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ABSTRACTS OF TALKS

(the information in brackets next to the titles refers to session numbers)

Dual Frames under Constraints [M-10A]

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Given a frame of a finite dimensional Hilbert space, we study the existence of a dual frame that satisfies certain constraints. We explore the conditions under which a dual frame can include imposed directions, and discuss the pros and cons of two methods to recover the desired dual.

Recovery of Signals from Saturated Linear Measurements [M-6A]

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We consider the problem of recovering a signal using a collection of linear sensors which have some specified range. The sensors will output the maximum value whenever they measure something above the range, and the sensors will output the minimum value whenever they measure something below the range. We present this as a non-linear problem about frames for Hilbert spaces, and we will provide some of the foundational mathematical theory. In particular, we prove that the frame algorithm can be adapted to this non-linear setting to provide an algorithm which recovers the signal and converges exponentially fast. This is joint work with Daniel Freeman, Dorsa Ghoreishi, and Brody Johnson.

Explicit Packing Asymptotes on some Minkowski Measurable Sets of Dimension $D \in (0, 1)$ [M-1B]

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With regard to a work in progress, we study the asymptotics of best packing on a class of fractal subsets of the real line satisfying a certain monotonicity assumption. Particularly, we produce explicit first order asymptotics for a collection of Minkowski measurable sets Γ covering every Minkowski dimension $D \in (0, 1)$. After an appropriate normalization by the Minkowski content $M(\Gamma, D)$, the resulting limits possess a desirable universality property with respect to the dimension. To pose our problem, we heavily rely on a characterization of Minkowski measurability due to Lapidus and Pomerance. Our main optimization tool is then a duality theorem from linear programming.

Explicit Exponential Bases on Disconnected Domains [M-14B]

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An exponential basis on a measurable domain of $\mathbb{R}^d$ is a Riesz basis in the form of $\{e^{2\pi i\lambda.x}\}_{\lambda\in\Lambda}$, where Λ is a discrete set of $\mathbb{R}^d$. The problem of proving (or disproving) the existence of such systems on measurable sets is still largely unsolved. In particular, the existence of exponential bases on unbounded domains is proved only in very few special cases. Moreover, for most of the domains for which the existence of exponential bases is proved, no explicit expression of such bases is given.

In my talk, I am providing simple constructions of exponential bases and precise estimates of their frame constants on certain bounded or unbounded measurable domains. Also, I hope to apply these results to the construction of exponential bases on more general domains.

On Landau-Kolmogorov-type Inequalities for Charges and their Applications [C-16B]

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We consider charges ν defined on Lebesgue measurable subsets of an open cone $C \subset \mathbb{R}^d$ that are absolutely continuous with respect to the Lebesgue measure μ. For such charges we obtain sharp Landau – Kolmogorov type inequalities that estimate the $L_\infty(C)$–norm of the Radon – Nikodym derivative $D_\mu \nu$ using the value of some seminorm of the charge ν and the $L_\infty(C)$–norm of the gradient $\nabla D_\mu \nu$ of this derivative. We also solve the problem of approximation of the operator D_μ by bounded ones on the class of charges ν such that the $L_\infty(C)$–norm of the vector-function $\nabla D_\mu \nu$ does not exceed 1, and the problem of optimal recovery of the operator D_μ on this class given the elements known with error.

In the case $C = \mathbb{R}^m_+ \times \mathbb{R}^{d-m}$, $0 \le m \le d$, we obtain inequalities that estimate the $L_\infty(C)$–norm of a mixed derivative of the function $f\colon C \to \mathbb{R}$ using the $L_\infty(C)$–norm of the function and the $L_\infty(C)$–norm of the gradient of the mixed derivative and prove sharpness of the obtained inequalities in the cases $m = 0, 1$. We also solve the problem of approximation of the mixed derivative operator by bounded operators on the class that is defined by restrictions on the $L_\infty(C)$–norm of the gradient of the mixed derivative; and the problem of optimal recovery of the mixed derivative operator on this class with elements known with error.

Taikov-type Inequalities on Equipped Hilbert Spaces and their Applications [C-16B]

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Let H_0 be a Hilbert space over the field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ with the norm $\|\cdot\|_{H_0}$ and the scalar product $(\cdot,\cdot)_{H_0}$, and H_+ be a dense subset of H_0. Let H_+ be a Hilbert space over the field $\mathbb{K}$ with respect to the norm $\|\cdot\|_{H_+}$ and the scalar product $(\cdot,\cdot)_{H_+}$. We assume that $\|u\|_{H_0} \le \|u\|_{H_+}$ for every $u \in H_+$. We then consider equipped chain $H_- \supset H_0 \supset H_+$ of Hilbert spaces, where H_- is the completion of H_0 with respect to the "negative" norm $\|\alpha\|_{H_-} = \sup\left\{\frac{|(\alpha,u)_{H_0}|}{\|u\|_{H_+}} : u \in H_+ \setminus \{\theta\}\right\}$. By construction, H_- is isomorphic to the space of anti-linear bounded functionals in space H_+. One can think of H_+ as the space of "smooth" elements and of H_- as the space of "generalized" elements.

Let $T_0 : H_0 \to H_0$ be a closable operator with domain $\mathcal{D}(T_0) \supset H_+$. Given $\alpha \in H_-$ consider the functional $g_{\alpha,T_0} : H_+ \to \mathbb{K}$, defined by $g_{\alpha,T_0}(\cdot) = (\alpha, T_0(\cdot))_{H_0}$. The *Taikov-type inequality* is the inequality that estimates the value of $g_{\alpha,T_0}(u), u \in H_+$, in terms of H_+-norm of u. In this talk, we give some sufficient conditions that ensure boundedness of g_{α,T_0} and obtain explicit expression for the norm of this functional. We also present various applications of the new Taikov-type inequalities.

Local Approximation of Cross-Boundary Derivatives for the Hermite Interpolation over Clough-Tocher Spherical Triangulations [M-6B]

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We present a local method for solving a Hermite interpolation problem over a Clough-Tocher partition on the unit sphere. A subset of cubic spline coefficients is found by satisfying nodal interpolating conditions. The rest of the coefficients are estimated using a simple, efficient, and accurate approach. We show that this spline converges to the sampled function cubically. We conclude with numerical examples.

Nonlinear Approximation with Subsampled Rank-1 Lattices [M-14A]

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In this paper we approximate high-dimensional functions $f\colon \mathbb{T}^d \to \mathbb{C}$ by sparse trigonometric polynomials based on function evaluations. Recently it was shown that a dimension-incremental sparse Fourier transform (SFT) does not require the signal to be exactly sparse and is applicable in this setting. We combine this approach with subsampling techniques for rank-1 lattices. This way our approach benefits from the underlying structure in the sampling points making fast Fourier algorithms applicable whilst achieving the good sampling complexity of random points (logarithmic oversampling).

In our analysis we show detection guarantees of the frequencies corresponding to the Fourier coefficients of largest magnitude. In numerical experiments we make a comparison to full rank-1 lattices and uniformly random points to confirm our findings.

Near-Optimal Learning [M-14A]

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We study the problem of learning an unknown function f from given data about f. The learning problem is to give an approximation $\hat{f}$ to f that predicts the values of f away from the data. There are numerous settings for this learning problem depending on: (i) the model class assumption, i.e., what additional information we have about f; (ii) how we measure the accuracy of how well $\hat{f}$ predicts f; (iii) what is known about the data and data sites; (iv) whether the data observations are polluted by noise. In the presence of a model class assumption, we use the notion of Chebyshev radius of a set to give a mathematical description of the optimal performance possible, i.e., the smallest possible error of recovery. Under standard model class assumptions, we show that a near-optimal $\hat{f}$ can be found by solving a certain finite-dimensional over-parameterized optimization problem with a penalty term. Here, near-optimal means that the error is bounded by a fixed constant times the optimal error. This explains the advantage of over-parameterization which is commonly used in modern machine learning. The main results of this work prove that over-parameterized learning with an appropriate loss function gives a near optimal approximation $\hat{f}$ of the function f from which the data is collected. Quantitative bounds are given for how much over-parameterization needs to be employed and how the penalization needs to be scaled in order to guarantee a near optimal recovery of f. An extension of these results to the case where the data is polluted by additive deterministic noise is also given.

This is collaborative research with Andrea Bonito, Ronald DeVore, and Guergana Petrova from Texas A&M. It is supported in part by NSF DMS 2038080 and ARO W911NF2010318.

Sparse Recovery and Heat Kernels on Graphs [M-3A]

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Linear combinations of heat kernels give rise to a standard type of reproducing kernel Hilbert space that is popular in harmonic analysis and machine learning. Sparse recovery for the Gaussian case in Euclidean spaces has been examined in earlier work using elements from time-frequency and harmonic analysis. The results presented here focus on the setting of heat kernels on graphs. The key element for sparse recovery, identifying the individual terms in a linear combination of heat kernels, is the construction of a dual certificate, as in the literature on compressed sensing off the grid or superresolution. Instead of techniques from harmonic analysis, bounds on the heat kernel establish the existence of a dual certificate in the context of graphs.

Pointwise Optimal Recovery Method for Twice Differentiable Functions on a Box [M-3B]

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We study the problem of optimal global recovery of the class $W^2(P)$ of functions defined on a d-dimensional parallelepiped P and having a Lipschitz gradient over P (with a uniformly bounded Lipschitz constant). The known data about each function is given by its values and gradients at a fixed rectangular grid in P. We construct a spline method optimal on $W^2(P)$ in the worst-case error sense for recovery of the function value at every point of P (pointwise optimal method). It is optimal for global recovery of $W^2(P)$ in any monotone norm and its worst-case error function over $W^2(P)$ is the tensor sum of d univariate Euler splines.

On Akemann and Weaver Conjecture [M-13A]

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Akemann and Weaver showed Lyapunov-type theorem for rank one positive semidefinite matrices as an extension of Weaver's KS_2 conjecture that was proven by Marcus, Spielman, and Srivastava in their breakthrough solution of the Kadison-Singer problem. They conjectured that a similar result holds for higher rank matrices. We prove the conjecture of Akemann and Weaver by establishing Lyapunov-type theorem for trace class operators. In the process we prove a matrix discrepancy result for sums of hermitian matrices. This extends rank one result of Kyng, Luh, and Song who established an improved bound in Lyapunov-type theorem of Akemann and Weaver.

Universal Lower Bounds on Polarization of Spherical Codes and Designs [M-1B]

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We obtain universal lower bounds for N-point polarization problems for spherical codes and designs of fixed dimension, strength, and cardinality. The universality means that the main parameters of the bounds are independent on the potentials. Our bounds are valid for a large class of potentials that includes absolutely monotone functions of inner products.

We use the rich structure of the binary Golay code to show that the potentials of most of the known sharp codes attain our polarization universal lower bounds and to describe the points of minima. We consider in detail two famous cases, based on the Higman-Sims graph on 100 vertices and the MacLaughlin graph on 275 vertices.

Graph Barron Space and Graph Convolution Neural Networks [M-1A]

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The Barron space has been well accepted as the right space to understand direct and inverse approximation for two-layer neural network in the Euclidean space setting. In this talk, we introduce graph Barron space and study the direct and inverse approximation for two-layer graph convolution neural networks.

Local Regularization via Truncated Toeplitz Operators [M-14B]

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The canonical formulation of an inverse problem is to solve a linear operator equation $Au = f$, where A is a bounded operator with unbounded inverse. As such problems are ill-posed, they require a controlled construction and analysis ("regularization") of approximate solutions to ensure their stability in the presence of noise. In this talk, we revisit the method of *local regularization*, as developed by P.K. Lamm and her collaborators, to solve $Au = f$ with data f and a Volterra convolution operator A on a Lebesgue space. We provide a new perspective on the method, based on joint work with C.D. Brooks and J. Barbara Llanes, through the lens of truncated Toeplitz operators with bounded analytic symbols.

Polynomial Embeddings of Unilateral Weighted Shifts in 2–variable Weighted Shifts [M-14B]

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Given a bounded sequence ω of positive numbers, and its associated unilateral weighted shift W_ω acting on the Hilbert space $\ell^2(\mathbb{Z}_+)$, we consider natural representations of W_ω as a 2–variable weighted shift, acting on the Hilbert space $\ell^2(\mathbb{Z}_+^2)$. Alternatively, we seek to examine the various ways in which the sequence ω can give rise to a 2–variable weight diagram, corresponding to a 2–variable weighted shift. Our best (and more general) embedding arises from looking at two polynomials p and q nonnegative on a closed interval $I \subseteq \mathbb{R}_+$ and the double-indexed moment sequence $\{\int p(r)^k q(r)^\ell d\sigma(r)\}_{k,\ell\in\mathbb{Z}_+}$, where W_ω is assumed to be subnormal with Berger measure σ such that supp $\sigma \subseteq I$; we call such an embedding a (p,q)–embedding of W_ω. We prove that every (p,q)–embedding of a subnormal weighted shift W_ω is (jointly) subnormal, and we explicitly compute its Berger measure.

We apply this result to answer three outstanding questions:

(i) Can the Bergman shift A_2 be embedded in a subnormal 2–variable spherically isometric weighted shift $W_{(\alpha,\beta)}$? If so, what is the Berger measure of $W_{(\alpha,\beta)}$?

(ii) Can a contractive subnormal unilateral weighted shift be always embedded in a spherically isometric 2–variable weighted shift?

(iii) Does there exist a (jointly) hyponormal 2–variable weighted shift $\Theta(W_\omega)$ (where $\Theta(W_\omega)$ denotes the classical embedding of a hyponormal unilateral weighted shift W_ω) such that some integer power of $\Theta(W_\omega)$ is not hyponormal?

As another application, we find an alternative way to compute the Berger measure of the Agler j–th shift A_j ($j \geq 2$). Our research uses techniques from the theory of disintegration of measures, Riesz functionals, and the functional calculus for the columns of the moment matrix associated to a polynomial embedding.

High-Dimensional Approximation, Compositional Sparsity, and Deep Neural Networks [P-9]

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The need to recover or approximate functions of many variables is ubiquitous in numerous application contexts such as machine learning, uncertainty quantification, or data assimilation. In all these scenarios the so-called *Curse of Dimensionality* is an intrinsic obstruction that has been a long standing theme in approximation theory. It roughly expresses an exponential dependence of "recovery cost" on the spatial dimension. It is well-known that being able to avoid the Curse depends on both, the structure of the particular "model class" of functions one wishes to approximate and on the approximation system that is to be used. In this talk we highlight recent results concerning the interplay between these two constituents. For small spatial dimensions approximation complexity is in essence determined by the smoothness of the approximand, e.g. in terms of Sobolev- or Besov-regularity which is effectively exploited by approximation systems that rely on spatial localization. By contrast, in high-dimensions, more global structural sparsity properties determine the ability to avoid the Curse, unless one imposes excessively high smoothness degrees.

Inspired by the highly nonlinear structure of Deep Neural Networks (DNNs) and the Kolmogorov-Arnold Superposition Theorem, we focus in particular on a new notion of "*tamed compositional sparsity*" that leads to new types of model classes for high-dimensional approximation. The relevance of such classes is perhaps best illustrated in the context of *solution manifolds* of parameter-dependent families of partial differential equations (PDEs). Specifically, the framework accommodates "inheritance theorems": compositional sparsity of problem data (like parameter-dependent coefficient fields) are inherited by solutions. In particular, we briefly discuss transport equations. In fact, it is well-known that common model reduction concepts for an effective approximation of corresponding parameter-to-solution maps fail for this type of PDEs. Nevertheless, given compositionally sparse data, corresponding solution manifolds can be shown to belong to compositional approximation classes whose manifold-widths defy the Curse of Dimensionality. Corresponding concrete approximation rates, realized by DNNs, exhibit only a low algebraic dependence on the (large) parametric dimension. We conclude by briefly discussing the bearing of these findings on other problem types and ensuing research directions.

Near Whitney Extensions, Non-rigid Alignment of Point Cloud Data, and Optimal Transport [M-12A]

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The Whitney near extension problem for finite sets in $\mathbb{R}^d$, $d \geq 2$ asks the following: Let $\phi : E \to \mathbb{R}^d$ be a near distortion on a finite set $E \subset \mathbb{R}^d$ with certain geometry. How to decide whether ϕ extends to a smooth, one to one and onto near distortion $\Phi : \mathbb{R}^d \to \mathbb{R}^d$ which agrees with ϕ on E and with Euclidean motions in $\mathbb{R}^d$? In this work (joint with Charles Fefferman), we analyze this problem in detail and show a special case relates to the problem of non-rigid alignment of distinct point cloud data in $\mathbb{R}^d$. The alignment maps (near extensions) are constructed from near distorted diffeomorphisms, for example Slow twists and Slides, which are locally and globally rigid and agree with Euclidean motions in $\mathbb{R}^d$. In many real-world applications, the deformation between two sets of point cloud data is inherently non-rigid. For instance, in medical imaging, the point cloud data could come from the surface of a tissue (e.g., liver), which can undergo large non-rigid deformations. Solving the correspondence between points in the source and target point clouds is closely related to the celebrated optimal transport (OPT) problem. In short, for two empirical distributions and given a transportation cost, the OPT problem seeks the optimal assignment between the samples of the two distributions such that the expected transportation cost between the assigned samples is minimized. In the second part of this talk, we discuss work (joint with Soheil Kolouri, Yikun Bai and Huy Tran) which studies an OTP framework for accelerated non-rigid registration between large scale point clouds using Slow twists and Slides.

Simplicial Patterns in Mildly Repulsive Aggregation Dynamics [M-10B]

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Intriguing emergent behaviours arising from simple pairwise interactions between particles are actively studied in disciplines from biology to game theory. Mathematically, such interactions are described by the aggregation equation, the dynamics of which we study through the lens of a Lyapunov energy functional $\mathcal{E}_W$ on the space of probability measures $\mathcal{P}(\mathbb{R}^n)$. Such energy functionals are parametrized by potentials $W : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, where $W(x, y)$ is the interaction between two particles of unit mass located at points x and y, respectively. Potentials of the form $W_{\alpha,\beta}(x, y) = |x - y|^\alpha/\alpha - |x - y|^\beta/\beta$ for $\alpha > \beta > -n$ are known as attractive-repulsive power-law potentials and, despite their simplicity, generate fascinating dynamics which have been the object of much recent study.

For potentials $W_{\alpha,\beta}$ in the mildly repulsive regime $\alpha > \beta \geq 2$, Lim and McCann proved the existence of a threshold function $\alpha_{\Delta^n}(\beta)$ such that, for all $\alpha > \alpha_{\Delta^n}(\beta)$, $\mathcal{E}_{W_{\alpha,\beta}}$ is uniquely minimized (up to rotation and translation) by the uniform distribution on the vertices of a unit n-simplex. In my talk, I will discuss recent work by myself, Lim, and McCann, wherein we provide stronger structural results on this threshold function. Moreover, I will discuss our upper and lower bounds on this threshold function which are asymptotically sharp as $n \to \infty$, considerations motivating our definition of these bounds, and avenues for future progress.

Meshless Finite Difference Methods and Overlap Splines [P-15]

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Meshless finite difference methods are the finite difference methods on irregular nodes, with stencils obtained by optimizing numerical differentiation using polynomials or radial basis functions. These techniques are inherently meshless and isogeometric, and efficient in applications. I will report on several recent computational results for the elliptic equations on complicated 3D domains and on manifolds, elliptic interface problems, Stokes equation, level set methods on evolving in time surfaces, transport equations and scalar conservation laws, obtained in collaboration with C. Bracco, C. Giannelli, D. Kuzmin, D. T. Oanh, M. Safarpoor, A. Sestini, A. Sokolov, N. M. Tuong, S. Turek and A. Westermann. In addition, I will demonstrate a link to the collocation methods for the spaces of "overlap splines," which promises a progress on the theoretical understanding of these methods.

Applications of Lax-Milgram Theorem to Problems in Frame Theory [M-13B]

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We apply Lax-Milgram theorem to characterize scalable and piecewise scalable frame in finite and infinite-dimensional Hilbert spaces. We also introduce a method for approximating the inverse frame operator using finite-dimensional linear algebra which, to the best of our knowledge, is new in the literature.

Szegő Class and the Scattering Problem [M-6B]

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The probability measure μ on the unit circle $\mathbb{T}$ generates the five-diagonal (CMV) matrix $\mathcal{C}$ that is unitarily equivalent to the multiplication by z in $L^2_\mu(\mathbb{T})$. Let the Lebesgue measure m correspond to $\mathcal{C}_0$. The analysis of polynomials orthogonal with respect to μ shows that the existence of classical wave operators $\lim_{n\to\pm\infty} \mathcal{C}^n\mathcal{C}_0^{-n}$ is equivalent to measure μ being in the Szegő class, i.e., $\int_{\mathbb{T}} \log \mu' dm > -\infty$. In that talk, we will discuss the analogs of that statement for Krein strings and other models. Based on the joint work with R. Bessonov from St. Petersburg State University.

Data Embeddings from Transport Theory [M-12A]

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Applications of Optimal Transport (OT) theory have gained popularity in several fields such as machine learning and signal processing. In this talk, we will address this point by introducing embeddings or transforms based on OT. First, we will introduce the so-called Cumulative Distribution Transform (CDT), its version for signed signals/measures (Signed CDT), and the Linear Optimal Transport Embedding (LOT). Due to the underlying conservation of mass law, these new signal representations have suitable properties for decoding information related to certain signal displacements. We will demonstrate this by describing a Wasserstein-type metric in the embedding space and showing applications in classifying (detecting) signals under random displacements, parameter estimation problems for certain types of generative models, and interpolation. Moreover, we will emphasize that these techniques allow faster computation of the classical Wasserstein between pairs of probability measures/densities. However, even though the balanced mass requirement is crucial in classical OT, it also limits the performance of these transforms/embeddings. Therefore, we will finally move to Optimal Partial Transport (OPT) theory and propose a new embedding that generalizes LOT. The presentation will cover parts of joint works with A. Aldroubi, Y. Bai, S. Kolouri, I. Medri, S. Thareja, and G. Rohde.

Numerical Integration for Subdivision IGA [M-12B]

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Due to built-in refinability, subdivision algorithms are a promising tool for the construction of function spaces in the context of Isogeometric Analysis. In this talk, we briefly summarize the opportunities and challenges of this approach for the surface and volume cases and then focus on numerical quadrature. Concretely, for the assembly of the Galerkin system it is necessary to compute integrals on subdivision surfaces and volumes. Since errors in the integration impair the accuracy of simulation, these integrals should be calculated efficiently and with high precision. We discuss strategies for that purpose in the surface case and present first experimental results.

Planar Splines on a Subdivision with a Single Interior Edge [M-13B]

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We derive a formula for the dimension of the space $S^r_d(\Delta)$ of C^r splines on a polygonal subdivision Δ in the plane that has a single interior edge. The formula is valid for any order of smoothness r and any degree d, and depends only on the number of interior edges and the number of distinct slopes at the two interior vertices. Our formula extends work of Tohăneanu, Mináč, and Sorokina in the case when there is a single interior edge and three distinct slopes at the two interior vertices. In particular, we show that Schumaker's lower bound gives the correct dimension of the spline space for $d \geq 2r+1$ and we analyze supersmoothness across the interior edge. We use techniques from commutative algebra for these results.

Iteratively Consistent One-Bit Phase Retrieval [M-16A]

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Phase retrieval is the task of identifying an unknown vector in a Hilbert space from measurements that consist of taking the norm of linear maps applied to the unknown vector. We are specifically concerned with *one-bit* phase retrieval, where each such measurement is coarsely quantized to a two-element alphabet, e.g. $\{0, 1\}$. We introduce an iterative reconstruction algorithm for one-bit phase retrieval, `ICQPhase`, that seeks to enforce consistency with each quantized measurement in turn. We demonstrate experimentally that `ICQPhase` achieves mean squared error of order $1/m^2$ in a variety of settings where m is the number of quantized phase retrieval measurements, and provide a theoretical analysis that confirms this in the case of two-dimensional real vectors.

Spectral Universality of Regularized Linear Regression with Nearly Deterministic Sensing Matrices [M-16A]

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Spectral universality refers to the empirical observation that asymptotic properties of a high-dimensional stochastic system driven by a structured random matrix are often determined only by the spectrum (or singular values) of the underlying matrix - the singular vectors are irrelevant provided they are sufficiently "generic". Consequently, the properties of the underlying system can be accurately predicted by analyzing the system under the mathematically convenient assumption that the singular vectors as uniformly random (or Haar distributed) orthogonal matrices. This general phenomenon has been observed in numerous contexts, including statistical physics, communication systems, signal processing, statistics, and randomized numerical linear algebra. We study this universality phenomenon in the context of high-dimensional linear regression, where the goal is to estimate an unknown signal vector from noisy linear measurements specified using a sensing matrix. We prove a spectral universality principle for the performance of convex regularized least squares (RLS) estimators for this problem. Our contributions are two-fold: (1) We introduce a notion of a universality class for sensing matrices, defined through nearly deterministic conditions that fix the spectrum of the matrix and formalize the heuristic notion of generic singular vectors; (2) We show that for all sensing matrices in the same universality class, the dynamics of the proximal gradient algorithm for the regression problem, and the performance of RLS estimators themselves (under additional strong convexity conditions) are asymptotically identical. In addition to including i.i.d. Gaussian and rotational invariant matrices as special cases, our universality class also contains highly structured, strongly dependent, and even nearly deterministic matrices. Examples include randomly signed incoherent tight frames and randomly subsampled Hadamard transforms. Due to this universality result, the performance of RLS estimators on many structured sensing matrices with limited randomness can be characterized using the rotationally invariant sensing model with uniformly random (or Haar distributed) singular vectors as an equivalent yet mathematically tractable surrogate.

Metric Approximation of Set-Valued Functions [P-2]

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This talk is a review of ideas in the approximation of Set-Valued functions, mapping a closed interval to the space of all non-empty, compact subsets of $\mathbb{R}^d$. These ideas have evolved in a series of papers, written jointly, first with Elza Farkhi and Alona Mokhov, and later also with Elena Berdysheva. We were inspired by the first paper on the approximation of these set-valued functions, authored by Z. Artstein in the late eighties. Most papers on that subject have studied the class of set-valued functions with values restricted to convex sets, based on operations such as Minkowski sum of sets and the Auman integral. The new tool in Artstein's paper is the "metric average", with which a "piecewise linear" interpolant is constructed. In our papers we introduced several more metric tools, such as metric chains, metric selections, weighted metric integral, with which we adapt to set-valued functions classical, sampled-based, linear approximation operators, Fourier approximants, and several integral operators. The metric adaptation yields approximants with similar approximation order as in the real-valued case, for continuous SVFs, and for SVFs of bounded variation.

Geometric Method for Manifold Approximation for High-Dimensional Noisy Scattered Data [M-14A]

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In this talk, we present a method for denoising and approximating a low-dimensional manifold in a high-dimensional space. Given a noisy point-cloud situated near a low dimensional manifold, the proposed solution distributes points near the unknown manifold in a noise-free and quasi-uniformly manner, by leveraging a generalization of the robust L1-median to higher dimensions. We prove that the non-convex computational method converges to a local stationary solution with a bounded linear rate of convergence if the starting point is close enough to the local minimum. Next, using the proposed framework we address the problem of nonlinear regression on Riemannian manifolds. We demonstrate the effectiveness of our framework by considering different manifold topologies with various amounts of noise, including a case of a manifold of different dimensions at different locations. Joint work with David Levin.

Learning Interaction Variables and Kernels from Observations of Agent-Based Systems [M-1B]

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Dynamical systems across many disciplines are modeled as interacting particles or agents, with interaction rules that depend on a very small number of variables (e.g. pairwise distances, pairwise differences of phases, etc...), functions of the state of pairs of agents. Yet, these interaction rules can generate self-organized dynamics, with complex emergent behaviors (clustering, flocking, swarming, etc.). We propose a learning technique that, given observations of states and velocities along trajectories of the agents, yields both the variables upon which the interaction kernel depends and the interaction kernel itself, in a nonparametric fashion. This yields an effective dimension reduction which avoids the curse of dimensionality from high-dimensional observation data (states and velocities of all the agents).

We demonstrate the learning capability of our method to a variety of first-order interacting systems.

On Learning with Bounded Loss Functions [M-8A]

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In machine learning, bounded loss functions have been more and more frequently used owing to their robustness to outliers and heavy-tailed noise. However, the understanding of bounded loss functions, especially from a theoretical viewpoint, is still limited due to their nonconvexity. In this talk, I will report some of our recent efforts made in this regard. First, I will show that in the context of empirical risk minimization, bounded loss functions can be interpreted from a minimum distance estimation viewpoint. Second, results on the prediction ability of estimators resulting from bounded loss functions will also be provided and discussed.

Doubly Transitive Equiangular Tight Frames that Contain Regular Simplices [M-3B]

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An equiangular tight frame (ETF) is a finite sequence of equal norm vectors in a Hilbert space of lesser dimension that achieves equality in the Welch bound, and so has minimal coherence. The binder of an ETF is the set of all subsets of its indices whose corresponding vectors form a regular simplex. An ETF achieves equality in Donoho and Elad's spark bound if and only if its binder is nonempty. When this occurs, its binder corresponds to the set of all linearly dependent subsets of it of minimal size. Moreover, if members of the binder form a balanced incomplete block design (BIBD) then its incidence matrix can be phased to produce a sparse representation of its dual (Naimark complement). A few infinite families of ETFs are known to have this remarkable property. We relate this property to the recently introduced concept of a doubly transitive equiangular tight frame (DTETF). In particular, we show that the binder of any DTETF is either empty or forms a BIBD. We then apply this general theory to certain known infinite families of DTETFs.

Numerical Convolution of Probability Densities using Knot Addition Method for Non-uniform Polynomial Interpolation [M-8A]

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Exact convolution of non-uniform piecewise polynomial interpolant approximations of Inverse Gaussian densities is compared with conventional uniform pitch discrete convolution techniques. Interpolant knots are chosen by successively adding knots at the points of maximum pointwise error. For moderate μ around 1 discrete convolution is shown to outperform nonuniform interpolation but as μ goes both large and small, nonuniform interpolation gives better convergence despite worse asymptotic complexity.

On the Optimal Recovery of Graph Signals [M-10A]

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Learning a smooth graph signal from partially observed data is a well-studied task in graph-based machine learning. We consider this task from the perspective of optimal recovery, a mathematical framework for learning a function from observational data that adopts a worst-case perspective tied to model assumptions on the function to be learned. Earlier work in the optimal recovery literature has shown that minimizing a regularized objective produces optimal solutions for a general class of problems, but did not fully identify the regularization parameter. Our main contribution provides a way to compute regularization parameters that are optimal or near-optimal (depending on the setting), specifically for graph signal processing problems. Our results offer a new interpretation for classical optimization techniques in graph-based learning and also come with new insights for hyperparameter selection.

Stability Measurements for Phase Retrieval [M-6A]

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A frame $(x_j)_{j\in J}$ for a Hilbert space H allows for a linear and stable reconstruction of any vector $x \in H$ from the linear measurements $(\langle x, x_j\rangle)_{j\in J}$. However, there are many applications such as X-ray crystalography and coherent diffraction imaging where one is only able to obtain the magnitudes of some linear measurements. In these contexts, one must do phase retrieval to reconstruct a vector. We say that the frame $(x_j)_{j\in J}$ does phase retrieval if it is possible to reconstruct any vector $x \in H$ (up to a unimodular scalar) from the values $(|\langle x, x_j\rangle|)_{j\in J}$, and we say that the frame does C-stable phase retrieval if this reconstruction map is C-Lipschitz. That is, $(x_j)_{j\in J}$ does C-stable phase retrieval if

$$\min_{|\lambda|=1} \|x - \lambda y\|_H \le C\big\|\big(|\langle x, x_j\rangle| - |\langle y, x_j\rangle|\big)_{j\in J}\big\|_{\ell_2(J)} \qquad \text{for all } x, y \in H. \tag{1}$$

One may naturally consider different measurements of stability of phase retrieval by considering different norms in (1). We will present multiple situations where the stability of phase retrieval with respect to different Banach space norms may have useful applications to phase retrieval for frames of Hilbert spaces. This talk covers joint work with Dorsa Ghoreishi and joint work with Ben Pineau, Timur Oikhberg, and Mitchell Taylor.

Nonparametric Bivariate Density Estimation for Missing Censored Lifetimes [M-8B]

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Estimation of the joint density of two censored lifetimes is a classical problem in survival analysis, but only recently the theory and methodology of efficient nonparametric estimation have been developed. A familiar complication in survival analysis is that in real data censored lifetimes and indicators of censoring may be missing. For the model of missing completely at random, an efficient bivariate density estimator is proposed, and a practical example is presented.

Neural Control of Parametric Solutions for High-dimensional Evolution PDEs [M-16A]

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We develop a novel computational framework to approximate solution operators of evolution partial differential equations (PDEs). By employing a general nonlinear reduced-order model, such as a deep neural network, to approximate the solution of a given PDE, we realize that the evolution of the model parameter is a control problem in the parameter space. Based on this observation, we propose to approximate the solution operator of the PDE by learning the control vector field in the parameter space. From any initial value, this control field can steer the parameter to generate a trajectory such that the corresponding reduced-order model solves the PDE. This allows for substantially reduced computational cost to solve the evolution PDE with arbitrary initial conditions. We also develop comprehensive error analysis for the proposed method when solving a large class of semilinear parabolic PDEs. Numerical experiments on different high-dimensional evolution PDEs with various initial conditions demonstrate the promising results of the proposed method.

Stable Phase Retrieval under Purturbations [M-6A]

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Frames, like orthonormal bases, give a continuous, linear, and stable reconstruction formula for vectors in a Hilbert space. However, frames allow for redundancy, and this makes frames much more adaptable for theory and applications. Phase retrieval is an application of frame theory which is prominently used in X-ray crystallography and coherent diffraction imaging where only the intensity of each linear measurement of a signal is available and the phase information is lost. The goal of phase retrieval is to recover these lost phases up to some universal global factor. Notably, phase retrieval requires the redundancy of a frame, and is not possible with a basis. When the recovery of any vector form the magnitude of frame coefficients is C-Lipschitz, we say that the frame does C-stable phase retrieval. It is known that stable phase retrieval holds under small perturbation but the stability bound gets worst. We will provide new quantitative bounds on how the stability constant for phase retrieval is affected by a small perturbation of the frame vectors.

Lower Estimate on Square Function of an Indicator Set [C-16B]

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It has been a long standing fact that the distribution of the continuous Hilbert transform of an indicator set only depends on the measure of the set, both when restricted inside and outside of it. However, the counterpart results in the discrete case have not been studied at all.

With that motivation, we are able to obtain a lower estimate on discrete square functions of indicator sets, that only depend on the measure of the the set itself. The easiest setting for this study is using the language of discrete martingales. We extend this to wavelet square functions, address the extra complications that arise in this setting and point out some additional interesting observations on the distribution of the square function itself. Let Sf be a discrete wavelet square function. One can use Haar wavelets as a toy example. Then, there is a constant $\eta > 0$ so that for all sets $V \subset \mathbb{R}$ of finite measure, we have $\int_V (S\mathbf{1}_V)^2 \, dx \geq \eta |V|$. Finally, we discuss some related open questions.

Unraveling the Genetic Basis of Cancer Risk: Leveraging Statistical and Deep Transfer Learning Approaches [M-8A]

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Genetic studies aim to identify causal genetic variants and genes associated with disease risk. One effective approach is to use statistical methods like transcriptome-wide association studies (TWAS) to uncover potential disease susceptibility genes. Another approach involves using deep convolutional neural networks and attention mechanisms such as Enformer, to annotate regulatory activities of genetic variants by learning from extensive epigenetic data in different human tissues and cells. However, the current methods lack tailored learning of transcription-occupancy and other epigenetic signals in disease-associated target tissues. In my talk, I will present our recent work and discuss potential opportunities to integrate multi-omics data using these approaches to enhance our understanding of the genetic basis underlying cancer risk.

Wasserstein Isometric Mapping [M-13A]

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We will discuss an algorithm called Wasserstein Isometric Mapping (Wassmap), a nonlinear dimensionality reduction technique that provides solutions to some drawbacks in existing global nonlinear dimensionality reduction algorithms in imaging applications. Wassmap represents images via probability measures in Wasserstein space, then uses pairwise Wasserstein distances between the associated measures to produce a low-dimensional, approximately isometric embedding. We show that the algorithm is able to exactly recover parameters of some image manifolds including those generated by translations or dilations of a fixed generating measure. We will discuss computational speedups to the algorithm such as use of linearized optimal transport or the Nyström method. Testing of the proposed algorithms on various image data manifolds show that Wassmap yields good embeddings compared with other global and local techniques. This is joint work with Nick Henscheid, Shujie Kang, Alex Cloninger, Caroline Moosmüller, and Varun Khurana.

Ridgeless Interpolation with Shallow Univariate ReLU Networks [M-16A]

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In this talk I will present a complete answer to the question of how neural networks use training data to make predictions on unseen inputs in a very simple setting. Namely, for a fixed dataset $D = (x_i, y_i), i = 1, ..., N$ with x_i and y_i being scalars, consider the space of all one layer ReLU networks of arbitrary width that exactly fit this data and, among all such interpolants, achieve the minimal possible ℓ_2-norm on the neuron weights. Intuitively, this is the space of "ridgeless ReLU interpolants" in that sense that it consists of ReLU networks that minimize the mean squared error over D plus an infinitesimal ℓ_2- regularization on the neuron weights. I will give a complete characterization of how such ridgeless ReLU interpolants can make predictions on intervals (x_i, x_{i+1}) between consecutive datapoints. I will then explain how to use this characterization to obtain, uniformly over the infinite collection of ridgeless ReLU interpolants of a given dataset D, tight generalization bounds under the assumption $y_i = f(x_i)$ with f a Lipschitz function.

Inverse Potential Problems in Divergence Form for Measures Supported in the Plane [M-8B]

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We consider inverse problems for the Poisson equation with source term the divergence of an $\mathbb{R}^3$-valued measure with planar support; that is, the potential Φ satisfies

$$\Delta\Phi = \nabla \cdot \boldsymbol{\mu},$$

and $\boldsymbol{\mu}$ is to be reconstructed knowing (a component of) the field $\nabla\Phi$ on a set disjoint from the support of $\boldsymbol{\mu}$. Such problems arise in several electro-magnetic contexts in the quasi-static regime, for instance when recovering a remanent magnetization from measurements of its magnetic field. We develop methods for recovering $\boldsymbol{\mu}$ based on total variation regularization and provide sufficient conditions for the unique recovery of $\boldsymbol{\mu}$ when the magnetization has a support which is sparse in the sense that it is purely 1-unrectifiable.

A Study of Two Approaches for Polylogarithm Functions: Generalized Ramanujan Integrals and Special Functions [M-13B]

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Inspired by Ramanujan's integral $\int_{v=0}^{1} \frac{\log u}{v} dv$, where $v = u^n - u^{n-1}$, we study its generalization $\int_{v=0}^{1} \log g(t) \frac{dv(t)}{v(t)}$, where $v(t) = \frac{g(t)^n}{g(t^{-1})}$, $n \geq 0$, and its extension $\int_{u=1}^{z} \frac{\log u}{v} dv$, where $u = g(t)$ and $v(t) = \frac{g(t)^n}{g(t^{-1})}$, as well as their close connection with polylogarithm function $Li_2(z)$. By means of the above study, we may construct some identities and evaluate some values of $Li_2(z)$. Another approach is shown in a paper just published by Paul Butzer, Clemens Markett, and the speaker on complex order polylogarithmic functions using Euler fractions, Stirling, Bernoulli, and Euler functions.

A Cutting Plane Approach to Constrained Spline Approximation [M-8B]

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We consider a family of constrained spline approximation problems from the perspective of semi-infinite convex optimization. We recall the cutting plane method for domain refinement and present a branch-and-bound-like cutting plane *oracle*. We also present applications that motivated this pursuit.

A Spline Method for G^r Smooth Interpolating Curves and Surfaces of Arbitrary Triangle Mesh [M-6B]

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In this presentation, a new method for interpolating curves and surfaces using multivariable splines will be demonstrated. The approach includes an exploration of G^r smoothing surfaces and an algorithm for creating smoothing curves and surfaces. These curves and surfaces are constructed using piecewise polynomials without subdividing the mesh. The algorithm can also produce curves and surfaces with any degree of smoothness, provided that the appropriate degree of polynomials for interpolation is supplied.

Matrix Completion with Cross-Concentrated Sampling [M-13A]

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Uniform sampling has been extensively studied in the matrix completion literature, while CUR sampling approximates a low-rank matrix through row and column samples. However, both sampling models lack flexibility for real-world applications in various circumstances. In this presentation, I will discuss my recent work, in which we introduce a novel and easy-to-implement sampling strategy, Cross-Concentrated Sampling (CCS). CCS combines the strengths of uniform sampling and CUR sampling, providing extra flexibility that can potentially save sampling costs in applications. Additionally, we provide a sufficient condition for CCS-based matrix completion. We also propose a highly efficient non-convex algorithm, called Iterative CUR Completion (ICURC), for the CCS model. Numerical experiments demonstrate the empirical advantages of CCS and ICURC over uniform sampling and its baseline algorithms on both synthetic and real-world datasets.

Sparse Spectral Methods for Solving High-Dimensional and Multiscale Elliptic PDEs [M-13A]

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In his monograph “Chebyshev and Fourier Spectral Methods”, John Boyd claimed that, regarding Fourier spectral methods for solving differential equations, [t]he virtues of the Fast Fourier Transform will continue to improve as the relentless march to larger and larger [bandwidths] continues [pg. 194]. This talk will discuss attempts to further the virtue of the Fast Fourier Transform (FFT) as not only bandwidth is pushed to its limits, but also the dimension of the problem. Instead of using the traditional FFT however, we make a key substitution from the sublinear-time compressive sensing literature: a high-dimensional, sparse Fourier transform (SFT) paired with randomized rank-1 lattice methods. The resulting sparse spectral method rapidly and automatically determines a set of Fourier basis functions whose span is guaranteed to contain an accurate approximation of the solution of a given elliptic PDE. This much smaller, near-optimal Fourier basis is then used to efficiently solve the given PDE in a runtime which only depends on the PDEs data/solution compressibility and ellipticity properties, while breaking the curse of dimensionality and relieving linear dependence on any multiscale structure in the original problem. Theoretical performance of the method is established with convergence analysis in the Sobolev norm for a general class of nonconstant diffusion equations, as well as pointers to technical extensions of the convergence analysis to more general advection-diffusion-reaction equations. Numerical experiments demonstrate good empirical performance on several multiscale and high-dimensional example problems, further showcasing the promise of the proposed methods in practice.

Optimal Line Packings and Strongly Regular Graphs [M-3B]

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An equiangular tight frame is a type of optimal packing of lines. In the real case, after selecting a unit vector in each line, they naturally give rise to graphs. If the vectors are chosen carefully, and the resulting graph is regular, then it is actually strongly regular. We will give a few examples where this choice can be made, giving rise to new infinite families of strongly regular graphs. We also show how this discrete problem can be relaxed to a continuous minimization problem and obtain yet another new strongly regular graph.

Stability of Iterated Filter Banks [M-13A]

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This talk will outline recent results of the authors on the frame properties of finitely and infinitely iterated dyadic filter banks. The main result describes a sufficient condition under which the infinitely iterated dyadic filter bank associated with a specific class of finitely supported filters is stable.

Weak Form Approach to Identifying Differential Equation [M-1A]

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We consider identifying differential equation using numerical techniques (IDENT) from one set of noisy observation. We assume that the governing PDE can be expressed as a linear combination of different linear and nonlinear differential terms. In this talk, extending from IDENT and robust IDENT, we will discuss using weak form for differential equation identification. We consider both ODE and PDE models. Numerical results show robustness against higher level of noise and higher order derivative in underlying equation.

Interpolation and Approximation on Sparse Grids [M-8B]

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Applied and computational mathematics problems, such as numerical approximation, differentiation, integration, and optimization, often involve the values and derivatives of underlying functions at points on a regular grid. For high-dimensional problems with many grid points, the size of these data sets is easily beyond the performance and storage limitations of computers. To circumvent these limitations, it is useful to develop strategies which reduce the size of the data sets without affecting accuracy too much. In this talk, we present algorithms for the approximation of functions f on $\Omega := [0,1]^d \subset \mathbb{R}^d$ for dimension $d > 1$. Rather than choosing the usual *uniform (regular) grids* with n^d grid points, we choose certain *sparse grids* which have far fewer grid points, on the order of $\mathcal{O}(n\,(\log n)^{d-1})$. On the other hand, while the approximation of smooth functions on full grids is of the order $\mathcal{O}(h^{2m})$ for $h = 1/(n-1)$, the rate of approximation on sparse grids is of the order $\mathcal{O}(h^{2m}\,(\log h^{-1})^{d-1})$. Hence, while the number of grid points is dramatically less for sparse grids, the rate of approximation only slightly lags that of the full grid. In this talk, we describe the *combination techniques* using splines and quasi-interpolants for the approximation of smooth surfaces, and their application to optimization.

PCA is not Dead: Vectorized Persistent Homology and Flag Medians [M-16A]

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In this talk two methods will be presented which both make use of principal component analysis. The first involves applying persistent homology to atmospheric data and then vectorizing the output. Surprisingly, principal component analysis (PCA) enables a very low dimensional representation of this data that enables clustering. A low-level explanation of persistent homology will be presented for those not familiar with topological data analysis. The second is a novel method inspired by the Weiszfeld algorithm to find prototypes of subspaces of possibly different dimensions which are robust to outliers. This algorithm involves solving a particular series of weighted PCA problems. Some applications to computer vision will be presented.

Optimal Partial Transport in Machine Learning [M-12A]

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Optimal transport (OT) has emerged as a powerful tool for comparing probability measures, finding widespread use in data science, statistics, machine learning, signal processing, and computer vision. Despite its effectiveness in measuring dissimilarities between probability measures, OT theory has limitations in its ability to compare general multi-dimensional signals with possibly negative values. This talk focuses on addressing some of these limitations in two parts.

During the first part of the talk, we will review the optimal partial transport (OPT) problem, which enables the comparison of positive measures with varying total mass. We will present our recently proposed algorithm for calculating OPT between positive measures on the real line, and then utilize this efficient algorithm to construct the sliced optimal partial transport metric for positive measures defined in the d-dimensional Hilbert space. Additionally, we will demonstrate the application of this metric in various challenging point-set registration problems.

In the second part of the talk, we will review the concept of transport-L_p (TL_p) distances, which allow for the comparison of general multi-dimensional signals. We will then extend TL_p distances and provide their sliced and partial extensions as powerful and scalable computational tools for comparing general multi-dimensional signals. We will demonstrate several applications of the proposed distances in machine learning.

Norm Retrieval from Few Spatiotemporal Samples [M-6A]

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In this work, we use phaseless measurements collected via dynamical sampling to determine the norm of a vector. We specifically examine the cases where we have insufficient information to solve the stronger phase retrieval problem.

Multivariate Splines and Their Applications [P-5]

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I will first explain the concept of multivariate splines and explain the significance of Larry Schumaker's contributions to the approximation theory of multivariate splines. Then I will explain the spline collocation method to numerical solutions of partial differential equations. Next, I will explain how to use trivariate splines for smooth surface construction. Several smooth surfaces will be shown. Finally, I will explain how to use multivariate splines for smoothing KB-splines which are based on Kolmogorov representation theorem, and demonstrate that the smoothed version of KB-splines can approximate high dimensional functions very well.

Point Source Equilibrium Problems with Connections to Weighted Quadrature Domains [M-10B]

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We investigate equilibrium measures in the plane in the presence of point charges added to an external field. After some general comments we focus on the external field $Q(z) = |z|^{2p}$. The support of the equilibrium measure after activating finitely many point charges is described via quadrature domains with respect to weighted area measures $|z^{2p}|dA_z$ and complex boundary measures $|z|^{-2p}dz$.

Transport Subspace Models and Invariance Encoding [M-12A]

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Transport-based metrics and related embeddings have recently been used to model data classes where nonlinear structures or variations are present. We will describe several transport transforms and their mathematical properties related to convexification under various algebraic generative modeling assumptions, enabling efficient modeling of data classes as subspaces in the transform domain. Such modeling also gives rise to simple machine learning algorithms with the ability to incorporate meaningful invariances, which are robust to out-of-distribution samples (generalizability). We will show applications in time series classification and face recognition under varying illumination conditions. This talk is based on joint work with Akram Aldroubi, Yan Zhuang, Hasnat Rubaiyat, Gustavo Rohde, M Shifat Rabbi, and Xuwang Yin.

Optimal Recovery from Inaccurate Data in Hilbert Spaces: Regularize, but what of the Parameter? [M-16A]

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In Optimal Recovery, the task of learning a function (or a quantity depending on the function) from observational data is tackled by adopting a worst-case perspective tied to an explicit model assumption made on the functions to be learned. Working in the framework of Hilbert spaces, we considers a model assumption based on approximability and the observational inaccuracies modeled via additive errors bounded in ℓ_2. Earlier works have demonstrated that regularization provide algorithms that are optimal in this situation, but did not fully identify the desired hyperparameter. In this talk, I will provide a way to compute the optimal regularization parameter in both a local scenario and a global scenario. We also mention some partial results with assuming observational errors bounded in ℓ_1.

A Two-stage Iterative Method for Sparse Generalized Eigenvalue Problem [M-1A]

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We consider the sparse generalized eigenvalue problem, which aims to approximate the leading generalized eigenvector using a sparse vector. Several well-known statistical learning methods can be formulated as this problem. Existing algorithms for solving it often converge to a local optimum, compromising their accuracy. To address this issue, we propose a two-stage iterative method that employs a local algorithm in the first stage and improves its output in the second stage. Our numerical experiments conducted under multiple settings demonstrate that the proposed algorithm outperforms existing methods in terms of accuracy while maintaining comparable computation time.

Advances in Phaseless Sampling of the Short-time Fourier Transform [M-6A]

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Short-time Fourier transform (STFT) phase retrieval refers to the reconstruction of a function $f \in L^2(\mathbb{R})$ from phaseless samples of the form

$$|V_g f(\Lambda)| := \{|V_g f(\lambda)| : \lambda \in \Lambda\},$$

where $V_g f$ denotes the STFT of f with respect to a window function $g \in L^2(\mathbb{R})$, and $\Lambda \subseteq \mathbb{R}^2$ is a set of sampling locations. We present recent advances in STFT phase retrieval and focus on the question for which window functions g and which sets Λ is every f uniquely determined (up to a global phase) by the samples $|V_g f(\Lambda)|$. It turns out, that the phaseless sampling problem differs from ordinary sampling in a rather fundamental way: if $\Lambda = A\mathbb{Z}^2$ is a lattice then uniqueness is unachievable, independent of the choice of the window function and the density of the lattice. On the basis of this discretization barrier, we present possible ways to overcome it. We show that a restriction of the function class from $L^2(\mathbb{R})$ to certain subspaces of $L^2(\mathbb{R})$ yields uniqueness of the problem if one samples on sufficiently dense lattices. The proofs are based on completeness properties of systems of translates and approximation by shift-invariant spaces. Finally, we highlight that without any restriction of the function class, a discretization is still possible: a multi-window system with 4 window functions yields lattice uniqueness in $L^2(\mathbb{R})$. This result constitutes the first general uniqueness result for the STFT phase retrieval problem. This is joint work with Philipp Grohs.

Generalized Nash Equilibria and their Existence [M-10B]

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https://tlim0213.github.io/

I will go over the classical concept of Nash Equilibrium and its existence result in this talk. Then I talk about how it could be extended so that each player's strategy profile is described by probability measures on a compact convex domain. I demonstrate the existence of corresponding Nash equilibria.

Low-energy Points on the Sphere and the Real Projective Plane [M-3B]

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In the last decades, the problem of evenly distributing points on manifolds like spheres and projective spaces has attracted the attention of the mathematical community due to its theoretical interest and its numerous practical applications, constituting nowadays a very active field of research.

In this talk I will tackle the problem of distributing points on the usual two-dimensional sphere and on the real projective plane. More precisely, I will present a generalization of a family of points on $\mathbb{S}^2$, the Diamond ensemble, containing collections of N points on $\mathbb{S}^2$ with very low logarithmic energy for all $N \in \mathbb{N}$. In addition, I will also show how the ideas for distributing points on $\mathbb{S}^2$ can be extended to the real projective plane, thereby obtaining lower and upper bounds for the Green and logarithmic energies which constitute the best results in that regard thus far.

A Lifted ℓ_1 Framework for Sparse Recovery [M-1A]

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Motivated by re-weighted ℓ_1 approaches for sparse recovery, we propose a lifted ℓ_1 (LL1) regularization that can be generalize to several popular regularizations in the literature. During the course of reformulating the existing methods into our framework, we discover two types of lifting functions that can guarantee that the proposed approach is equivalent to the ℓ_0 minimization. Computationally, we design an efficient algorithm via the alternating direction method of multiplier (ADMM) and establish the convergence for an unconstrained formulation. Experimental results are presented to demonstrate how this generalization improves sparse recovery over the state-of-the-art.

Simplex Spline Bases on Triangular and Simplicial Partitions [P-4]

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In the first part of the talk we give a review of simplex splines and their use as a basis for spaces of piecewise polynomials.

In the second part we consider the Alfeld split which generalizes the CloughTocher split of a triangle. To describe it, let $T_s := \langle\{p_1, p_2, ..., p_{s+1}\}\rangle$ be a simplex in $\mathbb{R}^s$ for some $s \geq 2$. Using the barycenter p_T of T_s we can split T_s into $s+1$ subsimplices

$$T_{s,j} := \langle\{p_1, p_2, ..., p_{s+1}, p_T\} \setminus \{p_j\}\rangle, \quad j = 1..., s+1.$$

On T_s we derive a simplex spline basis for the space

$$S^1_{d,s} := \{f \in C^1(T_s) : f_{|T_{s,j}} \in P^s_d,\ j = 1..., s+1\},$$

where $d = 2s - 1$, and P^s_d is the space of polynomials of total degree $\leq d$ in s variables. We also show a Marsden like identity for $s \leq 20$.

Learning Interaction Laws in Particle- and Agent-based Systems [M-10A]

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We consider systems of interacting agents or particles, which are ubiquitous in science, from modeling of particles in Physics to prey-predator and colony models in Biology, to opinion dynamics in social sciences. While these systems may exhibit complex behaviors, oftentimes the laws of interactions between the agents are quite simple, for example they depend only on pairwise interactions, and perhaps even only on pairwise distance in each such interaction. We consider the following inference problem for a system of interacting particles or agents: given only observed trajectories of the agents in the system, can we learn what the laws of interactions are? We would like to do this without assuming any particular form for the interaction laws, i.e. they might be any function of pairwise distances. We discuss when this problem is well-posed, we construct estimators for the interaction kernels with provably good statistically and computational properties, and discuss extensions to second-order systems, more general interaction kernels, and stochastic systems. We measure empirically the performance of our techniques on various examples, that include extensions to agent systems with different types of agents, second-order systems, families of systems with parametric interaction kernels, and settings where the interaction kernels depend on unknown variables. We also conduct numerical experiments to test the large time behavior of these systems, especially in the cases where they exhibit emergent behavior.

Tchebycheffian Splines and Isogeometric Methods [P-7]

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Tchebycheffian splines are smooth piecewise functions whose pieces are drawn from (possibly different) Tchebycheff spaces, a natural generalization of algebraic polynomial spaces. They enjoy most of the properties known in the polynomial spline case. In particular, under suitable assumptions, Tchebycheffian splines admit a representation in terms of basis functions, called Tchebycheffian B-splines (TB-splines), completely analogous to polynomial B-splines. A particularly interesting subclass consists of Tchebycheffian splines with pieces belonging to null-spaces of constant-coefficient linear differential operators. They grant the freedom of combining polynomials with exponential and trigonometric functions with any number of individual shape parameters. Moreover, they have been recently equipped with efficient evaluation and manipulation procedures.

Isogeometric analysis is a methodology for the analysis of problems governed by partial differential equations. It aims to simplify the interoperability between geometric modeling and numerical simulation by constructing a fully integrated framework for computer-aided design and finite element analysis.

In this talk we review the main properties of TB-splines with pieces belonging to null-spaces of constant-coefficient linear differential operators and we analyze the differences and the similarities with the polynomial case. We discuss the use of TB-splines as a substitute for standard polynomial B-splines and rational NURBS in isogeometric methods and we show that they offer a wide and robust environment for the isogeometric paradigm beyond the limits of the rational NURBS model.

C^2 Cubic Spline Quasi-Interpolants on Arbitrary Triangulations [M-12B]

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Splines on triangulations have widespread applications in many areas, ranging from finite element analysis and physics/engineering applications to computer graphics and entertainment industry. Highly smooth spline spaces are often preferred.

When dealing with a general triangulation, to obtain splines of high smoothness in a stable manner sufficiently large degrees have to be considered. An alternative is to use lower-degree macro-elements that subdivide each triangle into a number of subtriangles (or more generally subdomains).

In [Lyche, T., Manni, C., Speleers, H., *Construction of C^2 Cubic Splines on Arbitrary Triangulations*. Found. Comput. Math. 22, 13091350 (2022)] the construction and the representation of C^2 cubic splines on a suitable refinement of an arbitrary triangulation has been addressed, by using simplex splines as local bases.

Spline quasi-interpolants (QIs) are widely used tools both in approximation theory and applications. In this talk, starting from the results in [Lyche, T., Manni, C., Speleers, H., *Construction of C^2 Cubic Splines on Arbitrary Triangulations*. Found. Comput. Math. 22, 13091350 (2022)], we provide the explicit construction of different C^2 cubic spline QIs on general triangulations which are locally defined on each triangle through a simplex spline basis. Several numerical experiments are considered to underline their pros and cons as well as their approximation performance in challenging situations.

Non-recoverable Signals via Fourier Partial Sums [M-14B]

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Given an analytic function f on the open unit disc $\mathbb{D}$, or an integrable function on its boundary $\mathbb{T} = \partial\mathbb{D}$, our first attempt to approximate f is via its partial Taylor sums $s_n(f) = \sum_{k=0}^{n} \hat{f}(k)z^k$ or partial Fourier sums $s_n(f) = \sum_{k=-n}^{n} \hat{f}(k)e^{ikt}$. If this first direct approach fails, we exploit several well-developed summation methods, e.g., Abel, Borel, Cesàro, Hausdorff, Hölder, Lindelöf, Nörlund, etc., to come up with an appropriate combination of the partial sums which converges to the original function. More explicitly, we consider the weighted sums $\sigma_n(f) = \sum_{k=0}^{n} w_{nk}\hat{f}(k)z^k$ or $\sigma_n(f) = \sum_{k=-n}^{n} w_{nk}\hat{f}(k)e^{ikt}$. While, in many cases, this procedure is a success story, we may naturally wonder if for each space an appropriate summability method via partial Taylor or Fourier sums can be always designed. We show that this is not always feasible. We construct a Hilbert space of integrable functions on $\mathbb{T}$ such that 1) trigonometric polynomials are dense in $\mathcal{H}$, 2) odd trigonometric polynomials are not dense in the subspace of odd functions in $\mathcal{H}$. Hence, as an outcome, there is a signal $f \in \mathcal{H}$ such that with <u>no</u> lower-triangular summability method, one can recover f from its partial Fourier sums $s_n(f)$.

Polarization and Greedy Energy on the Sphere [M-1B]

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We shall discuss a "greedy" approach to producing sequences on the sphere $\mathbb{S}^d$, where at each step of the algorithm the point that minimizes the Riesz s-energy is added to the existing set of points. Such a sequence has very good distribution properties, and we shall investigate how it behaves with respect to a few different notions of equidistribution. We show that for the range $0 < s < d$, the greedy sequence with respect to the Riesz s-energy achieves optimal second-order behavior (up to constants). As part of the proof, we establish that the second-order term of the max-min polarization with respect to Riesz s-kernels for $0 < s < d$ is of order $N^{s/d}$. Using the Stolarsky principle relating the L_2-spherical cap discrepancy of a point set with the pairwise sum of distances, we also show a simple upper bound on the L_2-discrepancy for the greedy sequence when $s = -1$ and discuss numerical results that indicate the true discrepancy is much lower.

The Eigenvalue Distribution of Time-frequency Limiting Operators in Higher Dimensions [M-12A]

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Most wireless communications rely on a fixed time band and frequency-limited signals, despite the uncertainty principle showing they are technically incompatible. One important question in the signal processing is how to construct signals that are not only restricted to a given frequency band but also have minimal energy outside a time band.

This question has been investigated in dimension $d = 1$ for the signals of one-variable, by a series of Bell Labs papers by H. Landau, H. Pollak, D. Slepian, H. Widom and I. Daubechies between 1960-1980. The higher-dimensional case has a crucial aspect in many applications, particularly in scientific imaging problems such as cryoelectron microscopy (cryo-EM) and MRI, and certain optimal orthogonal systems that are approximately space-limited and bandlimited functions in two or more variables play a vital role in achieving accurate and efficient representation of complex multi-dimensional data. In this talk, we will be presenting an extension of the $1-$dimensional results to multiple dimensions. To achieve this, we utilize a Whitney decomposition technique along with a modified version of the Coifman-Meyer local sine basis. However, there are still many open questions and challenges that need to be addressed to fully exploit the potential of this approach.

Neural Collapse with Unconstrained Features [M-1B]

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Neural collapse is an emergent phenomenon when minimizing empirical risk in deep learning that was recently discovered by Papyan, Han, and Donoho. We propose a simple "unconstrained features model" in which neural collapse also emerges empirically. By studying this model, we provide some explanation for the emergence of neural collapse in terms of the landscape of empirical risk.

Approximations and Learning in the Wasserstein Space [M-13A]

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Detecting differences and building classifiers between distributions, given only finite samples, are important tasks in a number of scientific fields. Optimal transport and the Wasserstein distance have evolved as the most natural concept to deal with such tasks, but have some computational drawbacks. In this talk, we describe an approximation framework through local linearizations that significantly reduces both the computational effort and the required training data in supervised learning settings.

Climate Change and Approximation Theory [M-8B]

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The climate crisis is here, and approximation theory is helping to tackle it! This talk will highlight a variety of ways mathematicians, scientists, and engineers are using approximation theory to enhance understanding of climate change, inform climate adaptation, decarbonize energy production, and electrify everything. I will share application examples which use techniques such as radial basis functions, splines, wavelets, and machine learning. This talk will be of interest to researchers who have a desire to work on industry-relevant problems and will offer early-stage researchers an insight into careers in industry that use approximation theory.

Geodesic Distance Riesz Energy on the Projective Space [M-3B]

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In 1959, Fejes Tóth conjectured that for N lines through the origin in $\mathbb{R}^d$, the pairwise sum of the acute angles between them is maximized by repeated copies of the orthonomal basis. This conjecture remains open for $d \geq 2$, and can be restated both as an energy optimization problem on $\mathbb{S}^d$, and as the energy optimization of the geodesic distance $\theta(x, y)$ on $\mathbb{RP}^d$. We use the projective plane formulation of the problem to obtain results for the Riesz energy $\theta(x, y)^\alpha$. We show that the for $-d < \alpha \leq -(d-2)$, the relevant Gegenbauer and Jacobi polynomial expansions of $\theta(x, y)^\alpha$ have nonnegative coefficients, hence energy minimization results in uniform distribution. In addition, we connect the $d = 1$ case of this problem to optimization of the geodesic distance on the sphere, and run numerical experiments for the case $0 < \alpha < 1$ on $\mathbb{S}^2$.

Divergence of Taylor Series in de Branges-Rovnyak Spaces [M-14B]

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In this talk, I will present sufficient conditions for the existence of a function in a given de Branges-Rovnyak space for which the Taylor series is unbounded in norm or diverges in norm. The result is a consequence of a refined version of the boundedness principle established by Müller and Vrsovsky. This is a joint work with Thomas Ransford.

Rank-aware Orthogonally Weighted Regularization for Joint Sparse Recovery [M-13A]

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In this talk, we present a novel method for joint sparse recovery called orthogonally weighted $\ell_{2,1}$ regularization, which takes into account the rank of the solution matrix. Our method differs from existing regularization-based approaches by exploiting the full rank of the row-sparse solution matrix. We prove the rank-awareness of our method, establish the existence of solutions to the optimization problem, and provide an efficient algorithm for solving it. We analyze the convergence of our algorithm and present numerical experiments to demonstrate the effectiveness of our method.

Sensor Networks and Quantized Fusion Frames [M-10A]

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Fusion frames provide a vector-valued generalization of frame theory and were introduced as a tool for distributed sensor networks and other data fusion problems. Motivated by this, we construct high-order low-bit Sigma-Delta ($\Sigma\Delta$) quantizers for the vector-valued setting of fusion frames. We prove that these $\Sigma\Delta$ quantizers can be stably implemented to quantize fusion frame measurements on subspaces W_n using $\log_2(\dim(W_n) + 1)$ bits per measurement. Signal reconstruction is performed using a version of Sobolev duals for fusion frames, and numerical experiments are given to validate the overall performance. This includes joint work with Jiayi Jiang, Felix Krahmer, and Zhen Gao.

Dual-Graph Regularized Foreground Background Separation [M-3A]

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Foreground-background separation (FBS) has been widely used in many applications, such as video surveillance and robotics. Due to the presence of the static background, a motion video can be decomposed into a low-rank background and a sparse foreground. Many regularization techniques that preserve low-rankness of matrices can therefore be imposed on the background. In the meanwhile, geometry-based regularizations, such as graph regularizations, can be imposed on the foreground. In this talk, I will present a dual-graph regularized FBS method based on weighted nuclear norm regularization and discuss its fast algorithm based on the matrix CUR decomposition. Numerical experiments on realistic human motion data sets are used to demonstrate the proposed effectiveness and robustness in separating moving objects from background, and the potential in robotic applications.

The Complete Parametrization of the Length Sixteen Wavelets [C-16B]

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In this paper, a complete parametrization of the length sixteen wavelets is given for the dilation coefficients of the trigonometric polynomials, $m(\omega)$, that satisfy the necessary conditions for orthogonality, that is $m(0) = \sqrt(2)$ and $|m(\omega)|^2 + |m(\omega + \pi)|^2 = 2$. This parametrization has seven free parameters and has a simple compatibility with the shorter length parametrizations for some specific choices of the free parameters. These wavelets have varying numbers of vanishing moments and regularity, but continuously transform from one to the other with the perturbation of the free parameters.

Data Representation With Optimal Transport [M-12A]

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Finding a useful mathematical model to represent data (functions, signals, images) can facilitate the solution of numerous problems including signal or image detection, estimation, filtering, reconstruction, compression, and classification. Fourier transforms, for example, render convolution operations into multiplications in Fourier domain, thereby simplifying the solution of linear shift-invariant systems. Wavelet transforms, on the other hand, are well-suited for detecting and analyzing signal transients at different resolutions. Given the emergence of modern machine learning problems such as signal and image classification, the demand for nonlinear signal and image representation techniques has significantly increased.

One emerging nonlinear representation technique uses optimal transport techniques. The idea is based on using optimal transport theory to establish a unique relation (a transport map) between a fixed function (or measure), and any function (measure) one wishes to represent. The unique transport map that pushes the reference function (measure) onto the function (measure) of interest can be interpreted as a new representation for data (functions, signals, and images). Like Fourier and Wavelet transforms, this emerging transport-based representation is invertible, and has interesting mathematical properties that can help "convexify" data classes. Therefore, they are able render signal processing problems (e.g. estimation, classification) that are difficult (e.g. nonlinear) to solve in signal domain simpler (e.g. linear) to solve in transport representation domain. This talk will highlight this emerging mathematical representation technique, describing numerous signal and image analysis (e.g. classification) applications along the way.

Statistics for Coulomb Gases at Low Temperatures, Sampling and Interpolation [M-10A]

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The Coulomb gas consists of a large number of repelling point charges confined by an external potential. At very low temperatures a certain almost deterministic pattern can be expected to emerge (freezing regime). I will present results on the plane consistent with this intuition by describing the separation, discrepancy and equidistribution of the particles, at the microscopic scale and with respect to the so-called equilibrium measure.

To prove the results, we investigate sampling and interpolation properties of weighted holomorphic polynomials with respect to random draws of the planar Coulomb gas (Boltzmann-Gibbs distribution). We then develop a variant of Landau's theory for bandlimited functions that depends on novel nonasymptotic estimates for Toeplitz operators on the range of the erfc-kernel (sometimes called Faddeeva or plasma dispersion kernel). The talk is based on joint work with Yacin Ameur (U. Lund) and Felipe Marceca (King's College London).

Operator Theoretic Approaches to System Identification and Dynamic Mode Decompositions [C-16B]

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The system identification (or approximation) problem is the determination of an accurate approximation of a dynamical system from observations of the state or output. In this talk we discuss the role operators and reproducing kernel Hilbert spaces play in nonlinear system identification and modeling problems. This is directly connected to the Koopman operator framework that has recently become popular. I will present the advantages that come with the kernel perspective, as well as a pointwise convergent result that comes from the use of compact operators.

Systems with Riesz Interactions in the Mean-Field Regime [M-10B]

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We present recent results on the large particle number and large time effective behavior of conservative or gradient dynamics for particle systems with mean-field interactions governed by a Coulomb or more general Riesz potential and subject to possible noise modeling thermal fluctuations. The talk will discuss modulated energy/free energy techniques for studying the rate of mean-field convergence, how the rate deteriorates with time, and how fluctuations around the mean-field limit behave.

Predicting Fluid Particle Trajectories without Flow Computation: A Data-driven Approach [M-1A]

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Conventional methods for tracking fluid particles involves costly flow computations. More specifically, we need to solve a system of Partial Differential Equaions (PDEs) governing the flow at each time point and a system of Ordinary Differential Equations (ODEs) to march forward in time. We show that given sufficient trajectory data, a Deep Neural Network (DNN) can be employed to "learn" the flow map, a function that maps a particle's initial position to its future position after a time lag. Compared to conventional methods, this new approach is "data hungry" rather than computationally intensive; and once trained, the DNN can be deployed as a reduced-order model to perform fast, real-time, and on-demand trajectory calculation without PDE or ODE solves.

Computing with Splines on Curved Triangulations [P-11]

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Splines on ordinary triangulations are very effective tools in approximation theory, and have important applications in a variety of settings, including data fitting and the numerical solution of boundary-value problems for PDEs. They are most useful when the domains have polygonal boundaries. However, in practice many of the problems of interest are defined on domains with curved boundaries. In a recent paper with one of my students, we introduced the concept of *curved triangulations* and show how to work with polynomial splines on such triangulations using the well-developed Bernstein-Bézier techniques which are so fundamental for computing with splines on ordinary triangulations. In this talk we explain the methods, and show several explicit numerical examples for both data fitting and boundary-value problems.

Distribution of Primes and Approximation on Weighted Dirichlet Spaces [M-14B]

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We study zero-free regions of the Riemann zeta function ζ related to an approximation problem in the weighted Dirichlet space D_{-2} which is known to be equivalent to the Riemann Hypothesis since the work of Bez-Duarte. We prove, indeed, that analogous approximation problems for the standard weighted Dirichlet spaces D_α when $\alpha \in (-3, -2)$ give conditions so that the half-plane $\{s \in C : \Re(s) > -\frac{\alpha+1}{2}\}$ is also zero-free for ζ. Moreover, we extend such results to a large family of weighted ℓ^p-spaces of analytic functions. As a particular instance, in the limit case $p = 1$ and $\alpha = -2$, we provide a new proof of the Prime Number Theorem.

Multiscale Approximation with Manifold-valued Data [M-14A]

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Manifold-valued functions are of the form $F\colon \Omega \to \mathcal{M}$, where $\Omega \subseteq \mathbb{R}^d$ is a bounded domain and $\mathcal{M}$ is a Riemannian manifold. The problem of approximating and representing functions with manifold values has risen in various research areas, ranging from signal processing and modern statistics to essential applications such as brain networks, structural dynamics, and more.

In this talk, I will briefly survey a few instances of multiscale approximation problems of manifold-valued functions. First, we will describe the problems and introduce our method for solving them. Then, we numerically demonstrate the results, including applying our techniques for denoising and abnormalities detection.

Polynomial Interpolation on Arbitrary Varieties [M-12B]

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A traditional interpolation problem consists of a collection of distinct points $v_1, ..., v_n \in \mathbb{C}^d$ and data $p_1, ..., p_n \in \mathbb{C}$. The problem is to find a polynomial $p \in \mathbb{C}[x_1, ..., x_d]$ such that $p(v_j) = p_j$ for all $j = 1, ..., n$. In this work we study the following extension of the problem: Given distinct affine varieties $\mathcal{V}_1, ..., \mathcal{V}_n \subset \mathbb{C}^d$ and n polynomials $p_1, ..., p_n \in \mathbb{C}[x_1, ..., x_d]$ find one polynomial $p \in \mathbb{C}[x_1, ..., x_d]$ such that $p \mid \mathcal{V}_j = p_j \mid \mathcal{V}_j$ for all $j = 1, ..., n$. (Here $p \mid \mathcal{V}_j$ denotes the restriction of p onto variety $\mathcal{V}_j$). The existence of such an interpolation depends on the relationship between the varieties $\mathcal{V}_1, ..., \mathcal{V}_n$ and polynomials $p_1, ..., p_n$. For instance, if the varieties $\mathcal{V}_1, ..., \mathcal{V}_n$ are all disjoint then the interpolation polynomial exists for any $p_1, ..., p_n$. The proofs are all simple and depend only on The Hilbert Nullstellensatz.

A Constructive Approach for Computing the Proximity Operator of the p-th Power of the ℓ_1-norm [M-3A]

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This talk is about computing the proximity operator of the power function of the ℓ_1 norm. By examining the properties of the proximity operator of the power function of the ℓ_1 norm, we will present a simple and well justified approach to compute the proximity operator of $\|\cdot\|_1^p$ with any power $p > 1$. We also discuss how the structure of $\|\cdot\|_1^p$ represents a class of relative sparsity promoting functions.

Applying Kolmogorov Superposition Theorem to Break the Curse of Dimensionality [M-6B]

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We explain how to use Kolmogorov Superposition Theorem (KST) to break the curse of dimension when approximating continuous multivariate functions. We first show that there is a class of functions called K-Lipschitz continuous and can be approximated by a ReLU neural network of two layers to have an approximation order $O(d^2/n)$, and then we introduce the K-modulus of continuity of multivariate functions and derive the approximation rate for any continuous function $f \in C([0,1]^d)$ based on KST. Next we introduce KB-splines by using uniform B-splines to replace the K-outer function and their smooth version called LKB-splines to approximate high dimensional functions. Our numerical evidence shows that the curse of dimension is broken in the following sense. When using the standard discrete least squares method (DLS) to approximate a continuous function f over $[0,1]^d$, one expects to use a dense data set P, a large number of data locations and function values over the locations. Based on the LKB splines, the structure of K-inner functions leads to a sparse solution of the linear system associated with the DLS. Furthermore, there exists a magic set of points from P with much small in size such that the rooted mean squares error (RMSE) from the DLS based on the magic set is similar to the RMSE of the DLS based on the original set P. In addition, the number of LKB-splines used for approximation is the same as the size of the magic data set. Hence we need not a lot of basis functions and a lot of data locations and function values to approximate a high dimensional continuous function f when f is not very oscillated.

Locally-Verifiable Sufficient Conditions for Exactness of the Hierarchical B-spline Discrete de Rham Complex in n Dimensions [M-12B]

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Discretization of the de Rham complex of differential forms helps construct stable numerical methods for saddle point problems (including those for electromagnetism and fluid mechanics). For such stable numerical methods, the discrete de Rham complex should be cohomologically equivalent to the continuous complex. In this talk, we present locally-verifiable sufficient conditions for such a cohomological equivalence for the hierarchical B-spline de Rham complex of discrete differential forms defined on a hypercube in n-dimensional Euclidean space.

Minimal Versus Generalized Minimal Projections [M-14A]

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Let $V \subset X$ be a (closed) subspace of a Banach space X. The set of all continuous projection X onto V is denoted by

$$\mathcal{P}(X,V) := \{P \in \mathcal{L}(X,V) : P|_V = id\}.$$

We say that P_o is a **minimal projection** (MP) if it has the smallest possible norm, that is if

$$\|P_o\| = \lambda(V,X) := \inf\{\|P\| : P \in \mathcal{P}(X,V)\}.$$

Given a projection P, we can approximate x by Px and the error of such approximation is directly related to the norm of P. Here we try to relax the projection condition by introducing the intermediary subspace Z. Let $V \subset Z \subset X$. Put

$$\mathcal{P}_V(X,Z) := \{P \in \mathcal{L}(X,Z) : P|_V = id\},$$

each element of the above is called a **generalized projection**. We say that P_o is a **minimal generalized projection** (MGP) if it has the smallest possible norm, that is if

$$\|P_o\| = \lambda_Z(V,X) := \inf\{\|P\| : P \in \mathcal{P}_V(X,Z)\}.$$

The question arises, to what extent we can improve the approximation of x by Px if we consider generalized projections. The related question is how smaller the norms of the generalized projections are in relation to $\lambda(V,X)$.

The classic example considers the Fourier projections F_n in $X = \mathcal{C}_0(2\pi)$ or $X = L_1[0, 2\pi]$ onto $V = \Pi_n$, the space of all trigonometric polynomials of a degree $\leq n$. F_n is a minimal projection of norm $\approx \frac{4}{\pi^2} \ln n$. Introducing the intermediary subspace

$$V = \Pi_n \subset Z = \Pi_{2n-1} \subset X$$

we can construct de la Vallée Poussin generalized projections $H_n = \frac{F_n + F_{n+1} + \ldots + F_{2n-1}}{n} \in \mathcal{P}_{\Pi_n}(X, \Pi_{2n-1})$ that have uniformly bounded norms.

Decentralized Algorithms for Spatially Distributed Systems [M-3A]

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In a centralized system, all processing and decision-making is handled by a single entity, which can become a bottleneck as the system grows in size and complexity. Decentralized algorithms distribute the processing load across multiple nodes, allowing the system to scale much more effectively. Many decentralized algorithms distribute the global objective function (usually the sum of many local objective functions) across multiple nodes such that each node only handle its own local objective function. The state variable is "copied" to each node and communication between neighboring nodes helps them reach a consensus in the convergence of iterations. However, the computational cost for each agent can still be high if the common state variable has a large dimension. To address this, we would further divide the global state variable into multiple local state variables, so that each node only handles a few components of the global state variable. In particular, we will analyze its performance on spatially distributed systems.

Piecewise Divergence-free and Harmonic Finite Elements [M-6B]

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We show how to use Bernstein-Bézier form to construct macro-elements satisfying certain differential equations. The associated finite element solutions are then the Galerkin projections in smaller vector spaces. Our approach results in smaller systems of equations and better condition numbers. The number of unknowns on each element is reduced significantly when compared to traditional finite elements. We prove their optimal order of convergence, and confirm our findings by numerical tests.

Constructing C^1 Cubic Powell-Sabin B-Splines with Super-Smoothness [M-12B]

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The construction of smooth spline functions of low polynomial degree on general triangulations has shown to be a challenging problem. Its complexity originates from the fact that a small change in geometry of the triangulation can have an impact on the degrees of freedom of the spline function. There exist numerous techniques to tackle this problem, such as specific refinements of the triangulation (e.g., Clough-Tocher or Powell-Sabin splits), increased smoothness in parts of the domain, or simply restrictions to special classes of triangulations.

In this talk, we combine several of the aforementioned techniques in order to shape smooth cubic splines suitable for approximation and not too complex to analyze. We start from the full C^1 cubic spline space defined on a Powell-Sabin 6-refinement of the triangulation, which admits a B-spline representation constructed in a geometrically intuitive way. Then we reduce the number of degrees of freedom by prescribing additional C^2 smoothness conditions, particularly on locally structured parts of the triangulation, and we investigate how to recombine the basis functions of the original B-spline representation in order to adjust them to the reduced subspaces. We illustrate the application of these super-smooth B-splines in the context of least squares approximation and finite element approximation for second and fourth order boundary value problems.

Normalizied Circular Bernstein-Bezier Polynomials [M-12B]

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Alfeld, Neamtu, and Schumaker introduced Circular Bernstein-Bezier (CBB-) polynomials, trigonometric polynomial analogs of the well-known Bernstein-Bezier polynomials. In their paper, the authors present a theory of CBB-polynomials that parallels the classical polynomial case, including barycentric coordinates, a deCasteljau algorithm, subdivision, and degree raising. In this talk a different trigonometric analog of the Bernstein polynomials will be presented. These trigonometric polynomials have the partition of unity property and are therefore named Normalized Circular Bernstein-Bezier polynomials. This work was done as part of an undergraduate research project.

Wiener Filters and Inverse Filters for Graph Signal Processing [M-10A]

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In this talk, we discuss Wiener filters for stationary graph signals and distributed algorithms to implement Wiener/inverse filters on networks in which agents are equipped with a one-hop communication subsystem for direct data exchange only with their adjacent agents.

Locally Sparse Representations via Structured Triangulations [M-3B]

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Given a Delaunay triangulation of a set of m points in $\mathcal{R}^d$ represented as $\mathbf{A} = [\mathbf{a}_1, ..., \mathbf{a}_m]$ and a point $\mathbf{y} \in \mathcal{R}^d$, we study the following optimization problem

$$\min_{\mathbf{x} \in \Delta_m} \sum_{j=1}^{m} x_j ||\mathbf{y} - \mathbf{a}_j||_2^2 \quad \text{subject to} \quad \mathbf{y} = \mathbf{A}\mathbf{x},$$

where $\Delta_m = \{\mathbf{z} \in \mathcal{R}^m | z_i \geq 0 \; \forall i \text{ and } \sum_{j=1}^m z_j = 1\}$ is the probability simplex. We give interpretations of the discrete energy objective. We show that the solutions to the above problem are locally sparse. Further, we characterize the stability of the solutions using the Cayley-Menger determinant.

Discrete Periodic Energy Problems [M-10B]

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Discrete point configurations which are periodic with respect to some lattice admit a natural notion of energy, often called periodic energy. We'll introduce periodic energy and present new findings that certain low cardinality configurations, coming from the equitriangular lattice A_2, optimize periodic energy for a general class of interaction kernels. We'll also show a connection between a particularly interesting sequence of periodic energy problems and the important conjecture that A_2 is universally optimal. Our main tools are the Delsarte-Yudin linear programming method along with some basic facts regarding polynomial interpolation.

Identification Problem in Second-Order Nonlinear Hyperbolic PDE with Initial and Boundary Data [C-16B]

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Identification problems are among the oldest and most important mathematical problems in science and engineering. However, the field of identification problems has seen rapid development in the last two decades due to massive increases in computing power and the development of powerful numerical techniques.

In this presentation, we look at an identification problem for physical parameters associated with a second-order nonlinear hyperbolic model with initial and boundary data. The existence, uniqueness, and continuous dependency of the model's weak solution are established. The method of transposition is used to demonstrate the solution map's Gâteaux differentiability. The optimal parameters are determined. The computational algorithm and numerical results are presented.

REM Sleep Stage Identification With Raw Single Channel EEG [M-8A]

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This paper focuses on creating an interpretable model for automatic Rapid Eye Movement (REM) sleep stage scoring for a single-channel electroencephalogram (EEG). Many methods attempt to extract meaningful information to provide to a learning algorithm. This method attempts to let the model extract the meaningful interpretable information by providing a smaller number of time-invariant signal filters for 5 frequency ranges using 5 CNN algorithms. A bi-directional gated recurrent unit (GRU) algorithm is applied to the output to incorporate time transition information. Training and tests are run on the well-known sleep-EDF-expanded database. The best results produced 97% accuracy, 93% precision, and 89% recall.

Identifying Low-Dimensional Structure for Functional Compression [M-1A]

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Traditionally, the inherent structure in data is leveraged for compression and later recovery. However in many cases, we may ultimately be interested in some function of the data rather than the data itself. By taking advantage of additional redundancy introduced by the function in the form of equivalence classes, we can achieve greater compression and efficiency . In this talk, I will describe novel approaches to this problem inspired by compressed sensing, optimization theory, and cognitive science.

Generalized Splines in Degree Two and the Dimension of Bivariate Spline Spaces [M-13B]

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We consider the collection of piecewise polynomials of degree 2 and smoothness 1 defined over any reasonably-behaved partition of a simply connected domain in $\mathbb{R}^2$. We describe how to compute the dimension of this spline space in the generic case, using the planar graph that is combinatorially dual to the partition together with the theory of generalized splines. We describe how this is related to Schumaker's lower bound.

Multivariate Splines on Oranges [M-13B]

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In the talk, we will focus on splines defined on a special kind of partitions which we call "oranges". These partitions are composed of a finite number of simplices which are of the same dimension and such that they all share a common lower dimensional face. For any fixed maximal polynomial degree and a given order of global smoothness, we prove that the dimension of the spline space on an orange can be computed as a sum of the dimension of spline spaces on simpler lower-dimensional partitions. The examples and results combine both algebraic and Bernstein-Bézier methods for splines.

Equilibrium Measures of Riesz Energies with External Fields [M-10B]

D. Chafaï, R. W. Matzke, E. B. Saff, M. H. Vu*, and R. S. Womersley
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In this talk, we focus on minimal energy problems of the Riesz kernels $\frac{1}{s}|x-y|^{-s}$ under the effect of external fields $\frac{\gamma}{\alpha}|x|^{\alpha}$ in $\mathbb{R}^d$. We show that when $-2 < s < d-3$ and $\alpha \geq \alpha_{s,d}$, the uniform measure on a sphere with certain radius is the minimizer. Furthermore, the constant $\alpha_{s,d}$ is a sharp bound for α to guarantee that the equilibrium measure is supported on a sphere.

What's New in Wavelet Phase Retrieval? [M-6A]

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Phase retrieval refers to mathematical problems in which one aims to recover the phases of a large structured collection of complex numbers from their moduli: if the collection has enough redundancy, it is possible to reconstruct the complex phases. In this talk, we investigate the problem of recovering real-valued signals from the magnitudes of their wavelet frame coefficients. We will demonstrate that such signals can be uniquely recovered (up to global phase) from certain multi-wavelet frame coefficients based on the Poisson wavelet. Notably, our results do not require imposing any bandlimiting constraints or a priori knowledge on the signals.

Approximation Results for Gradient Descent Trained Shallow Neural Networks [M-14A]

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Approximation theory for neural networks has been extensively studied in recent years showing that they can match or even surpass most other methods, including splines, finite elements or wavelets. However, in most of these results, the network weights are hand picked and it remains unclear if the results are matched by networks that are trained by general purpose algorithms, like gradient descent. On the optimization question much progress has been made in over-parametrized regimes, with more weights than samples. These regimes do not contain typical problems in approximation theory or scientific computing that use continuous error norms corresponding to infinite sample limits, resulting in under-parametrized problems.

Some form of smoothness or regularity is required by all approximation methods to achieve quantifiable convergence rates. In the presentation, we show that these requirements can be used to eliminate the over-parametrization assumption for the optimizers and thus provide neural network approximation results for networks that are trained by gradient descent. The redundancy in over-parametrized regimes reappears as a loss in convergence rate. We present first results for shallow networks in one dimension and then some abstractions of the results that are suitable for multiple dimensions.

Understanding the Thomson Problem from a Machine Learning Perspective [M-1B]

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One of the enduring problems in Potential Theory is the Thomson Problem, which asks to find which configurations of n electrons constrained to the surface of a unit sphere produce the minimal potential. We investigate the method of approximating such configurations using a projected gradient descent method and the limitations of such an approach as well as mention some open questions we have in this regard.

Bias Correction for Distributed Machine Learning [M-8A]

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Distributed machine learning system is able to efficiently process large scale data. The divide and conquer approach has been proven a simple but effective method to implement distributed machine learning. We propose a bias correction approach to improve the performance of distributed kernel regression while preserving its theoretical optimality. We also propose effective strategies for distributed classification and other machine learning tasks.

Distributed Least Square Monte Carlo for American Option Pricing [M-13B]

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Fast and accurate pricing of the American option is critical to the stability and development of the financial market. The least-square Monte Carlo (LSMC) method is an effective method to solve the American option pricing problem. This paper discusses the improvement of LSMC with distributed computing technology and compares it with parallel computing. Distributed computing has several advantages compared with parallel computing including reducing the total computational complexity, avoiding complicated matrix transformation, and improving data privacy and accuracy. The LSMC is suitable for distributed computing. This research aims to show how distributed regression technology can improve the efficiency and accuracy of LSMC to provide a new method for American option pricing.

A High Dimensional Cramér-von Mises Test [M-8A]

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We develop a Cramér-von Mises type test for testing distributions of high dimensional continuous data and establish an asymptotic theory for quadratic functions of high-dimensional stochastic processes. To obtain cutoff values of our tests, we introduce two different procedures to implement high-dimensional Cramér-von Mises test in practice: a plug-in calibration method and subsampling method. Theoretical justification and numerical studies of both approaches are provided. The method is applied to test the marginal normality of residuals from a high-dimensional vector autoregression of a macro-economic dataset.

On Multipoint Padé Approximants whose Poles Accumulate on Contours that Separate the Plane [M-6B]

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The multipoint Padé approximants to Cauchy integrals of analytic non-vanishing densities will be discussed in the situation where the (symmetric) contour attracting the poles of the approximants does separate the plane.

An Algebraic Framework for Geometric Continuous Splines [M-13B]

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Geometrically continuous splines (G-splines) are piecewise polynomial functions defined on a collection of patches which are stitched together. Compared to the traditional parametric splines, G-splines are more flexible in construction of shapes with complicated topology. In this talk, we provide an algebraic framework for studying G-splines. An immediate consequence of introducing this framework is the application of algebraic methods to estimate the dimension of G-spline spaces and to construct a basis for given G-spline space. This talk is based on a program joint with Angelos Mantzaflaris, Bernard Mourrain and Nelly Villamizar.

Graph Framelet Neural Networks [M-3A]

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Deep learning has revolutionized modern society and peoples daily life during the past decade ranging from automated driving, online shopping, AI assistant, to smart surveillance systems, medical diagnosis, drug discovery, and so on. Graph neural networks (GNNs) are powerful deep learning methods for machine learning tasks on graph-structured data, e.g., node classification and link prediction in social networks and citation networks. Graph framelet systems, like the traditional wavelet systems for Euclidean data (signals, images, videos, etc.), provide a powerful tool for multiresolution analysis of graph-structured data. Based on coarse-grained chains and graph Fourier transforms, we establish fast graph framelet transforms and demonstrate how to build graph framelet systems that can be efficiently and effectively utilized for the design of graph neural network architecture. We demonstrate state-of-the-art performances of the graph framelet neural networks (GFNNs) in some graph deep learning tasks.