Asymptotic Behavior of Rigid Bodies with a Liquid-Filled Gap

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We consider the system constituted by a rigid body $\mathcal{B}$ having a hollow cavity which (strictly) contains a homogeneous rigid ball $\mathcal{B}_R$. The gap between these rigid bodies is completely filled by a viscous incompressible fluid (simply called *liquid*), and whose motion is governed by the Navier-Stokes equations. We assume that the whole system *rigid bodies with a liquid-filled gap* is constrained to move (without friction) around the center, G, of the ball $\mathcal{B}_R$. No external forces are applied on the whole system which then moves driven by its inertia once an initial angular momentum is applied.

For a large class of configurations for the liquid and the solid $\mathcal{B}$, we show that the long-time behavior of weak solutions (*à La Leray-Hopf*) corresponding to initial data having (arbitrary) finite kinetic energy is characterized by a steady state in which the rigid body $\mathcal{B}$ is rotating with a constant angular velocity, whereas the liquid and the ball $\mathcal{B}_R$ are in a rest state relative to $\mathcal{B}$. More precisely, the velocity of the liquid relative to $\mathcal{B}$ tends to zero as time approaches to infinity, and the system rigid bodies with a liquid-filled gap rotates as a whole rigid body with constant angular velocity around one of the principal axes of the tensor $\boldsymbol{I}_1 + \boldsymbol{I}_{\mathcal{L}}$, where $\boldsymbol{I}_1$ and $\boldsymbol{I}_{\mathcal{L}}$ are the inertia tensors with respect to G of the rigid body $\mathcal{B}$ and of the liquid, respectively.