# Moduli of continuity for solutions to degenerate phase transitions

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**Abstract**

The classical two-phase Stefan problem is an archetypal free boundary problem that models a phase transition at constant temperature. It consists of solving the heat equation (or nonlinear variants of it) in the solid and liquid phases, coupled with the so-called Stefan condition at the *a priori* unknown interface separating them. This condition corresponds to an energy balance, prescribing the proportionality between the jump of the heat flux across the free boundary and its local velocity.

In its weak form, for which any explicit reference to the free boundary is absent, the nonlinear problem can be formulated as

$$\partial_t \beta(u) - \operatorname{div}(|Du|^{p-2} Du) \ni 0 \quad \text{weakly in } E_T, \tag{1}$$

for a function $u : E_T \to \mathbb{R}$ representing the temperature. Here, $E_T \stackrel{\text{def}}{=} E \times (0, T]$, for some open set $E \subset \mathbb{R}^N$, $N \in \mathbb{N}$, $p > 2$, and $T > 0$, whereas $\beta(\cdot)$ is the maximal monotone graph defined by

$$\beta(s) = \begin{cases} s & \text{if } s > 0, \\ [-\nu, 0] & \text{if } s = 0, \\ s - \nu & \text{if } s < 0, \end{cases} \tag{2}$$

for a positive constant $\nu$, the latent heat of the phase transition, representing the aforementioned proportionality ratio.

We substantially improve in two scenarios the current state-of-the-art modulus of continuity for weak solutions to (1)–(2): for $p = N \geq 3$, we sharpen it to

$$\boldsymbol{\omega}(r) \approx \exp(-c|\ln r|^{\frac{1}{N}}),$$

and for $p > \max\{2, N\}$, we derive an unexpected Hölder modulus.

This is a joint work with Naian Liao (Universität Salzburg, Austria) and José Miguel Urbano (KAUST, Saudi Arabia and University of Coimbra, Portugal)