Functorial Field Theories and Ring Spectra

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Abstract

One important role of mathematics is to serve as a language for other sciences. There are many success stories, on the more theoretical side general relativity (Lorentz geometry, Einsteins equation) and quantum mechanics (functional analysis, Schrdingers equation) come to mind. However, new mathematical formalisms still need to be discovered for the physically relevant quantum field theories.

A very promising approach, via "functorial field theories", was initiated by Witten, Atiyah and Segal. In this mini-course, we will explain this approach from first principles and discuss several structural analogies to ring spectra studied in algebraic topology. This is ongoing work with Stephan Stolz and many others. For example, Sigma-models lead to spaces of field theories, deformations correspond to homotopies, central charge to the degree (or more generally, the twist) of a cohomology class. Gauge theories give equivariant cohomology classes and best of all, quantization is related to push-forward (or Gysin) maps. In the future, this could be a way in which algebraic topology contributes to making some of the ill-defined Feynman path integrals rigorous.