NCGOA 2023: Von Neumann Algebras

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organized by
Dietmar Bisch (Vanderbilt Universty) and Jesse Peterson (Vanderbilt University)

Abstracts

Bat-Od Battseren (RIMS, Kyoto University)

Von Neumann equivalence and $\mathbf{M_d}$-type approximation properties

Abstract: M_d type approximation properties are recently introduced weaker versions of amenability. When $d = 2$, these are just weak amenability, approximation property, and weak Haagerup property. In this talk, I will explain that these properties are stable under von Neumann equivalence, Measure Equivalence, and W*-equivalence. We will also see that these properties are inherited from lattices.

Quan Chen (Ohio State University)

A categorical Connes' $\chi(M)$

Abstract: Popa introduced the tensor category Chi(M) of approximately inner, centrally trivial bimodules of a II_1 factor M, generalizing Connes' $\chi(M)$. We construct a unitary braiding on Chi(M), generalizing Jones' kappa invariant on $\chi(M)$. Given a finite depth inclusion $M_0 \subseteq M_1$ of non-Gamma II_1 factors, we show that the braided unitary tensor category $\mathrm{Chi}(\mathrm{M}_\infty)$ is equivalent to the Drinfeld center of the standard invariant, where M_∞ is the inductive limit of the associated Jones tower. I will talk about Connes' short exact sequence of $\chi(M \rtimes G)$ and the in-progress work on the gauging theory of Chi(M) if time permits. This is joint work with Corey Jones and David Penneys.

Ionut Chifan (University of Iowa)

Wreath-like product groups and rigidity aspects of their von Neumann algebras

Abstract: Wreath-like products are a new class of groups, which are close relatives of the classical wreath products. Examples of wreath-like product groups arise as quotients of non-elementary (relative) hyperbolic groups via deep methods in geometric group theory. As a consequence, unlike classical wreath products, many wreath-like products have Kazhdan's property (T). In this talk, we will present several rigidity and structural results for von Neumann algebras associated with wreath-like product groups.

For a large class of property (T) wreath-like product groups, we show all isomorphisms between their von Neumann algebras arise from isomorphisms or multiplicative characters of the underlying groups; in particular, this provides the first positive evidence towards a well-known conjecture of V.F.R. Jones. Combining this with a new group theoretic development, we establish a converse to a classical result of A. Connes: any countable group can be realized as the outer automorphism group of a property (T) group von Neumann algebra. For a fairly large family our property (T) von Neumann algebra we are able to describe explicitly all possible embeddings between them. Pairing this with a result in group

theory, it yields examples of property (T) von Neumann algebras whose endomorphisms are all inner, their fundamental semigroup is trivial and their Jones' invariant is contained in the set of positive integers.

In the second part, we will highlight new families of property (T) wreath-like products which are completely recognizable from both their von Neumann algebras, L(G) and their reduced C*-algebras, $C_r{}^*(G)$. Then, we will explain how wreath-like products can be used to show the category of property (T) factors is embedding universal. This has several applications to the structure of II_1 factors arising from model theoretic considerations.

This is based on several works with Daniel Drimbe, Adrian Ioana, Denis Osin and Bin Sun.

Changying Ding (Vanderbilt University)

Biexact von Neumann algebras

Abstract: The notion of biexactness for groups was introduced by Ozawa in 2004 and has since become a major tool used for studying solidity of von Neumann algebras. We introduce the notion of biexactness for von Neumann algebras, which allows us to place many previous solidity results in a more systematic context, and naturally leads to extensions of these results. We will also discuss examples of solid factors that are not biexact. This is a joint work with Jesse Peterson.

Ben Hayes (University of Virginia)

Applications of free entropy to the study of tracial von Neumann algebras

Abstract: Voiculescu's free entropy dimension theory is a powerful tool to prove many indecomposability problems in II_1-factors, particularly for free group factors. It led to the first proof of abscence of Cartan subalgebras as well as primeness of free group factors. The 1-Bounded entropy was implicitly defined by Jung and explicitly by myself as a refinement of free entropy dimension which can be shown to be an invariant. I will explain how this invariant is related to many recent problems of interest, such as the Peterson-Thom conjecture. Time permitting, I will also discuss other topics such as strong 1-boundedness of Property (T) von Neumann algebras (due to myself, Jekel, and Kunnawalkam Elayavlli).

Roberto Hernandez Palomares (University of Waterloo)

Discrete inclusions of C*-algebras

Abstract: We will describe a class of inclusions of C*-algebras that admits a notion of a standard invariant akin to that of subfactors. We consider unital inclusions of C*-algebras with a conditional expectation, and characterize the abstract property of C*-discreteness precisely in terms of inclusions obtained from a generalized crossed product construction. These crossed products arise from what we call a C*-quantum dynamics and a generalized Q-system, which are given formally by an action of a unitary tensor category on a C*-algebra and a C*-algebra object from that category. Our formalism includes many infinite-index inclusions, and during the talk, we will review synthetic and natural examples of C*-discrete inclusions. As an application, we will describe a generalized Galois correspondence for C*-algebras, describing a certain lattice of intermediate C*-subalgebras to a given C*-discrete inclusion in terms of a lattice of C*-algebra objects. Time allowing, we will present an application of our constructions to the Elliott Classification Program.

Peter Huston (Vanderbilt University)

Skein modules, dome algebras, and string-net models for topological order

Abstract: Skein modules for unitary fusion categories give rise to the Ocneanu tube algebra, whose representations form the Drinfeld center of the fusion category, as well as (2+1)D string net models of topological order. The tube algebra acts as local operators on anyonic excitations in such string net models. Chiral (2+1)D topological orders arise on the boundaries of invertible (3+1)D topological orders. Mathematically, such a situation corresponds to a fusion category enriched over a modular tensor category. Enriched skein modules can be used to construct the dome algebra, whose representations form the enriched center, and string-net models of the (2+1)D boundaries, where the dome algebra again classifies anyonic excitations.

Corey Jones (North Carolina State University)

Quantum cellular automata on nets of algebras

Abstract: Quantum cellular automata (QCA) are automorphisms with bounded spread of the quasi-local C*-algebra of a spin system on a lattice $L \subseteq \mathbb{R}^n$. They provide models for discrete-time unitary dynamics, and there is significant interest in the physics community in classifying QCA up to finite depth circuits. Motivated by the recent emergence of floquet topological phases, we introduce a generalization of QCA and the classification problem for arbitrary nets of operator algebras on a lattice. We will explain how to use ideas from algebraic quantum field theory to build new topological invariants in this setting. We apply this theory to a 1D spin system with on-site group symmetry to show that surprisingly, the group of QCA modulo symmetric circuits can be non-abelian.

Kate Juschenko (University of Texas at Austin)

Topological amenability and Kesten's criteria

Abstract: We will discuss results connecting the uniform Liuoville property of group actions on the classes of a countable Borel equivalence relation with amenability of this equivalence relation. We also discuss extensions of Kesten's theorem to certain amenable topological groups and prove a version of this theorem for amenable groups with the SIN property. Furthermore, we discuss potential implications of generalizations of Kesten's theorem for concentration inequalities on the inverted orbits of random walks on the classes of an amenable equivalence relation. This is joint with Max Chaukhari and Martin Schneider.

Merhdad Kalantar (University of Houston)

Generalized complexification for operator algebras and operator spaces

Abstract: Complexification is the main technical tool in the study of real spaces, it allows to pass to a canonical complex space which shares almost all structural properties with the original real space. We give a generalization of this process (that also generalizes 'quaternionification' of real spaces), and extend some of the main results, including Ruan's uniqueness theorem for "reasonable" operator space complexifications. This gives a more conceptual understanding of these notions, and allows to unify main results regarding complexification and quaternionification of real spaces.

This is joint work with David Blecher.

Srivatsav Kunnawalkam Elayavalli (IPAM, UCLA)

On the story of matrices of high finite order: Then (1942) and now (2022)

Abstract: Matrix ultraproducts are large but fascinating non separable beasts. In light of the Connes embedding problem, which asks if matrix ultraproducts are embedding universal for separable II_1 factors, their study is of utmost importance to both intrinsic considerations in operator algebras, and also to deep external problems in many other areas including quantum information theory. I will begin by discussing a utterly remarkable yet largely neglected paper of von Neumann from 1942 that can be seen a precursor to the modern systematic study of asymptotic representation theory, especially manifesting in the theory of random matrices. This result is proved using a technique which he named the 'volumetric method'. The first result that allowed one to track the isomorphism problem of matrix ultraproducts among other ultrapowers of II_1 factors is this 1942 theorem of von Neumann, that shows that the ultrapower of a factor with property Gamma can never be isomorphic to any matrix ultraproducts. In this talk I will discuss my recent 2022 result joint with I. Chifan and A. Ioana that on the flipside there are examples of non Gamma factors whose ultrapowers can never be isomorphic to any matrix ultraproducts. The core idea of proof uses a sophisticated refinement of the 'volumetric method' in Voiculescu's free entropy theory in the form of the 1-bounded entropy of Jung and Hayes, and crucially features other modern ideas such as Popa's free malleable deformation, Property T and weak spectral gap.

Brent Nelson (Michigan State University)

Von Neumann dimension for faithful normal strictly semifinite weights

Abstract: The notion of von Neumann dimension for a tracial von Neumann algebra (M, τ) has been used extensively throughout the theory, including to define other numerical invariants from Jones' index of a subfactor to Connes and Shlyakhtenko's ℓ^2-Betti numbers of von Neumann algebras. The latter relies on work of Lück showing that Murray and von Neumann's original definition could be extended to purely algebraic M-modules, and more recently Petersen further extended von Neumann dimension to pairs (M, τ) where τ is a faithful normal semifinite tracial weight. In this talk, I will introduce a yet further extension of this theory to pairs (M, φ) where φ is a faithful normal strictly semifinite weight. Here strict semifiniteness means the restriction of φ to the centralizer subalgebra $M^\varphi \subset M$ is still semifinite (note this condition is automatic for faithful normal states), and by work of Takesaki this is equivalent to the existence of a φ-invariant faithful normal conditional expectation $\mathcal{E}_\varphi \colon M \to M^\varphi$. Consequently, one can consider the Jones basic construction for the inclusion $M^\varphi \subset M$, and this is the key ingredient in our definition of von Neumann dimension for the pair (M, φ). I will discuss properties of this dimension and how it can be used to recover the index for certain inclusions of type III factors. This is based on joint work with Aldo Garcia Guinto and Matthew Lorentz.

David Penneys (Ohio State University)

Local topological order and boundary algebras

Abstract: We introduce a set of local topological order (LTO) axioms for spin systems on the lattice $\mathbb{Z}^k$, which produces a net of algebras on a codimension 1 sublattice $\mathbb{Z}^{k-1}$ which we call the net of boundary algebras. The LTO axioms are satisfied for the Toric Code and Levin-Wen models. The nets of boundary algebras for these models are bounded-spread isomorphic to nets of algebras

coming from unitary fusion categories. For these fusion categorical nets, Corey Jones' category of DHR bimodules recovers the Drinfeld center, leading to a bulk-boundary correspondence where the bulk topological order is described by representations of the boundary net.

Emily Peters (Loyola University Chicago)

Categorifying connections

Abstract: The road between subfactors and fusion categories is well-trodden in both directions; subfactors provide examples of fusion categories not seen elsewhere, and fusion category techniques can be applied in, eg, the construction and classification of subfactors. In this talk we translate the main ideas of Ocneanu's paragroup theory into tensor categorical language; we begin by explaining why connections on bipartite graphs correspond to a pair of functors from the Temperley-Lieb-Jones categories into a graph category, and commutator data.

Eric Rowell (Texas A&M University)

The loop braid group and charge conserving Yang–Baxter operators

Abstract: The loop braid group LB_n is the motion group of n free loops in S^3, with generators the "leapfrog" motions and the symmetric exchanges. This can be viewed as a 3-dimensional analogue of the braid group, which is the motion group of points in a disk. The representation theory of LB_n is largely unexplored, but is of interest in the study of 3D topological phases as well as in algebra and topology. In recent work with Celeste Damiani and Paul Martin we defined finite dimensional quotients of the loop braid group algebras, analogous to the Hecke algebra construction from braid group algebras. We also found (conjecturally faithful) local representations via a "charge conserving" Yang-Baxter operator. In a follow up paper with Martin we classified all charge conserving Yang-Baxter operators, and determined, in a 3rd paper with Fiona Torzewska, when these give rise to representations of the loop braid group. I will summarize this trilogy, emphasizing the surprising combinatorics we encountered along the way.

Thomas Sinclair (Purdue University)

A notion of index for finite-dimensional operator systems

Abstract: Inspired by a well-known characterization of the Jones index of an inclusion of II_1 factors due to Pimsner and Popa, we define an index-type invariant for inclusions of finite-dimensional operator systems. We will compute examples of this invariant, and explain how it generalizes the Lovasz theta invariant to general matricial systems in a manner that is closely related to the quantum Lovasz theta invariant defined by Duan, Severini, and Winter.

Noah Snyder (Indiana University)

Oligomorphic groups, measures, and the Delannoy category

Abstract: The prototypical example of a symmetric tensor category is Rep(G) for G a compact group. The other main source of symmetric tensor categories are Deligne's interpolation categories (S_t, GL_t, and O_t for t a complex number) which extend a family of group representation categories defined at positive integers to all complex numbers. These categories are typically described diagrammatically

in the style of Jones's Planar Algebras, where the objects are points on a line and the morphisms are linear combinations of certain diagrams. Rep(G) is a "concrete" category, meaning its objects are vector spaces with certain structure, while the interpolation categories are "abstract" categories since the objects themselves are just points with no internal structure. A natural question is whether one can find a concrete realization of these interpolation categories. This question should be familiar from subfactor theory, where abstract tensor categories can be realized concretely as categories of bimodules over a factor. For the interpolation categories, since some objects have negative dimension, it is impossible to use an operator algebraic construction.
In deep recent work, Harman and Snowden construct Deligne's interpolation category S_t concretely as a category of certain kinds of representations of the infinite symmetric group. More generally they develop a theory of well-behaved measures on Oligomorphic permutation groups and in certain circumstances use this to construct a symmetric tensor category in a way that generalizes Deligne's S_t. These measures are unusual in that they're allowed to take negative values! In addition to Deligne's S_t there are several other interesting examples of Oligomorphic groups with well-behaved measures, The simplest new example is the group of order-preserving bijections of the real line with the measure given by Euler characteristic. In joint work with Harman and Snowden we give a detailed description of this new symmetric tensor category, which has a number of novel properties. In this talk I aim to sketch the general theory of Harman-Snowden and then illustrate how it works in this special case coming from the line, which we call the Delannoy category because the dimensions of Hom spaces are given by Delannoy numbers.

Hui Tan (UCSD)

On existentially closed II_1 factors

Abstract: A separable II_1 factor M is existentially closed if and only if for any separable II_1 factor N containing M, there is an embedding of N into an ultrapower of M that restricts to the diagonal embedding on M. I will talk about the structure of bimodules of an existentially closed separable II_1 factor. I will also discuss the notion of existential closedness within the class of full factors. This is joint work with Adrian Ioana.

Hans Wenzl (UCSD)

Module categories and subfactors from quantum groups

Abstract: Any inclusion of type II_1 factors $N \subset M$ produces a tensor category of $N - N$ bimodules and a right module category of $M - N$ bimodules. Conversely, any module category of a suitably nice tensor category gives examples of subfactors. We give a survey of recent progress in the classification of module categories for fusion categories coming from quantum groups.

Feng Xu (UC Riverside)

Rigorous results about entropies in QFT

Abstract: I will discuss recent work on rigorous results about entropies in QFT, with particular emphasis on singular limits of relative entropies in CFT.

Jun Yang (Harvard University)

Limit multiplicities and von Neumann dimensions

Abstract: Given a semi-simple real Lie group G and a tower of discrete subgroups in it, we review the limit multiplicity problem, which deals with the distribution of irreducible representations of a semi-simple real Lie group G in the space of L^2 functions on the quotient. When the quotient is compact, the Selberg trace formula can be applied to describe the multiplicities. However, for the non-compact case, the multiplicity problem is much more complicated. We will apply the Arthur-Selberg trace formula with an approach from von Neumann algebras, and state the results on towers of arithmetic subgroups in SL(n,$\mathbb{R}$), in which case the limit multiplicities are the von Neumann dimensions.