Colloquium. Academic Year 19-20

Thursdays 4:10pm on Zoom, unless otherwise noted

Colloquium Chair (2020-2021): Doug Hardin

September 5, 2019 (Thursday), 4:10 pm

Orthogonal Fourier Analysis on Domains

Mihalis Kolountzakis, University of Crete
Location: Stevenson 5211
We all know how to do Fourier Analysis on an interval or on the entire real line. But what if our functions live on another subset of Euclidean space, let's say on a regular hexagon in the plane? Can we use our beloved exponentials, functions of the form $e_\lambda(x) = {\rm exp}(2\pi i \lambda \cdot x)$ to analyze the functions defined on our domain? In other words, can we select a set of frequencies such that the corresponding exponentials form an *orthogonal basis* for L^2 of our domain? It turns out that the existence of such an orthogonal basis depends heavily on the domain. So the answer is yes, we *can* find an orthogonal basis of exponentials for the hexagon, but if we ask the same question for a disk, the answer turns out to be no. B. Fuglede conjectured in the 1970s that the existence of such an exponential basis is *equivalent* to the domain being able to *tile* space by translations (the hexagon, that we mentioned, indeed can tile, while the disk cannot). In this talk we will track this conjecture and the mathematics created by the attempts to settle it and its variants. We will see some of its rich connections to geometry, number theory and harmonic analysis and some of the spectacular recent successes in our efforts to understand exponential bases. Tea at 3:33 pm in Stevenson 1425. (Host: Akram Aldroubi)

September 12, 2019 (Thursday), 4:10 pm

Decay Estimates for the Wave Equation on Manifolds

Jacob Sterbenz, University of California at San Diego
Location: Stevenson 5211
We'll discuss long time decay estimates for the wave equation on a general class of asymptotically flat metrics. In the case of nontrapping metrics, when the operators are symmetric with slow time variation and a zero energy spectral condition, we'll discuss local energy decay modulo finite dimensional dynamics. For a more general class of metrics, including black holes, we'll discuss a general vector field method which takes local energy decay as an assumption. When used in combination, and also with the recent work of Dafermos, Rodnianski, and Shlapentokh-Rothman, these results establish a general asymptotic theory for both linear and null form equations on a wide variety of backgrounds. This is joint work with Jason Metcalfe, Jesus Oliver, Daniel Tataru. Tea at 3:33 pm in SC 1425. (Contact Person: Alex Powell)

September 19, 2019 (Thursday), 4:10 pm

The Strong Cosmic Censorship Conjecture in General Relativity

Jonathan Luk, Stanford University
Location: Stevenson 5211
The Einstein equations in general relativity admit explicit black hole solutions which have the disturbing property that global uniqueness fails. As a way out, Penrose proposed the strong cosmic censorship conjecture, which says that this phenomenon of global non-uniqueness is non-generic. We will discuss this conjecture and some recent mathematical progress. This talk is based on joint works with Mihalis Dafermos, Sung-Jin Oh, Jan Sbierski and Yakov Shlapentokh-Rothman. Tea at 3:33 pm in Stevenson 1425. (Contact Person: Jared Speck)

September 26, 2019 (Thursday), 4:10 pm

Around Cantor Uniqueness Theorem

Alexander Olevskii, Tel Aviv University
Location: Stevenson 5211
After a short introduction to Riemann's uniqueness theory I will discuss the following problem, going back to early 60-s: Can a (non-trivial) trigonometric series
\sum c(n) e^inx ,   c(n) = o(1),
have a subsequence of partial sums which converges to zero everywhere? Joint work with Gady Kozma. Tea at 3:33 pm in SC 1425. (Host: Akram Aldroubi)

October 10, 2019 (Thursday), 4:10 pm

Irreducibility in Complex Dynamics

Sarah Koch, University of Michigan
Location: Stevenson 5211
A major goal in complex dynamics is to understand dynamical moduli spaces; that is, conformal conjugacy classes of holomorphic dynamical systems. One of the great successes in this regard is the study of the moduli space of quadratic polynomials; it is isomorphic to $\mathbb C$. This moduli space contains the famous Mandelbrot set, which has been extensively studied over the past 40 years. Understanding other dynamical moduli spaces to the same extent tends to be more challenging as they are often higher-dimensional. In this talk, we will begin with an overview of complex dynamics, focusing on the moduli space of quadratic rational maps, which is isomorphic to $\mathbb C^2$. We will explore this space, finding many interesting objects along the way. We will then focus on special algebraic curves, called "Milnor curves" in this space. In general, it is unknown if Milnor curves are irreducible over $\mathbb C$. Because these curves are smooth, this is equivalent to asking whether they are connected. We will exhibit an infinite collection of Milnor curves that are connected. This is joint work with X. Buff and A. Epstein. Tea at 3:30pm in SC 1425. (Contact Person: Spencer Dowdall)

October 17, 2019 (Thursday), 4:10 pm

Quantum Computation and Universal Algebra

Matthew Moore, University of Kansas
Location: Stevenson 5211
Digital computers perform calculations on individual strings of bits. Quantum computers extend classical digital computers by making use of quantum phenomena to perform calculations on bits which are in a state of superposition. For some classes of problems, this allows for superpolynomial speedup over classical algorithms. In the first half of the talk we give an introduction to quantum computation and to the various classes of problems exhibiting superpolynomial speedup. In the second half, we propose generalizations of these problems to the domain of universal algebras and present some results for the quantum tractability of these generalizations. Tea at 3:30pm in SC 1425. (Contact person: Ralph McKenzie)

November 14, 2019 (Thursday), 4:10 pm

Invariants of Rings via Equivariant Homotopy

Teena Gerhardt, Michigan State University
Location: Stevenson 5211
Algebraic K-theory is an invariant of rings which illustrates a fascinating interplay between algebra and topology. Defined using topological tools, this invariant has important applications to algebraic geometry, number theory, and geometric topology. One fruitful approach to computing algebraic K-theory uses equivariant homotopy theory, a branch of algebraic topology which studies topological objects with a group action. In this talk I will give an introduction to algebraic K-theory and its applications, and talk about modern methods to compute algebraic K-theory.

February 6, 2020 (Thursday), 4:10 pm

The Stark Conjectures and Equiangular Complex Lines

Jeffrey C. Lagarias, University of Michigan
Location: Stevenson 1206
Hilbert's 12-th Problem concerns the construction of class fields using special values of analytic functions, especially modular forms. The Stark Conjectures (work of my advisor Harold M. Stark) conjecturally construct various class fields, particularly real quadratic fields, using special values of L-functions at s = 0. We review Hilbert's 12th problem and the Stark conjectures and describe an unexpected connection of the Stark Conjectures to the (conjectured) existence of maximal systems of complex equiangular lines in Cn, known as SIC- POVM's in quantum information theory. The latter is work of my student Gene Kopp (PhD 2017).

February 13, 2020 (Thursday), 4:10 pm

Orthogonality Relations for GL(n)

Dorian Goldfeld, Columbia University
Location: Stevenson 1206
Orthogonality is a fundamental theme in representation theory and Fourier analysis. In the case of a finite abelian group G, the orthogonality relation for characters of G was used by Dirichlet in 1837 to prove that there are infinitely many primes in an arithmetic progressions a,a+d,a+2d,a+3d,… provided a,d are co-prime positive integers. This type of orthogonality relation occurs on GL(1) over the adele group of $\mathbb{Q}$. When considering automorphic representations for GL(n) with n>1, however, the automorphic representations are infinite dimensional and it is not so clear how to even formulate an orthogonality relation. We shall survey what is known (including applications to number theory) and introduce new results for the real group GL(4,$\mathbb{R}$). This talk is based on recent joint work with Eric Stade and Michael Woodbury and is aimed at a general audience.

February 20, 2020 (Thursday), 4:10 pm

Emergent Behavior in Collective Dynamics

Eitan Tadmor, University of Maryland
Location: Stevenson 1206
A fascinating aspect of collective dynamics is the self-organization of small-scales and their emergence as higher-order patterns — clusters, flocks, tissues, parties. The emergence of different patterns can be described in terms of few fundamental “rules of interactions”. I will discuss recent results of the large-time, large-crowd dynamics, driven by anticipation that tend to align the crowd, while other pairwise interactions keep the crowd together and prevent over-crowding. In particular, I address the question how short-range interactions lead to the emergence of long-range patterns, comparing different rules of interactions based on geometric vs. topological neighborhoods.

February 27, 2020 (Thursday), 4:10 pm

End Sums of Open Manifolds

Craig Guilbault, University of Wisconsin-Milwaukee
Location: Stevenson 1206
Connected sums and boundary connected sums play important roles in the study of closed manifolds and manifolds with boundary, respectively. When working with open manifolds, a third variety of connected sum—the “end sum”—is useful. Each of these operations involves a number of arbitrary choices, making well-definedness of the resulting manifold a significant question. With regards to the end sum operation, we will discuss familiar situations where all goes smoothly (the notion of semistability plays a role here) and others where significant problems arise. Our analysis leads naturally into the subtle and interesting theory of infinitely generated abelian groups. The work to be presented in this talk is joint with Jack Calcut and Patrick Haggerty.

April 23, 2020 (Thursday), 4:00 pm

Arbitrarily Small Perturbations of Integrable KAM Systems Which are metric(!) Entropy Expansive; and Exponentially Many Collisions in Hard Ball Gas Systems

Dmitri Burago, Penn State University
Virtual Colloquium via Zoom

1. One of the greatest achievements in Dynamics in the XX century is the KAM Theory. It says that a small perturbation of a non-degenerate completely integrable system still has an overwhelming measure of invariant tori with quasi-periodic dynamics. What happens outside KAM tori remains a great mystery. The main quantitate invariants so far are entropies. It is easy, by modern standards, to show that topological entropy can be positive. It lives, however, on a zero measure set. We are now able to show that metric entropy can become infinite too, under arbitrarily small C^{infty} perturbations, answering an old-standing problem of Kolmogorov. Furthermore, a slightly modified construction resolves another long–standing problem of the existence of entropy non-expansive systems. In these modified examples positive metric entropy is generated in arbitrarily small tubular neighborhoods of one trajectory. Joint with S. Ivanov and Dong. Chen.
2. Just 20 years ago the topic of my talk at the ICM was a solution of a problem which goes back to Boltzmann and has been formulated mathematically by Ya. Sinai. The conjecture of Boltzmann-Sinai states that the number of collisions in a system of n identical balls colliding elastically in empty space is uniformly bounded for all initial positions and velocities of the balls. The answer is affirmative and the proven upper bound is exponential in n. The question is how many collisions can actually occur. On the line, one sees that there can be $n(n-1)/2$ collisions, and this is the maximum. Since the line embeds in any Euclidean space, the same example works in all dimensions. The only non-trivial (and counter-intuitive) example I am aware of is an observation by Thurston and Sandri who gave an example of 4 collisions between 3 balls in R^2. Recently, Sergei Ivanov and me proved that there are examples with exponentially many collisions between n identical balls in R^3, even though the exponents in the lower and upper bounds do not match.

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