Multibump solutions of nonlinear Schrödinger equations with steep potential well and indefinite potential

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Abstract

We are concerned with the existence of single- and multi-bump solutions of the equation $-\Delta u + (\lambda a(x) + a_0(x))u = |u|^{p-2}u,\ x \in \mathbb{R}^N$; here $p > 2$, and $p < \frac{2N}{N-2}$ if $N \geq 3$. We require that $a \geq 0$ is in $L^\infty_{loc}(\mathbb{R}^N)$ and has a bounded potential well Ω, i.e. $a(x) = 0$ for $x \in \Omega$ and $a(x) > 0$ for $x \in \mathbb{R}^N \setminus \bar{\Omega}$. Unlike most other papers on this problem we allow that $a_0 \in L^\infty(\mathbb{R}^N)$ changes sign. Using variational methods we prove the existence of multibump solutions u_λ which localize, as $\lambda \to \infty$, near prescribed isolated open subsets $\Omega_1, \ldots, \Omega_k \subset \Omega$. The operator $L_0 := -\Delta + a_0$ may have negative eigenvalues in Ω_j, each bump of u_λ may be sign-changing.