Coalitions

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This is a post-peer-review, pre-copyedit version of an article published in *The New Palgrave Dictionary of Economics*, Second Edition. The definitive publisher-authenticated version,

Wooders, Myrna and Frank H. Page, Jr. "Coalitions." The New Palgrave Dictionary of Economics. Second Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan, 2008. The New Palgrave Dictionary of Economics Online. Palgrave Macmillan. 12 November 2013]

is available online at:
<http://www.dictionaryofeconomics.com/article?id=pde2008_C000179> doi:10.1057/9780230226203.0250>.

The New Palgrave Dictionary of Economics Online

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From The New Palgrave Dictionary of Economics, Second Edition, 2008
Edited by Steven N. Durlauf and Lawrence E. Blume

Abstract

Coalitions appear in an incredible diversity of economic and game-theoretic situations, ranging from marriages, social coalitions and clubs to unions of nations. We discuss some of the major approaches to coalition theory, including models treating why and how coalitions form, equilibrium (or solution) concepts for predicting outcomes of models allowing coalition formation, and current trends in research on coalitions. We omit a number of related topics covered elsewhere in this dictionary, such as matching and bargaining.

Keywords

f-core; abstract games; admissible set; asymmetric information; bargaining; bargaining set; basins of attraction; clubs; coalitions; cooperative games; cores; differential information; domination; epsilon core; extensive form games; far-sighted stability; hedonic games; implicit coalitions; incomplete information; *information sharing* ; inner core; irreversibilities; kernel; law of demand; law of supply; linear programming; link formation; local public goods; Myerson value; Nash equilibrium; Nash program; network formation; non-cooperative games; non-transferable utility games; Owen equilibrium; Owen set; pairwise stability; partnered core; private information; public goods; Shapley value; small group effectiveness; solution concepts; strong stability; subgame perfection; superadditivity; supernetworks; tau value; Tiebout hypothesis; transferable utility games; von Neumann–Morgenstern stable set

Article

The traditional notion of a coalition is a group of players who can realize some set of outcomes for its own membership. How to define this set of outcomes is a fundamental question and its definition is typically either avoided, by assuming that the set of outcomes is given, or treated simultaneously with a solution concept. Alternatively, some process may be given that plays a role in determining the set of outcomes that are achievable by each coalition.

How to define a coalition is an even more fundamental question. Typically a coalition is taken as a subset of players of a game. Yet we often perceive that individuals belong to overlapping coalitions. For example, an individual may belong to the Citizens Coalition for Responsible Media, Immunization Action Coalition and the Democratic Party. We also perceive that coalitions may be *temporary* alliances of groups of people, factions, parties, or nations. For most of this article, however, we view a coalition as simply a subset of players of a game.

When both the concepts of a coalition and its attainable set of outcomes have been defined, the question arises of how the gains from coalitional activities are to be allocated among the members of any coalition that might form, bringing us to the notion of a solution concept. A solution concept is a rule which must be satisfied by any allocation or attainable outcome that is viewed as stable or as an equilibrium. Given a description of the primitives of a situation (a game, economy, or social situation, for example) a solution concept may be viewed as predicting which outcome(s) will emerge. Implicitly, a solution concept involves assumptions about the behaviour of individuals or groups of individuals. Even in situations where a particular solution concept seems compelling, however, there may be no attainable outcomes satisfying the requirements of the solution concept. This problem, and the fact that no single solution concept seems to fit all situations, means that there are competing notions of solution concepts.

In this article we discuss issues of coalitions, the outcomes attainable by coalitions and the division of the benefits of coalition formation among the members of a coalition. Many of the fundamental questions that still intrigue researchers have their roots in the early literature of game theory. We will sketch some of the main concepts in the literature on coalitions, going back to von Neumann and Morgenstern's celebrated volume, with its notion of dominance, and also sketch some of the current approaches to questions of coalitional activities. We conclude by noting some new approaches to what a coalition might be and do and directions that research may be taking.

Domination

What a coalition can achieve, or, even more fundamentally, what a coalition can improve upon for its own membership is a fundamental question. This was realized already by von Neumann and Morgenstern (1953), who introduced the notion of domination. An imputation x (or payoff vector, listing a payoff for each participant in the society) *dominates* another imputation y with respect to a coalition S if the members of S are convinced or can be convinced that they have a positive motive for bringing about y and believe that they can do so. The coalition S is called *effective* (for x). Note that it is possible there is another payoff vector y', a coalition S' that is effective for y', and y' dominates y with respect to S' (but not with respect to S) and in general, the relation ‘dominates’ may not be transitive.

Solution concepts

A number of solution concepts based on notions of domination and effectiveness of coalitions have been defined. Three especially prominent concepts are the von Neumann–Morgenstern stable set, the Shapley value, and the core. A set V of payoff vectors, where each vector is a listing of payoffs to players in a game, is a *von Neumann–Morgenstern stable set* if (*a*) no payoff vector in V is dominated by another payoff vector in V and (*b*) every payoff vector not in V is dominated by some vector in V. The *core*, introduced in Gillies and Shapley in 1953 (see the Logistics Research Project, 1957, which contains descriptions of the presentations of D. Gillies and L.S. Shapley, where the core was introduced), consists of those payoff vectors x that are feasible and undominated. The formulation of Gillies (1959) of the core of an abstract game can be widely applied. An *abstract game* consists of a set of alternatives for each coalition and a dominance relationship. The *Shapley value*, introduced in Shapley (1953), assigns to each player his *expected* marginal contribution to coalitions and is also used in numerous applications. Alternative notions of the core and of the value include the *Owen value* (Owen, 1977), the *τ-value* (Tijs, 1981), the *inner core* (Myerson, 1995; Qin, 1994; and references therein), and the *partnered core* (Albers, 1979; Bennett, 1983; Reny and Wooders, 1996a).

Let us consider a simple example. Let $N=\{1,2,3\}$ be the player set. Suppose that any one player can earn zero, any two players can earn one dollar and the three players together can earn $M\geq 0$ dollars. Suppose $M=1$; then the von Neumann–Morgenstern stable set consists of the payoff vectors $(\frac{1}{2},\frac{1}{2},0)$, $(\frac{1}{2},0,\frac{1}{2})$, and $(0,\frac{1}{2},\frac{1}{2})$. Any payoff vector (z_1, z_2, z_3) is in the core if $z_i\geq 0$ for all $i\in N$ and $z_i+z_j\geq 1$ for every pair *i*, *j*. This implies that, unless $M\geq\frac{3}{2}$, the core is empty. The Shapley value is defined for *superadditive games*, games with the property that the set of payoff vectors achievable by any union of disjoint coalitions is at least as large as the set of payoff vectors achievable by the coalitions independently. Superadditivity, for our example, implies that $M\geq 1$, in which case the Shapley value consists of the payoff vector $(\frac{M}{3},\frac{M}{3},\frac{M}{3})$.

The bargaining set, introduced by Aumann and Maschler (1964), is based on threats and counter-threats. A payoff vector *x* is in the *bargaining set* if for every credible objection there is a credible counter-objection. That is, if there is a payoff vector *y* that dominates *x* with respect to a coalition *S* then there is another payoff vector *y′* and coalition *S′* that is effective for *y′* and *y′* is at least as good as *x* for the members of *S′* who are not in *S* and at least as good as *y* for members of both *S* and *S′*. There are a number of related concepts. The *kernel*, introduced in Davis and Maschler (1965), requires that objections and counter-objections have equal strengths. For our example above, the point $(\frac{M}{3},\frac{M}{3},\frac{M}{3})$ is also in the bargaining set and in the kernel. Recent research on concepts of the bargaining set has been spurred by the *Mas-Colell bargaining set* (Mas-Colell, 1989) which adapts the bargaining set to economies with a continuum of agents and proves equivalence of the outcomes of the bargaining set and the core in an exchange economy.

Another interesting notion is the *admissible set*, introduced in Kalai and Schmeidler (1977). (See also references therein and Shenoy, 1980.) Take as given a set of feasible alternatives, denoted by *S*, a dominance relation M and the transitive closure of *M*, denoted by $\widehat{M}$. The *admissible set* is the set $A(S,M)=\{x\in S: y\in S \text{ and } y\,\widehat{M}x \text{ imply } x\,\widehat{M}y\}$. The admissible set describes those outcomes that are likely to be reached by any dynamic process that respects preferences. Note that the admissible set concept can be applied to a host of game-theoretic situations, ranging from non-cooperative games, where a coalition consists of an individual player, to fully cooperative games, where any coalition can be allowed to form. As shown by Kalai and Schmeidler, under certain conditions the admissible set coincides with the set of Nash equilibria and, for cooperative games, the admissible set coincides with the core. More recently, it has been shown that the admissible set consists of the union of basins of attraction, and a von Neumann–Morgenstern set consists of one member of each basin (Page and Wooders, 2006).

Behaviour of coalition members

What a coalition can achieve also depends on the *behaviour of the members of the coalition*. For example, potential coalition members may bargain over the distribution of the gains to coalition formation and outcomes in the core may not be achievable as equilibria of non-cooperative bargaining processes (an important point made by John Nash, 1953, leading to the *Nash program*). Chatterjee et al. (1993) demonstrate this point very well for transferable utility (TU) games, which describe what a coalition can achieve by simply a number, in interpretation, an amount of money, for example. As stressed by Xue (1998), it may matter whether players are farsighted or myopic in their thinking about forming coalitions. Myopic players take as given the actions of others and behave accordingly. In choosing their actions, farsighted players, in contrast, take into account the reactions of other players to their actions and thus the eventual consequences of their actions. See also Diamantoudi and Xue (2003) who study the far-sighted core of a hedonic game – a game where, instead of payoff sets for coalitions, preferences are given for each individual over all coalitions in which he is contained – and Mauleon and Vannetelbosch (2004) who both allow ‘spillovers’ between coalitions and farsightedness of players, and demonstrate sufficient conditions for there to exist stable outcomes. (Two important papers in the game theoretic literature studying farsightedness, but not coalition formation, are Chwe, 1994, and Harsanyi, 1974.)

Players may also take into account ‘asymmetric dependencies’ within coalitions. A solution displays an asymmetric dependency if one player needs the presence of a second player to realize his payoff in the solution, but the second player does not need the presence of the first. When a player i is dependent on another player j in this sense, but j is not dependent on i, then j is in a position to attempt to obtain a larger share of the surplus from i. Consider, for example, a two-person divide-the-dollar bargaining game. Any division giving the entire dollar to one participant displays an asymmetric dependency; the player receiving the dollar is dependent on the player receiving zero. The player receiving zero is not compelled to join the two-person coalition to receive his part of the payoff. In contrast, to achieve the payoff of 50 cents for each player the two-person coalition is compelled to form – the players are partnered. The partnered core, introduced in Albers (1979) and Bennett (1983) for TU games and in Reny and Wooders, (1996a) for non-transferable utility games (where the set of payoffs achievable by a coalition are described by vectors listing a payoff for each member of the coalition) consists of those outcomes in the core with the property that, to achieve his payoff, no individual needs another individual who does not need him. Even in well-behaved exchange economies there may be no outcomes in the core that are not partnered; that is, all outcomes in the core may be vulnerable to the threat of secession by some coalition of players. Page and Wooders (1996) provide an example.

Behaviour of non-coalition members

In many situations, what a coalition can achieve depends on assumptions about the *behaviour of non-coalition members* (sometimes called the ‘complementary coalition’, although there is no requirement that the complementary coalition actually forms an alliance); for example, individuals may steal, or drop garbage in the backyards of others, or there may be widespread pollution. Two alternative definitions of the core, from Aumann and Peleg (1960), highlight the dependence of the core on the assumptions made about what outcomes are perceived as feasible by coalitions: the α-core, consisting of those outcomes that a coalition can guarantee for its membership, and the β-core, consisting of those outcomes that a coalition cannot be prevented from achieving for its membership. In some situations, such as private goods economies without externalities or in some recent models of economies with clubs or local public goods, these two notions are equivalent, but, as noted by Shapley and Shubik (1969a), in the presence of externalities between coalitions these concepts may yield different outcomes.

Members of a coalition may also be directly affected by the structure of alliances among non-members of the coalition. This consideration underlies the Lucas and Thrall (1963) concept of *a partition function form game*, where the attainable total payoff to a coalition depend on the structure of coalitions formed by the complementary player set.

In the approach of Chander and Tulkens (1995; 1997), to predict the set of outcomes that it can achieve, a coalition presumes that the outside players will adopt their individually best reply strategies, leading to their notion of the gamma core. In the sense that the non-coalition members are treated as forming one-person coalitions, the Chander–Tulkens approach is more restrictive than that of Lucas and Thrall. When it is assumed that coalitions can freely merge or break apart and are farsighted, however, Chander (2007) demonstrates that, subsequent to a deviation by a coalition, the non-members will have incentives to break apart into singletons, thus providing a justification for the Chander–Tulkens approach.

Other approaches to the question of what a coalition can achieve for its membership have also appeared in the literature. Some recent contributions allow theft or pillage by non-coalition members; see, for example, Jordan (2006), where the payoffs attainable by a coalition are determined endogenously, and references therein.

In application, questions of the behaviour of the non-coalition members have been especially important in industrial organization and environmental economics; see, for example, Yi (1997) and Bloch (1996); see Bloch (2005) and Carraro (2005) for discussions of relevant literature.

Information sharing within coalitions

When players have private information new and difficult issues arise. Chief among these is the issue of *information sharing within coalitions*. How can members of a coalition be induced to share their private information truthfully? Or, if it is not shared truthfully, how much information will be shared and how much of it will be believed? In his seminal paper, Wilson (1978) introduced two notions of the core for situations with private information, namely, the coarse core and the fine core; later Yannelis (1991) introduced the private core. Each of these core notions corresponds to assumptions about the extent to which private information of individual players is shared within coalitions. These issues are further addressed in Allen (2006), who treated core concepts in exchange economies, and Page (1997), who extended Allen's results to infinite dimensional commodity spaces. There is also the question of what informational time frame should be used in defining a solution concept. Following the informational distinctions introduced by Holmstrom and Myerson (1983) in extending the notion of Pareto efficiency to economies with private information, we can ask whether the solution concept should be *ex ante* (that is, defined relative to *ex ante* probability beliefs concerning the future information state of the economy – and therefore before players know their private information), whether it should be interim in nature (that is, defined relative to each possible profile of players' private information – and therefore after each player knows his private information but before players know the information of others), or whether it should be *ex post* (that is, defined relative to each possible information state of the economy – and therefore after each player knows the information state of the economy).

Following a mechanism design approach, Forges, Mertens and Vohra (2002) address the issue of honest information revelation within coalitions by focusing on coalitionally incentive-compatible direct mechanisms. A coalitional direct mechanism is a mapping from the set of information profiles of coalition members into coalitional allocations. A coalitional direct mechanism is *incentive compatible* if no coalition member has an incentive to lie about his private information –on the assumption that other coalition members report their private information truthfully (that is, truthful reporting is a Nash equilibrium of the coalitional revelation game induced by the mechanism). Formulating the coalitional mechanism design game as a TU game in characteristic function form, they demonstrate non-emptiness of the incentive compatible '*ex ante* core'. Other contributions which analyse interim core notions include Ichiishi and Idzik (1996), Hahn and Yannelis (1997), Vohra (1999), Volij (2000), Demange and Guesnerie (2001), Dutta and Vohra (2005) and Myerson (2007). See Forges, Minelli and Vohra (2002) for a survey.

The core with incomplete information is gaining prominence in applications, such as political economy (see, for example, Serrano and Vohra, 2006).

Coalition formation

Other important questions are *how coalitions form and how coalition structures influence the behaviour of individuals within coalitions*. Several approaches are possible. Coalition formation and individual behaviour can be viewed as outcomes of market mechanisms or as outcomes of assumed cooperation within groups that may form. Alternatively, coalition formation and individual behaviour can be viewed as outcomes induced from non-cooperative behaviour. More recently coalition formation and individual behaviour within coalitions have been modelled in network settings.

The market/cooperative game approach

As suggested by Tiebout (1956) and Buchanan (1965), individuals may take as given prices for membership in coalitions (clubs, firms, jurisdictions, and so on). Tiebout conjectured that if public goods are 'local' (that is, public goods are subject to congestion and individuals can be excluded from the public goods provided in jurisdictions in which they are non-members), then the possibility of individuals moving to the jurisdictions where their wants are best satisfied subject to their budget constraints and to taxes creates a competitive 'market-like' outcome. A part of the outcome is a partition of individuals into jurisdictions. Buchanan (1965) stressed the importance of collective activities in a model of clubs with optimal club size; to illustrate, considering our example above where any two players can earn one dollar, if $M < \frac{3}{2}$, then two is the optimal club size. One way to formulate the Tiebout hypothesis (Pauly, 1970; Wooders, 1978; 1980) is to model the economy as one where individuals pay prices to join coalitions/clubs/jurisdictions and to demonstrate equivalence of the core and the set of outcomes of price-taking equilibrium. The results of these early papers have been greatly extended and refined; see, for example, Conley and Wooders (2001); Ellickson et al. (2001) and, for a survey, Conley and Smith (2005). The spirit of the main results is that, whenever *small group effectiveness* holds – that is, whenever all or almost all externalities can be internalized within relatively small groups of individuals (clubs, jurisdictions, firms, trading coalitions, and so on) or, in other words, whenever all or almost all gains to collective activities can be realized with some partition of the total player set into relatively small coalitions – then economies with many participants are 'market like' in the sense that price-taking economic equilibrium exists and the set of equilibrium outcomes is equivalent to the core of the economy.

The results for models of economies with local public goods and clubs suggest results for cooperative games with endogenous coalition structures. Under small group effectiveness, cooperative games with many players are 'market games' (as defined in Shapley and Shubik, 1969b) and thus can be represented as economies where all individuals have concave, continuous utility functions (Wooders, 1994a; 1994b). (That the conditions of Wooders, 1983, imply that games with many players are market games was first noted by Shubik and Wooders, 1982, and the concavity of the limiting per capita payoff function was first explicitly noted in 1987 by Robert Aumann in his entry game theory in the first edition of this dictionary, which is reproduced in the present edition).

A simple example may provide some intuition. Suppose any two players can earn \$1.00, as in our earlier example, but now suppose that there are *n* players in total. If *n* is odd, then the core is empty, but for large *n* each player can receive nearly \$0.50 so certain approximate cores are non-empty and the approximation is 'close'. In defining an appropriate approximate core concept the modeller can either suppose that there are some costs to coalition formation, which can be allowed to go to zero as *n* becomes large, or that a relatively small set of players can be ignored. Now, more generally, suppose instead that the payoff to a coalition with *m* members is a real number *v*(*m*). Suppose the game is essentially superadditive – the total payoff achievable by $m + m'$ players is greater than or equal to $v(m) + v(m')$. Then the only condition required to ensure non-emptiness of approximate cores of games with many players is that there is a bound *K* such that $\frac{v(m)}{m} \le K$ for all *m*, which implies small group effectiveness. The limiting concave utility function alluded to above is $u(n) = \sup \frac{v(m)}{m} n$. See also Robert Aumann's discussion of Wooders's (1983) result in game theory. Some other market properties of a game with many participants are that: Outcomes in the core or approximate cores treat most similar players nearly equally (Wooders. 1983; Shubik and Wooders, 1982; and for the most recent results, Kovalenkov and Wooders 2001a). The Shapley value is in an approximate core (Wooders and Zame 1987). A 'law of scarcity' holds; that is, increasing the abundance of one type of player leads to a decrease in the core payoffs to individual players of the same of similar types (Scotchmer and Wooders, 1988; and, for recent results and references, Kovalenkov and Wooders 2005b; 2006). The law of scarcity is in the spirit of the law of demand and law of supply of private goods economies but differs in that an additional player in a game creates both creates additional demand (for the cooperation of others) and additional supply (of players of the same type).

To illustrate further the intimate relationships between markets and economies with group activities such as clubs and/or local public goods, we will discuss Owen (1975), who treats a production economy where individuals are endowed with resources that may be used in production. Rather than selling their resources to firms, individuals form coalitions and use the resources owned by the coalition to produce output which can then be sold at given prices. Owen places conditions on the model – specifically linear production functions – that ensure non-emptiness of the core of the derived game, whose coalitions consist of owners of resources. From the fundamental theorem of linear programming, associated with any point in the core of the game there is a price vector for resources, which is analogous to a competitive equilibrium price vector for resources except that the budget constraint need not be satisfied by individuals but instead only by coalitions. Owen demonstrates that, when the economy is replicated, the core converges to the set of Owen equilibrium prices. The Owen set and the Owen equilibrium prices have been studied in a number of papers – for example, Kalai and Zemel (1982), Samet and Zemel (1984), Granot (1986) and Gellekom et al. (2000). (There is also some relationship to the literature on oligopoly and cost-sharing; see, for example, Sharkey, 1990, and Tauman, Urbano and Watanabe, 1997.)

It is easy to interpret the resources in Owen's model as attributes of individuals, such as their intelligence, skill level, wealth, ability to dance the tango, and so on. (Of course, labour is typically an input into a production process.) We can also easily interpret a coalition that forms as a club. For example, the club may be a dinner club, where each person brings himself – his personality, his gender, and so on – and also perhaps contributes a dish for the meal. The benefits to membership in a club depend on the attributes of its members – whether they are charming, whether they are good cooks. A difficulty in applying Owen's model to economies with clubs, jurisdictions, or any sort of essential group activity is that his results require linearity of the production function. However, as Owen remarks, concavity of preferences and production possibilities, as in Debreu and Scarf (1963), suffices for all his results except uniqueness of Owen equilibrium prices. But the concavity of limiting per capita payoff functions under the conditions of essential superadditivity and small group effectiveness of Wooders (1983; 1994a; 1994b) implies that in large games with clubs or coalitional activities the economy is representable as a market economy where individuals have concave preferences. Essential superadditivity simply allows a set of players to partition itself and achieve the outcomes achievable by the collective activities of the members of each element of the partition. Finiteness of the supremum of per capita payoffs (per capita boundedness) rules out average (per individual player) payoff from becoming infinitely large. Recent research investigates the relationship between club economies and games in more detail (see, for recent surveys, Wooders, 1994b; Kovalenkov and Wooders, 2005a; Conley and Smith, 2005).

Closely related in important ways to the market approach are approaches that assume cooperative behaviour on the part of members of the coalitions that form. As in the market approach, what a coalition can achieve is taken as defined, a solution concept assumed (which in some cases includes a partition of the set of players into groups that can achieve their part of the outcome), and the existence and properties of outcomes satisfying the requirements of the solution concept are examined. Classic contributions to this literature, besides those mentioned above, include Aumann and Maschler (1964), Aumann and Shapley (1974), Shapley (1971), and Hart and Kurz (1983). More recent contributions include, among others, Demange (1994), Bogomolnaia and Jackson (2002), Banerjee, Konishi and Sonmez (2001), Le Breton, Ortuno-Ortin and Weber (2006), and Bogomolnaia et al. (2007). These interesting works deepen insight into the question of conditions on models ensuring there is some outcome satisfying the requirements of solutions having desirable properties, especially the core.

Necessary and sufficient conditions for non-emptiness of cores are demonstrated by Bondareva (1963) and Shapley (1967) for games with transferable utility and, most recently, by Predtetchinski and Herings (2004) and Bonnisseau and Iehle (2007) for non-transferable utility games.

A small but growing literature, initiated by the assignment games of Gale and Shapley (1962), Shapley and Shubik (1972) and Aumann and Drèze (1974), addresses the question of what conditions on permissible coalition structures will ensure that a game has a non-empty cores, independently of the sets of attainable outcomes of the game. Early papers providing such conditions are Kaneko and Wooders (1982) and Le Breton, Owen and Weber (1992). Recent papers have treated sufficient conditions for non-emptiness of the core of a hedonic game, where preferences are defined directly over coalitions (Bogomolnaia and Jackson, 2002; Banerjee, Konishi and Sonmez, 2001; Papai, 2004) while Iehle (2006) provides necessary and sufficient conditions. Demange (2004) demonstrates that imposing a hierarchical structure on the set of players, limiting the coalitions that can form, will ensure existence of an efficient outcome that is stable in the sense that no admissible coalition, called a team, could improve upon the outcome for its members. A hierarchical structure is represented by a pyramidal network. A team is a group of individuals who can communicate through the channels created by the hierarchical structure.

A related branch of literature focuses on conditions ensuring that groups of agents do not break away from a coalition. Le Breton and Weber (2001), Haimanko, Le Breton and Weber (2004), and Drèze, Le Breton and Weber (2007) investigate models with heterogeneous individuals and conditions ensuring existence of secession-proof outcomes, that is, outcomes that are immune to breakaways by subgroups of individuals and are thus in the core. For a different approach motivated by the idea that if a group secedes from a larger group then it does not necessarily stand alone, see Reny and Wooders (1996b), who use the solution concept of the partnered core. See also Alesina and Spolaore (1997) who demonstrate that, in a model of public good provision with a continuum of consumers who are differentiated by their preferred location for a facility and voting within each community, in equilibrium there are too many coalitions (nations).

Non-cooperative game approach

Coalitions can arise as equilibrium outcomes of either static or dynamic non-cooperative games. In the non-cooperative literature on clubs or local public goods, it may be assumed that there is a fixed set of jurisdictions, each providing some level of a public good for its residents. Individuals who move to a jurisdiction pay the average cost of public good provision. Alternatively, individuals may be required to pay a proportion of their income towards financing the public good produced by the jurisdiction. Individuals each chose a jurisdiction in which to live. The main questions are whether a non-cooperative equilibrium (Nash equilibrium in pure strategies) exists and its properties, such as whether, in equilibrium, members of the same jurisdiction have similar wealths. Contributions to this literature include Greenberg and Weber (1986), Demange (1994), Konishi, Le Breton and Weber (1997; 1998), Gravel and Thoron (2007). See also Demange (2005), who discusses literature involving both cooperative and non-cooperative approaches. Based on the concept of coalition-proofness (Bernheim, Peleg and Whinston, 1987) Conley and Konishi (2002) obtain existence of an efficient, migration-proof equilibrium for local public good (club) economies with many but a finite number of players. Casella (1992) and Casella and Feinstein (2002) consider the effects of the possibilities of trade in private goods in the formation of clubs/jurisdictions.

In a number of papers on dynamic games of coalition formation, a payoff set is given for each coalition. Suppose for simplicity that, for each coalition *S*, there is a unique attainable payoff vector $\{x^i(S) : i \in S\}$. If players are randomly ordered and if according to the ordering each player lists those players he would like as members of his coalition, then one possible solution to such a game of non-cooperative coalition formation would be a partition of the total player set into coalitions where for each coalition *S* in the partition the members of *S* all choose *S* and each player $i \in S$ receives the payoff $x^i(S)$. If player *i* belongs to no such coalition, then he receives some default payoff $x^i(\{i\})$. This sort of approach was introduced in Selten (1981). Perry and Reny (1994) provide a non-cooperative implementation of the core for TU games. In the Perry–Reny model proposed, time is continuous. This ensures that there is always time to reject a non-core proposal before it is consummated. Which coalitions will form typically depends crucially on the rules of the game. The Perry–Reny implementation is meant to reflect the standard motivation for the core as closely as possible. Hart and Mas-Colell (1996) implement the *consistent value* (Maschler and Owen, 1992) for NTU games, which, for TU games, is equivalent to the Shapley value. Bloch (1996) treats games where, as in the Lucas–Thrall model, the payoff achievable by a group of players may depend on the entire coalition structure of the remaining players. Ray and Vohra (1997; 1999) study coalitional agreements and coalitional bargaining in partition function games. See Bandyopadhyay and Chatterjee (2006) for a survey of coalition formation based on non-cooperative bargaining. See also Myerson (1995), Seidmann and Winter (1998), Mauleon and Vannetelbosch (2004), among others.

Networks and coalition formation

Because networks allow for a detailed specification of interactions between individuals and between coalitions, abstract games over networks have a greater potential to capture the subtleties of bargaining and negotiation than do the abstract coalitional form games of von Neumann–Morgenstern and Gillies and Shapley. A seminal contribution to this line of research is the paper by Myerson (1977). Myerson begins by assuming that the worth of each possible coalition depends on the structure of cooperation between individuals as given by a graph where nodes represent individuals and links between nodes represent interactions between individuals. As in much of the subsequent literature Myerson imposes an allocation rule, a rule specifying how the worth of a coalition is to be shared among its members. The worth of any connected (linked) set of players is divided according to the rule. The specific rule chosen by Myerson is a variant of the Shapley value, now known as the Myerson value. As Myerson shows, this is the only rule satisfying both component efficiency (in sum, the members of each component of the network receive the worth of that component as a coalition) and a fairness property that requires any two players to benefit equally from the formation of a link. Aumann and Myerson (1988) work with extensive form games, where players choose links strategically and allow players to look ahead and to take into account the end effects of their actions. In their model, once a link is formed, it cannot be broken. The equilibrium concept is non-cooperative subgame perfection. Once players have formed links, the payoffs to players are determined by the Myerson value.

Jackson and Wolinsky (1996) also treat link formation between individual players. A network satisfies their pairwise stability condition if no two players could benefit by creating a link between them and no one player could benefit by cutting a link with another player. Based on the Jackson–Wolinsky model, numerous papers have now looked at costs and benefits of link formation between players and equilibrium outcomes; see Dutta, van den Nouweland, and Tijs (1998) for example, and van den Nouweland (2005) for some recent results and a review. Herings, Mauleon and Vannetelbosch (2006) introduce notions of pairwise farsighted stability. Jackson and van den Nouweland (2005) introduce the concept of a strongly stable network. A network is strongly stable if no coalition could benefit by making changes (additions or deletions) to the links of coalition members. As Jackson and van den Nouweland show, the existence of strongly stable networks is equivalent to non-emptiness of the core in a derived cooperative game. See also Jackson and Watts (2002), who use linking networks and stochastic dynamics to study the evolution of networks.

Other recent works addressing questions of coalition formation in networks make assumptions concerning what a coalition believes it can achieve. These contributions include Watts (2001), who assumes that dominance must be direct, in the sense that a coalition will act to change a network from g to g' only if it perceives an immediate gain. In contrast, Page, Wooders and Kamat (2005) consider indirect dominance where a network g dominates another network g' if there is a coalition S that believes it can trigger a series of changes beginning with the network g and ending with the network g' that is preferred by all members of S. Whether dominance is direct or indirect is of crucial importance, as illustrated in Diamantoudi and Xue (2003) and Page and Wooders (2007), among others. Consider, for example, a situation with two jurisdictions, say J_1 and J_2 and seven people. Each person would like to live in the jurisdiction with the fewest residents. With direct dominance, any partition of the people between the two jurisdictions with three people in one jurisdiction and four in the other is stable. In contrast, with indirect dominance, the situation changes; players can be more optimistic. Suppose that initially there are four people in jurisdiction J_1 and three in J_2. Two people in J_1 may move into J_2 in the belief that, since J_2 has become so crowded, three people will leave J_2 and move to J_1, with the result that the two initial movers will be better off.

Using supernetworks, introduced in Page, Wooders and Kamat (2005), where nodes represent networks and directed arcs represent coalitional moves and coalitional preferences, networks can also provide a simple representation of the rules of network formation and hence the rules of coalition formation. Network formation rules play a crucial role in determining coalitional outcomes. To illustrate, in the literature on markets and on cooperative games, it is assumed that coalitions can exclude individuals. It may be, however, that groups (or coalitions) are subject to ‘free entry’ – any group of players can freely join another group without the consent of those being joined. This has long been important in the literature on economies with clubs/local public goods; compare, for example, the models of Konishi, Le Breton and Weber (1998) and Demange (1994) with that of Conley and Wooders (2001). As a special case, networks can also accommodate a systematic analysis of coalition formation and payoff division when there are potential irreversibilities. For example, given the informational environment, it may be that the only coalitions which can form are sub-coalitions of existing coalitions. Or the rules of network formation may not allow cycles.

How to define a coalition

The traditional approach of cooperative game theory models a coalition as an alliance of players who take as given a well-defined set of possible outcomes or payoffs. The alliance, when considering whether to ‘block’ a proposed outcome, is faced with the alternative of standing alone. In reality, however, we observe that individuals belong to multiple, possibly overlapping alliances. This fact has received remarkably little attention in the literature. Some papers in the club literature allow individuals to belong to multiple clubs for the purposes of local public good provision and private good production within each club, including Shubik and Wooders (1982), Ellickson et al. (2001) and Allouch and Wooders (2006). Roughly, if there is only a finite set of sorts of clubs, bounded in size, (Ellickson et al.) or if ‘per capita payoffs’ are bounded (Allouch and Wooders), then in large economies the core and the set of price taking equilibrium outcomes are equivalent. An interesting application of the idea of overlapping coalitions is developed in Conconi and Perroni (2002), who assume that a country can enter into different alliances, where each alliance to which it belongs is concerned with a different issue.

The definition of a coalition also becomes an issue when the total player set is an atomless continuum. There are two approaches. One approach, introduced in Aumann (1964), is to model a coalition as a subset of positive measure. Major theorems using this approach and relating to coalitions demonstrate equivalence of the core and outcomes of price-taking equilibrium of models of economies Another approach is to describe a coalition as a finite set of players, as in Keiding (1976). This has the advantage that individuals may interact with other individuals, and permits matching or marriage models, for example. An obvious difficulty with such an approach is that, at the heart of economics, is the problem of relative scarcities. Think of the diamond–water paradox; even though water is essential for life itself, it is abundant and thus inexpensive, while diamonds are relatively inessential but scarce and thus expensive.

To see the difficulty in retaining relative scarcities while allowing finite coalitions, suppose, for example, that the points in the interval [0,2] represent boys and the points in the interval [3,4] represent girls so that there are ‘twice’ as many boys as girls. Suppose the only effective coalitions consist of either boy, girl pairs (i, j) where $i \in [0,2]$ and $j \in [3,4]$, or singletons – a matching model. Consider the set of coalitions $\{(i,j) : j = 3 + \frac{1}{2}i\}$; this set describes a partition of the total player set and marries each boy to a girl; clearly this partition is not consistent with the relative scarcities given by Lebesgue measure. Indeed, since there are one-to-one mappings of a set of positive measure onto a set of measure zero, it is even possible to have partitions of the total player set into boy–girl pairs and singletons that match each boy to a girl while leaving a set of girls of measure 1 unmatched! A solution to this problem was proposed by Kaneko and Wooders (1986) with the introduction of *measurement-consistent* partitions. A simple formulation of measurement consistency has recently been provided (Allouch, Conley and Wooders, 2006), and we use it here. Define an *index set* for a partition of a continuum of players as one member from each element of the partition. A partition of players into finite coalitions is ‘measurement-consistent’ if every index set for the partition has the same measure. The partition given above is not measurement-consistent while the partition $\{(i,j) : j = 3 + i \quad i \in [0,1] \cup \{\{i\} : i \in (1,2]\}\}$ is measurement-consistent. While in models of exchange economies, the core with finite coalitions (the *f*-core) and the Aumann core yield equivalent outcomes, in the presence of widespread externalities, such as global pollution, the *f*-core coincides with the set of competitive equilibrium prices while the Aumann core may be empty and, even if non-empty, may have an empty intersection with the set of equilibrium outcomes; the concepts of the Aumann core and the *f*-core are distinct with the *f*-core apparently most closely related to the set of competitive equilibrium prices (Kaneko and Wooders, 1986; Hammond, Kaneko and Wooders, 1989; Kaneko and Wooders 1994). Other works using the *f*-core approach include Berliant and Edwards (2004) and Legros and Newman (1996; 2002). These papers illustrate the advantage of the *f*-core approach in that it enables analysis of activities within groups (firms or clubs, or other organizations) that may contain any finite number of individuals but are negligible relative to the entire economy.

An interesting difference between the Aumann-core and the *f*-core is that, while the Aumann-core has been axiomatized by Dubey and Neyman (1984), the authors stress that the axiomatization is completely different than axiomatizations for the core in cooperative games with only a finite number of players. In contrast, Winter and Wooders (1994) provide an axiomatization for the core of a game with finite coalitions that applies whether the player set is finite or an atomless continuum.

Conclusions

This article began with some of the first works on coalitions in the literature of game theory and concluded with recent work on coalitions and networks. It becomes apparent that the concepts of early works underlie much of even the most recent research. We see at least a part of the future of coalition theory in network modelling of socio-economic coalitions and in more behavioural approaches to coalition theory, involving ‘implicit’ and ‘tacit’ coalitions. Language and the ability to communicate well are clearly involved; see multilingualism and references there. Instead of being bound together by commitments and contracts, members of an implicit coalition may be bound together by common language, culture, objectives or by common group memberships and, even though there may be no explicit agreement, members of an implicit coalition might act together, as if they were a coalition. This raises questions of to what extent individuals, who share common group memberships as in Durlauf (2002) for example, are an implicit coalition and whether such individuals have tendencies to form more explicit coalitions. While much has been done on coalitions, there remains much to do.

See Also

- bargaining
- core convergence
- game theory
- multilingualism
- network formation

We are grateful to Harold Kuhn for his generous assistance in tracking down the origins of the concept of the core.

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How to cite this article

Wooders, Myrna and Frank H. Page, Jr. "coalitions." The New Palgrave Dictionary of Economics. Second Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan, 2008. The New Palgrave Dictionary of Economics Online. Palgrave Macmillan. 12 September 2008 <http://www.dictionaryofeconomics.com/article?id=pde2008_C000179> doi:10.1057/9780230226203.0250