Boundary Estimates for Quasilinear Parabolic Equations

Let E be an open set in $\mathbb{R}^N$, and for $T > 0$ let E_T denote the cylindrical domain $E \times (0, T]$. Consider quasi-linear, parabolic differential equations of the form

$$u_t - \operatorname{div} \mathbf{A}(x, t, u, Du) = 0 \quad \text{weakly in } E_T \tag{1}$$

where the function $\mathbf{A} : E_T \times \mathbb{R}^{N+1} \to \mathbb{R}^N$ is only assumed to be measurable and subject to the structure conditions

$$\begin{cases} \mathbf{A}(x, t, u, Du) \cdot Du \geq C_o |Du|^p \\ |\mathbf{A}(x, t, u, Du)| \leq C_1 |Du|^{p-1} \end{cases} \quad \text{a.e. } (x, t) \in E_T \tag{2}$$

where C_o and C_1 are given positive constants, and $p > \frac{2N}{N+1}$. The prototype of such a class of parabolic equations is the well–known parabolic p–laplacian

$$u_t - \operatorname{div} |Du|^{p-2} Du = 0 \quad \text{weakly in } E_T. \tag{1$_o$}$$

If E is a Lipschitz domain and $u = 0$ on a portion of the lateral boundary $\partial E \times (0, T)$, I will show that both in the degenerate $p > 2$, and in the singular super-critical range $\frac{2N}{N+1} < p < 2$, solutions satisfy proper Carleson estimates on such a portion.

This joint work with Benny Avelin (University of Umeå, Sweden) and Sandro Salsa (Polytechnics of Milan, Italy), extends well-known results for linear parabolic equations with bounded and measurable coefficients.