Income Distribution and Firm Formation

Elaine Bennett* and Myrna Wooders**

*School of Management, State University of New York, Buffalo, New York 14212; and

**Department of Economics, State University of New York, Stony Brook, New York 11794

Abstract. This paper investigates equilibria associated with alternative income-distribution schemes in economies where form formation is endogenous. The income realizable by a firm depends upon the membership of the firm. An entrepreneurial equilibrium, where agents form and/or join firms so as to maximize their individual incomes, has firm structures that maximize aggregate income but does not have good existence properties. Two alternative equilibrium concepts that have better existence properties than the entrepreneurial equilibrium but weaker optimality properties are considered. The results are applied to the debate on the optimality of average-income-per-worker maximization within a market-socialist firm.

1 Introduction

The purpose of this paper is to investigate the effects of income distribution schemes on firm formation, and, in particular, to point out some problems inherent in the development of decentralizable income-distribution schemes whose equilibria have Pareto-optimal *firm structures* (partitions of the set of agents into firms).

The following simple example illustrates the relevance of the income-distribution scheme to the achievement of Pareto-optimal firm structures.

Consider an economy with two types of agents, called "skilled" and "unskilled," and one produced good. Each skilled agent, working alone, can produce an income (i.e., revenue minus costs of inputs other than labor) of 5 units of output per week, while unskilled agents can each produce an income of only one unit per week. A skilled and an unskilled agent, working together, can produce an income of 8 units per week. For maximum productivity firms should be organized by combining skilled and unskilled workers within each firm (rather than having each type work on their own). Suppose the workers are part of a society that has decided on an egalitarian distribution of income within firms, but has not changed the income-maximizing nature of its members. Both skilled and unskilled workers would expect 4 units per week from their joint production. Since skilled workers can produce 5 units per week in firms that consist only of skilled workers, we would not expect firms with both skilled and unskilled workers to voluntarily come into being in this society. The optimal organization of firms is then not in the individual self-interest of the workers as entrepreneurs.

The question of how the income arising from joint production should be shared among those contributing to that production has received considerable attention. One point of view is that the income should be distributed according to some notion of equity or fairness. As the above example illustrates, such income-distribution schemes might not lead to optimal firm structures. A second point of view is that all who engage in production should receive the value of their marginal product. When there are constant returns to scale for each type of labor in the production process, paying each worker the value of her/his marginal product will exactly exhaust the income from production by the product-exhaustion theorem. However, when for some types of labor there are either increasing or decreasing returns to scale, the aggregate value of the marginal products of labor will either exceed or fall short of the total income from production. In consequence, distributing the values of the marginal products is not always a feasible rule for distributing the income from production. In addition, unless all agents have access to the same technology (unlike the situation in the above example), even if there are constant returns to scale for each firm, rewarding each agent by the value of her/his marginal product leaves unanswered the questions of how a firm structure is determined and whether or not it is optimal. A third point of view is that all factors of production, including labor, should be paid their "market" prices, and that the entrepreneur who organizes production, or owns the technology, should receive the residual income after all other factors have been paid. This view has been the subject of investigation in the context of coalition production economies (cf. Boehm, 1973; Ichiishi, 1977; and Sonderman, 1974). However, the assumptions used in this literature obscure some interesting features of the models.[1]

The investigation of the effects of alternative income-distribution schemes in general is beyond the scope of this paper. Instead, we set up a framework for analyzing the effects of alternative income-distribution schemes on the formation of firms and consider properties of several equilibrium concepts. First we consider a partial equilibrium that can be viewed as the outcome of a process where "entrepreneurs" or "groups of citizens" can join and/or form firms, and, in equilibrium, no group of agents could benefit by forming a new firm. An income distribution associated with this partial

[1] These papers have either assumed strongly increasing returns to coalition size or "balancedness" of the aggregate technology set. Both assumptions imply that equilibria exist, thus avoiding one of the central issues of this paper. This is further discussed in Wooder (1979a,b).

equilibrium is called an equilibrium-reservation-price vector (an ERP). It is shown that this type of equilibrium has several desirable properties, but might not be realizable as an actual equilibrium, called an entrepreneurial equilibrium, because of a destabilizing form of structural underemployment. Consequently we develop another equilibrium, called the α-equilibrium, that is derived from the partial-equilibrium concept. The α-equilibrium is shown to always exist and to possess both desirable and undesirable properties. The entrepreneurial equilibrium is then modified, so that firm-formation costs are taken into account. This equilibrium concept is called an ε-equilibrium. When firm-formation costs are positive, ε-equilibria are shown to exist for all sufficiently large economies. In addition, the ε-equilibria converge to the entrepreneurial equilibria as firm-formation costs become small and the economy grows large. Although the ε-equilibrium concept is interesting for large economies, for any given economy with fixed firm-formation costs an ε-equilibrium might not exist.

We conclude the paper by relating our work to other literature on equilibria of labor-managed economies with free entry of firms and on the debate as to whether or not the market-socialist economy, where workers seek to maximize average income per worker, has optimal equilibria.

2 The Model

In this section, we describe the class of economies considered and introduce the central theoretical concepts.

In our model, we study firm formation in isolation, ignoring both potential interactions between firm affiliation and consumption and the effect of agents' consumption on the income of firms. An *economy* is specified by (N, v) where N is the collection of agents and v is a function that assigns to every potential firm (i.e., coalition, or subset of agents) the maximum income it could achieve by forming a firm. Let $N = \{1, ..., n\}$ denote the set of agents in the economy, and let $v(S)$ denote the maximum income realizable by a coalition S if it forms a firm. It it assumed that $v(S) \geq 0$ for all coalitions S. Each agent can participate in at most one firm at a time, so *feasible firm structures* (that is, the different ways that agents can organize themselves into firms) are partitions of the set of agents. Such a partition is denoted by P and $S \in P$ is a firm in the partition P.

In a labor-managed market-socialist economy, the members of S can be viewed as worker-managers, and $v(S)$ as the revenue resulting from the sale of goods minus the costs of marketed (nonlabor) inputs. In a capitalist society, $v(S)$ can be regarded as the returns to the entrepreneurs in S. Alternatively, we could regard each agent as owning an initial bundle of capital goods and $v(S)$ as the returns to the agents in S when they pool their capital in a firm. Other interpretations are also possible but will not be discussed here.

An *income distribution* is a vector $x = (x^1, ..., x^n) \in R^n_t$, and x is a *feasible income* *distribution* if there is a firm structure P such that

$$\sum_{i \in N} x^i \leq \sum_{S \in P} v(S).$$

The ith coordinate of the vector x is interpreted as the income received by the ith agent.

A firm structure P is *Pareto-optimal* if, for all firm structures P',

$$\sum_{S \in P} v(S) \geq \sum_{S' \in P'} v(S').$$

In other words, a firm structure is Pareto-optimal if no other firm structure yields a higher aggregate income. Similarly, x is a *Pareto-optimal income distribution* if x is a feasible income distribution and

$$\sum_{i \in N} x^i \geq \sum_{S' \in P'} v(S')$$

for all firm structures P'. Clearly, Pareto optimality is a desirable property for both firm structures and income distributions.

In many economies the income from the firm accrues only to the agents participating in the firm. An income distribution possessing this property is said to be coalitionally feasible. Formally x is a *coalitionally feasible* income distribution if for some firm structure P,

$$\sum_{i \in S} x^i = v(S)$$

for all $S \in P'$.

Given an income-distribution scheme, it would be desirable for the scheme to lead to an outcome (i.e., an "equilibrium") so that, relative to the scheme, no group of agents could benefit by forming a new firm. In other words, we would like an income distribution to have some "stability" properties. We have not investigated the stability of income-distribution schemes in general; instead we later discuss stability properties of the equilibria introduced in the next section.

In the remainder of this section we consider a class of income distributions, called equilibrium reservation-price vectors, that are stable in the sense that for all income distributions x in the class, $\sum_{i \in s} x^i \geq v(S)$ for all $S \subset N$, and that also have the property that for each $i \in N$ there exists $S \subset N$ such that $i \in S$ and

$$x^i = v(S) - \sum_{j \in S_{j \neq i}} x^j.$$

The first of these conditions guarantees that no group of agents can benefit by forming a new firm. The second requires that for each agent i there is some coalition S, containing i, that is capable of paying i the income x^i. It is not required that equilibrium-reservation-price vectors (ERPs) are feasible income distributions. Nevertheless, the following discussion and the results of the next section motivate interest in ERPs.[2]

Formally, $p \in R^n_+$ is an *equilibrium-reservation-price vector* (an ERP), if

$$\begin{aligned} (a)\ \textstyle\sum_{i\in S} p^i &\geq v(S) \text{ for all } S \subset N, \\ (b) \text{ for each } i &\in N, \text{ there is an } S \subset N \text{ such that } i \in S \text{ and such that} \\ \textstyle\sum_{j\in S} p^j &= v(S). \end{aligned}$$

Given an ERP, p, let

$$A_i(p) = \{S : i \in S and \textstyle\sum_{j\in S} p^j = v(S)\}.$$

The coalitions contained in $A_i(p)$ are those coalitions satisfying (b) of the definition of an ERP. Elements of $A_i(p)$ are called *optimal firm-formation strategies for i relative to p.*

ERPs can be viewed as outcomes of a tatonnement process. Let $p \in R^n_+$ be given, where the ith coordinate of p, p^i, is interpreted as the payment (or income) required by the ith agent to induce her/him to join a coalition, or, in other words, the price of the ith agent. Just as in the Walrasian tatonnement process, where agents take prices as given, here agents take prices of other agents as given.

Each agent then determines the maximum income she/he could realize by forming a firm, after paying the other agents in the firm their stated prices; in other words, the jth agent chooses $S \subset N$, where $j \in S$, so as to maximize

$$v(S) - \sum_{i\in S_{i\neq j}} p^i.$$

This maximum income then becomes the reservation price of the jth agent. We call this a "reservation" price since, given prices of other agents, income-maximizing agents will not join a firm unless they are paid at least this reservation price; otherwise, they will attempt to hire other agents at their stated prices and form a new firm. A vector, p, is an ERP if each agent i's reservation price, given p, is simply p^i.[3]

Since an ERP has the property that $\sum_{i\in S} p^i \geq v(S)$ for all $S \subset N$, it follows immediately that

$$p^j \geq v(S) - \sum_{i\in S_{i\neq j}} p^i \text{ for all } S \subset N,$$

so no agent j could form a new firm, pay the other agents in that firm their stated prices, and realize a greater income than her/his equilibrium reservation price p^j.

[2] ERPs are discussed in Wooders (1979a) and Bennett (1979c) where they are called "quasi-equilibrium prices" and "rational aspirations" respectively.

[3] We remark that in Wooders (1979a) the existence of an ERP is shown to follow from a fixed-point theorem.

From (b) of the definition of an ERP, there is some $S \subset N$ so that

$$p^j = v(S) - \sum_{i \in S_{i \neq j}} p^i$$

so the jth agent's ERP is the maximum income j could achieve by forming a firm. Consequently, the equilibria of this process coincides with the set of ERPs.

To illustrate the properties of ERPs, we consider an economy with identical agents so that $v(S)$ depends only on the number of agents in S. Let the number of agents in S be denoted by s and define $v(s) = v(S)$. Presumably, as the number of agents in S increases, $v(s)$ initially increases at an increasing rate, then increases at a decreasing rate and then eventually decreases in magnitude (this corresponds to the "three stages of production" as we vary the labor input). We assume that some number of agents (call it s^*) maximizes the average income $v(s)/s$ of participating agents (the value of the average product of labor). If there are at least s^* agents in the economy, then one equilibrium reservation price for the economy assigns to each agent this maximum average income $v(s^*)/s^*$ (see Fig. 1). If s^* is the number that uniquely maximizes the average income of agents, and the number of agents in the economy exceeds s^*, then the ERP, where $p^i = v(s^*)/s^*$, is a feasible income distribution if and only if the number of agents in the economy is an integer multiple of s^*, or, in other words, if and only if the agents can be exactly partitioned into firms of size s^*, any firm structure will contain at least one firm incapable of paying its members $v(s^*)s^*$; this necessarily results in instability.

Also, we remark that $v(s^*)/s^*$ is the maximum value of the average product of the agents, and if a firm has s^* agents, then the value of the average product equals the value of the marginal product of an agent; thus members in such firms are paid the value of their marginal products.

3 Equilibria

3.0.1 A. Entrepreneurial Equilibria[4]

An *entrepreneurial equilibrium* is an income distribution x and a firm structure P for which

$$\text{(a) } \textstyle\sum_{i \in S} x^i = v(S) \text{ for all } S \subset P \text{ and}$$
$$\text{(b) there does not exist } S' \subset N \text{ such that } v(S') > \textstyle\sum_{i \in S'} x^i.$$

The first condition of the entrepreneurial equilibrium is coalitional feasibility; the income earned by any firm must accrue only to the members of the firm. The second

[4]We use the term "entrepreneurial" since all agents are potentially entrepreneurs (i.e., firm formers) and in equilibrium there is no central-government intervention. The equilibrium could be viewed as one for a capitalist or an Illyrian society. The theorems of this subsection and that on the ε-equilibrium are proven in Wooders (1979a) or follow easily from results in Wooders (1979a).

is a stability property; no firm S' could form and achieve an income sufficiently large so that all members of the firm could have a higher income than that specified for them by the income distribution x.

The entrepreneurial equilibrium can be viewed as the result of the actions of entrepreneurs in capitalist economies or "groups of citizens" in market-socialist economies.

It is easy to see that the entrepreneurial income distribution and firm structure has desirable optimality and stability properties.

THEOREM 1.

If x is an entrepreneurial income distribution, then x is Pareto-optimal.

If x were not Pareto-optimal, then for some P' we would have $\sum_{S' \in P'} v(S') > \sum_{S \in P} v(S) = \sum_{i \in N} x^i$. From this it follows that for some $S' \in P'$, $v(S') > \sum_{i \in S'} x^i$, contradicting (b) of the definition of the equilibrium.

Our next two theorems establish the relationships between ERPs and entrepreneurial equilibria, and between optimal firm-formation strategies and entrepreneurial equilibria.

THEOREM 2.

The economy (N, v) has an entrepreneurial equilibrium if and only if some ERP is feasible for the economy. Furthermore, if x is an equilibrium income distribution then x is an ERP and $\sum_{i \in N} x^i \leq \sum_{i \in N} p^i$ for all ERPs, p. (An equilibrium income distribution x is an ERP and the total income associated with every other ERP is greater than or equal to the total income associated with x.)

THEOREM 3.

The economy (N, v) has an entrepreneurial equilibrium if and only if for some ERP, say p, and some partition P, $S \in A_i(p)$ for all $i \in S$ and for all $S \in P$.

The second theorem states that unless agents can be partitioned into optimal firms relative to some ERP, the conditions of the entrepreneurial equilibrium cannot be satisfied by any income distribution. In our example following the definition of ERPs, it was demonstrated that it might not be possible to partition agents into optimal firms relative to an ERP. If agents cannot be partitioned into optimal firms relative to some ERP, then any partition has the property that some agents are "underemployed" (i.e., are not in optimal firms and thus are unable to produce at their maximal average income). This is what was meant when we referred to "structural underemployment" in the Introduction.

Without further assumptions on (N, v), we cannot guarantee that an entrepreneurial equilibrium exists. Consequently, in the next subsections we investigate alternative equilibrium concepts.

3.1 B. The α-Equilibrium[5]

To motivate the following definition of an α-equilibrium, consider an economy for which an entrepreneurial equilibrium does not exist. For such an economy, for every

[5] The results concerning the α-equilibrium are proven in Bennett (1978).

ERP, p, each agent has a set of optimal firms relative to p, but no possible partition allows all agents to belong to optimal firms. No matter which equilibrium reservation price is chosen and how agents are "assigned" to optimal firms, there are always agents left over. These agents cause instability. If we can specify what the "leftover" agents' second-best firm-formation strategy is, then we can consider "equilibria" where all agents belong either to an optimal or a second-best firm among the feasible alternatives.

Suppose an equilibrium reservation price is given, that as many optimal firms are formed as is possible, and there are leftover agents. We assume that the leftover agents' next-best strategy is to form the firms in which they can obtain the largest possible fraction of their reservation price. Let $\alpha(S)$ be the fraction of the reservation price that a firm S "can afford," i.e.,

$$\begin{aligned} \alpha(S) &= \frac{v(S)}{\sum_{i \in S} p_i}, \text{ if } \sum_{i \in S} p_i \neq 0, \\ &= 0, \text{ if } \sum_{i \in S} p_i = 0. \end{aligned}$$

For firms where the equilibrium reservation price is feasible, $\sum_{i \in S} p_i = v(S)$. These firms are optimal firms for each agent in them ($S \in A_i(p)$ for every $i \in S$), and for these firms $\alpha(S) = 1$. For all other firms $0 \leq \alpha(S) < 1$.

Definition.

An economy (N, v) has an *α-equilibrium* relative to (the ERP) p if there exists a firm structure P and a feasible income distribution $d \in R^n_+$ such that

$$\begin{aligned} \text{(a) } d_i &= \alpha(S)p_i \text{ for each } i \text{ in } S \in P, \text{ and} \\ \text{(b) there is no } S' &\subset N \text{ such that } \alpha(S')p_i > d_i \text{ for every } i \text{ in } S'. \end{aligned}$$

Implicit in (b) of the definition of an α-equilibrium is the idea that a firm will not form unless *every* agent i can receive a higher fraction of p_i than she/he is receiving in the income distribution d. In addition, in each firm S, all agents in S receive the *same* fraction, $\alpha(S)$, of their ERPs. This rules out certain kinds of strategic behavior; in particular, it rules out bargaining over the percentage of a firm's income received by each agent in that firm. In other words, relative prices of agents within each firm are taken as given.

We remark that if we had required that every agent in S' had to be at least as well off and some agent in S' strictly better off, then an α-equilibrium would exist for an economy if and only if an entrepreneurial equilibrium exists. The definition we employ weakens the notion of equilibrium sufficiently so that, for any economy, one of these weaker equilibria exist.

THEOREM 4.

Let (N, v) be an economy and p be an ERP for (N, v). Then there exists an α-equilibrium for (N, v) relative to p.

THEOREM 5.

For the economy (N, v), assume that there exists an entrepreneurial equilibrium with the associated ERP p and firm structure P. Then there exists an α-equilibrium for the economy (N, v), relative to p, whose associated industry structure is P.

The theorem basically says that the set of α-equilibria contains all entrepreneurial equilibria.

The next theorem presents a problem with the notion of α-equilibria. It shows that an α-equilibrium firm structure is not necessarily Pareto-optimal. This is due to a more general problem discussed in Bennett (1979b).

THEOREM 6.

Consider a strictly superadditive economy ($v(S \cup T) > v(S) + v(T)$ whenever $S \cap T = \phi$). Let P be the firm structure associated with an α-equilibrium for some strictly positive ERP. If P is not an entrepreneurial firm structure, then P is not Pareto-optimal.

On this discordant note we turn to a second approach to weakening the notion of equilibrium.

4 C. The ε-Equilibrium

The underlying motivation for considering ε-equilibria is that firm formation is costly, and that, in economies where entrepreneurial equilibria (with zero firm-formation costs) do not exist, a weaker equilibrium, where no additional firm formation is profitable net of the firm-formation costs, may exist. We assume that the cost of firm formation is proportional to the number of agents in the potential firm. Let ε be the per capita cost of firm formation so that the cost of forming a firm with k participants is just $k\varepsilon$.[6]

An ε-equilibrium is an income distribution x and a firm structure P for which

(a) $\sum_{i \in N} x^i = \sum_{S \in P} v(S)$, and

(b) $\sum_{i \in S'} x^i \geq v(S') - \varepsilon\, |S'|$ for all $S' \subset N$, where $|S'|$ denotes the number of agents in S.

From condition (b), it follows that, for small ε, an ε-equilibrium distribution is "almost" coalitionally feasible.

Our results for the ε-equilibrium are for economies with a "large" number of agents, and a fixed number of "types" of agents. Two agents i and j, are *identical* if, for all $S \subset N$ such that $i \notin S$ and $j \notin S$, $v(S \cup \{i\}) = v(S \cup \{j\})$.In other words, two agents are identical if, for any firm S containing neither i nor j, the joining of either i or j to S will equally augment the income of S. Agents who are identical in this sense are referred to as being of the same *type*.

[6]In Wooders (1979b) a more general formulation of a firm-formation cost function is used.

For this section, we use the framework of a replica economy. It is assumed that there are T types of agents in the economy. The *set of agents of the rth replica economy* is denoted by $N_r = \{(1,1), ..., (1, m_1 r), ..., (t,q), ..., (T, m_T r)\}$, where (t,q) represents the qth agent of type t. There are $m_t r$ agents of type t in all. Consequently, the set of agents of the rth replica economy can also be represented by the vector $rm = r(m_1, ..., m_t, ..., m_T)$.

Given a coalition $S \subset N_r$, we let $s = (s_1, ..., s_T)$ represent the *profile* of S, where s_t is the number of agents of type t contained in S. From our definition of identical agents and the assumption that agents of the same type are identical, it follows that $v(S) = v(S')$ whenever S and S' have the same profile, which means that both coalitions contain the same number of agents of every type. We define $v(s) = v(S)$ when s is the profile of S.

Equilibrium reservation prices and optimal firm-formation strategies are defined for the economy (N_r, v) just as they were in Section 2. We restrict our attention to ERPs with the *equal-treatment property*: an ERP, $p = (p^{11}, ..., p^{tq}, ..., p^{m_T r})$, where p^{tq}denotes the reservation price of agent (t,q), has the equal-treatment property when $p^{tq} = p^{tq'}$for all $(t,q),(t,q') \in \{(t,1), ..., (t, m_t r)\}$ (i.e., all agents of the same type have the same reservation price). We represent the set of equal-treatment ERPs by $ERP \subset R_+^T$, where $p = (p_1, ..., p_T) \in ERP$ if $p' = (p^{11}, ..., p^{tq}, ..., p^{Tm_T r})$ is an ERP with $p^{tq} = p_t$ for all q and all t.

We assume that there are optimal firms that become "small" relative to the size of the economy as r becomes "large." Formally, we assume that for any r, and any $p \in ERP$, there is a coalition $S \in A_{tq}(p)$, where the profile of S is less than or equal to the profile of N_1. For the case where all agents are identical, as in the example of Section 2, this assumption is that the function $v(s)/s$ has a maximum, as in Fig. 1. We rule out cases as in Fig. 2, where there are always increasing average returns to coalition size (in Fig. 2, $|S|$ denotes the number of agents in S).

Given the above assumption, it can be shown that the set of ERPs with the equal-treatment property for (N_r, v) is equal to that for $(N_{r'}, v)$ for all r, r'.

The first theorem of this section relates the set ERP to entrepreneurial equilibria for a replica economy.

THEOREM 7.

When $r \geq 2$, and x is an entrepreneurial equilibrium income distribution, then x is an ERP with the equal-treatment property.

We remark that if p is in ERP, then

$$p_{t'} = \max_s \{(v(s) - \Sigma_{t \neq t'} s_t p_t / s_{t'} : s = (s_1, ..., s_T), s_{t'} \neq 0\},$$

so that in an entrepreneurial equilibrium for a replica economy, within each firm the average income of agents of type t' is maximized, given that agents of other types are being paid their ERPs. Consequently, the marginal contribution to the total income of the firm of an agent of type t equals the average contribution of an agent of type t.

The following theorem states that, given firm-formation costs parameterized by ε, an ε-equilibrium exists for all sufficiently large economies.

THEOREM 8.

Given $\varepsilon > 0$, there is an r' sufficiently large, so that, for all $r \geq r'$, an ε-equilibrium exists for (N_r, v).

The proof of the theorem (see Wooders, 1979a) proceeds by showing that for all r sufficiently large, we can construct a firm structure P where most agents are in optimal firms relative to $p \in ERP$. These agents can be taxed (or pay underemployment insurance) and the resulting revenue can be distributed to those agents not in optimal firms (the underemployed) so that the partition and the resulting distribution is approximately an ε-equilibrium. The next theorem states that for large economies with small firm-formation costs, every ε-equilibrium income distribution is approximately an equal-treatment ERP.

THEOREM 9.

Given λ and $\delta > 0$, there is an r^* and an ε^* so that for all $r \geq r^*$ and all ε where $0 \leq \varepsilon \leq \varepsilon^*$, if $x = (x^{11}, ..., x^{m_T r})$ is an ε-equilibrium, then there is some $p \in ERP$ such that the percentage of agents of type t whose income, x^{tq}, differs by more than δ from p_t is less than λ for each type t.

An exact entrepreneurial equilibrium, as defined in Section 2, has desirable properties of Pareto optimality and stability and can be viewed as arising from the attempts of agents to form and/or join firms without central control. For small ε, the ε-equilibrium "approximates" an exact equilibrium and also seems a reasonable equilibrium concept. However, for an ε-equilibrium to obtain, some central-government intervention (transfers from optimal firms to nonoptimal firms) might be required. In addition, firm-formation costs might in reality be large, in which case an ε-equilibrium could still be far from optimal.

5 Conclusions

The results of this paper (and those of Wooders (1979b and 1979c), in which prices for marketed goods are determined endogenously) provide a general-equilibrium model of a labor-managed economy as discussed in Vanek (1970) and Ward (1967).

For exact equilibria, the ability of groups of agents to form firms ensures that in equilibrium each agent is paid her/his equilibrium reservation price. As the example of Section 2 illustrates, each firm admits members so as to maximize the average income of the members of the firm. This is, in equilibrium, equivalent to paying all agents their marginal contribution to the income of the firm (since marginal contribution equals average contribution).

Since an equilibrium firm structure is Pareto-optimal, hiring agents within a firm so as to maximize the average income of each type, given "prices" of other types, is consistent with Pareto optimality. We remark that the discussions of average-income maximization of the labor-managed firm have been carried out in the framework of a partial-equilibrium model, whereas previous general-equilibrium models of the labor-managed firm have required convexity or superadditivity assumptions (see, for example, Dreze, 1975), which render the question of the optimality of average-income maximization almost irrelevant.

If there are significant firm-formation costs, however, an ε-equilibrium might not be optimal and the equality of average and marginal contributions of an individual to the firm of which she/he is a member might not occur. An ε-equilibrium only approximates an exact equilibrium for small ε and large economies.

The α-equilibrium provides a partial solution; it can be shown that for large economies most agents are in optimal firms. However, it does require knowledge of an ERP by the central authority and, in the absence of an equilibrium, an ERP cannot be observed but must either be calculated from knowledge of the technologies available to all coalitions, or inferred from what actually is occurring in an economy.

This paper has posed a warning. When income distribution schemes, such as quota-bonus schemes, are implemented over an entire society rather than only within one given firm, the effects of the scheme on firm formation decisions should not be ignored. When firms are formed predominantly by individual and group action, skewing the patterns of income distribution within firms for reasons of equity and/or to create desirable incentives within each firm can lead to nonoptimal firm formation.

Our (not very original) conclusion is that more research is needed; both more theoretical research into the effects of income-distribution schemes on firm formation, and more empirical research into actual firm-formation processes and, in particular, the costs of firm formation. If the costs of firm formation are prohibitively high, some sort of governmental intervention might be desirable to facilitate the formation of new firms. Also, theoretical research is needed into how governments might best facilitate new firm formation, and, in socialist economies, whether centrally controlled capital can be allocated so as to lead to approximately optimal firm formation.

References

[1] Bennett, Elaine, "Profit Distribution Rules and Firm Formation: Notes on the α-Process." S.U.N.Y.-Buffalo, School of Management Working Paper Series (forthcoming) 1979a.

[2] Bennett, Elaine, "Symmetry, Stability and Optimality: An Impossibility Result." S.U.N.Y.-Buffalo, School of Management Working Paper No. 413, June 1979b.

[3] Bennett, Elaine, "Coalitional Stability in Characteristic Function Games." Ph.D. Thesis, Northwestern University 1979c.

[4] Boehm, Volker, "Firms and Market Equilibria in a Private Ownership Economy." *Zeitschrift fur Nationalokon.* **33**, 1, 2:87-102, 1973.

[5] Dreze, Jacques, "The Pure Theory of Labour-Managed and Participatory Economies, Part 1: Certainty." CORE Discussion Paper No. 7122, 1975.

[6] Ichiishi, Tatsuro, "Coalition Structure in a Labor-Managed Market Economy." *Econometrica* **45**:341-360, Mar. 1977.

[7] Sondermann, Dieter, "Economies of Scale and Equilibria in Coalition Production Economies." *J. Econ. Theory* **8**, 3:259-291, July 1974.

[8] Vanek, Jaroslav, *The General Theory of Labor-Managed Market Economies.* Ithaca, N.Y.: Cornell Univ. Press, 1970.

[9] Ward, Benjamin, *The Socialist Economy.* New York: Random House, 1967.

[10] Wooders, Myrna, "A Characterization of Approximate and Exact Equilibria and Cores in a Class of Coalition Economies." Mimeo., June 1979a.

[11] Wooders. Myrna, "Existence of Approximate Equilibria of Coalition Production Economies." Stony Brook Economic Research Bureau Working Paper No. 209, Jan. 1979b.

[12] Wooders. Myrna, "Convergence of Approximate Equilibria of Coalition Production Economies." Stony Brook Economic Research Bureau Working Paper 1979c.