JOURNAL OF URBAN ECONOMICS **41,** 421–440 (1997)
ARTICLE NO. UE962008

Equivalence of the Core and Competitive Equilibrium in a Tiebout Economy with Crowding Types*

John P. Conley[†] and Myrna H. Wooders[‡]

[†]*Department of Economics, University of Illinois, Champaign, Illinois 61820; and*
[‡]*Department of Economics, University of Toronto, Toronto, Ontario M5S 1A1, Canada*

We propose a new model of a local public goods economy with differentiated crowding. The new feature is that taste and crowding characteristics of agents are distinguished from one another. We prove that if the economy satisfies strict small group effectiveness then the core is equivalent to the set of Tiebout equilibrium outcomes. Equilibrium prices are defined to depend solely on crowding characteristics. This implies that only publicly observable information, and not private information such as preferences, is needed to induce agents to sort themselves into efficient jurisdictions. Thus, our model allows us to satisfy Bewley's (T. Bewley, *Econometrica*, **49,** 713–740, 1981) anonymity requirement on taxes in his well-known criticism of the Tiebout hypothesis. © 1997 Academic Press

1. INTRODUCTION

Lindahl equilibrium requires that agents with different tastes pay different prices for public goods. Samuelson [22] notes that agents may therefore find it in their best interests to conceal their true preferences. This observation leads Samuelson to conclude that it may be impossible to achieve a Lindahl equilibrium outcome in a market context. In his seminal paper, Tiebout [28] proposes a solution. He observes that many types of public goods are ‘‘local’’ rather than ‘‘pure,’’ and suggests that competition among local jurisdictions for members will lead to a market-like outcome. Agents will find it optimal to reveal their preferences through their choice of jurisdiction. As a consequence, the free-rider problem disappears and the outcome is efficient.

* We are grateful to Jan Brueckner, Vicky Barham, and Sergiu Hart for helpful comments. We also gratefully acknowledge support from the National Science Foundation and the Social Sciences and Humanities Research Council of Canada, respectively. We take responsibility for any remaining errors.

0094-1190/97 $25.00
Copyright © 1997 by Academic Press
All rights of reproduction in any form reserved.

Tiebout's claim has sparked a great deal of debate. Bewley [6], for example, offers counterexamples to several interpretations of Tiebout's basic hypothesis. In particular, Bewley shows that in some cases anonymous prices may support only inefficient allocations and in other cases anonymous prices that decentralize efficient allocations may not exist at all. Bewley's argument goes to the heart of the question. His main point is that any meaningful confirmation of the Tiebout hypothesis must show that efficient allocations can be decentralized through a price system which is *anonymous* in the sense that it does not depend on agents' preferences. Otherwise the price system violates the usual assumption in market economies that tastes are private information and leaves Samuelson's criticism unanswered.

We conclude from Bewley that efficient allocations in a local public goods economy cannot, in all cases, be decentralized. Other researchers, however, have shown that, if a small number of natural conditions are imposed on the economic environment, positive results may be recovered. Wooders [29], for example, shows that when agents crowd each other nondifferentially and small groups are strictly effective,[1] the core is equivalent to the set of Tiebout equilibrium states. Recall that nondifferentiated crowding, which has also been called ''anonymous crowding'' in the previous literature, specifies that agents are affected only by the size of the jurisdiction in which they live, and not by its composition (the identities or types of agents in their jurisdiction). Small groups are strictly effective if all gains from scale in population can be realized by groups bounded in size, and this bound is strictly smaller than the entire population of agents of each type.

An important feature of the results given in Wooders [29] is that the price system decentralizing the core is anonymous. These results provide strong support for the Tiebout hypothesis in large economies with nondifferentiated crowding.[2] Related contributions that confirm this finding are Boadway [7], Berglas and Pines [5], Scotchmer and Wooders [26], and Barham and Wooders [1], among many others.

While small group effectiveness is a comparatively mild restriction, nondifferentiated crowding is not. For example, we care not only about the

[1] Requiring small groups to be strictly effective is natural in this context. In Conley and Wooders [13], the authors argue that small group effectiveness is almost equivalent to the definition of the type of local public goods economy Tiebout described in his original paper. We provide a formal definition in section three.

[2] An alternative view is that the ''Tiebout Hypothesis'' relates to the homogeneity of efficient jurisdictions. It is our interpretation that Tiebout's interest in this question stemmed from his belief that having a single type of agent in each jurisdiction was a necessary condition to maximize per capita utility and therefore for optimality. This question is taken up in the context of the current model in the conclusion and in Conley and Wooders [14].

number of people at a dance, but also about the ratio of men to women. Peer-group effects and labor complementaries in production are other well-known causes of differentiated crowding that have been widely discussed in the literature. See for example, Berglas [4] (who introduces the idea of differentiated crowding), McGuire [21], Brueckner [8], Brueckner and Lee [9], de Bartolome [2], Schwab and Oates [23], Benabou [3], and Epple and Romano [16].[3]

Wooders [30, 33] formalizes a general equilibrium model of an economy with differentiated crowding and shows that when small groups are effective, as an economy increases in size while the costs of group formation decrease, the core converges to the set of Tiebout equilibrium states.[4] The major problem with the differentiated crowding literature is that decentralizing optimal decision-making requires that prices be defined to depend on the tastes of agents. Unless different types are charged or compensated for their positive or negative effects on others, they will not take into account the externalities they produce when they decide to join a given jurisdiction. Thus, it appears that since we must know the type of each agent before we can assign him personalized prices, the Tiebout hypothesis in its full force fails in economies with differentiated crowding.

The main goal of this paper is to introduce a model with public goods and differentiated crowding in which decentralization of the core does not require prices that are dependent on preferences of agents. The key difference between the model presented here and previous differentiated crowding models is that we make a distinction between two separate sets of characteristics of agents. The first set consists of tastes and endowments. These are unobservable and do not enter into the objectives or constraints of other agents. The second set includes crowding characteristics that enter into utility or production functions and therefore affect the welfare of others. We assume that crowding characteristics are observable and allow their effects to be either positive or negative. For example, we may be able to distinguish males from females, but we cannot usually tell which agents prefer country music to jazz.

To demonstrate that only crowding characteristics need be observable to obtain first best outcomes, we begin by defining the notion of an anony-

[3] The Epple and Romano and the Brueckner papers are especially closely related to our work. These two papers also make a distinction between agents' tastes and external effects although in the context of specific applied models.

[4] In Wooders [35] the proof given in Wooders [30, 33] is extended to show the same convergence results with Lindahl prices (and admission prices). Wooders [30] simplifies the model and results by assuming, as in Wooders [29], that small groups are effective, that is, all gains to collective activities can be realized by groups bounded in size—the other papers require only boundedness of average feasible utilities (per capita boundedness). The special case of one private good and one public good is discussed in Scotchmer [24].

mous Tiebout admission price equilibrium. This includes an equilibrium price system which specifies a price that each particular crowding type must pay to be allowed to join any given jurisdiction with a given level of public good. Thus, our equilibrium notion satisfies Bewley's [6] requirement that decentralization be accomplished through prices that do not depend on private information.

Our main result is that when small groups are strictly effective, the set of core states is equivalent to the set of Tiebout equilibrium states. We also show that the First Welfare Theorem holds, but that the Second Welfare Theorem is generally false. We prove that if small groups are strictly effective all core states have the equal treatment property. Finally, we note that when neither crowding nor taste types are observable, decentralization is generally not possible. In this case, market failure is due to adverse selection. But even private goods economies fail to be efficient when agents hide information that enters directly into the constraints or objectives of others, so this is not a problem particularly associated with Tiebout economies. We conclude that Tiebout's hypothesis is correct even when crowding is differentiated, and the informational requirements of Tiebout equilibrium are no more stringent than those of Walrasian equilibrium in private goods economies.

2. THE MODEL

In order to make our arguments more transparent, we restrict attention in the current paper to a model with one public good, one private good, and a finite number of agents with quasi-linear preferences. These results can be generalized to nontransferable utility economies with many public goods and asymptotically to economies with many private goods using techniques developed by Wooders (see, for example, Wooders [33, 35]).

Agents are defined by two characteristics. There are T different sorts of tastes or preference maps, denoted by $t \in \{1, \ldots, T\} \equiv \mathscr{T}$, and C different sorts of crowding types, denoted $c \in \{1, \ldots, C\} \equiv \mathscr{C}$. We assume no correlation between c and t. Imagine, for example, a dance at which men and women crowd each other differently. Some individuals like country music and some like jazz. There are men and women with each type of preference. The tastes and endowments of individuals are private information, but their crowding characteristics are publicly observable.[5] We can easily imagine a situation in which an agent might try to hide his true preference

[5] This is stricter than Bewely's [6] demand that preferences be private information but that endowments be publicly observable. The desirability of this additional restriction might be motivated by pointing out how difficult it is for the IRS to get honest reporting of income. In any event, the price system we describe below does not need to discriminate between agents on the basis of endowment.

type in order to take advantage of the allocations or prices available to agents of the other type. The main question we will address is whether it is possible to get an efficient decentralizing price system in which prices can be based only on publicly observable characteristics $\mathscr{C}$ and not the personal preferences of agents $\mathscr{T}$.

The population of agents is denoted by $N = (N_{11}, \dots, N_{ct}, \dots, N_{CT})$, where N_{ct} is interpreted as the total number of agents with crowding type c and taste type t in the economy. A *jurisdiction* is a group of agents which collectively produce and consume a common level of public good. A jurisdiction is represented by a vector $m = (m_{11}, \dots, m_{ct}, \dots, m_{CT})$, where m_{ct} is interpreted as the number of agents with crowding type c and taste type t in the jurisdiction m. The set of all feasible jurisdictions is denoted by $\mathscr{N}$. We will denote by $\mathscr{N}_c$ the set of feasible jurisdictions that contain at least one agent of crowding type $c \in \mathscr{C}$. Formally,

$$\mathscr{N}_c \equiv \{m \in \mathscr{N} \mid \text{there exists } t \in \mathscr{T} \text{ such that } m_{ct} > 0\}.$$

Two jurisdictions, m and $\hat{m}$, have the *same crowding profile* if for all $c \in \mathscr{C}$, $\sum_t m_{ct} = \sum_t \hat{m}_{ct}$. That is, two jurisdictions have the same crowding profile if the number of agents of each given crowding type is the same in both jurisdictions. A *partition* $n = \{n^1, \dots, n^K\}$ of the population is a collection of jurisdictions such that $\sum_k n^k = N$.[6] It will sometimes be necessary to refer to individual agents $i \in \{1, \dots, I\} \equiv \mathscr{I}$. Observe that $I = \sum_{c,t} N_{ct}$. Let $\theta \colon \mathscr{I} \to \mathscr{C} \times \mathscr{T}$ be a function that indicates the type of a given individual. Thus, if agent i is of crowding type c and taste type t, then $\theta(i) = (c, t)$. With a slight abuse of notation, if individual i is a member of jurisdiction m, we shall write $i \in m$.

We consider an economy with one private good x and one public good y. Assume that agents are members of exactly one jurisdiction. Each agent $i \in \mathscr{I}$ of taste type t is endowed with ω_t of the private good and has a quasi-linear utility function

$$u_t(x, y, m) = x + h_t(y, m),$$

where $i \in m$ and y is the quantity of public good produced in jurisdiction m.[7] The only condition we impose on utility functions other than quasi-linearity is *taste anonymity in consumption* (TAC).

[6] As with generic jurisdictions m, the notation n_{ct}^k denotes the number of agents of crowding type c and taste type t in the jurisdictions n^k which is an element of the partition n.

[7] Formally, this implies that agents with the same tastes but different crowding characteristics have the same endowments. This is without loss of generality since there is no requirement that agents of taste type t have different preferences from agents of type t'. Thus, we can consider agents of the same crowding type with the same preferences but different endowments to be different taste types.

TAC: For all $m, \hat{m} \in \mathscr{N}$, if for all $c \in \mathscr{C}$ it holds that $\sum_t m_{ct} = \sum_t \hat{m}_{ct}$, then for all $y \in \Re_+$, and for all $t \in \mathscr{T}$ it holds that $h_t(y, m) = h_t(y, \hat{m})$.

This is a formal statement of the notion that agents care only about the crowding types and not the taste types of the agents in their jurisdiction.

The cost in terms of private good of producing y public good for a jurisdiction with membership m is given by the function

$$f(y, m).$$

We do not impose any condition of convexity, continuity or even monotonicity on the cost function. In keeping with the spirit of the model, we assume that only the crowding profile of agents in a jurisdiction affects the cost of producing the public good. We call this *taste anonymity in production* (TAP).

TAP: For all $m, \hat{m} \in \mathscr{N}$, if for all $c \in \mathscr{C}$ it holds that $\sum_t m_{ct} = \sum_t \hat{m}_{ct}$, then for all $y \in \Re_+$ it holds that $f(y, m) = f(y, \hat{m})$.

The assumptions TAC and TAP are maintained in all that follows and no further mention of them will be made. The main point of this paper is to show that when we impose these two assumptions, the core can be decentralized by an anonymous price system. Without these assumptions on an economy, it is easy to find counterexamples to our results.

A *feasible state of the economy* (X, Y, n) is a partition n of the population, an allocation $X = (x_1, \dots, x_I)$ of private goods, and public good production plans $Y = (y^1, \dots, y^K)$ such that

$$\sum_k \sum_{c,t} n_{ct}^k \omega_t - \sum_i x_i - \sum_k f(y^k, n^k) \geq 0.$$

Denote the set of feasible states by F. We will also say that $(\bar{x}, \bar{y})$ is a *feasible allocation for a jurisdiction* m if

$$\sum_{c,t} m_{ct} \omega_t - \sum_{i \in m} \bar{x}_i - f(\bar{y}, m) \geq 0.$$

A jurisdiction $m \in \mathscr{N}$ producing a feasible allocation $(\bar{x}, \bar{y})$ *can improve upon* a feasible state $(X, Y, n) \in F$ if, for all $i \in m$:[8]

$$u_t(\bar{x}_i, \bar{y}, m) > u_t(x_i, y^k, n^k),$$

[8] Since agents are assumed to have quasi-linear preferences, this is equivalent to requiring that no one agent be made better off, while keeping other agents at least as well off.

where $\theta(i) = (c, t)$ for some $c \in \mathscr{C}$, and agent $i \in n^k \in n$ in the original feasible state. A feasible state $(X, Y, n) \in F$ is in the *core* of the economy if it cannot be improved upon by any jurisdiction.

3. TIEBOUT EQUILIBRIUM AND THE CORE

In this section, we define a notion of Tiebout equilibrium with admission prices and study its relationship to the core. Jurisdictions base prices only on crowding types of agents since crowding type is the only publicly observable characteristic. No price discrimination is allowed between different taste types.

Our equilibrium notion requires an admission price for each crowding type for every possible jurisdiction with every possible level of public good.[9] At first glance, there might seem to be a similarity between the definition of the core and Tiebout equilibrium. The fact that all alternative jurisdictions are available through the price system may suggest that admission prices convey information about the feasible opportunities to improve on the equilibrium state. It is important to emphasize that this is not the case. There is nothing in the definition of these Tiebout supporting prices that requires them to bear any relationship to the actual cost of providing a public good to a group, or the marginal crowding cost imposed by the various types on jurisdictions. This follows the traditional definition of competitive equilibrium prices for private goods economies.[10] In both cases it is a result, and not an assumption, that in equilibrium supporting prices reflect marginal costs. This in turn implies that all supportable allocations are Pareto optimal. In our case, when small groups are strictly effective, we are able to go one step further and show core equivalence.

A price system ρ_c for agents of crowding type $c \in \mathscr{C}$ gives an admission price for every jurisdiction an agent of this type is able to join and for every possible public good level. An agent is able to contemplate joining any jurisdiction that contains at least one member of his crowding type. For example, no matter how much Wynton Marsalis may wish it, it is impossible for him to join an all-female band. Once he joins, it is no longer an all-female band since it includes at least one man. Thus, we should provide admission prices for bands that include at least one male, but it makes no sense to provide an admission price to Wynton Marsalis for female-only bands. Formally, a *Tiebout price system for crowding type* c is a

[9] See Conley and Wooders [14] for an alternative equilibrium concept supported by a finite set of anonymous prices and discussion of other equilibrium concepts.

[10] When we take the points in the commodity space to be pairs (y, m) consisting of descriptions of jurisdictions and levels of public good, our approach is similar to the differentiated commodities literature. See, for example, Mas-Colell [18].

mapping:

$$\rho_c \colon \Re_+ \times \mathscr{N}_c \to \Re.$$

A *Tiebout price system* is simply the collection of price systems, one for each crowding type, and is denoted by ρ.

Even though the price system defined above is constrained to charge each agent of the same crowding type the same prices, we add one extra condition to make it fully anonymous.[11] Observe that ρ gives an admission price for every jurisdiction m, and that included in this description is the *taste profile* of the jurisdiction. Since we assume that tastes are not observable, a system has *fully anonymous prices* (FAP) only if jurisdictions with the same crowding profile are priced identically. Formally,

FAP: For all $m, \hat{m} \in \mathscr{N}$, if for all $c \in \mathscr{C}$ it holds that $\sum_t m_{ct} = \sum_t \hat{m}_{ct}$ then for all $y \in \Re_+$ it holds that $\rho(y, m) = \rho(y, \hat{m})$.

A *Tiebout equilibrium* is a feasible state $(X, Y, n) \in F$ and a price system ρ such that:

1. For all $n^k \in n$, all individuals $i \in n^k$ such that $\theta(i) = (c, t)$, all alternative jurisdictions $m \in \mathscr{N}_c$, and for all levels of public good production $y \in \Re_+$,

$$\omega_t - \rho_c(y^k, n^k) + h_t(y^k, n^k) \geq \omega_t - \rho_c(y, m) + h_t(y, m).$$

2. For all potential jurisdictions $m \in \mathscr{N}$ and all $y \in \Re_+$,

$$\sum_{c,t} m_{ct}\, \rho_c(y, m) - f(y, m) \leq 0.$$

3. For all $n^k \in n$,

$$\sum_{c,t} n^k_{ct}\, \rho_c(y^k, n^k) - f(y^k, n^k) = 0.$$

Condition (1) says that all agents maximize utility given the price system. Note that the price schedule available to an agent depends only on his crowding type. Condition (2) requires that given the price system, no firm can make positive profits by entering the market and offering to provide

[11] We thank Jan Brueckner for this observation. An alternative interpretation which would make it unnecessary to impose FAP is that pricing jurisdictions with identical crowding profiles differently does not violate anonymity since these prices are still commonly available to all regardless of tastes (cf. Wooders [29]). It turns out that this is not an issue for admission price systems since we demonstrate that the core may be decentralizing with prices satisfying FAP. It is not immediate, however, that this is possible for all types of price systems.

any sort of jurisdiction. Condition (3) requires that all equilibrium jurisdictions make zero profit, and so cover their costs.[12]

At first it may seem natural that the price system of a Tiebout equilibrium discriminates only on the basis of the effects an agent has on others, but correct pricing of these external effects is only part of the task of the system. Efficient provision of the public good requires that the Samuelson conditions be satisfied: the sum of the marginal willingness to pay of all agents in a given jurisdiction must equal the marginal cost. Thus, an anonymous price system must also induce agents to reveal taste information through their choice of jurisdiction, just as Tiebout envisioned. The satisfaction of the Samuelson conditions in such economies is treated in more detail in Conley and Wooders [14]. There, the authors look at Lindahl price equilibrium in which each agent of a given crowding type pays a lump sum participation price to join a jurisdiction, and may then purchase as much public good as he likes at a fixed per-unit Lindahl price. It would also be possible to consider equilibrium concepts similar to valuation or cost-share equilibrium (see Mas-Colell and Silvestre [19]) or equilibrium notions defined for economies with variable usage of club goods (see Scotchmer and Wooders [26]) for the crowding types model. We leave these questions for future research.

Our first theorem shows that all Tiebout states are contained in the core. For the sake of readability, most proofs are relegated to the appendix.

THEOREM 1. *If the state $(X, Y, n) \in F$ and the price system ρ constitute a Tiebout equilibrium, then (X, Y, n) is in the core.*

Proof. See appendix. ∎

The First Welfare Theorem is an immediate corollary of Theorem 1.

COROLLARY 1.1. *All Tiebout states are Pareto optimal.*

Proof. By Theorem 1, Tiebout states are contained in the set of core states. Since these core states are Pareto optimal, Tiebout states must also be Pareto optimal. ∎

Without stronger assumptions, the Second Welfare Theorem generally does not hold. This is because the core may be empty for all reallocations of initial endowments even when the economy satisfies strict small group

[12] Sergiu Hart has pointed out to us that condition (3) is implied by condition (2) and the definition of feasibility. We continue to state condition (3) because we wish to emphasize that equilibrium jurisdictions make zero profit, and thus, club formation is competitive.

effectiveness (formally defined below).[13] Since all Tiebout equilibrium states are in the core, when the core is empty it is obviously impossible to support *any* of the Pareto optimal allocations as Tiebout equilibria. The emptiness of the core is not overly troubling in this case, and may be solved either by going to the continuum, (Kaneko and Wooders [17]) or by considering ϵ-cores, as in Wooders [31] and Shubik and Wooders [27]. There is no technical difficulty in extending these results to economies with crowding types.

We now turn our attention to economies in which small groups are effective. An economy satisfies *strict small group effectiveness* (SGE), if there exists a positive integer B such that:

1. For all core states (X, Y, n) and all $n^k \in n$, it holds that $\sum_{ct} n^k_{ct} \leq B$.
2. For all $c \in \mathscr{C}$ and all $t \in \mathscr{T}$, it holds that $N_{ct} > B$.

The first condition requires that any state which includes at least one jurisdiction with more than B agents can be improved upon. In other words, coalitions larger than B do strictly worse that coalitions with B agents or fewer. The second condition says that there are at least B agents of each type in the economy.

This is a relatively strong formalized version of the sixth assumption in Tiebout's original paper. Alternative definitions of strict small group effectiveness include assuming that all feasible utility vectors can be realized with partitions of the agents into jurisdictions containing no more than B members or that for sufficiently large replications of the economy, further replications do not increase per capita utilities. A less restrictive version, small group effectiveness, would require that groups bounded in size are able to achieve all or almost all per capita gains. More formally, given any ϵ greater than zero, there is an integer $B(\epsilon)$ such that groups can be constrained to be of size less then or equal to $B(\epsilon)$ with a loss due to this constraint of at most ϵ per capita. If sufficiently many agents of each type appear in the economy, this form of SGE is equivalent to the mild condition that per capita payoffs are bounded, as in Wooders [31], to show nonemptiness of approximate cores.[14] Given this, our view is that the choice of form of SGE is largely a matter of convenience and so we choose a version that contributes to the simplicity of our proofs.

[13] Recall that if the core of a quasi-linear economy with one private good is empty for *any* set of initial endowments, it must also be empty for all reallocations.

[14] See Wooders [34] for a discussion of the relationships between several forms of strict SGE and SGE. It is interesting to note that in the standard differentiated crowding model, the equal treatment core is equivalent to the set of nonanonymous admission price equilibrium states regardless of whether SGE holds (see Conley and Wooders [13], Section 7, for discussion). While this result carries over to the crowding types model developed in the current paper, the equal treatment core is not equivalent, in general, to the *anonymous* admission price equilibrium unless SGE holds.

We are now ready to state the remainder of our economic results. Theorem 2 establishes that SGE implies that two agents with the same taste and crowding type receive the same utility level in any core state. That is, the core has the equal treatment property.[15]

THEOREM 2. *Let (X, Y, n) be a core state of an economy satisfying SGE and having population N. For any two individuals $i, \hat{i} \in \mathscr{I}$ such that $\theta(i) = \theta(\hat{i}) = (c, t)$, if $i \in n^k$ and $\hat{i} \in n^{\hat{k}}$, then $u_t(x_i, y, n^k) = (u_t, \hat{x}_{\hat{i}}, \hat{y}, n^{\hat{k}})$.*

Proof. See appendix. ∎

The next theorem is especially important. It establishes that SGE implies that in all core states, in any given jurisdiction, the implicit private goods contribution to public goods provision of a given crowding type is the same across all taste types. In other words, the implicit admission prices to jurisdictions are anonymous in the sense that they depend only on crowding characteristics, and not on tastes of agents.

THEOREM 3. *Let (X, Y, n) be a core state of an economy satisfying SGE and having population N. Suppose that for some jurisdiction $n^k \in n$, for some crowding type $c \in \mathscr{C}$, and for two taste types $t, \hat{t} \in \mathscr{T}$, both $n^k_{ct} > 0$, and $n^k_{c\hat{t}} > 0$. Then for $i, \hat{i} \in n^k$ such that $\theta(i) = (c, t)$ and $\theta(\hat{i}) = (c, \hat{t})$, it holds that $\omega_t - x_i = \omega_{\hat{t}} - x_{\hat{i}}$.*

Proof. See appendix. ∎

We are now able to prove the main result of the paper. Theorem 4 states that if the economy satisfies SGE then all core states can be supported as Tiebout equilibrium states.

THEOREM 4. *If an economy satisfies SGE, then for all states (X, Y, n) in the core there exists a price system ρ such that (X, Y, n) and ρ form a Tiebout equilibrium.*

Proof. See appendix. ∎

The equivalence of the core and anonymous Tiebout equilibrium states is an immediate consequence.

THEOREM 5 (Core-Equilibrium Equivalence with Anonymous Pricing). *If an economy satisfies SGE then the set of states in the core of the economy is equivalent to the set of Tiebout equilibrium states.*

[15] The equal treatment property of the core for games and economies satisfying strict small group effectiveness holds generally; see Wooders [31, Theorem 3, and 34 and references therein].

Proof. By Theorem 1, all Tiebout states are in the core, and by Theorem 4, all core states can be decentralized as Tiebout equilibria. ∎

Note that core equivalence holds even though there are only a finite number of agents. In contrast, results for private goods economies (see Debreu and Scarf [15]) and economies with pure public goods (see Conley [11]) depend on limiting arguments. The reason for this difference is that in Tiebout economies with one private good, blocking opportunities are exhausted by relatively small coalitions. This causes agents to form the relatively small coalitions that characterize such economies. In pure private and pure public goods economies, on the other hand, blocking opportunities are exhausted only in the limit if at all. As a consequence, the core continues to shrink as the economy gets large, but it is optimal for all agents to stay in one large jurisdiction.

4. CONCLUSION

This paper has introduced an apparently small modification to the standard model of differentiated crowding economies. Specifically, we formally distinguish between the tastes and crowding effects of agents through the simple device of adding a second index to the definition of each agent's type. This modification, however, allows us to show that even when crowding is differentiated, if small groups are effective, it is not necessary to know private information such as preferences when defining a price system that induces agents to allocate themselves efficiently to jurisdictions. Provided crowding characteristics are publicly observable, agents will respond by "voting with their feet," thus avoiding the free rider problem and implementing the Tiebout equilibrium.

One may reasonably object that it is not always possible to observe the crowding types of agents. For example, one may discover only after he signs a lease that a new roommate is given to singing arias from *La Traviata* while sleep-walking. In such cases, the core may not be decentralizable. But such situations cause the same sorts of problems in the private goods case as well. For example, if an insurance company cannot tell which agents are likely to eat wisely and exercise regularly, it cannot price insurance efficiently. The full information core cannot be decentralized if prices do not take agents' eating and exercise habits into account. In both cases, the agents' tastes are completely irrelevant to the problem of supporting first-best allocations. Only the characteristics that enter into the objectives or constraints of others are important.

Our model also allows us to address several long-standing questions in a new light. For example, it is well established that the core of a Tiebout economy with nondifferentiated crowding must be taste homogeneous.[16] (See, for example, Wooders [29], Berglas and Pines [5], and Barham and Wooders [1] for a recent discussion). When crowding is differentiated, the results are not so clear. Recent contributions show that gains from collaboration in production can offset gains from homogeneity of taste. In particular, McGuire [21] and Brueckner [8] explore comparative static properties of economies with complementaries in production and peer-group effects. When crowding types are separate from taste types it would seem that this motivation to mix evaporates. It is possible to take advantage of the full array of crowding characteristics while segregating agents into taste-homogeneous jurisdictions. It is somewhat surprising, therefore, that taste-homogeneity turns out not to be optimal in general. Conley and Wooders [12], however, show the taste-heterogeneous coalitions may be strictly better on a per capita utility basis than taste homogeneous coalitions with the same profiles of crowding types. A weaker homogeneity result, however, does hold.

In this paper we have presented a model that for the first time clearly supports Tiebout's hypothesis in the differentiated crowding case while maintaining the private information structure that is standard in market economies. This should be of value to applied practitioners since it suggests that when small groups are effective, admission prices such as property taxes can be based solely on publicly observable characteristics like number of children, size of house, and number of cars, and not on unobservable tastes. From a theoretical standpoint, there are many areas for future research. Most important are issues surrounding the taste-homogeneity of the core. The problems of core existence and the conditions under which the Second Welfare Theorem holds are also open. In short, distinguishing taste from crowding characteristics makes it possible to explore questions that traditionally have been of interest to local public goods economists in a realistic market environment.

APPENDIX

THEOREM 1. *If the state $(X, Y, n) \in F$ and the price system ρ constitute a Tiebout equilibrium, then (X, Y, n) is in the core.*

Proof. Suppose not. Then the Tiebout equilibrium state can be improved upon by some jurisdiction $m \in \mathscr{N}$, providing a feasible allocation $(\bar{x}, \bar{y})$. Consider an arbitrary agent $i \in m$, where $\theta(i) = (c, t)$. Suppose

[16] By taste homogeneity we mean that in all core states every jurisdiction will contain agents who have the same demands for public goods and crowding. Thus, it may be the case that at equilibrium prices, agents with different preferences will choose to join the same jurisdiction, but it will never be the case that a jurisdiction with several different taste types can do strictly better on a per capita basis than a taste homogeneous jurisdiction.

that in the Tiebout equilibrium state, i is a member of the jurisdiction $n^{k_i} \in n$. By definition, in the Tiebout equilibrium state, agent i's consumption of private good is

$$x_i \equiv \omega_t - \rho_c(y^{k_i}, n^{k_i}).$$

Suppose that jurisdiction m forms and agents pay admission prices given by the Tiebout pricing system instead of receiving the consumption levels they are assigned in the improving allocation. Denote these "Tiebout" consumption levels by

$$\tilde{x}_i \equiv \omega_t - \rho_c(\bar{y}, m).$$

Since (X, Y, n) is a Tiebout state and, by condition (1) of the definition of Tiebout equilibrium agents maximize utility under Tiebout prices, we know that

$$\omega_t - \rho_c(y^{k_i}, n^{k_i}) + h_t(y^{k_i}, n^{k_i}) \geq \omega_t - \rho_c(\bar{y}, m) + h_t(\bar{y}, m).$$

Substituting and summing over all agents in m yields

$$\sum_{i \in m} x_i + \sum_{c,t} m_{ct} h_t(y^{k_i}, n^{k_i}) \geq \sum_{i \in m} \tilde{x}_i + \sum_{c,t} m_{ct} h_t(\bar{y}, m).$$

But by the definition of improving jurisdictions, for all $i \in m$,

$$u_t(\bar{x}_i, \bar{y}, m) > u_t(x_i, y^{k_i}, n^{k_i}),$$

or equivalently,

$$\bar{x}_i + h_t(\bar{y}, m) > x_i + h_t(y^{k_i}, n^{k_i}).$$

Summing this over agents in m yields

$$\sum_{i \in m} \bar{x}_i + \sum_{c,t} m_{ct} h_t(\bar{y}, m) > \sum_{i \in m} x_i + \sum_{c,t} m_{ct} h_t(y^{k_i}, n^{k_i}).$$

This implies that

$$\sum_{i \in m} \bar{x}_i + \sum_{c,t} m_{ct} h_t(\bar{y}, m) > \sum_{i \in m} \tilde{x}_i + \sum_{c,t} m_{ct} h_t(\bar{y}, m),$$

which allows us to conclude that

$$\sum_{i \in m} \bar{x}_i > \sum_{i \in m} \tilde{x}_i.$$

However, by the definition of improving jurisdictions, it holds that

$$\sum_{c,t} m_{ct} \omega_t - \sum_{i \in m} \bar{x}_i - f(\bar{y}, m) \geq 0.$$

By the definition of a Tiebout equilibrium, it holds that

$$\sum_{c,t} m_{ct}\, \rho(\bar{y}, m) - f(\bar{y}, m) \equiv \sum_{c,t} m_{ct}\, \omega_t - \sum_{i \in m} \tilde{x}_i - f(\bar{y}, m) \leq 0.$$

Together, these two imply

$$\sum_{i \in m} \bar{x}_i \leq \sum_{i \in m} \tilde{x}_i,$$

a contradiction. ▮

THEOREM 2. *Let (X, Y, n) be a core state of an economy satisfying SGE. For any two individuals $i, \hat{i} \in \mathscr{I}$ such that $\theta(i) = \theta(\hat{i}) = (c, t)$, if $i \in n^k$ and $\hat{i} \in n^{\hat{k}}$, then $u_t(x_i, y^k, n^k) = u_t(\hat{x}_{\hat{i}}, y^{\hat{k}}, n^{\hat{k}})$.*

Proof. Suppose not. By SGE, for all $n^k \in n$, it holds that $\sum_{ct} n^k_{ct} \leq B$ and for all $c \in \mathscr{C}$, and all $t \in \mathscr{T}$, it holds that $N_{ct} > B$. Thus, we know that there are at least two jurisdictions containing at least one agent of type (c, t). Since there exists at least one pair of individuals of type (c, t) who are not equally treated in the core state, there must also exist at least one a pair of individuals of type (c, t) who are also not equally treated and who are members of *different jurisdictions* in the core partition n. We may therefore assume without loss of generality that $n^k \neq n^{\hat{k}}$ and $u_t(x_i, y^k, n^k) > u_t(x_i, y^{\hat{k}}, n^{\hat{k}})$.

Consider now a new jurisdiction $\overline{m}$ consisting of agent $\hat{i}$ and of all the agents in n^k except agent i. Let the public good production $\bar{y}$ for $\overline{m}$ be the same as the public good production in jurisdiction n^k, and let the private good allocation, $\bar{x}$ for $\overline{m}$, be as follows: for all $j \neq \hat{i}$, $\bar{x}_j \equiv x_j$, and for agent $\hat{i}$, $\bar{x}_{\hat{i}} \equiv x_i$. Note that since (X, Y, n) is a core state, $(\{x_i\}_{i \in n^k}, y^k)$ must be a feasible allocation for jurisdictions n^k. Otherwise, this jurisdiction would be receiving a net subsidy from the remaining agents in the population, and in this case the remaining population could improve upon this state by distributing this surplus among themselves instead. Since by construction, the crowding profile, production of public good, and net consumption of private good is the same for these two jurisdictions $\overline{m}$ and n^k, by TAP $(\bar{x}, \bar{y})$ must also be a feasible allocation for jurisdiction $\overline{m}$. Then by TAC, for all $j \in \overline{m}$ such that $j \neq \hat{i}$ we have,

$$u_{\tilde{t}}\left(\bar{x}_j, \bar{y}, \overline{m}\right) = u_{\tilde{t}}\left(x_j, y^k, n^k\right),$$

where $\theta(j) = (\tilde{c}, \tilde{t})$ for some $\tilde{c} \in \mathscr{C}$ and for agent $\hat{i} \in \overline{m}$ we have

$$u_t\left(\bar{x}_{\hat{i}}, \bar{y}, \overline{m}\right) > u_t\left(x_{\hat{i}}, y^k, n^k\right).$$

Thus, by equally distributing the surplus private good received by agent $\hat{i}$, the jurisdiction m would be able to improve upon (X, Y, n), contradicting the hypothesis that this state is in the core. ∎

THEOREM 3. *Let (X, Y, n) be a core state of an economy satisfying SGE and having population N. Suppose that for some jurisdiction $n^k \in n$, for some crowding type $c \in \mathscr{C}$, and for two taste types $t, \hat{t} \in \mathscr{T}$, $n^k_{ct} > 0$, and $n^k_{c\hat{t}} > 0$. Then for $i, \hat{i} \in n^k$ such that $\theta(i) = (c, t)$ and $\theta(\hat{i}) = (c, \hat{t})$, it holds that $\omega_t - x_i = \omega_{\hat{t}} - x_{\hat{i}}$.*

Proof. Suppose not. Without loss of generality, assume $\omega_t - x_i > \omega_{\hat{t}} - x_{\hat{i}}$. By the argument given in the first paragraph of the proof of Theorem 2, SGE implies that there exists a jurisdiction $n^{\hat{k}} \in n$ such that $n^{\hat{k}} \neq n^k$ containing an individual $\tilde{i} \in n^{\tilde{k}}$, such that $\theta(\tilde{i}) = (c, t)$. Since $\tilde{i}$ has same crowding type as $\hat{i}$, by TAC the other members of jurisdiction n^k would be just as well off if agent $\tilde{i}$ were to replace agent $\hat{i}$ in coalition n^k provided agent $\tilde{i}$ contributes at least $\omega_{\hat{t}} - x_{\hat{i}}$ to public good production. But by Theorem 2, all agents of any given type (c, t) are treated equally in the core. Therefore, since agent i is willing to join jurisdiction n^k and contribute $\omega_t - x_i$, and i and $\tilde{i}$ are of the same crowding and taste type, agent $\tilde{i}$ must be exactly as well off when he moves to jurisdiction n^k and contributes $\omega_t - x_i$ as he was in his original jurisdiction $n^{\tilde{k}}$. But $\omega_t - x_i - (\omega_{\hat{t}} - x_{\hat{i}}) > 0$, so it is possible to distribute this surplus over the agents now in the jurisdiction to make them better off than they were in the core state, a contradiction. ∎

THEOREM 4. *If an economy satisfies SGE, then for all states (X, Y, n) in the core there exists a price system ρ such that (X, Y, n) and ρ form a Tiebout equilibrium.*

Proof. Since the economy satisfies SGE, by Theorem 2 all agents of the same type are equally treated regardless of their choice of jurisdiction. Denote the utility received by agents of crowding type c and taste type t in the core state (X, Y, n) by U_{ct}. For all $c \in \mathscr{C}$ and $t \in \mathscr{T}$, denote the willingness of an agent of type (c, t) to pay to join a jurisdiction with profile $m \in \mathscr{N}_c$ producing public good level y as

$$p_{ct}(y, m) \equiv \omega_t + h_t(y, m) - U_{ct}.$$

Define the price system by

$$\rho_c(y, m) = \max_t p_{ct}(y, m).$$

Note that for any jurisdiction $n^k \in n$ that appears in the core partition and provides its members with the allocation (x, y^k), and for any given individual $i \in n^k$ such that $\theta(i) = (c, t)$, it holds that $U_{ct} = x_i + h_t(y^k, n^k)$. Thus,

$$p_{ct}(y^k, n^k) \equiv \omega_t - x_i.$$

But by Theorem 3, for all $i, \hat{i} \in n^k \in n$ such that $\theta(i) = (c, t)$, and $\theta(\hat{i}) = (c, \hat{t})$, it holds that $\omega_t - x_i = \omega_{\hat{t}} - x_{\hat{i}}$. Therefore,

$$\rho_c(y^k, n^k) = \omega_t - x_i. \tag{1}$$

1. We start by showing that the price system constructed here is anonymous in the sense that it satisfies the condition FAP. Consider any pair of jurisdictions $m, \hat{m} \in \mathscr{N}$ such that for all $c \in \mathscr{C}$ it holds that $\sum_t m_{ct} = \sum_t \hat{m}_{ct}$, and take any two agents $i \in m$ and $\hat{i} \in \hat{m}$ such that for some $c \in \mathscr{C}$, $\theta(i) = (c, t)$, and $\theta(\hat{i}) = (c, \hat{t})$. Recall that

$$p_{ct}(y, m) \equiv \omega_t + h_t(y, m) - U_{ct},$$

$$\rho_c(y, m) = \max_t p_{ct}(y, m),$$

and similarly for $\hat{m}$. But since for all $c \in \mathscr{C}$ it holds that $\sum_t m_{ct} = \sum_t \hat{m}_{ct}$ and by TAC all agents are indifferent between these two jurisdictions, if $y = \hat{y}$, then

$$\max_t p_{ct}(\hat{y}, \hat{m}) = \max_t p_{ct}(y, m).$$

Thus, if $y = \hat{y}$ then

$$\rho_c(\hat{y}, \hat{m}) = \rho_c(y, m).$$

2. Next we show that when agents maximize utility under these prices, they choose to join the jurisdictions to which they are assigned in the core state. By construction, an agent i satisfying $\theta(i) = (c, t)$ and $i \in n^k$ receives a utility level

$$\omega_t + h_t(y^k, n^k) - \rho_c(y^k, n^k) = \omega_t + h_t(y^k, n^k) - (\omega_t - x_i) \equiv U_{ct}.$$

Now consider the utility the agent could achieve by joining an arbitrary jurisdiction $m \in \mathscr{N}_c$ producing public good level y. By construction, for all $t \in \mathscr{T}$ such that $m_{ct} > 0$, it holds that

$$\omega_t + h_t(y, m) - U_{ct} = p_{ct}(y, m) \leq \rho_c(y, m).$$

Thus,

$$\omega_t - \rho_c(y, m) + h_t(y, m) \leq U_{ct}.$$

That is, no agent can get more utility by going to a jurisdiction other than the one to which he is assigned in the core state. Therefore, these prices satisfy condition (1) of the definition of the Tiebout equilibrium.

3. Now we show that no jurisdiction can make positive profits under these prices. Take an arbitrary jurisdiction $m \in \mathscr{N}$ and public good level y. Construct a new jurisdiction $\tilde{m}$ where for every crowding type $c \in \mathscr{C}$ we distinguish one particular taste type $\tilde{t} \in \mathscr{T}$ that has the property $p_{c\tilde{t}}(y, m) = \rho_c(y, m)$, and set $\tilde{m}_{c\tilde{t}} = \sum_t m_{ct}$, and $\tilde{m}_{ct} = 0$ for all $t \neq \tilde{t}$. That is, for every crowding type c, we replace each agent of taste type t with one of the agents of taste type $\tilde{t}$ who is willing to pay the most to join the jurisdiction.

By SGE it holds that $\sum_{c,t} \tilde{m}_{ct} \leq B$, and by construction,

$$\sum_{c,t} \tilde{m}_{ct}\, \rho_c(\tilde{y}, \tilde{m}) = \sum_{c,t} \tilde{m}_{ct}\big(\omega_t + h_t(\tilde{y}, \tilde{m}) - U_{ct}\big).$$

Thus, by SGE, $\tilde{m} \in \mathscr{N}$, and so $\tilde{m}$ is a potentially improving jurisdiction. However, since (X, Y, n) is a core state, if we sum across agents in $\tilde{m}$ we find

$$\sum_{c,t} \tilde{m}_{ct}\big(\omega_t + h_t(\tilde{y}, \tilde{m})\big) - f(\tilde{y}, \tilde{m}) \leq \sum_{ct} \tilde{m}_{ct} U_{ct}.$$

Since by TAP if $y = \tilde{y}$ it holds that $f(\tilde{y}, \tilde{m}) = f(y, m)$, we can rearrange the expression above to obtain

$$\sum_{c,t} \tilde{m}_{ct}\big(\omega_t + h_t(\tilde{y}, \tilde{m}) - U_{ct}\big) \leq f(y, m).$$

Since by construction

$$\sum_{c,t} \tilde{m}_{ct}\, \rho_c(\tilde{y}, \tilde{m}) = \sum_{c,t} m_{ct}\, \rho_c(y, m),$$

we conclude that

$$\sum_{c,t} m_{ct}\, \rho_c(y, m) \leq f(y, m).$$

Thus for these coalitions, condition (2) of the definition of the Tiebout equilibrium is satisfied.

4. Finally, we show that the jurisdictions in a core partition generate enough revenue at these prices to pay for the public good level they provide. From Eq. (1), for all $n^k \in n$ that appear in the core state, and for all $i \in n^k$ such that $\theta(i) = (c, t)$,

$$\rho_c(y^k, n^k) = \omega_t - x_i.$$

From part (3) above, we know

$$\sum_{c,t} n_{ct}^k \rho_c(y^k, n^k) = \sum_{c,t} n_{ct}^k(\omega_t - x_i) \le f(y^k, n^k).$$

Suppose that $\sum_{c,t} n_{ct}^k(\omega_t - x_i) < f(y^k, n^k)$. Then the jurisdiction n^k spends more on public good provision than it collects from the agents it includes. Since the state (X, Y, n) is feasible by definition, there must be some other jurisdiction that spends less on public good provision than it collects from its members. But then this jurisdiction could improve on the state (X, Y, n) by redistributing this surplus to its members, contradicting the supposition that (X, Y, n) is a core state. We conclude that

$$\sum_{c,t} n_{ct}^k \rho_c(y^k, n^k) = \sum_{c,t} n_{ct}^k(\omega_t - x_i) = f(y^k, n^k),$$

and so condition (3) of the definition of Tiebout equilibrium is satisfied by these prices. ∎

REFERENCES

1. V. Barham and M. Wooders, "First and Second Welfare Theorems for Economies with Collective Goods," University of Ottawa Working Paper (1994).
2. C. de Bartolome, Equilibrium and in efficiency in a community model with peer-group effects, *Journal of Political Economy*, **98,** 110–133 (1990).
3. R. Benabou, Workings of a city: Location, education, and production, *Quarterly Journal of Economics*, **105,** 619–652 (1993).
4. E. Berglas, Distribution of tastes and skills and the provision of local public goods, *Journal of Public Economics*, **6,** 409–423 (1976).
5. E. Berglas and D. Pines, Clubs, as a case of competitive industry with goods as a variable quality, *Economics Letters*, **5,** 363–366 (1980).
6. T. Bewley, A critique of Tiebout theory of local public expenditure, *Econometrica*, **49,** 713–740 (1981).
7. R. Boadway, A note on the market provision of club goods, *Journal of Public Economics*, **13,** 131–137 (1980).
8. J. Brueckner, Tastes, skills and local public goods, *Journal of Urban Economics*, **35,** 201–220 (1994).
9. J. Brueckner and K. Lee, Club theory with a peer-group effect, *Regional Science and Urban Economics*, **19,** 399–420 (1989).
10. H. Cole and E. Prescott, "Valuation Equilibrium with Clubs," Federal Reserve Bank of Minneapolis Staff Report 174 (1994).
11. J. Conley, Convergence theorems on the core of a public goods economy: Sufficient conditions, *Journal of Economic Theory*, **62,** 161–185 (1994).
12. J. Conley and M. H. Wooders, "Taste-Homogeneity of Optimal Jurisdictions in a Tiebout Economy with Crowding Types," University of Illinois Working Paper (1994).
13. J. Conley and M. H. Wooders, "Anonymous pricing in public goods economies," *in* "Topics in Public Finance" (D. Pines, E. Sadka, and I. Zilcha, Eds.), Cambridge (1997).

14. J. Conley and M. H. Wooders, "Anonymous Lindahl Pricing in a Tiebout Economy with Crowding Types," University of Illinois Working Paper (1994).
15. G. Debreu and H. Scarf, A limit theorem on the core of an economy, *International Economic Review*, **4,** 285–293 (1963).
16. D. Epple and R. Romano, "Competition between Private and Public Schools, Vouchers and Peer Group Effects," unpublished manuscript (1994).
17. M. Kaneko and M. H. Wooders, The core of a game with a continuum of players and finite coalitions: The model and some results, *Mathematical Social Sciences*, **2,** 105–137 (1986).
18. A. Mas-Colell, A model of equilibrium with differentiated commodities, *Journal of Mathematical Economics*, **2,** 263–295 (1975).
19. A. Mas-Colell and J. Silvestre, Cost share equilibria: A Lindahlian approach, *Journal of Economic Theory*, **47,** 239–256 (1989).
20. M. McGuire, Group segregation and optimal jurisdictions, *Journal of Political Economy,* **82,** 112–132 (1974).
21. M. McGuire, Group composition, collective consumption, and collaborative production, *American Economic Review,* **81,** 1391–1407 (1991).
22. P. Samuelson, The pure theory of public expenditure, *Review of Economics and Statistics,* **36,** 387–389 (1954).
23. R. Schwab and W. Oates, Community composition and the provision of local public goods: A normalize analysis, *Journal of Public Economics*, **44,** 217–237 (1991).
24. S. Scotchmer, Public goods and the Invisible Hand, *in* "Modern Public Finance," (J. M. Quigley and E. Smolensky, Eds.), pp. 93–125 Harvard Univ. Press, Cambridge/ Massachusetts/ London (1994).
25. S. Scotchmer and M. H. Wooders, "Optimal and Equilibrium Groups," Harvard Discussion Paper 1251 and incomplete revisions with the title "Competitive Equilibrium and the Core in Club Economies with Nonanonymous Crowding (1986).
26. S. Scotchmer and M. H. Wooders, Competitive equilibria and the core in economies with anonymous crowding, *Journal of Public Economics*, **34,** 159–174 (1987).
27. M. Shubik and M. H. Wooders, Approximate cores of replica games and economies. I. Replica games, externalities, and approximate cores, *Mathematical Social Sciences*, **6,** 27–48 (1983).
28. C. Tiebout, A pure theory of local expenditures, *Journal of Political Economy*, **64,** 416–424 (1956).
29. M. H. Wooders, Equilibria, the core, and jurisdiction structures in economies with a local public good, *Journal of Economic Theory*, **18,** 328–348 (1978).
30. M. H. Wooders, "A Limit Theorem on the ϵ-Core of an Economy with Public Goods," National Tax Institute of Japan Discussion Paper (1981).
31. M. H. Wooders, The ϵ-core of a large replica game, *Journal of Mathematical Economics,* **11,** 277–300 (1983).
32. M. H. Wooders, Stability of jurisdiction structure in economies with local public goods, *Mathematical Social Sciences*, **15,** 29–49 (1988).
33. M. H. Wooders, A Tiebout theorem, *Mathematical Social Sciences*, **18,** 33–55 (1989).
34. M. H. Wooders, Large Games and Economies with Effective Small Groups, *in* "Game Theoretic Approaches to General Equilibrium Theory" (J-F. Mertens and S. Sorin, Eds.) Kluwer, Amsterdam (1994).
35. M. H. Wooders, Equivalence of Lindahl equilibrium with participation prices and the core, *Economic Theory*, in press (1996).