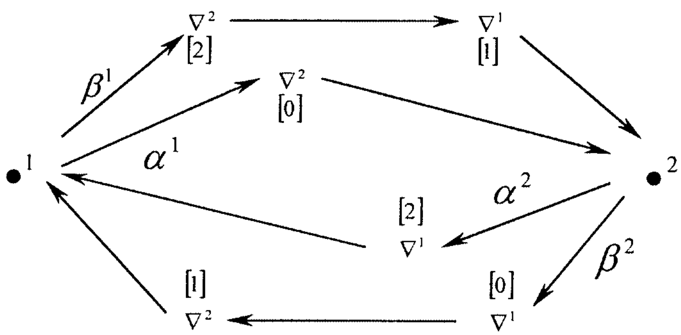

FIGURE 2

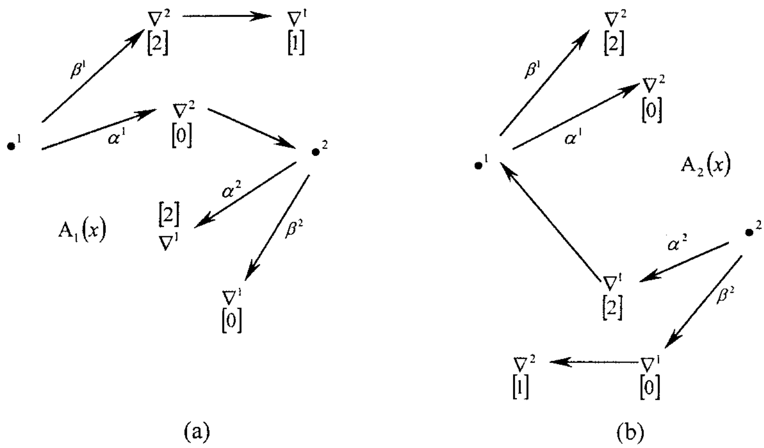

FIGURE 3

For this example, Fig. 3 shows the graph $A_i(x)$ for $i = 1$ and $i = 2$. Note that the graph is acyclic and, for this example, also a tree. An interpretation is that Player 1, from his decision point $\bullet^1$, sees the game given by Fig. 3a. He views himself as the first mover. The payoffs given at the absorbing points of A_1 are payoffs for Player 1. The same interpretation holds for Player 2.

It may be useful to observe that from the viewpoint of each player the game begins at his decision point. The activity that is relevant to him begins with his entry into the game. Although in this example there is only one entry point for each player (the first decision point for that Player after one of his exit points), typically there may be several. When we cut the graph at the exit points of Player i we may create several disjoint graphs similar to Fig. 3a (or b) for Player i. These can be interpreted as the Player entering into different states of the game.

Figure 4 describes the game in normal form. The normal form game is derived from Fig. 3. Looking, for example, at Fig. 3a we see that if Player 1 chooses β^1 he gets a payoff of 1. If he chooses α^1 and Player 2 chooses α^2, then Player 1 gets a payoff of 2. If Player 2 chooses β^2 then Player 1 gets 0.

From Fig. 3b we can describe the normal form of the game from the viewpoint of Player 2. To illustrate, if Player 2 chooses α^2 and Player 1 chooses β^1, then Player 2 gets a payoff of two.

It is clear that the normal form game does not have an equilibrium in pure strategies. The only equilibrium is for Player 1 to choose α^1 and for Player 2 to choose α^2, both with probability 1/2.

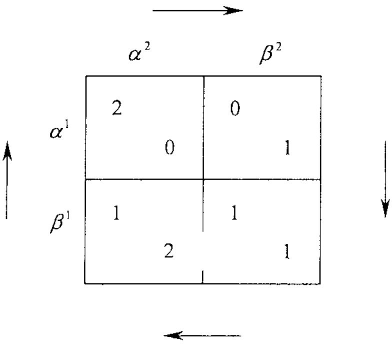

FIGURE 4

3.1. *Payoffs, Strategies, and Equilibria*

We will define equilibrium only in behavioral strategies. Our approach is similar to that of Selten (1981). (Selten and Wooders, 1992, use a similar approach and provide an example of the sorts of situations motivating this paper.)

Let $G = (X, Z, E, P, A, \pi, h)$ be a cyclic game.

Strategies

A *local strategy* b_x at a point $x \in X \backslash X_0$ is a probability distribution over $A(x)$. The probability assigned to $y \in A(x)$ is denoted by $b_x(y)$. For every $x \in X \backslash X_0$ let B_x be the set of all local strategies.

A *global strategy* b is a function that assigns a local strategy b_x to every $x \in X \backslash X_0$. Let B be the set of all global strategies for G.

Remark. The global strategies defined above are *stationary* in the sense that behavior depends only on x and not on the whole past history.

Expected Payoffs

For every $x \in X$ and every global strategy $b \in B$ we define player i's expected payoff to b at x, denoted by $H_i(b \mid x)$. We first introduce some auxiliary definitions.

Recall that for $m = 1, 2, \ldots,$ a sequence $x_1, x_2, \ldots, x_m$ with $x_k \in X$ is called a chain from x_1 to x_m if we have $x_{k+1} \in A(x_k)$ for $k = 1, \ldots, m-1$. The chain is *admissible for player i* if $x_1, x_2, \ldots, x_{m-1} \notin E_i$; in other words, the chain is admissible if the sequence does not pass through an exit point for player i.

For every $x \in X$ and for $i = 1, \ldots, n$, let $Z_i(x)$ be the set of all payoff points z for player i such that there exists at least one admissible chain from x to z. The probability of a chain $x_1, x_2, \ldots, x_m$ is the product, for

$k = 1, \ldots, m-1$, of all $b_{x_k}(x_{k+1})$ for all x_k where $x_k \in X \backslash X_0$ times the product of the probabilities assigned to points x_k in X_0. Let $p(x, z, b)$ be the sum of all the probabilities of chains from x to $z \in Z_i(x)$. Note that in the case $x \in E_i$ we have $p(x, x, b) = 1$. Player i's expected payoff is defined as follows:

$$H_i(b \mid x) = \sum_{z \in Z_i(x)} p(x, z, b) h_i(z)$$

for $i = 1, \ldots, n$.

Stationary Equilibrium Points

For $i = 1, \ldots, n$ a choice $y \in A(x)$ at a decision point $x \in X_i$ is *optimal at x with respect to b* if it holds that:

$$H_i(b \mid y) = \max_{w \in A(x)} H_i(b \mid \omega).$$

A global strategy is a *stationary equilibrium point of G* if for every $x \in X \backslash X_0$, the following is true for the local strategy b_x assigned to x by b : Every $y \in A(x)$ with $b_x(y) > 0$ is optimal at x with respect to b.

The definition above of an equilibrium point is based on a local optimality condition. It can be shown (and has been in similar frameworks) that local optimality implies global optimality. In other words, if, at each of his decision points, given the choices of the other players a player makes a choice that is locally optimal at that point, then he could not improve upon his expected payoff at any of his entry points by changing his choice at several of his decision points. We will state a Proposition to this effect. First, we introduce the following definition: A *deviation c* of Player i from b, denoted by b/c, is a global strategy which differs from b only at positions $x \in X_i$, i.e., we have $b_x = c_x$ for all $x \in X \backslash X_i$.

PROPOSITION 1. *If b is a stationary equilibrium point of G, then for $i = 1, \ldots, n$ it holds that*

$$H_i(b/c \mid x) \leq H_i(b \mid x)$$

for every $x \in X_i$ and for every deviation c of Player i from b.

Subgame Perfection

An equilibrium point is *subgame perfect* if it induces an equilibrium point on every subgame (Selten, 1965, 1975). For the special case of an extensive form game of perfect information the stationarity of the equilibrium point in the sense of the definition given above implies subgame perfectness since the optimality condition refers to the local payoff at x. This local payoff is optimized regardless of whether x is reached by equilibrium play. The definition of subgame perfection, however, may be without impact for cyclic games since a cyclic game may well have no subgames.

PROPOSITION 2. *For every cyclic game G there exists a stationary equilibrium point.*

Proof. Our strategy of proof is simply to construct an "agent normal form" from G and then invoke the existence of equilibrium theorem of Nash.

For each i and for each $x \in X_i$ assign an index number, say ij, to x so that for each $j = 1, \dots, |X_i|$, there is one and only one member x_{ij} of X_i with assigned label ij. Let b denote a global strategy and for x assigned the index number ij define $H_{ij}(b \mid x) = H_i(b \mid x)$.

Consider the normal form game with player set $\{ij : i = 1, \dots, n$, and $j = 1, \dots, |X_i|$ for each $i\}$ and with, for each agent ij, the payoff to the pure strategy b_{ij} for the agent ij given by $H_{ij}(b_{ij} \mid c_{km}, km \neq ij)$, where c_{km} is a pure strategy for km for each agent $km \neq ij$. This normal form game has an equilibrium in mixed strategies. Let γ denote an equilibrium point for the normal form game. Now we claim that γ is a stationary equilibrium point for the game G. If not, then some player i would benefit from deviating at some decision point, with all other strategies held constant. This means that at least one of his agents must not have been optimizing, given the strategy choice of the other players (more precisely, their agents). This contradicts the fact that γ was an equilibrium for the normal form game. ■

4. A CYCLIC ENTRY GAME

Our final example is an application to oligopoly theory. We consider a situation where there is some fixed demand for a product. In each period, a firm has the possibility of entering the industry. A firm "lives" two periods. For example, the firm may need to buy a machine that lasts only two periods. The payoff to an entering firm depends on the market situation when it enters, in particular, whether or not there is already a firm in the industry. In the second period of the life of a firm, the payoff to the firm depends on whether a new firm enters the industry that period.

We describe the example further with Fig. 5. A decision point for Player i is indicated by a black point $\bullet^i$, a payoff point that is not an exit point is indicated by $\triangle^i$ with the payoff written alongside in brackets $[\cdot]$, and an exit point is indicated by ∇^i, with the payoff again written alongside in brackets $[\cdot]$.

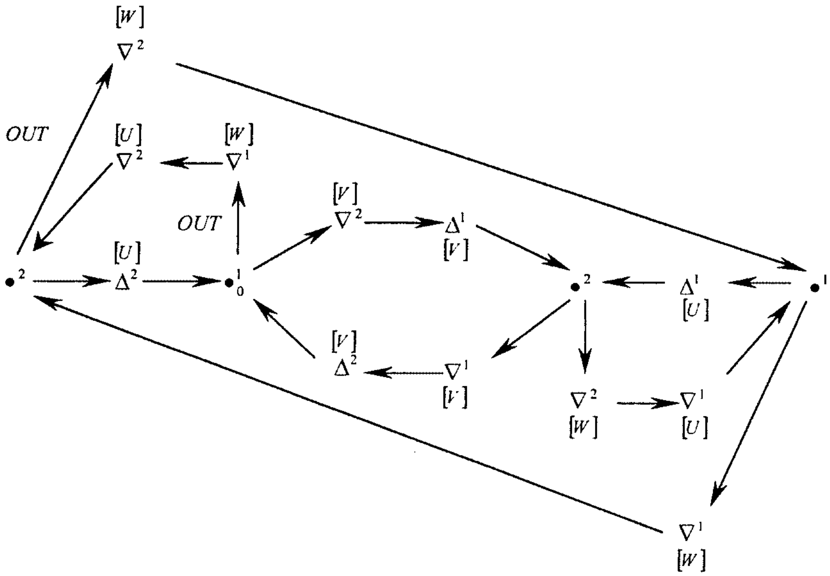

FIGURE 5

If a player chooses not to enter the industry, he receives a payoff of W. If there is only one firm in the industry, that firm earns a profit of U. If there are two firms in the industry then each earns a profit of V.

Let us first describe the outer rectangle of the cyclic game diagram in Fig. 5. If a player, say 2, chooses not to enter the industry he receives a payoff of W, leaves the game, and the move passes to Player 1. If Player 1 also chooses not to enter the industry, then he too receives a payoff of W and leaves the game. The move then passes to the (new) Player 2. If both players continue to choose not to establish a firm the move passes around the outer rectangle indefinitely.

Now let us turn to the inner six-sided figure, beginning with the move $\bullet_0^1$ for Player 1, his left-most decision point. At $\bullet_0^1$ the situation is one where Player 2 is the incumbent and Player 1 must decide whether to enter. If he chooses "IN" then the move passes to a payoff point for Player 2, where he receives V. Note that this payoff point is an exit point for 2; he exits from the game. The next situation is a payoff point for Player 1. Note that the payoff point for Player 1 is not an exit point; he stays in the game for his second period. The move now passes to (the new) Player 2, who can choose to enter the industry or leave the game. If he chooses "IN" then the move passes to a payoff point, which is also an exit point, for Player 1, where he receives the payoff of V and leaves the game. The move then passes to a payoff point for Player 2; he also receives V. We return to the situation $\bullet_0^1$. This can continue indefinitely, as long as both Players (or, in other words, player types), continue to choose "IN."

Let us now look at what happens when we are at the point $\bullet_0^1$ and Player 1 chooses not to enter the game. In this case, Player 1 chooses "OUT," and receives a payoff of W, and Player 2 receives a payoff of U at his exit payoff point. The move passes to an entry point for a new Player 2.

Observe that each player has an "outside" decision point at a corner of the rectangle and an "inside" decision point in the interior of the rectangle. Let $i1$ be Player i's agent at his inside decision point and let $i2$ be his agent at the outside decision point. In addition, let α_i be agent $i1$'s probability of choosing "IN" and let β_i be agent $i2$'s probability of choosing "IN." A global strategy for the example can be described as a quadruple $(\alpha_1, \alpha_2, \beta_1, \beta_2)$.

PROPOSITION 3. *For*

$$U + V > W > 2V$$

the game of Fig. 5 *has exactly three stationary equilibrium points, namely* $(1, 0, 1, 1), (0, 1, 1, 1)$ *and* $(\alpha, \alpha, 1, 1)$ *with*

$$\alpha = \frac{U + V - W}{U - V}.$$

Proof. The payoffs of the agents ij are described by the following table.

Agent	Payoffs for IN	Payoffs for OUT
11	$V + \alpha_2 V + (1 - \alpha_2)U$	W
12	$U + \alpha_2 V + (1 - \alpha_2)U$	W
21	$V + \alpha_1 V + (1 - \alpha_1)U$	W
22	$U + \alpha_1 V + (1 - \alpha_1)U$	W

It follows from $U + V > W > 2V$ that agents 12 and 22 receive at least $U + V$ for IN. Thus, we must have $\beta_1 = \beta_2 = 1$.

Assume $\alpha_{i.} = 0$. Then it is optimal for the other player to play $\alpha_j = 1$ in view of $U + V > W$. For $\alpha_i = 1$ it is optimal to play $\alpha_j = 0$ in view of $W > 2V$. This shows that the game of Fig. 5 has exactly two stationary equilibria in pure strategies, namely $(1, 0, 1, 1)$ and $(0, 1, 1, 1)$.

Now assume that at a stationary equilibrium one player, say player i, uses a mixed local strategy with $0 < \alpha_i < 1$ at his inside decision point. This has the consequence that $0 < \alpha_j < 1$ holds for the other player j since player i's best replies to $\alpha_j = 0$ and $\alpha_j = 1$ are pure. Since player i must be indifferent between IN and OUT at his inside decision point we must have

$$V + \alpha_j V + (1 - \alpha_j)U = W.$$

This yields

$$\alpha_j = \frac{U + V - W}{U - V}.$$

In view of $U + V > W$ the numerator is positive. The inequality $U + V > 2V$ implies that the denominator also is positive. From $W > 2V$ we have $-2V > -W$. It follows by addition of $U + V$ on both sides that the numerator is smaller than the denominator. Thus, we have $0 < \alpha_j < 1$. The symmetry of the game has the consequence

$$\alpha_1 = \alpha_2 = \alpha.$$

Note that symmetry of the equilibrium is not assumed but is obtained in the above argument as a consequence of the assumption that one player, i, is genuinely mixing. It is also clear that $(\alpha, \alpha, 1, 1)$ is a stationary equilibrium point. As we have seen, it is the only stationary equilibrium point besides the two pure stationary equilibrium points. ■

Interpretation. U is the per period payoff of a supplier who is alone in the market and V is the per period payoff of each one of two competing suppliers. In view of $W > 2V$ the market is not big enough for the suppliers. The pure strategy equilibrium describes a situation in which always only one player is in the market, the same one in every period. This player is willing to enter no matter what and thereby deters the other player from entry. Whenever he enters he receives a rent of $2U - W$, above his alternative profit W from staying out.

The symmetric mixed strategy equilibrium $(\alpha, \alpha, 1, 1)$ is the only stationary equilibrium in genuinely mixed strategies. At his inside decision point a player is indifferent between IN and OUT. This is due to the fact that the potential successor of his competitor will enter with probability α. At an inside decision point, the equilibrium profit is zero, but not at an outside decision point. Sometimes a player has an opportunity to enter an empty market. If one looks at the change from inside to outside decision points as a Markov chain, one obtains the transition diagram shown in Fig. 6.

Let p be the stationary probability of "outside." We have

$$p = (1 - \alpha)(1 - p)$$

and therefore

$$p = \frac{1 - \alpha}{2 - \alpha}.$$

Obviously, p is smaller than $1/2$.

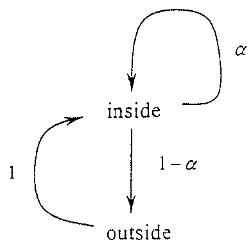

FIGURE 6

We next compare the mixed stationary equilibrium points and the pure ones with respect to the average rent, defined as the average surplus over W of a potential entrant's expected payoff.

At the pure stationary equilibria half of the potential entrants enter. Therefore the average rent R_1 at the pure stationary equilibrium points is as follows:

$$R_1 = \frac{1}{2}(2U - W).$$

At the mixed stationary equilibrium no surplus over W is achieved by entry at an inside decision point. At an outside decision point expected payoffs are U in the first period and $\alpha V + (1-\alpha)U$ in the second period. Therefore the average rent R_2 at the mixed stationary equilibrium is as follows:

$$R_2 = p(U + \alpha V + (1-\alpha)U - W).$$

From the first equation of the proof of the Proposition, it follows that

$$\alpha V + (1-\alpha)U = W - V$$

and therefore

$$R_2 = p(U - V).$$

Since $p < \frac{1}{2}$ it holds that:

$$R_2 < \frac{1}{2}(U - V).$$

Moreover, from $2V < W < U + V$ it follows that:

$$2R_2 < (U - V) < (2U - W) = 2R_1.$$

We can conclude that the average rent at the mixed strategy stationary equilibria is less than the average rent at the pure strategy stationary equilibrium. In this sense competition is stronger at the mixed strategy equilibria than at the pure strategy equilibrium, but in contrast to what we might expect from the theory of contestable markets (Baumol *et al.*, 1982), rents are not necessarily completely eliminated.

5. CONCLUSIONS

A limitation of our model is that we require complete information. It is an open question whether results such as those of Jackson and Kalai would hold in the context of a cyclic game with incomplete information. As Jackson and Kalai (1997), p. 104–105 write "The fact that recurring-game players are disjoint across states leads, under natural conditions, to convergence to trembling hand perfect equilibrium. . . . We should emphasize that this result is unique to the recurring setting and is not true in repeated settings." Thus, whether the Jackson and Kalai result would apply to cyclic games is quite unclear.

A research interest which led to the current paper is related but distinct from the motivaton of Jackson and Kalai. Casual observation suggests that players frequently learn from others who are in the game at the same time. For example, children can learn from parents and assistant professors can learn from full professors and from each other. With a suitable information structure, it may be possible to study this sort of learning in the framework of cyclic games.

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