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Some corrections to claims about the literature in Engl and Scotchmer (1996)

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Abstract

Some assertions in Engl and Scotchmer [J. Math. Econ. 26 (1996) 209] concerning prior literature are corrected. In addition, I discuss the differences between the convergence results of Engl and Scotchmer [J. Math. Econ. 26 (1996) 209], and those of this author, alone and with Martin Shubik, dating from 1980. Our prior and concurrent results show that (approximate) ε-cores of games with many players treat most similar players nearly equally; that is, approximate cores of large games have the equal treatment property. The convergence result of Engl and Scotchmer shows that, in per capita terms, ε-core payoffs to sufficiently large *groups* of players can be approximated by equal-treatment payoffs.

JEL classification: C7; D4; D7

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1. Motivation and summary

In an article published on 5 May 1996, in the *Boston Globe*, a major US newspaper, David Warsh wrote that this author [1] : **(a): "was accused of duplicating—in an unrefereed book—the results of University of California economists Greg Engl and Suzanne Scotchmer—in effect, depriving her colleagues of credit due. Wooders has replied indirectly, writing of her "impression . . . that most claims of misappropriation begin with insufficient knowledge and/or appreciation of the literature". (Warsh 1996a,b).**

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[1] Other publications, in joint research with Paulo Monteiro and Frank Page and in various journals, treating scientific matters mentioned in Warsh's article are listed on my web page. See also a recent paper by Dana et al. (1999).

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In view of this publicity, it seems appropriate to set the record straight about relationships between my research, alone and with others, and that of Engl and Scotchmer (E&S). Wider and more important issues of conduct in science are discussed in the concluding section of this paper. My intention is that this paper should contribute to fostering a climate conductive to the advancement of research, in which question of attribution, and of relationships between models and results, are treated scientifically. Dealing with anolamies in this climate by publications in scientific journals will contribute to uninhibited and vigrous flourishing of research. In the following, I will discuss some of the more salient points of similarity between the works of E&S and my own papers and show that the prior, concurrent, and subsequent results of this author (MW) do not duplicate those of E&S (1996). While for some particular cases, results in E&S (1996) follow from my prior research—for example, nonemptiness of approximate cores—it is an open question whether the convergence results of E&S imply those of this author or whether those of this author imply the E&S convergence results. Also, in view of the relevance of the misleading claims in E&S concerning the dates of the results of this author and their incorrect claims to priority, these are also directly addressed.

While the inaccuracies in E&S (1996) in combination with ambiguities in their description of the literature and the publicity given this matter motivate the corrections in this paper and careful reference to my own prior research, there is much prior research that is related. Some seminal research in the area of games with side payments includes: Shubik (1959) which shows, for an example with two types of players, that cores of large market games (games with side payments with the property that all subgames have non-empty cores) converge to equal-treatment cores; [2] Shapley and Shubik (1966), which shows that, under apparently mild assumptions on utility functions, approximate cores of large exchange economies with transferable utility are non-empty; Bondareva (1962) and Shapley (1967), which introduce the concept of balancedness and show that a game with side payments is balanced if and only if it has a non-empty core, and Shapley and Shubik (1969), which shows that games derived from convex exchange economies with quasi-linear utilities are equivalent to market games. These papers, along with Aumann (1964), Aumann and Shapley (1974), and Tiebout (1956) are, in my view, seminal to the line of research discussed in the paper. Some other important background papers are noted later. But this is not the place for a survey back to the beginnings of cooperative game theory and large economies with clubs—my objective here is to set the record straight about the relationship between E&S (1996) and my earlier and concurrent research (alone and with others). [3]

In brief, that part of my research closely related to the E&S papers was initiated in MW (1977) and continued through a number of papers, alone and with others. *Contrary to the claims of E&S* (1996) concerning the prior literature:

1. A number of prior papers obtaining results closely related to those of E&S use only the assumptions of per capita boundedness (PCB)—finiteness of the supremum of average

[2] The equal treatment core consists of those payoffs in the core that treat identical players identically.

[3] For the reader interested in a more comprehensive, but extremely short, survey of some aspects of this line of research, see MW (1999).

payoff—and superadditivity (see, e.g. MW (1980, 1983); Shubik and Wooders (1982)); E&S are far from the first.

2. The results of MW (1992a) and a number of prior papers are not restricted to games that satisfy the same sort of scale assumption as in Scotchmer and Wooders (1989) [4] but instead include convergence and non-emptiness results for approximate cores using only the conditions of PCB and superadditivity (the same examples as in (1) apply).
3. The results of MW (1992a) showing convergence of approximate cores to equal-treatment outcomes are not subsequent to those of E&S (1996). For games with a fixed distribution of player types, these results first appeared (MW, 1977, 1979a,b, 1980) and subsequently, for economies with multiple private goods and multiple memberships in clubs, in Shubik and Wooders (1982). The results were next shown to hold uniformly for all sufficiently large games derived from pregames in MW (1992a) under the assumption of *small group effectiveness* (SGE) dictating that all or *almost all* gains to coalition formation can be realized by groups of players bounded in size. As shown in MW (1994), when there are sufficiently many players of each type that appears in the games, then SGE is equivalent to PCB (in addition, convergence of ε-core equal-treatment payoffs was shown in Wooders and Zame (1987) under boundedness of marginal contributions to coalitions and in MW (1991) under the weaker condition of SGE).

First, inaccuracies in claims in E&S (1996) are pointed out and then relationships between papers are discussed.

2. Per capita boundedness

In exchange economies, there are typically natural bounds on equilibrium payoffs (cf. Debreu and Scarf, 1963). For cooperative games to resemble markets, a first question is how average payoffs must to limited for approximate cores to be non-empty and to converge to equal-treatment payoffs. For games with a fixed distribution of player types, only per capita boundedness of average payoff is required. To the best of our knowledge, the first results of this nature appear in MW (1979b, 1980). See also Shubik and Wooders (1982).

E&S (1996) write: **(b): "The only assumption on feasible payoffs that we use are superadditivity and finiteness of per capita payoffs. This is in contrast to the literature on large games (SIC) (see, e.g. Wooders and Zame (1984)); Scotchmer and Wooders (1988); and other papers summarized in MW (1992a)) which use a "scale" assumption to bound the size of coalitions with blocking power**" (SIC) (E&S, 1996, p. 210). [5]

The above quotation contains errors. The same conditions of superadditivity and finiteness of per capita payoffs, called *per capita boundedness* in my research, are the only ones in MW (1979b, 1980, 1983); Kaneko and Wooders (1986); Shubik and Wooders (1982, 1983a,

[4] Apparently, by a scale assumption E&S (1996) mean that all gains to collective activities can be realized by groups of players bounded in size.

[5] The (a) after 1992 was inserted by this author. In addition to superadditivity and per capita boundedness E&S (1996) also requires a differentiability assumption. This is in contrast to all the papers of this author, alone and with others, with the exception of Winter and Wooders (1990) but a minor matter.

1983b, 1986), and also are the only conditions used for some results, including convergence results, in MW (1991, 1992a, 1994), among others. [6]

The results of MW (1979b), showing non-emptiness of approximate cores of large games with side payments under the conditions of superadditivity and per capita boundedness are in the published literature. Recall that a (TU) game is ε-balanced if and only if it has a nonempty ε-core. Indeed, Yakar Kannai writes: **(c): "A general framework for the study of market games (and other kinds of games) was introduced in MW (1979b). MW (1979b) noted that v has a non-empty ε-core iff v is ε-balanced. Remarkably, superadditivity and bounded per-capita payoffs suffice for ε-balancedness, as the theorem of MW (1979b) shows"** (Kannai, 1992, pp. 387–388). [7]

Aumann (1987) also discusses the side payments case of MW (1983) and reports that it requires only superadditivity and per capita boundedness, PCB. Aumann (1987), Kannai (1992) and MW (1983) are all cited in various Working Papers due to E&S.

The second error in **(b)** is the assertion that MW (1992a) uses a scale assumption bounding the sizes of effective coalitions. In a subsequent paper, E&S (1997) this scale assumption is clarified: **(d): "The scale property, which was introduced by Scotchmer and Wooders (1989), implies that if a payoff can be blocked then it can be blocked by a coalition smaller than a given maximum size"** (E&S, 1997, p. 541).

This is actually an alternative description of condition $(*)$ of MW (1977, 1979a) and the MES condition of MW (1983) and other prior papers. In fact, in MW (1991, 1992a,b, 1994), a *number* of assumptions and results are discussed and compared, including per capita boundedness. In addition, in MW (1992a) convergence of approximate cores is shown under the strictly fewer assumptions than in E&S (1996).

3. The equal-treatment property of the core

The equal-treatment property of the core—that identical agents are treated identically by every allocation in the core—for replicated private goods exchange economies has been known since Debreu and Scarf (1963). A number of papers have investigated the robustness of this conclusion; see Green (1972); Khan and Polemarchakis (1978) and especially Hildenbrand and Kirman (1973) which shows that in large economies, unequal-treatment allocations in the core disappear. Since cores and approximate cores of games are 'stand-ins' for the competitive equilibrium, the result that approximate cores of large games satisfy equal-treatment is important for the 'competitiveness' of large cooperative games. For the most recent results of this nature for large cooperative games, see Kovalenkov and Wooders (2000).

[6] Some of these papers are for games without (and with) side payments. In these papers, per capita boundedness appears as boundedness of the set of equal-treatment payoffs, which is of course equivalent to per capita boundedness for games with side payments. MW (1991) was intended to be the beginning of a book. Most of the results of that paper were incorporated into MW (1992a), presented at the Stony Brook NATO Advanced Study Institute held in 1991. MW (1994) has a submission date of July 1989 and thus, is well prior to the first typescript version of E&S (1996) known to me.

[7] The function v is the worth function, assigning worths to coalitions as functions of the numbers of players of each type in the coalition. I have added the "b" to 1979.

Another erroneous claim in E&S (1996) appears in footnote 9: **(e): "Subsequent to circulation of this paper, MW (1992a) has circulated an "equal-treatment" theorem for the ε-core for players with attributes which relies on a scale assumption. The equal-treatment payoffs are rewards to bundles of attributes (SIC) (i.e. players have 'types'), and not a linear function on attributes as here"** (E&S, 1996, p. 218). My equal-treatment results for the ε-core date back to MW (1977, 1979a, 1980, 1983). These 1977 results relied on a scale assumption (equivalent to that used in Scotchmer and Wooders (1988). The 1979a versions of my 1977 results are reported in Bennet and Wooders (1979) and *even in* Scotchmer (1986) and Scotchmer and Wooders (1986). My equal-treatment results for the ε-core using only per capita boundedness and superadditivity date back to a 1980 typescript. These were reported in my 1982 Cowles Foundation Discussion Paper with Martin Shubik and are highlighted in the version of Shubik and Wooders published in 1999.[8] In MW (1991, 1992a, 1994) the restriction to sequences with a fixed distribution of player types in the prior papers is relaxed and uniform convergence is demonstrated.

In MW (1992a, Section 4), commodities (or attributes) are simply re-named 'players'. Thus, as discussed in that section, it is immediate, from my approximate core convergence results for games—e.g., MW (1980, 1991)—that approximate cores converge to prices for commodities/attributes. These prices, giving per-unit payoffs for attributes, constitute the equal-treatment attribute core—a linear function on the space of commodities/attributes. (There are limitations in attempting to apply this to economies with clubs, as in Conley and Wooders, 1995, 1997, for example, since attributes, like personality and charm, cannot be separated from a player. This is discussed further below.)

My understanding is that MW (1991), extending some of my earlier convergence results to hold uniformly, and the 1991 unpublished version of E&S (1996) were concurrent. Versions of both papers were presented at a conference in Tampere, Finland, attended by this author and Scotchmer.[9] Again, of course, the claim that I use a scale assumption in my 1992a paper is in error.

4. Convergence of approximate cores

Non-emptiness of approximate cores of large exchange economies with quasi-linear utilities was first shown in Shapley and Shubik (1969); there the authors conjectured that under the same conditions, approximate cores would converge to competitive payoffs. The

[8] These early results all treated sequences of games with a fixed distribution of player types. In MW (1991, 1992a, 1994) this restriction was relaxed and uniform convergence was demonstrated. The results in Shubik and Wooders (1999) are all from their 1982 Discussion paper.

[9] In correspondence with third parties, I have been told that Scotchmer claims I heard her paper with Engl presented at a conference. Indeed, that is true. And Scotchmer heard my paper presented. At that time, E&S were not using the same assumptions as in my work, but instead assumed a uniform convergence. This was changed in the 1993 version of their paper. Even though my original results are well prior to those of E&S, nevertheless, their paper is cited in my related papers that are concurrent to theirs (including in MW (1992a)).

convergence results of MW (1980) and Shubik and Wooders (1982) apply to the model of Shapley and Shubik (1969) and require no additional conditions. [10]

The convergence results of MW (1992a), my 1977, 1979 and 1980 results, and my 1982 results with Martin Shubik are in the spirit of convergence in probability—under the conditions of superadditivity and PCB, approximate cores of large games treat most individual players/attributes of the same type approximately equally. Those of E&S (1992, 1993, 1996) are more in the spirit of convergence in mean—in per capita terms approximate core payoffs to sufficiently large groups of players/attributes are close to equal-treatment approximate core payoffs. Considering here only the special case where players are characterized by their types, the E&S result assumes PCB, superadditivity, a thickness assumption (more on this later), and a differentiability condition. Then given an approximate core payoff $x \in R^N$, there is a vector $p = (p_1, \ldots, p_T)$ such that for all sufficiently large sets S of players, the sum of the approximate core payoff x_i to the members of that set, say $x(S)$, divided by the number of players in that set, $|S|$, is approximately $p \cdot s/|S|$ and where $s = (s_1, \ldots, s_T)$ and s_t is the number of players of type t in S for $t = 1, \ldots, T$. That is, *on average*, the players in S are assigned approximately $p \cdot s|S|$. In brief, E&S (1996):

$$E\&S: \left| \frac{x(S)}{|S|} - \frac{ps}{|S|} \right| \text{ is "small" for sufficiently large coalitions } S.$$

In contrast, under the conditions of PCB and superadditivity, the results of MW (1977, 1979a, 1980, 1991, 1992a), and my 1982 paper with Shubik all show that in large games, any approximate core payoff vector x has the property that *most* players of type t receive *nearly* p_t each, that is, for each t, for most players i of type t (for all except a small percentage):

$$MW: |x_i - p_t| \text{ is "small"}$$

and p_t can be taken as the average approximate core payoff to players of type t, i.e.

$$p_t = \frac{\sum_{i:i \text{ is of type } t} x_i}{|\{i : i \text{ is of type } t\}|}.$$

The price vector p can also be taken as a core payoff of a limit game with the same proportion of players of each type. My results show that all ε-core payoffs treat (most) identical players almost equally in large games. Those of E&S show that payoffs to sufficiently large *groups* can be approximated by equal-treatment payoffs—their results do not explicitly say anything about payoffs to individual players. While my earlier results and may suggest the results of E&S (1992) [11] and those of that paper may suggest the extension of this

[10] There are numerous results in the literature on convergence of cores of private goods exchange economies with ordinal preferences; see Anderson (1992) for a survey. It may be possible that some of these apply to the Shapley and Shubik model.

[11] In fact, to this author the convergence result of E&S is more closely related to the *non-emptiness* results of MW (1980) and the convergence results of MW (1988, 1994) of games to markets since these results concern convergence of per capita payoffs $\psi^*(f)/|f|$ to a limiting (utility) function for games/economies with non-empty cores. Most important, as noted by Aumann (1987) in discussing my research, the limiting utility function is concave, a fact that also plays a role in E&S (1996).

author's results to the Shapley-Shubik (1966) framework of exchange economies (carried out in MW (1992a,b)) the results are quite distinct. Convergence in the Shapley-Shubik framework was explicitly recognized by the authors themselves in their 1966 paper. (A more complete discussion of convergence in this framework is presented in MW, 1993b).

In MW (1977, 1979a) the core of a limit game was defined and it was shown that approximate cores converge to the core of the continuum limit game and thus, to competitive payoffs. Scotchmer and Wooders (1986) *cite* my prior results on convergence of approximate cores to competitive (thus, equal treatment) payoffs. This again indicates that E&S were aware of the prior research.

Although under a stronger condition than superadditivity and per capita boundedness, it is noteworthy that Wooders and Zame (1987) show convergence of approximate core payoffs with the equal-treatment property to the core of the limit game as the numbers of players becomes large. (For games with types of players, the core of the limit game coincides with the linear function of E&S. [12]) These results are extended to hold under SGE in MW (1991, 1992a). In MW (1977, 1979a), the core of the limit game was also defined and the same sort of result obtained for games with a fixed distribution of player types (in addition to the results showing approximate cores converge to equal treatment payoffs). Scotchmer and Wooders (1986) refer to my prior results on convergence of approximate cores to competitive (thus, equal-treatment) payoffs.

5. Monotonicity and comparative statics

Another issue that may be relevant is monotonicity and comparative statics properties of approximate cores of large games. Again, in private goods exchange economies, such results are well known (cf. Hildenbrand (1994) and references therein). As recognized by Scotchmer (1986), [13] MW (1979a) stresses that, for large games satisfying exhaustion of

[12] Wooders and Zame (1987) use the condition of uniform boundedness of individual marginal contributions to coalitions— there is a constant K such that the increase in payoff to a coalition when an additional player is added to the coalition is bounded by K. This condition is stronger than required since it implies SGE but not the converse; see MW (1994).

[13] One of the results of my 1977, 1979a papers is that when all gains to improvement can be carried out by groups bounded in size, then for any initial player set, there is a replication of this player set so that the resulting game has a non-empty core. Moreover, all further replications of that replicated player set have a non-empty core. This result has been extended and used in a number of papers in game theory (see, e.g. Kaneko and Wooders (1982) and Myerson (1991)). In Scotchmer (1986), the author presents this result for (rational) proportions of player types, i.e. instead of taking a vector of integral numbers of players of each type as given, Scotchmer takes as given o vector of rational proportions of players of each type. She argues, for the special case of two types of players, that there is a replication of the vector of player-type proportions such that the replicated game has a non-empty core, as do all replications of that game. As is obvious and Scotchmer (1986) recognizes: *"This result does not differ substantially from MW (1979a,b) case of "integral" agents"*. This is the contribution of Scotchmer (1986), now cited in her subsequent research as a primary source. A difference between the papers of mine (alone and with other co-authors) and those of Scotchmer is that she, as in her 1986 paper, expresses exhaustion of gains to scale in terms of the limiting set of equal-treatment payoffs or the limiting per capita payoff as the player set is replicated. For games derived from pregames, as most of the research on large games cited in this paper, there is no substantial difference between the two formuations—one implies the other. In more general frameworks, as in Kovalenkov and Wooders (1999, 2000), the formulation favored by E&S plays no role.

gains to scale, if the numbers of players of one type increases while the numbers of other types are held constant, the payoff to the type that has increased in number does not increase. Indeed, Scotchmer (1986) writes: **(f): "An idea emphasized by MW (1979a,b)**... **is that the utility achieved by each agent in the core depends on his scarcity** .... **If type-a becomes more abundant, his utility in the core will generally decrease"**.

And, as also indicated in the same paragraph, if type-a becomes more abundant, his utility will not increase. A proof (actually, of a stronger result—more on this in next paragraphs) was provided in Scotchmer and Wooders (1988).[14] Versions of this same result—varying the percentage of one type while uniformly decreasing the percentages of players of all other types—next appeared in Scotchmer (1990) and then E&S (see the 1992 version of their paper, for example). These results are referred to by E&S (1992) and also Kovalenkov and Wooders (1999) as *comparative statics* results.

Scotchmer and Wooders (1988) actually proved the stronger result of monotonicity—under exhaustion of gains to scale, allowing the numbers of players of all types to change simultaneously, the product of a *vector of changes* in the numbers of players of each type in the population and a corresponding vector of changes in core payoffs is non-positive. In MW (1992a,b) this result is shown to hold for finite games under the condition of bounded effective group sizes, with a continuum of players (a result in terms of percentages), and approximately for large finite games satisfying SGE. The connection to the celebrated Law of Demand (cf. Hildenbrand (1994)) was also made in MW (1992a). In the 1993 version of their paper, using a Lemma from their earlier paper, E&S their result for vectors of changes, expressed in percentages. This should not be taken to suggest that E&S (1993) based their result on that of MW (1992a)—given their prior results it may not have been difficult to extend their results to hold for vectors of changes in population proportions. It is the case, however, that prior to 1993, based on the cited papers available to me, E&S had considered changes in the percentages of players of *only one type*, while the percentages of players of the other types were restricted to change by the same (compensating) amount. In view of David Warsh's article stating that this author had been accused of "duplicating" E&S, such changes in the various preliminary versions of E&S (1996) are necessary to clarify the timing of the appearance of different results.

As shown in Kovalenkov and Wooders (2000) in some generality, monotonicity and comparative statics with percentages of players of each type imply those with absolute numbers of players. The special features of the 'pregame framework', used in most of the papers cited herein on large games, are not required.

6. Conditions of small group effectiveness in the works of Scotchmer and of this author

As noted in the introduction, small group effectiveness is the condition that all or almost all gains to collective activities can be realized by relatively small groups of players. It has been known for some time that some sort of property limiting returns to size of coalitions holds for large exchange economies satisfying some mild conditions (cf. Grodal (1972);

[14] There is also an IMSSS version of this paper, dating from the summer of 1988.

Schmeidler (1972); Vind (1972); Kaneko and Wooders (1986, 1989); Hammond (1999)) for economies with a continuum of agents, and Mas-Colell (1979) and Kaneko and Wooders (1989) for asymptotic formulations. [15]

To show that when small groups of players are effective for the realization of all gains to collective activities, games with many players are 'market-like,' in MW (1977, 1979a) this author introduced a condition of 'strict small group effectiveness, then called '$*$'. The related condition of 'minimum efficient scale' (MES) was introduced in MW (1983). The name, MES, was used since this condition for games is analogous to the Novshek and Sonnenschein (1978) condition for production. Their condition requires that the average cost functions of firms have a minimum, not necessarily unique. Similarly, the MES condition for cooperative games requires that there is a smallest 'efficient' coalition sizes—but efficient coalitions may be arbitrarily large. Thus, the per capita payoff function may be '$\rangle$-shaped'.

It was understood early in research on large games that 'small group effectiveness' held for a broad class of games. In MW (1983) and subsequent papers, asymptotic results, showing that large games with a fixed distribution of player types and satisfying per capita boundedness, PCB, have non-empty cores were proven by approximating these games with games satisfying MES. Shubik and Wooders (1982), in fact, suggestively called PCB '*near* minimum efficient scale'. That games can be approximated by other games satisfying MES or strict SGE constitutes the SGE property. Motivated by a comment of Roger Myerson's, [16] this author introduced the condition of small group effectiveness in MW (1991, 1992a,b, 1994). SGE is equivalent to the condition that 'small player sets' cannot have significant impacts on aggregate payoff; asymptotically, small groups are negligible (Wooders, 1993a). As already noted, with 'thickness' of the total player set, SGE is equivalent to PCB (MW, 1994).

There are similarities and differences between the formulations of conditions limiting gains to collective activities of this author and those of Scotchmer (with this author and alone). First, let us compare strict small group effectiveness, from MW (1979a,b), and MES from MW (1983) (Theorem 3) with exhaustion of gains to scale in the papers of Scotchmer (with this author and alone). Roughly, the formulation of the condition of exhaustion of gains to scale in these papers is in terms of the limiting set of equal treatment payoffs. Exhaustion conditions, prior to her work with Engl, requires that all payoffs in this limiting set can be realized by groups bounded in size. In my earliest work (cf. MW (1979a,b)), condition $(*)$, it is required that any payoff that can be improved upon can be improved upon by groups bounded in size. The two conditions are equivalent.

Turning to small group effectiveness, SGE (non-strict), one problem that comes up in obtaining uniform results—results for *all* sufficiently large games rather than games with a fixed distribution of player types—is that "scarce types" may have big impacts. For example,

[15] Keiding (1976) shows that, in an economy with an uncountable set of agents, when coalitions are constrained to be finite, then the core and the competitive allocations are equal. The economic meaning of this is, however, unclear since Keiding's model, like prior papers in the literature with the same sorts of restrictions on coalition sizes, such as Kannai (1969), does not satisfy 'measurement consistency' and the relative scarcities of commodities are indeterminate. See MW (1984), available from the author on request, for a more complete discussion.

[16] Roger Myerson, in private correspondence, suggested that there should be some relationship between the conditions of my work (alone and with others) on large finite games and per capita boundedness as used in Kaneko and Wooders (1986). His comments are always inspiring.

suppose there are two types of agents. Every coalition consisting of type 1 players only can earn zero, while any coalition containing a player of type 2 can earn US$ 1 for each member of the coalition; players of type 2 know the secret of acquiring wealth. In the notation of E&S (1996),

$$V(n_1, n_2) = \begin{cases} 0 & \text{if } n_1 = 0 \\ n_1 + n_2 & \text{otherwise} \end{cases}$$

As long as the percentage of players of type 2 in the economy is bounded away from zero, then, given $\varepsilon > 0$ there is a bound on coalition sizes so that almost all (within ε-per capita) gains to coalition formation can be realized by groups of players bounded in size. Suppose, e.g. that we restrict to games where $(n_1/n_1 + n_2) > 1/10$. Then, for this example, *all* gains to collective activities can be realized by groups bounded in size by 10; simply partition the total player set so that no coalition contains more than 10 members and one player of type 2 is in each coalition.

One way to handle the scarce types problem is to make the Wooders and Zame (1984) assumption of boundedness of individual marginal contributions to coalitions. As shown in MW (1992a,b, 1994) SGE is the weakest assumption in the literature that solves the scarce type problem and allows uniform results. This assumption, however, is stronger than required. As an example in MW (1992a) illustrates, the Wooders and Zame assumption on marginal contributions implies SGE while SGE allows arbitrarily large marginal contributions.

The papers of E&S differ from the prior papers of Scotchmer (with this author and alone) in that their exhaustion condition, like SGE, concerns approximation. The E&S version of SGE, 'ε-exhaustion of gains to scale' simply rules out scarce types ($d(s, \delta\Delta) \geq \varepsilon > 0$)—a thickness condition. The condition of ε-exhaustion of gains to scale is obviously very closely related to SGE. Indeed, the proof of Theorem 4 of MW (1994) can be modified to show that with thickness, ε-exhaustion of gain to scale is equivalent to SGE.

Assuming PCB, E&S (1996) show that for any $\varepsilon > 0$ and $0 < \pi < 1$, if attributes are independently selected according to some probability distribution on attribute space, then for all sufficiently large games, with probability at least π, the game satisfies ε-exhaustion (their Proposition 1, page 224). The probability assumptions are used simply to make scarce types unlikely. Theorem 4 of MW (1994), *published* in 1994 by *Econometrica* (and originally submitted some years earlier) shows that when scarce types are ruled out (in E&S, the condition that $d(s, \delta\Delta) \geq \varepsilon > 0$), then PCB and SGE are equivalent. Thus, with thickness, the three concepts—PCB, SGE and ε-exhaustion for any $\varepsilon > 0$—are equivalent. The E&S Proposition 1 is essentially one direction of the apparently much earlier Theorem 4 of MW (1994).

Again, because these change are relevant, E&S (1996) Proposition 1 was not in the earlier versions of E&S available to this author (and cited herein). Thus, it would appear to be subsequent to the wide circulation and eventual publication of MW (1994).

7. The model itself

The claim that this author duplicated any of the work of E&S is puzzling. One final possibility is that perhaps E&S view their model itself as an original characterization of an

economy with clubs (or local public goods). The E&S model, however, as a special case of the model of Shapley and Shubik (1966)—an exchange economy with quasi-linear utilities. In particular, in the E&S version of the Shapley-Shubik model, all agents have the same utility/payoff function.

The E&S model is quite distinct from that of Conley and Wooders (1996, 1997), where players are characterized by their 'crowding types' or, in other words, their external effects on other (such as their skills, their genders, their ability to dance the tango, whether they are smokers or nonsmokers).[1] The Conley-Wooders (2001) model maintains the feature that the crowding type of a consumer, unlike his endowment of private goods, must be in the same club/jurisdiction/firm as the consumer. It should also perhaps be noted that the pricing system in the published version of E&S was not in their prior cited papers. The pricing system in Conley and Wooders (1997), which has become standard in subsequent literture, is distinct. (Conley and Wooders, 1996, published in that year, is a follow-up to their 1997 paper—the 1997 paper was originally completed in 1994.) The pricing system in E&S may by itself suggest that application of thier work to an economy with clubs, where players are affected by the crowding types of other players or, in other words, their external effects, requires quite special and restrictive assumptions, in comparison to those of Conley and Wooders (1996, 1997). Indeed, in general, hedonic pricing of attributes of agents in economies with clubs requires, for examples, taste homogeneity of optimal clubs/jurisdictions; see Conley and Wooders (1995, 1998) and, for a discussion of monotonicity in this context, Cartwright and Wooders (2001).

8. Conclusions

Returning again to David Warsh's comment, under his by-line, the statement appears that this author "was accused of duplicating—in an unrefereed book—the results of University of California economists Greg Engl and Suzanne Scotchmer". A copy of a letter, on a University of Berkeley letterhead, with the typed signature 'Greg Engl' and addressed to Wayne Shafer, then an editor for *Journal of Mathematical Economics*, stated that this author was circulating a paper 'duplicating' the E&S work (a scanned copy of the letter is attached to this author's web page and also available on request). The letter went on to state that the authors could now prove their result under a weaker condition, finiteness of the supremum of average payoff (identical to the assumption of per capita boundedness is this author's far earlier work, used to obtain related results). Copies of this letter were sent to many members of our profession.

It seems to be the case that false claims of misconduct are not uncommon and can have major impacts; see, e.g. Chapter 1 in Bell (1992), Kevles (1998) or the interview with Thereza Imanishi-Kari published in *Scientific American* in November 1996 [17] (there is an interesting book published by the National Academy of Sciences on conduct in science; it's on the web at http://books.nap.edu/catalog/4917.html). Certainly E&S were aware of relevant papers; see, e.g. their reference to Kannai in E&S (1994) and their reference to Part 2 of MW (1991) in E&S (1992). In some of their research, they cite (Aumann, 1987)

[17] A similar model was independently introduced in Cole and Prescott (1997).

and refer to him for first noting the concavity of the limiting per capita payoff function. And of course E&S reference (MW, 1992a), in its Working Paper form, so they were, at least in principle, informed that my convergence results date back to MW (1979b). As noted, the relationship between approximate exhaustion of gains to scale and per capita boundedness in E&S (1996), Proposition 1, is very similar to one direction of MW (1994), Theorem 4 and, for this author, has origins in research dating back to (1994), Theorem 4 and, for this author, has origins in research dating back to 1979—games satisfying PCB can be approximated by games satisfying MES or, in other words, exhaustion of gains to scale. The nonemptiness of appromimate cores of games with many players dates back to MW (1977, 1983). The result that large games with many players of each type or approximate type are market-like in the sense of convergence of approximate cores to price-taking equilibrium payoffs also dates back to MW (1977). But I would not claim that the E&S results duplicate either my earlier or concurrent research. It may have been appropriate and helpful to the reader if E&S had provided a correct and informative discussion of the relationship of their paper to prior literature but, while failing to do so may mislead the reader and leave him incorrectly informed about origins of some ideas and concepts, it does not constitute duplication.

My viewpoint is that allegations of misconduct should be made only with extreme care and not without careful substantiation by scientific evidence (if then). The place for scientific debates about relationships between papers is the scientific literature. If one result 'duplicates' another, then surely the first result should follow from the prior result, perhaps even with minor modifications of the assumptions or the result. On my part, while E&S (1996) use the same assumptions as my earlier (far earlier) work and obtain results in the same spirit, neither their convergence result nor mine (alone and with Shubik, and more recently, Kovalenkov) appears to follow easily from the other. The monotonicity results are all related, as discussed. From their references to Kannai (1992), Aumann (1987) and other papers it seems clear that E&S were aware of a number of their inappropriate claims in E&S (1996). In spite of this, E&S (1997) furthers the erroneous claims of E&S (1996). Thus, I would surely not claim that the E&S convergence result duplicates either my earlier or concurrent research.

Finally, with three exceptions, the papers referenced are all published by universities and available from the universities (a number can be obtained from the British Library. Several of my papers appear on line, attached to my web page). Let me first note that my 1984 paper about Hans Keiding's model appears only a part of a discussion of other literature. One exception relevant is MW (1979a). The same statements of results, however, appear in the 1977 Stony Brook Working Paper version. Also, versions of some of the results appear in Bennet and Wooders (1979) and the paper is referenced in Scotchmer and Wooders (1986). The other exception is MW (1980). The results of this paper, however, extend most of those of my 1977 SUNY–Stony Brook working paper to hold under per capita boundedness and the results of my 1980 paper are used in my 1982 Cowles Foundation Discussion Paper (with Martin Shubik). A paper not referenced is a 1991 typescript of E&S (1991), presented at the same conference as a preliminary version of MW (1991). The E&S typescript contains much the same material as the E&S (1992) working paper but the typescript uses somewhat stronger assumptions than the 1992 version (which itself uses apparently stronger assumptions than per capita boundedness and superadditivity).

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