Take home problems

Nashville Math Club

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1 Take-Home Problems

1. Show that the number of partitions into odd parts is the same as the number of partitions into distinct parts. For example, the partitions of 9 into odd parts are $9, 7+1+1, 5+3+1, 5+1+1+1+1, 3+3+3, 3+3+1+1+1, 3+1+1+1+1+1+1, 1+1+1+1+1+1+1+1+1$ and the partitions of 9 into distinct parts are $9, 8+1, 7+2, 6+3, 6+2+1, 5+4, 5+3+1, 4+3+2$. There are 8 partitions of each type.

2. (Andrews) The number of partitions of n in which only odd parts may be repeated equals the number of partitions of n in which no part appears more than three times. For example the partitions of $n = 5$ of the first type are $5, 4+1, 3+2, 3+1+1, 2+1+1+1, 1+1+1+1+1$, and the partitions of 5 of the second type are $5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1$. In both cases, there are 6 such partitions.

3. Prove that the greatest common divisor of F_n and F_{n+1} is 1 for all n.

4. (For those who know what mathematical induction is:) Find a formula for the sum of the first n Fibonacci numbers. (*Hint:* Try writing down the first few sums and see if you can come up with a guess for the answer. Then trying proving the result by induction.)

5. Define a sequence by $a_0 = 0$ and $a_{n+1} = 2a_n + 1$ for $n \geq 0$. Can you write down a closed formula for a_n? (*Hint:* Consider $GF(a_n)$ as we did when trying to find F_n.)