Introduction to measured group theory

Alex Furman

Abstract

Measured Group Theory concerns an area of mathematics that studies infinite countable groups using measure-theoretic tools, and studies ergodic theory of group actions, emphasizing the impact of group structure on the actions. The term Measured Group Theory also suggests an analogy with Geometric Group Theory.

In the minicourse I plan to cover the following topics:

1) A survey of rigidity of lattices in semi-simple Lie groups: Strong Rigidity (Mostow), Super-rigidity (Margulis), Cocycle Super-rigidity (Zimmer), a comparison to Popa's cocycle super-rigidity.

2) Measure Equivalence, its relation to lattices, Orbit Equivalence, and Quasi-isometries.

3) Strong ICC groups, tautness and rigidity results for ME.

4) ME rigidity for products of hyperbolic-like groups (after Monod-Shalom), using Weyl group approach of Bader-Furman.

Spectral gap and orbit equivalence rigidity for translation actions

Adrian Ioana

Abstract

A dense inclusion of a countable group Γ into a locally compact group G gives rise to a left translation action $\Gamma \curvearrowright G$. In this minicourse, I will present recent work motivated by the following question: to what extent does the equivalence relation on G of belonging to the same Γ-orbit remember the inclusion $\Gamma < G$? If G is compact, then this question is related to the spectral gap of the action. In particular, I will present a rigidity result which gives necessary and sufficient conditions for compact-translation actions with spectral gap to be orbit equivalent. On the other hand, if G is locally compact but not necessarily compact, then the motivating question is related to a certain local spectral gap property of the action. I will report on recent work with R. Boutonnet and A. Salehi Golsefidy in which we establish local spectral gap whenever Γ is a dense subgroup generated by algebraic elements of an arbitrary connected simple Lie group G. This extends to the non-compact setting works of Bourgain-Gamburd and Benoist-de Saxcé.

Some analysis techniques in II_1 factors

Sorin Popa

Abstract

I will present several analysis techniques in type II_1 factors, with applications to structural and classification results. This will include:

1. *Intertwining by bimodules* and *deformation-rigidity*, with some applications to the classification of Bernoulli group measure space factors and the structure of subalgebras of free group factors.

2. *Incremental patching*, which allows "simulating prescribed relations inside II_1 factors, with elements subject to constraints. Applications here will include: (at) the occurrence of approximate 3-independence in an arbitrary MASA of a II_1 factor, $A \subset M$, with free independence when A is singular; (b) the occurrence inside any subfactor $N \subset M$ of canonical "approximate subfactors, of free product type, with the same standard invariant as $N \subset M$ (leading to *reconstruction* results).

Finite-dimensional approximation properties of groups and their applications

Andreas Thom

Abstract

In this course I will explain how finite-dimensional approximation properties can help to understand infinite discrete groups and related objects such as group rings and the group von Neumann algebra. I will discuss Kaplanskys Direct Finiteness Conjecture, the Kervaire-Laudenbach Conjecture about equations in groups, aspects of Connes Embedding Problem, consequences of Lücks Approximation theorem for ℓ^2-Betti numbers, and a recent extension of it to sequences of lattices in totally disconnected groups.

Derivations on von Neumann algebras and L^2-cohomology

Vadim Alekseev

Abstract

Connes and Shlyakhtenko introduced L^2-Betti numbers for subalgebras of finite von Neumann algebras in hope to get a nice homological invariant connected to L^2-Betti numbers for groups. Later on, Andreas Thom introduced the cohomological approach to L^2-Betti numbers for von Neumann algebras, connecting them with derivations. In the talk, I'll present the development of the L^2-invariants for finite von Neumann algebras and explain the recent vanishing results for first continuous L^2-cohomology.

Spectral gap for translation actions on locally compact groups

Remi Boutonnet

Abstract

In this talk, I will present recent work with A. Ioana and A. Salehi-Golsefidi about a new notion of "local" spectral gap for infinite measure preserving actions. The main result states that many translation actions $\Gamma \curvearrowright G$, for Γ a countable dense subgroup in a simple Lie group G, have this property. This generalizes results of Bourgain-Gamburd and Benoist-de Saxcé to a non-compact setting. I will present applications to orbit equivalence rigidity and the Banach-Ruziewicz problem in connection with A. Ioana's minicourse.

Asymptotic shape of equivariant random metrics on nilpotent groups

Mike Cantrell

Abstract

In this talk we will present three seemingly different results about randomness in a finitely generated nilpotent group: an asymptotic shape theorem for First Passage Percolation (FPP); a generalization to random metrics of Pansus theorem that the unique asymptotic cone of a nilpotent group is a particularly nice nilpotent Lie group; a Subadditive Ergodic Theorem for nilpotent groups. The results are all related, and the proof involves sub-Riemannian geometry and Ergodic Theory.

Rigidity aspects in the von Neumann algebras associated with surface braid groups

Ionut Chifan

Abstract

In this talk I will present several recent results regarding the classification of the von Neumann algebras arising from actions of surface braid groups on probability spaces. For instance, we will show such algebras completely remember the initial group/action data. In addition, we show these algebras are prime, i.e., they cannot be decomposed as tensor products of diffuse von Neumann algebras. If time permits I will also discuss a few open problems in this area. This is based on several joint works with A. Ioana, Y. Kida, and S. Pant.

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Harmonic Maps on Groups

Darren Creutz

Abstract

Furstenberg's boundary theory allows us to characterize amenability in terms of the absence of bounded harmonic functions on the group. Building on joint work with Y. Shalom, I will present a similar method for characterizing Kazhdan's Property (T) in terms of the absence of certain harmonic maps on the group. Together, these results give some insight into a potential unified proof of Margulis' Normal Subgroup Theorem (and other Normal Subgroup Theorems).

Ergodic theorems for amenable groups

Alexander Fish

Abstract

We will talk on the validity of the mean ergodic theorem along left Flner sequences in a countable amenable group G. Although the weak ergodic theorem always holds along any left Flner sequence in G, we will provide examples where the mean ergodic theorem fails in quite dramatic ways. On the other hand, if G does not admit any ICC quotients, e.g. if G is virtually nilpotent, then we will prove that the mean ergodic theorem does indeed hold along any left Flner sequence. Based on the joint work with M. Bjorklund (Chalmers).

The Thompson groups F, T, V and their C^*- and von Neumann algebras

Uffe Haagerup

Abstract

In the talk I will give an introduction to the three Thompson groups F, T and V, and discuss some recent results: It is a long standing open problem whether the Thompson group F is amenable. Paul Jolissaint has shown that F is inner amenable and that its von Neumann algebra $L(F)$ has property Gamma. In a recent joint work with Kristian Knudsen Olesen, we prove that T and V are not inner amenable and $L(T)$ and $L(V)$ does not have property Gamma. We also prove that if the reduced C^*-algebra $C_r^*(T)$ of T is simple, then F is non-amenable. Moreover in collaboration with Maria Ramirez-Solano and Soren Haagerup we use extensive numerical computations to test the amenability problem for F by estimating the norms of certain elements of $C_r^*(F)$. Numerical computations alone cannot detect whether or not F is amenable, but the results we have obtained suggest that the most likely outcome is that F is non-amenable.

Unique prime factorization and bicentralizer problem for a class of type III factors

Cyril Houdayer

Abstract

I will show that any tensor product of free Araki-Woods factors retains the length of the tensor product as well as each tensor component up to unitary conjugacy, after permutation of the indices. This provides new Unique Prime Factorization (UPF) results in the framework of type III factors. In order to obtain these UPF results, I will explain a generalization of Popa's intertwining theorem for type III factors. I will also provide new structural results for bicentralizer algebras and show that Connes's bicentralizer problem has a positive solution for all (semi-)solid type III_1 factors. Joint work with Yusuke Isono.

The Furstenberg boundary and C^*-simplicity

Narutaka Ozawa

Abstract

A (discrete) group G is said to be C^*-simple if the reduced group C^*-algebra of it is simple. I will first explain Kalantar and Kennedy's characterization of C^*-simplicity for a group G in terms of its action on the maximal Furstenberg boundary. Then I will talk about my result with Breuillard, Kalantar, and Kennedy about examples and stable properties of C^*-simple groups.

Quasi-isometries of nilpotent groups

Henrik Densing Petersen

Abstract

A. Mal'cev showed in 1949 that every finitely generated, torsion free nilpotent group G embeds in a unique connected simply connected nilpotent Lie group as a cocompact lattice. The ambient Lie group is called the Mal'cev completion of G. In my talk, I will describe the main ideas to show that the Mal'cev completion completely classifies finitely generated torsion free nilpotent groups up to quasi-isometry. This is joint work with David Kyed.

Powers group methods for locally compact groups acting on trees

Sven Raum

Abstract

In this talk we report on our recent work on operator algebras associated with locally compact not necessarily discrete groups acting on a tree. Studying the action on the tree's boundary, we can apply Powers averaging method. We follow an idea of de la Harpe and Praux, who studied C*-simplicity of discrete HNN extensions. For a natural class of locally compact non-discrete groups, we prove C*-simplicity and uniqueness of specific KMS-weights on the reduced group C*-algebra. This answers the question of de la Harpe, whether there are non-discrete C*-simple groups. Our methods are also able to show factoriality and non-amenability of group von Neumann algebras of non-discrete groups, whose type we determine in terms of the modular homomorphism. Surprisingly, we need stronger assumptions on the group to show factoriality than C*-simplicity, which is is contrast to the situation for discrete groups.

Profinite actions and homology growth of aspherical Riemannian manifolds

Roman Sauer

Abstract

We provide upper bounds on the (torsion) homology growth of closed aspherical Riemannian manifolds with residual finite homology growth. An ingredient is the topological dynamics of the fundamental group acting on its completion with respect to a residual chain. We also illustrate connections to measurable variations of the simplicial volume.

Thomas Sinclair

TBA

Oligomorphic groups: representations, property (T), and measure-preserving actions

Todor Tsankov

Abstract

A permutation group G acting on a countable set M is called oligomorphic if its diagonal action on finite powers of M has only finitely many orbits. Such groups arise naturally in model theory as automorphism groups of omega-categorical structures and include the full infinite permutation group, the automorphism group of the linear order $(\mathbb{Q}, <)$, the automorphism group of the random graph, etc. It turns out that in a number of situations, those groups behave in a way similar to compact groups: for example, they all have property (T) and often their dynamical systems can be classified. In the talk, I will concentrate on unitary representations and measure-preserving actions. Some of the results I am going to present are joint work with David Evans.

Jones-Schmidt stability and relative property (T)

Robin Tucker-Drob

Abstract

A discrete probability measure preserving equivalence relation is said to be stable if it is isomorphic to its direct product with the odometer equivalence relation. This notion was introduced by Jones and Schmidt in their seminal 1987 treatment of strong ergodicity. I will discuss recent work on characterizing groups which freely generate stable orbit equivalence relations.

Representation theory for subfactors, λ-lattices and C*-tensor categories

Stefaan Vaes

Abstract

I present a joint work with Sorin Popa in which we develop a representation theory for λ-lattices, arising as standard invariants of subfactors, and for rigid C*-tensor categories, including a definition of their universal C*-algebra. We use this to give a systematic account of approximation and rigidity properties for subfactors and tensor categories, like (weak) amenability, the Haagerup property and property (T). I will also explain the relations between our representation theory and the recent approaches using unitary half braidings (Neshveyev-Yamashita), and representations of Ocneanu's tube algebra (Ghosh-Jones).

Weak amenability and Haagerup property for generalized Baumslag-Solitar groups.

Alain Valette

Abstract

A finitely generated group G is a generalized Baumslag-Solitar group of rank n if G acts cocompactly on a tree, with all edge and vertex stabilizers isomorphic to $\mathbb{Z}^n$. For such a group, there exists a *monodromy representation* $\mu : G \to GL_n(\mathbb{R})$. I will sketch the proof of the following (joint with Y. Cornulier)

Theorem: For G a generalized Baumslag-Solitar group of rank n, TFAE: a) G has the Haagerup property; b) G is weakly amenable with Cowling-Haagerup constant 1; c) The closure of $\mu(G)$ is an amenable subgroup of $GL_n(\mathbb{R})$

I will then explain how M. Carette exploited this result to prove that Haagerup property is not a quasi-isometric invariant.

Injectivity and disjointness for the radial masa

Chenxu Wen

Abstract

The study of the inclusion of amenable subalgebras inside II_1 factors leads to many important notions in the theory such as regularity, singularity, solidity, etc. Popa showed the first concrete examples of maximal amenable subalgebras inside a II_1 factor. Subsequent work on maximal amenable subalgebras has mostly revolved around a property due to Popa, called the asymptotic orthogonal property (AOP). Only recently, a new approach via the study of centralizers was developed by Boutonnet and Carderi. We show a stronger version of AOP which implies the "disjointness property" that any distinct maximal amenable subalgebra cannot have diffuse intersection with the radial masa. This confirms partially a conjecture of Jesse Peterson.

Dynamics and K-theory

Guoliang Yu

Abstract

I will introduce a notion of dynamic asymptotic dimension for group actions and discuss its application to K-theory and rigidity of manifolds. This is joint work with Erik Guentner and Rufus Willett.
