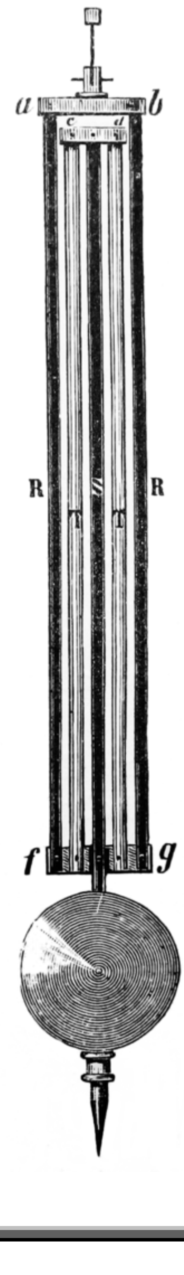
General Physics Laboratory I
PHYS 1601L

Laboratory Manual
Spring 2016



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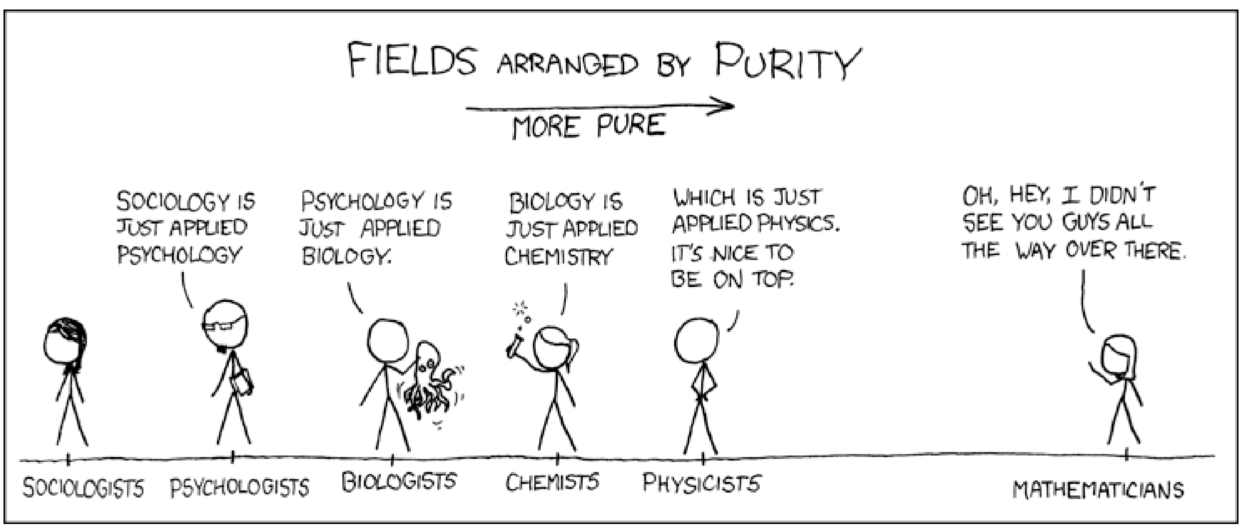
Acknowledgment

While this manual is (in at least the legal sense) my own work, I am very much indebted to my predecessors and colleagues, particularly Ken Shriver, Richard Helms, and Sherry Thompson. I am also grateful for much valuable feedback from my Teaching Assistants. About the only parts of the manual which are completely my own are the many mistakes. As you discover these errors, please point them out to your TA.

Oh, and I am also indebted to Randall Munroe, Wikipedia, and all who contribute to the Creative Commons. I plan to join them as soon as the lawyers let me.

Forrest T. Charnock



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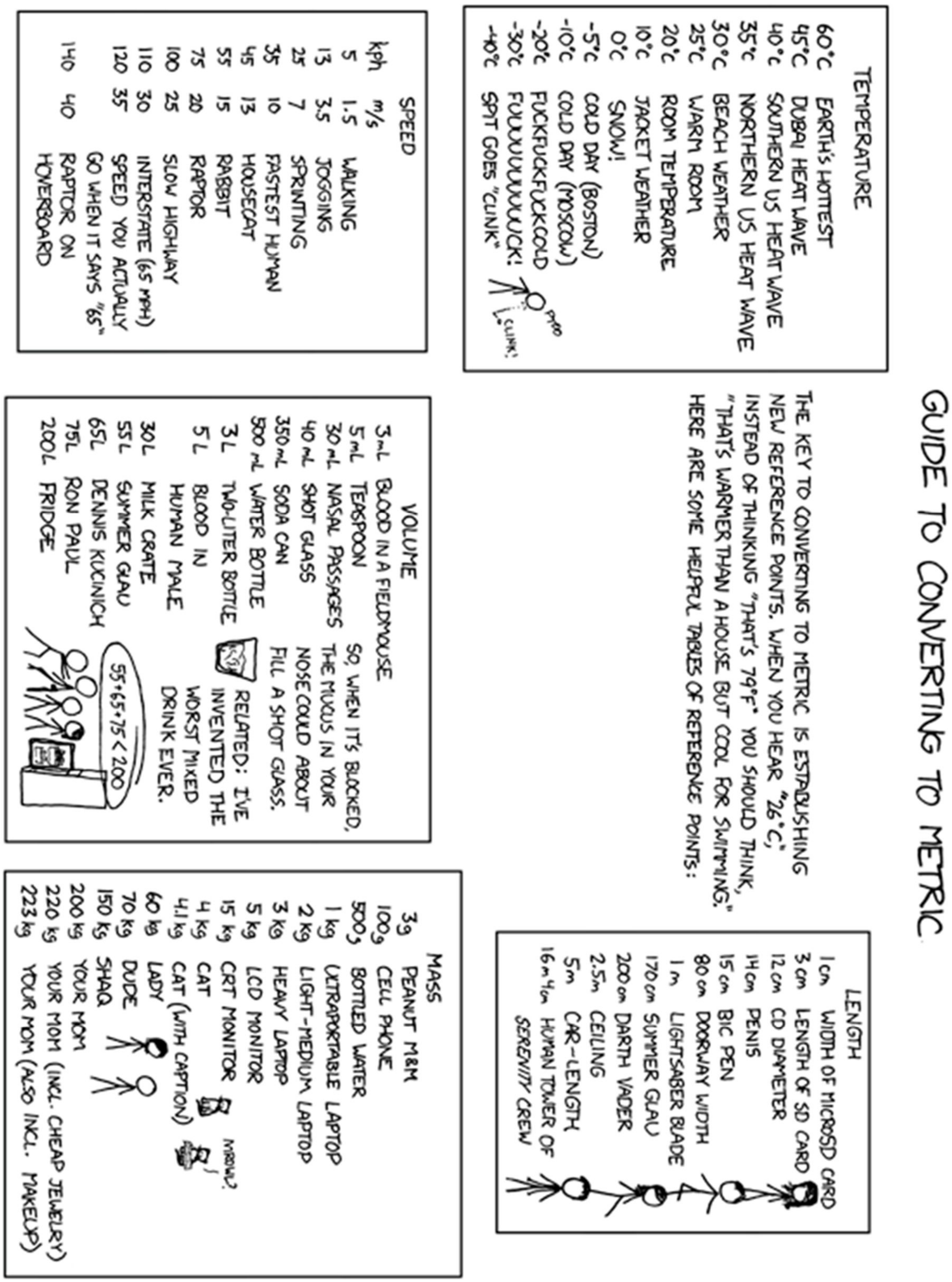
Front Illustration: *Diagram of a temperature compensating gridiron pendulum, invented by John Harrison, 1726. If you have never heard of John Harrison, you should. Google him.*


GUIDE TO CONVERTING TO METRIC
THE KEY TO CONVERTING TO METRIC IS ESTABLISHING NEW REFERENCE POINTS. WHEN YOU HEAR "26°C," INSTEAD OF THINKING "THAT'S 79°F" YOU SHOULD THINK, "THAT'S WARMER THAN A HOUSE BUT COOL FOR SWIMMING." HERE ARE SOME HELPFUL TABLES OF REFERENCE POINTS:
TEMPERATURE
60°C EARTH'S HOTTEST
45°C DUBAI HEAT WAVE
40°C SOUTHERN US HEAT WAVE
35°C NORTHERN US HEAT WAVE
30°C BEACH WEATHER
25°C WARM ROOM
20°C ROOM TEMPERATURE
10°C JACKET WEATHER
0°C SNOW!
-5°C COLD DAY (BOSTON)
-10°C COLD DAY (MOSCOW)
-20°C FUCKFUCKFUCKCOLD
-30°C FUUUUUUUUUCK!
-40°C SPIT GOES "CLINK"
PTOO
CLINK!
LENGTH
1cm WIDTH OF MICROSD CARD
3cm LENGTH OF SD CARD
12cm CD DIAMETER
14cm PENIS
15cm BIC PEN
80cm DOORWAY WIDTH
1m LIGHTSABER BLADE
170cm SUMMER GLAU
200cm DARTH VADER
2.5m CEILING
5m CAR-LENGTH
16m 4cm HUMAN TOWER OF SERENITY CREW
SPEED
kph m/s
5 1.5 WALKING
13 3.5 JOGGING
25 7 SPRINTING
35 10 FASTEST HUMAN
45 13 HOUSECAT
55 15 RABBIT
75 20 RAPTOR
100 25 SLOW HIGHWAY
110 30 INTERSTATE (65 MPH)
120 35 SPEED YOU ACTUALLY GO WHEN IT SAYS "65"
140 40 RAPTOR ON HOVERBOARD
VOLUME
3mL BLOOD IN A FIELDMOUSE
5mL TEASPOON
30mL NASAL PASSAGES
40mL SHOT GLASS
350mL SODA CAN
500mL WATER BOTTLE
3L TWO-LITER BOTTLE
5L BLOOD IN HUMAN MALE
30L MILK CRATE
55L SUMMER GLAU
65L DENNIS KUCINICH
75L RON PAUL
200L FRIDGE
SO, WHEN IT'S BLOCKED, THE MUCUS IN YOUR NOSE COULD ABOUT FILL A SHOT GLASS.
RELATED: I'VE INVENTED THE WORST MIXED DRINK EVER.
55+65+75 < 200
MASS
3g PEANUT M&M
100g CELL PHONE
500g BOTTLED WATER
1kg ULTRAPORTABLE LAPTOP
2kg LIGHT-MEDIUM LAPTOP
3kg HEAVY LAPTOP
5kg LCD MONITOR
15kg CRT MONITOR
4kg CAT
4.1kg CAT (WITH CAPTION)
MROWL?
60kg LADY
70kg DUDE
150kg SHAQ
200kg YOUR MOM
220kg YOUR MOM (INCL. CHEAP JEWELRY)
223kg YOUR MOM (ALSO INCL. MAKEUP)

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General Physics Laboratory I

PHYS 1601L

(Prior to Fall 2015, this lab was known as PHYS 116A.)

Contents

Useful Physical Constants[*]

Universal gravitational constant:	$G = 6.67384(80)\times10^{-11}\,\dfrac{m^3}{kg\,s^2}$
Speed of light (exact by definition)	$c = 2.99792458\times10^{8}\,\dfrac{m}{s}$
Avogadro's number	$N_A = 6.02214129(27)\times10^{23}\,\dfrac{1}{mol}$
Boltzmann constant	$k = 1.3806488(13)\times10^{-23}\,\dfrac{J}{K}$
Universal gas constant	$R = 8.3144621(75)\,\dfrac{J}{mol\,K}$
Acceleration due to gravity at Vanderbilt[†]:	$g = 9.7943(32)\,\dfrac{m}{s^2}$
Standard atmospheric pressure	$1\,atm = 1.01325\times10^{5}\,Pa$
Absolute zero	$0\text{ K} = -273.15°\text{C}$

[*] Values of fundamental constants are from NIST (physics.nist.gov/cuu). The number in parentheses is the standard uncertainty of the final digits of the main number. For example: 6.67384 ± 0.00080 = 6.67384(80)

[†] Dr. Medford Webster, Vanderbilt University

Introduction

The Sermon

The speed of light is 2.99792458×10^8 m/s. This is not science.

The Wikipedia entry on Newton's 2nd law of motion is not science.

Nor is the periodic table of the elements.

Science is not a collection of facts. (Not even true facts!) Rather, science is a **process** for figuring out what is really going on. *What is the underlying principle here? How does this relate to some other observation?* If you are not involved in such a process, you are not doing science. A brilliant, dedicated, A+ student memorizing a list of equations is not doing science. A baby dropping peas on the floor to see what happens: now *that's science!!* (Does oatmeal fall too? Let's find out!!)

This is a science lab. I expect you to do some science in it.

"*Yeah, yeah, Dr. Charnock, I've heard this sermon before.*"

Perhaps so, but I have seen too many brilliant and dedicated students who have learned to succeed in their other science classes by learning **lots of stuff**.[*] So, they come into physics planning to memorize every equation they encounter and are completely overwhelmed. **You cannot succeed in physics by learning lots of stuff.** There are simply too many physics problems in the world; you cannot learn them all.

Instead, **you should learn as little as possible!**[†] More than any other science, physics is about fundamental principles, and those <u>few</u> principles[‡] must be the focus of your attention. Identify and learn those fundamental principles and how to use them. Then you can **derive** whatever solution that you need. And that **process** of derivation is the **process** of science.

"OK, thanks for the advice for the class, but this is a lab!!"

It's still about fundamental principles. Look, each week you will come to lab and do *lots of stuff*. By following the instructions and copying (. . . oh, I mean *sharing* . . .) a few answers from your lab partners, you can blunder through each lab just fine. The problem is that the following week you will have a quiz, and you will not remember everything you did in that lab the week before.

When you are doing each lab, consciously relate your experiments to the underlying principles.

How did I measure this? Where did this equation come from? Why are we doing this?

On the subsequent quiz, instead of having to **remember what you did**, you can apply the principles to **figure out what you did**. Trust me. It really is easier this way.

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[*] To get through organic chemistry, sometimes you just have to memorize all those formulas.
[†] . . . but not less.
[‡] F = ma, conservation of energy and momentum, oscillations and waves. You will learn a few more in the second semester.

GOALS AND OBJECTIVES

Physics is about the real world, not some idealized Platonic world that only exists in your head. The purpose of this lab is to relate the theories and equations you are learning in the classroom to reality. Hopefully, we'll convince you that all that physics stuff actually does work. Of course, reality can be messy, and along the way you will learn to deal with experimental uncertainty, loose cables, bad sensors, sticky wheels, temperamental software, temperamental lab partners, your own awful handwriting, and the typos in this lab book.

Welcome to experimental physics!

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CORRELATION WITH LECTURE

Most of the topics covered in the lab will also be covered in your lecture, although not necessarily in the same sequence or at the same time during the semester. Given the scheduling of the different lecture sections (some are MWF and some are TR), and the different lab sections (the first lab is Monday at 1 PM, the last is Thursday at 4 PM), *perfect correlation of lecture and lab topics is not possible for all students at all times*. The TA will provide a brief overview of the physics concept being explored in the lab during the first part of each lab section.

Occasionally, to improve the correlation with the lecture, the order of the labs may be changed from the sequence in this lab book. If so, you will be informed by your TA. **Check your email.***

PREPARATION

Prior to coming to lab, you should read over each experiment. Furthermore, for each laboratory, you must complete a pre-lab activity printed at the beginning of each lab in this manual. The pre-

* Being an old fart, I don't Tweet.

lab should be completed before the lab and turned in at the **beginning** of the lab. See the course syllabus for more details. In some labs, you may also be required to complete experimental predictions and enter them in your lab manual before you come to lab. Sometimes, you must watch an online video. Your TA will discuss this with you when necessary.

Bring the following to the lab:

- Your lab manual and a ring binder to hold it.
- A notebook or some extra sheets of loose leaf paper.
- Your completed pre-lab.
- A pen, pencil and an eraser.*
- A scientific calculator. Graphing calculators are nice, but overpriced and not necessary. During quizzes, you will not be permitted to use your phone.

For some calculations, you may find a spreadsheet more appropriate. You are welcomed and encouraged to use such tools.

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PROCEDURE IN THE LABORATORY

In the laboratory, you will need to be efficient in the use of your time. We encourage a free exchange of ideas between group members and among students in the section, and we expect you to share both in taking data and in operating the computer, but **you should do your own work (using your own words) in answering questions in the lab manual and on the review questions handed out in lab.**

* You will definitely need the eraser.

HONOR CODE

The Vanderbilt Honor Code applies to all work done in this course. Violations of the Honor Code include, but are not limited to:

- Copying another student's answers on a pre-lab, lab questions, review questions, or quiz.
- Submitting data as your own when you were not involved in the acquisition of that data.
- Copying data or answers from a prior term's lab (even from your own, in the event that you are repeating the course).

GRADING

Your lab reports will be graded each week and returned to you the following week. Grades (including lab and quiz grades) will be posted on OAK.

- ***Mistakes happen!*** Check that the scores on OAK are correct. If you don't do this, *no one will.*
- Retain you lab reports so that any such errors can be verified and corrected.
- Details of grading may be found on the online syllabus

SYLLABUS: available online

https://my.vanderbilt.edu/physicslabs/documents/

How to Count Significant Figures[*]

For all *measured* quantities (excepting counted quantities[†]), there will always be an associated uncertainty. For example,

height of Mt. Everest[‡] = 8844.43 m ± 0.21 m

Understanding the uncertainty is crucial to understanding the quantity. However, it is usually not necessary to provide a precise uncertainty range as shown above. The simplest way to represent uncertainty is the method significant figures. Here, the ± is dropped and the uncertainty is implied by the figures that are shown. An individual digit is usually considered significant if its uncertainty is less than ±5. In the case of Mt. Everest, the uncertainty is greater than 0.05 m; thus making the "3" uncertain. Rounding to the nearest 0.1 meter, we can write

height of Mt. Everest = 8844.4 m.

This quantity has five significant figures. (Notice that a digit does not need to be precisely known to be significant. Maybe the *actual* height is 8844.2 m. Maybe it is 8844.6 m. But the Chinese Academy of Sciences is confident that it is NOT 8844.7 m. Hence, that final “4” is worth recording.)

In general, the rules for interpreting a value written this way are

- All non-zero digits are significant
- All zeros written between non-zero digits are significant
- All zeros right of the decimal AND right of the number are significant
- Unless otherwise indicated, all other zeros are implied to be mere place-holders and are not significant.

Consider the following examples. The significant digits are underlined

$\underline{1023}$

$\underline{1023}00$

$\underline{102300.00}$

$00\underline{1023.450}$

$0.00\underline{10230}$

---

[*] Even if you think you understand significant figures, read this anyway. Some of what you think you know may be wrong.
[†] For example: “There are **exactly** 12 eggs in that carton.”
[‡] 2005, Chinese Academy of Sciences, https://en.wikipedia.org/wiki/Mount_Everest

Occasionally, a zero that appears to be a mere place-holder is actually significant. For example, the length of a road may be measured as 15000 m ± 25 m. The second zero is significant. There are two common ways two write this.

- Use scientific notation (preferable): $1.500 \times 10^4\,\mathrm{m}$
- Use a bar to indicate the least significant figure: $150\bar{0}0\,\mathrm{m}$ or $150\underline{0}0\,\mathrm{m}$

Addition and Subtraction

If several quantities are added or subtracted, the result will be limited by the number with the largest uncertain decimal position. Consider the sum below:

$$\begin{array}{r} 123.4500 \\ 12.20 \\ 0.00023 \\ \hline 135.65\cancel{023} \\ 135.65 \end{array}$$

This sum is limited by 12.20; the result should be rounded to the nearest hundredth. Again, consider another example:

$$\begin{array}{r} 3210\underline{0}0 \\ 12.30 \\ -333 \\ \hline 32067\cancel{9.3} \\ 320680 \end{array}$$

In $3210\underline{0}0$, the last zero is not significant. The final answer is rounded to the ten's position.

Multiplication and Division

When multiplying or dividing quantities, the quantity with the fewest significant figures will determine the number of significant figures in the answer.

$$\frac{123.45 \times 0.0555}{22.22} = 0.308\cancel{34721} = 0.308$$

0.0555 has the fewest significant figures with three. Thus, the answer must have three significant figures.

To ensure that round off errors do not accumulate, ***keep at least one digit more*** than is warranted by significant figures during intermediate calculations. Do the final round off at the end.

How Do I Round a Number Like 5.5?

I always round up* (for example, 5.5 → 6), but others have different opinions†. Counting significant figures is literally an order-of-magnitude approximation, so it does not really matter that much.

How This Can Break Down

Remember, counting significant figures is NOT a perfect way of accounting for uncertainty. It is only a first approximation that is easy to implement and is often sufficient.

For transcendental functions (sines, cosines, exponentials, *etc*.) these rules simply don't apply. When doing calculations with these, I usually keep one extra digit to avoid throwing away resolution.

However, even with simple math, naively applying the above rules can cause one to needlessly loose resolution.

> Suppose you are given two measurements $1\underline{0}$m and 9s. You are asked to calculate the speed.
> With $1\underline{0}$m I will assume an uncertainty of about 0.5 out of 10 or about 5%.‡
> With 9s you have ***almost*** the same uncertainty (0.5 out of 9), but technically we only have one significant digit instead of two.
> If I naively apply the rules . . .
>
> $$\frac{1\underline{0}m}{9s} = 1.\cancel{1111}\frac{m}{s} = 1\frac{m}{s}$$
>
> . . . my answer has an uncertainty of 0.5 out of 1!!! 50%!!!

This is what I call the **odometer problem**: When you move from numbers that are close to rolling over to the next digit (0.009, 8, 87, 9752953, etc.) to numbers that have just barely rolled over (0.001, 1.4, 105, 120258473, etc.), the estimate of the uncertainty changes by a factor of 10.§ Here, we really need to keep a second digit in the answer.

$$\frac{1\underline{0}m}{9s} = 1.1\cancel{111}\frac{m}{s} = 1.1\frac{m}{s}$$

Notice: In the problem above, if the numbers are flipped, the odometer problem goes away:

$$\frac{9m}{1\underline{0}s} = 0.9\cancel{000}\frac{m}{s} = 0.9\frac{m}{s}$$

---

* . . ., and for good mathematical reasons, mind you. But still, it does not really matter that much.
† Google it, if you want to waste an hour of your life.
‡ Of course, I don't really know what the uncertainty is. It could be much larger, but bear with me anyway.
§ . . . and, vice-versa.

Oh Great! I thought this was supposed to be easy.

Well . . ., it is! But, you still have to use your head!

- Apply the rules.
- Look out for the "odometer problem".
- If warranted, keep an extra digit.
- Simple!

Remember: counting significant figures is literally an order-of-magnitude approximation. So, don't get too uptight about it. If you need something better than an order-of-magnitude approximation, see Lab 1.

What you should ***never*** do is willy-nilly copy down every digit from your calculator. If you are in the habit of doing that, STOP IT. You are just wasting your time and lying to yourself. If you ever claim that your cart was traveling at 1.35967494 m/s, expect your TA *to slap you down*. **That is just wrong!** What makes a result *scientific* is honesty, not precision.

To say that Mt. Everest is about 9000 m is perfectly true.
To say that Mt. Everest is 8844.4324 m is a lie.

Forrest T. Charnock
Director of the Undergraduate Laboratories
Vanderbilt Physics

The Greek Alphabet

The 26 letters of the Standard English alphabet do not supply enough variables for our algebraic needs. So, the sciences have adopted the Greek alphabet as well. You will have to learn it eventually, so go ahead and learn it now, particularly the lower case letters.

Alpha	A	α
Beta	B	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	E	ε
Zeta	Z	ζ
Eta	H	η
Theta	Θ	θ
Kappa	K	κ
Lambda	Λ	λ
Mu	M	μ
Nu	N	ν
Xi	Ξ	ξ
Omicron	O	o
Pi	Π	π
Rho	P	ρ
Sigma	Σ	σ
Tau	T	τ
Upsilon	Y	υ
Phi	Φ	ϕ or φ
Chi	X	χ
Psi	Ψ	ψ
Omega	Ω	ω

Name_______________________________ Section _______ Date_____________

Pre-Lab Preparation Sheet for Lab 1: Measurement, Uncertainty, and the Propagation of Uncertainty

(Due at the Beginning of Lab)

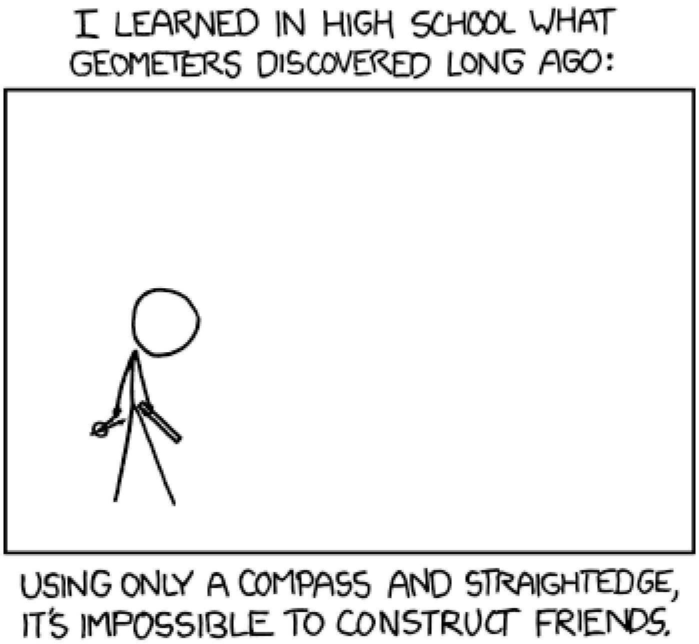
Directions:
Read the essay *How to Count Significant Figures*, then read over the following lab. Answer the following questions.

1. Applying the rules of significant figures, calculate the following

$123.4 + 120 + 4.822 - 21 =$

$$\frac{185.643 \times 0.0034}{3022} =$$

$$(523400 \times 0.0032) + 253 =$$



xkcd.com (But I want you to try anyway.)

Name ____________________ Date __________ Partners ________________
TA ________________ Section _______ ________________

Lab 1: Measurement, Uncertainty, and Uncertainty Propagation

"The first principle is that you must not fool yourself – and you are the easiest person to fool."

--Richard Feynman

Objective: To understand how to report both a measurement and its uncertainty.

Learn how to propagate uncertainties through calculations

Define, *absolute* and *relative* uncertainty, *standard deviation*, and *standard deviation of the mean*.

Equipment: meter stick, 1 kg mass, ruler, caliper, short wooden plank

DISCUSSION

Before you can really know anything, you have to measure something, be it distance, time, acidity, or social status. However, measurements cannot be "exact". Rather, all measurements have some uncertainty associated with them.* Ideally, all measurements consist of *two* numbers: the *value* of the measured quantity x and its *uncertainty*† Δx. The uncertainty reflects the reliability of the measurement. The range of measurement uncertainties varies widely. Some quantities, such as the mass of the electron $m_e = (9.1093897 \pm 0.0000054) \times 10^{-31}$ kg, are known to better than one part per million. Other quantities are only loosely bounded: there are 100 to 400 billion stars in the Milky Way.

Note that **we are not talking about "*human error*"! We are not talking about mistakes!** Rather, uncertainty is inherent in the instruments and methods that we use **even when perfectly applied.** The goddess Athena cannot not read a digital scale any better than you.

* The only exceptions are counted quantities. "There are exactly 12 eggs in that carton."

† Sometimes this is called the *error* of the measurement, but *uncertainty* is the better term. *Error* implies a variance from the correct value:

$$error = x_{measured} - x_{correct}$$

But, of course, we don't know what the correct value is. If we did, we would not need to make the measurement in the first place. Thus, we cannot know the error in principle! But we can measure the uncertainty.

Recording uncertainty

In general, uncertainties are usually quoted with no more precision than the measured result; and the last significant figure of a result should match that of the uncertainty. For example, a measurement of the acceleration due to gravity on the surface of the Earth might be given as

$$g = 9.7 \pm 1.2 \text{ m/s}^2 \qquad \text{or} \qquad g = 9.9 \pm 0.5 \text{ m/s}^2$$

But you should **never** write

$$g = 9.7 \pm 1.25 \text{ m/s}^2 \qquad \text{or} \qquad g = 9.92 \pm 0.5 \text{ m/s}^2.$$

In the last two cases, the precision of the result and uncertainty do not match.

Uncertainty is an inherently fuzzy thing to measure; so, it makes little sense to present the uncertainty of your measurement with extraordinary precision. It would be silly to say that I am (1.823643 ± 0.124992)m tall. Therefore, the stated uncertainty will usually have only one significant digit. For example

$$23.5 \pm 0.4 \qquad \text{or} \qquad 13600 \pm 700$$

However, if the uncertainty is between 1.0 and 2.9 (or 10 and 29, or 0.0010 and 0.0029, *etc.*) it may be better to have two significant digits. For example,

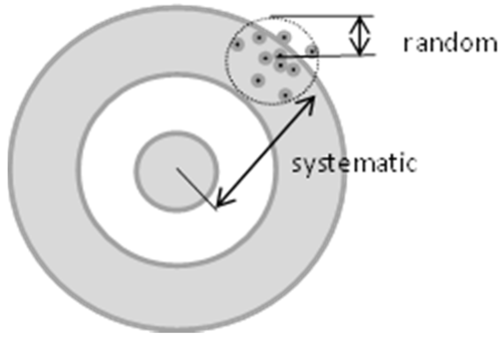
$$124.5 \pm 1.2$$

There is a big difference between saying ±1 and ±1.4. There is not a big difference between ±7 and ±7.4. (This is related to the odometer problem. See the above essay *How to Count Significant Figures*.)

Types of uncertainties

Random uncertainties occur when the results of repeated measurements vary due to truly random processes. For example, random uncertainties may arise from small fluctuations in experimental conditions or due to variations in the stability of measurement equipment. These uncertainties can be estimated by repeating the measurement many times.

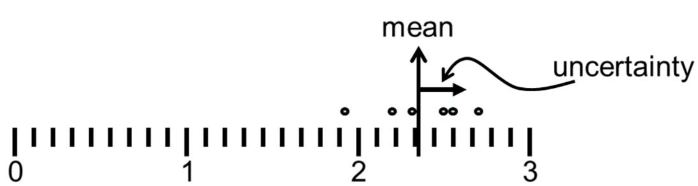
A *systematic uncertainty* occurs when all of the individual measurements of a quantity are biased by the same amount. These uncertainties can arise from the calibration of instruments or by experimental conditions. For example, slow reflexes while operating a stopwatch would systematically yield longer measurements than the true time duration.



Mistakes can be made in any experiment, either in making the measurements or in calculating the results. However, by definition, mistakes can also be avoided. Such blunders and major systematic errors can only be avoided by a thoughtful and careful approach to the experiment.

Estimating uncertainty

By repeated observation: Suppose you make repeated measurements of something: say with a stopwatch you time the fall of a ball. Due to random variations, each measurement will be a little different. From the spread of the measurements, you can calculate the uncertainty of your results.



Shortly, we will describe the formal procedure to do this calculation. (Oddly enough, truly random uncertainties are the easiest to deal with.)

By eye or reason: Sometimes, repeated measurements are not relevant to the problem. Suppose you measure the length of something with a meter stick. Meter sticks are typically ruled to the mm; however, we can often read them more precisely that that.

Consider the figure above. Measuring from the left side of each mark and considering the position uncertainties of <u>both ends</u> of the bar, I can confidently say that the bar is (1.76 ± 0.04) cm. Perhaps your younger eyes could read it with more confidence, but when in doubt it is better to overestimate uncertainty.

Could I do a better job by measuring several times? Not always. Sometimes with repeated measurements, it still comes down to "*Looks like (1.76 ± 0.04) cm to me.*" But that's ok. Your reasoned judgment is sufficient. **Science is defined by rigorous honesty, not rigorous precision!**

Vocabulary

Here we define some useful terms (with examples) and discuss how uncertainties are reported in the lab.

Absolute uncertainty: This is the magnitude of the uncertainty assigned to a measured physical quantity. It has the same units as the measured quantity.

Example 1. Again, consider the example above:

$$L = (1.76 \pm 0.04)\ \text{cm}$$

Here, the uncertainty is given in units of length: 0.04 cm. When the uncertainty has the same dimension as the measurement, this is an *absolute uncertainty*.

Relative uncertainty: This is the ratio of the absolute uncertainty and the value of the measured quantity. It has no units, that is, it is dimensionless. It is also called the *fractional uncertainty* or, when appropriate, the *percent uncertainty*.

Example 2. In the example above the *fractional uncertainty* is

$$\frac{\Delta V}{V} = \frac{0.04\ \text{cm}}{1.76\ \text{cm}} = 0.023 \qquad (1.1)$$

The *percent uncertainty* would be 2.3%.

Reducing random uncertainty by repeated observation

By taking a large number of individual measurements, we can use statistics to reduce the random uncertainty of a quantity. For instance, suppose we want to determine the mass of a standard U.S. penny. We measure the mass of a single penny many times using a balance. The results of 17 measurements on the same penny are summarized in Table 1.

Table 1. Data recorded measuring the mass of a US penny.

	mass (g)	deviation (g)
1	2.43	-0.088
2	2.49	-0.028
3	2.49	-0.028
4	2.58	0.062
5	2.52	0.002
6	2.55	0.032
7	2.52	0.002
8	2.64	0.122
9	2.55	0.032
10	2.46	-0.058
11	2.52	0.002
12	2.4	-0.118
13	2.58	0.062
14	2.61	0.092
15	2.49	-0.028
16	2.52	0.002
17	2.46	-0.058

The **mean** value $\overline{m}$ (that is, the average) of the measurements is defined to be

$$\overline{m} = \frac{1}{N}\sum_{i=1}^{N} m_i = \frac{1}{17}\left(m_1 + m_2 + \ldots + m_{17}\right) = 2.518g \tag{1.2}$$

The **deviation** d_i of the ith measurement m_i from the mean value $\overline{m}$ is defined to be

$$d_i = m_i - \overline{m} \tag{1.3}$$

Fig. 1 shows a histogram plot of the data on the mass of a US penny. Also on the graph is a plot of the smooth bell curve (that is a *normal distribution*) that represents what the distribution of measured values would look like if we took many, many measurements. The result of a large set of repeated measurements (when subject only to random uncertainties) will always approach a normal distribution which is symmetrical about $\overline{m}$.

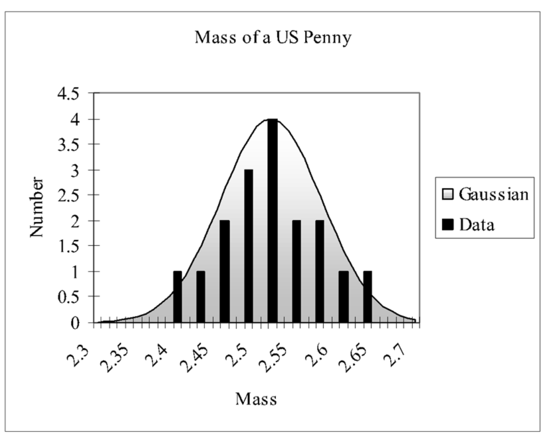


Figure 1. The Gaussian or normal distribution for the mass of a penny N=17, $\overline{m}$ =2.518 g, Δm=0.063 g.

OK, now I have all of these measurements. How accurate is any <u>*one*</u> *of these measurements?*

For this, we now define the **standard deviation** Δm as

$$\Delta m = \sqrt{\sum_{i=1}^{N} \frac{\left(m_i - \overline{m}\right)^2}{\left(N-1\right)}} = \sqrt{\sum_{i=1}^{N} \frac{\left(m_i - \overline{m}\right)^2}{16}} = 0.063g \tag{1.4}$$

For normal distributions, 68% of the time the result of an individual measurement would be within $\pm\,\Delta m$ of the mean value $\overline{m}$. Thus, Δm is the experimental uncertainty for an *individual* measurement of m.

The mean $\overline{m}$ *should have less uncertainty than any individual measurement. What is* ***that*** *uncertainty?*

The uncertainty of the final average is called the **standard deviation of the mean**. It is given by

$$\Delta\overline{m} = \frac{\Delta m}{\sqrt{N}} \tag{1.5}$$

With a set of N=17 measurements, our result is

$$\begin{aligned} \text{mass of a penny} &= \overline{m} \pm \Delta\overline{m} = \overline{m} \pm \frac{\Delta m}{\sqrt{N}} \\ &= 2.518g \pm \frac{0.063g}{\sqrt{17}} \\ &= \left(2.518 \pm 0.015\right)g \end{aligned} \tag{1.6}$$

Thus, *if our experiment is only subject to random uncertainties in the individual measurements*, we can improve the precision of that measurement by doing it repeatedly and finding the average. Note, however, that the precision improves only as $\frac{1}{\sqrt{N}}$. To reduce the uncertainty by a factor of 10, we have to make 100 times as many measurements. We also have to be careful in trying to get better results by letting $N \rightarrow \infty$, because the overall accuracy of our measurements will eventually be limited by systematic errors, which *do not cancel out* like random errors do.

Exercise 1:

a. With a caliper, measure the width and thickness of the plank. Make at least five measurements of each dimension and enter the result into Table 2.
b. With a ruler, measure the length of the wooden plank on your table as precisely as possible. Estimate the uncertainty, and enter the result into Table 2.

c. For both the width and thickness, calculate your final result (the *mean*) and the uncertainty (the *standard deviation of the mean*). Enter the final results below the table.

Table 2

width	deviation	thickness	deviation

width = ____

thickness = ____

length = ____

Propagation of uncertainties

Usually, to obtain a final result, we have to measure a variety of quantities (*say, length and time*) and mathematically combine them to obtain a final result (*speed*). How the uncertainties in individual quantities combine to produce the uncertainty in the final result is called the *propagation of uncertainty*.

Here we summarize a number of common cases. For the most part these should take care of what you need to know about how to combine uncertainties.*

Uncertainties in sums and differences:

If several quantities x_1, x_2, x_3 are measured with absolute uncertainties $\Delta x_1, \Delta x_2, \Delta x_3$ then the absolute uncertainty in Q (where $Q = x_1 \pm x_2 \pm x_3$) is

$$\Delta Q = |\Delta x_1| + |\Delta x_2| + |\Delta x_3| \tag{7}$$

In other words, *for sums and differences, add the **absolute** uncertainties.*

***Uncertainties in products and quotients*:**

Several quantities x, y, z (with uncertainties Δx, Δy, Δz,) combine to form Q, where

$$Q = \frac{x\,y}{z}$$

(or any other combination of multiplication and division). Then the fractional uncertainty in Q will be

$$\frac{\Delta Q}{|Q|} = \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta z}{z}\right| \tag{8}$$

In other words, *for products and quotients, add the **fractional** uncertainties.*

* These expressions for the propagation of uncertainty are an **upper limit** to the resulting uncertainty. In this case, you could actually do better. See Appendix C for details.

Exercise 2:

d. Calculate the total volume of the block and the associated uncertainty. Show your math below.

Exercise 3:

You can also find the volume of an object by measuring the volume water displaced by the object when it is submerged.

e. Measure the volume of water in the graduated cylinder and the associated uncertainty.
f. Submerge the block (holding it under the surface with a pen or pencil), and find the resulting volume and the associated uncertainty.
g. Calculate the volume of the plank and the associated uncertainty. Show your math below.
h. Is this answer consistent with your result from Exercise 3? Explain.

Exercise 4:

a. Calculate the total surface area of the block and the associated uncertainty. Show your math below.

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Pre-Lab Preparation Sheet for Lab 2:
Position, Velocity, and Acceleration in one-dimensional motion

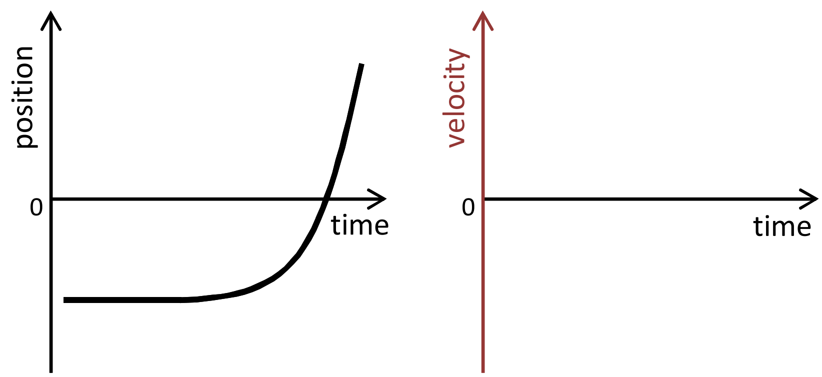
(Due at the Beginning of Lab)

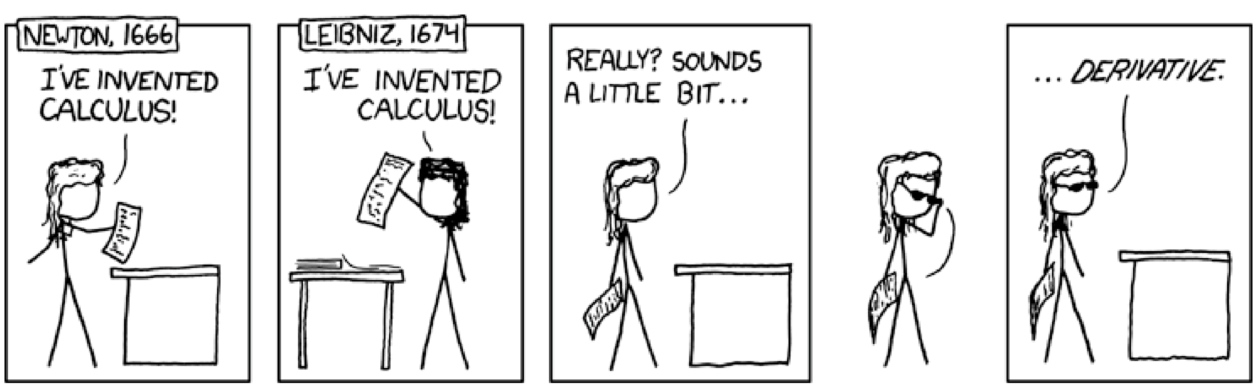
Watch the video *Introduction to Optimization and Curve Fitting* found at the following site:

https://my.vanderbilt.edu/physicslabs/videos/

Read over the lab and then answer the following questions

1. Given the following position curve, sketch the corresponding velocity curve.





xkcd.com

2. Imagine kicking a box across the floor: it suddenly starts moving then slides for a short distance before coming to a stop. Make a sketch of the position and velocity curves for such motion.

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Lab 2: Position, Velocity, and Acceleration in One-Dimensional Motion

"God does not care about our mathematical difficulties. He integrates empirically."
--Albert Einstein

Objectives:

- To understand graphical descriptions of the motion of an object.
- To understand the mathematical and graphical relationships among position, velocity and acceleration

Equipment:

- 2.2-meter track w/ adjustable feet and end stop
- A block to raise one end of the cart
- Motion sensor
- Torpedo level
- PASCO dynamics cart and friction cart

DISCUSSION

Velocity is the rate of change or time derivative of position.

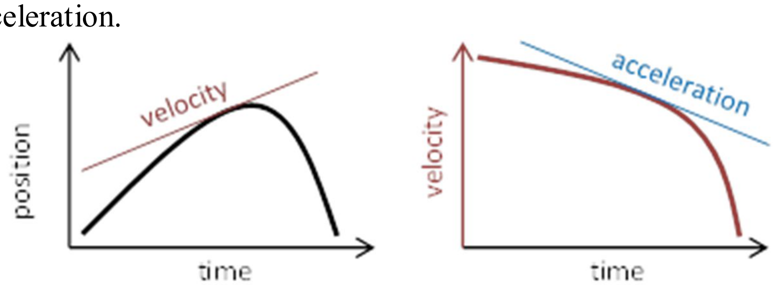
$$\vec{v} = \frac{d\vec{x}}{dt} \tag{1}$$

On a Cartesian plot of position vs. time, the slope of the curve at any point will be the instantaneous velocity.

Likewise, acceleration is the rate of change or time derivative of velocity (the 2nd derivative of position).

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2} \tag{2}$$

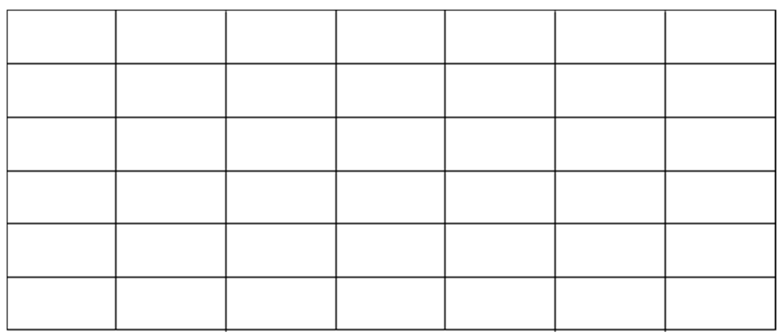
On a Cartesian plot of velocity vs. time, the slope of the curve at any point will be the instantaneous acceleration.



Thus, the shape of any one curve (position, velocity, or acceleration) can determine the shape of the other two.

Exercise 1: Back and Forth

a. Place the friction cart on the track. (That is the one with the friction pad on the bottom. Without letting go of the cart, quickly push it toward the detector by about a foot, then stop it for 1 or 2 seconds. Then quickly but smoothly return the cart to the starting point. Note the distance it travels, and sketch the position vs. time curve for the block on the plot below.



b. Now, open the Labfile directory found on your computer's desktop. Navigate to

A Labs/Lab2

and select the program Position. The PASCO Capstone program should open and present you with a blank position vs. time graph.

c. Click the **Record**  button (lower left side of the screen), and repeat the experiment above. Click **Stop** to cease recording data. Note how the PASCO plot compares to yours.

Note: The cart may bounce or stutter in its motion. If you don't get a smooth curve, delete the data* and repeat the run with more Zen†.

d. By clicking the scaling icon  (top left corner of the Graph window) you can better fill the screen with the newly acquired data.

e. Select the slope icon . A solid black line will appear on the screen. By dragging this line to points along the plot, you can measure the slope of the curve at those points. Using this tool, find the steepest part of the curve (that is, the largest velocity). Then, sketch the velocity curve for the block in the graph below. Add appropriate numbers to the x and y axes.

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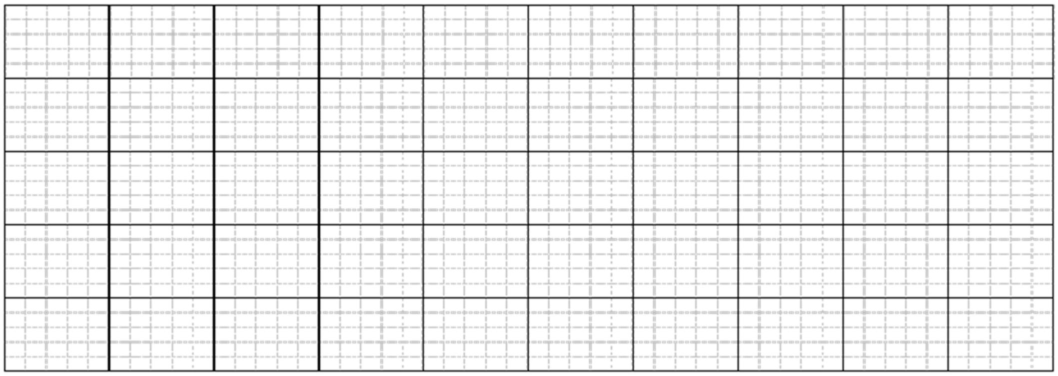
* To delete data: click the icon on the bottom of the screen.

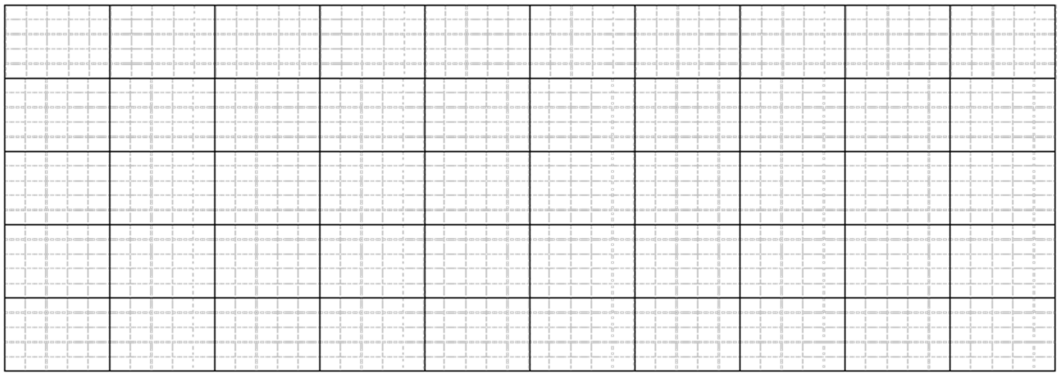
† "*This time, let go your conscious self and act on instinct.*" Obi-Wan Kenobi



f. How does the shape of the <u>position curve</u> determine the sign of the <u>velocity curve</u>?

g. Now, let's see how well you drew it! Double-click on the new plot icon  (middle top of the screen) and select **Velocity (m/s)** for the y-axis. Note the shape and position of the curve and see how well it matches your sketch. Also note how it aligns with the position curve.

h. Use the slope tool to find the changing slope along the velocity curve. With this information, sketch the acceleration curve for the block. Again, appropriately mark the axes.



i. Let's see what PASCO says about the acceleration. Again, create a new plot and select **Acceleration (m/s²)** for the y-axis. Compare it to your acceleration curve and PASCO's velocity curve.
j. How does the shape of the position curve determine the sign of the acceleration curve?

k. Print out the three PASCO plots. Annotate these plots to show the times when the push began, when the push ended, when it was slowing, and when it stopped.

Exercise 2: Skidding to a Stop

Delete your previous runs. (Top bar, Experiment, Delete ALL Data Runs). With a left click of the mouse, you can remove the slope tools.

a. Move the cart to end of the track opposite the detector.
b. Start recording data, then give the cart a quick, firm push so that it slides a few feet before coming to rest. Stop the data acquisition.

   By clicking the scaling icon , you can better fill the screen with the newly acquired data. You can also adjust the scale by clicking and dragging along the x or y axis, or zooming with the scroll wheel.

c. Again, if the data is not reasonably smooth, delete the data and repeat the experiment with more Zen.
d. Print out the curves and annotate on the graphs with the times when the push began, when the push ended, and when the cart was sliding on its own.

You should notice that as the cart is slowing down, the acceleration curve is nearly a constant flat line.

e. Given constant acceleration, what mathematical expression describes the velocity?

f. What mathematical expression describes the position?

You can verify that these expressions work by numerically fitting the data.

g. Click on the highlight tool  and a box for selecting data will appear on the screen. Adjust the size and position of the box to highlight the region of the velocity curve where the cart is slowing down. Then, select the fitting tool  and choose the appropriate expression to describe the data. Record the results of the fit below. (Note the uncertainty provided by the fit.)

h. Similarly, apply a numerical fit to the position data. Record the results below. Are the results consistent with the velocity and acceleration curves? (That is, do the uncertainties overlap?)

i. Similarly, find the average acceleration of this region.

Exercise 3: Up and Down

a. Place a block under one of the track stands to form a ramp. The detector must be on the raised end.
b. Place a low friction cart on the track and give it a push so that it rolls a few feet up the incline and then rolls back. After a few practice runs, run the detector and acquire motion data.
c. With a click and drag of the mouse, highlight that section of the data where the cart is freely rolling along the track. Then use the scaling tool  to zoom-in on that section of the data.
d. Print out these plots and annotate the graphs with the following information.
   When and where does the velocity of the cart go to zero?
   What is the acceleration when the velocity is zero?
   When and where does the acceleration of the cart go to zero?

e. Find the average acceleration going up the slope and down the slope. Record the results below.

f. How does the acceleration up the slope compare with the acceleration down the slope? What might account for the difference?

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Pre-Lab Preparation Sheet for Lab 3: Momentum

(Due at the Beginning of Lab)

Directions:
Read over the following lab, then answer the following questions.

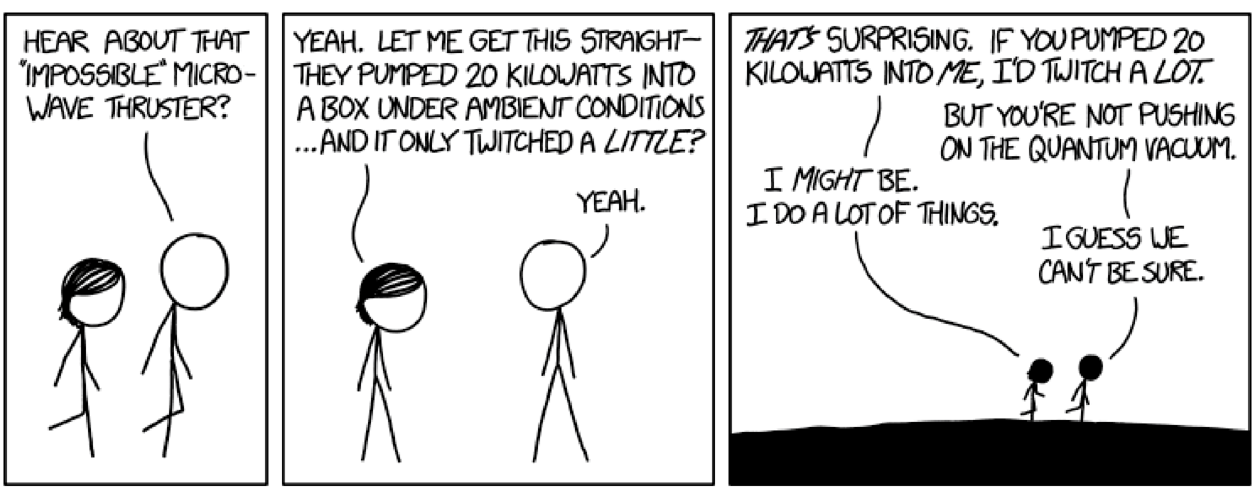
2. Wile E. Coyote steps off a cliff edge. He reasons that
   a. his momentum is zero,
   b. momentum is conserved,
      therefore . . .
   c. he cannot fall.

   Correct his reasoning. How is momentum conserved in this case?

3. Define *closed system*.

4. Two carts collide. Afterward, they have the same velocity. What kind of collision occurred?

5. Two carts have equal mass and form a closed system.
When t = 0, they have velocities v_1 and v_2.
When t = 1s, they have the same velocity v.
Assume both kinetic energy and momentum are conserved.
    a. How are v_1, v_2 and v related?
    b. What does this mean?



xkcd.com (If you don't get this, Google "resonant cavity thruster".)

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Lab 3: Momentum

When he was within six miles of the place,
There Number Four stared him straight in the face.
He turned to his fireman, said Jim you'd better jump,
For there're two locomotives that are going to bump.

--The Ballad of Casey Jones, variant by Eddie Newton and T. Lawrence Seibert

Objective: To observe conservation of momentum and conservation of kinetic energy (or not).

Equipment: 2m track, magnetic cart, plunger cart, two motion sensors, 2 (½ kg) cart masses

Introduction

The momentum of an object is

$$\vec{p} = m\vec{v} \tag{3}$$

Momentum is a vector. The direction of motion matters. The total momentum of a closed system is always conserved**.** No exceptions. If momentum of your system is changing, your system is not closed. Something else is meddling with it

The kinetic energy of an object is

$$K = \tfrac{1}{2}mv^2 \tag{4}$$

Kinetic energy is a scalar. The direction of motion is irrelevant.

The total energy of a closed system is always conserved. No exceptions. If the energy of your system is changing, your system is not closed. Something else is meddling with it.

But, energy has this inconvenient tendency of transforming from one type of energy to another. Kinetic energy may be transformed into potential energy, or chemical energy, or thermal energy, or magical energy[*] or . . . , and *vice versa*. While the total energy is conserved, the amount of any given species of energy may not be conserved.

Momentum, on the other hand, remains politely consistent. There is no *kinetic momentum*, or *potential momentum*, or . . . whatever. Just *momentum*, and it does not turn into anything else.[†]

Types of collisions

Object *a* (mass m_a, velocity $\vec{v}_{ai}$) collides with object *b* (mass m_b, velocity $\vec{v}_{bi}$). After the collision, the velocities are $\vec{v}_{af}$ and $\vec{v}_{bf}$.

We can define three basic types of collisions:

- **Totally inelastic collision**: Two or more objects collide and stick together after the collision. That is, their *relative* velocity is zero.
$$\vec{v}_{af} - \vec{v}_{bf} = 0$$
- **Totally elastic collision**: Two objects collide. Afterward, their *relative* velocity is unchanged, and kinetic energy is conserved.
$$\vec{v}_{ai} - \vec{v}_{bi} = \vec{v}_{af} - \vec{v}_{bf}$$
- **Partially elastic collision**: While the objects do not stick together after the collision, their relative velocities have changed, and kinetic energy is not conserved.
$$\vec{v}_{ai} - \vec{v}_{bi} \neq \vec{v}_{af} - \vec{v}_{bf} \neq 0$$

Exercise 1: Inventory

1. Mass each of the following:

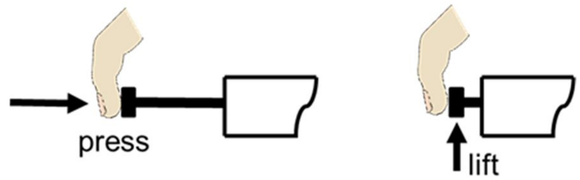
Cart 1: Cart 2:

Bar 1: Bar 2:

2. Level the track, then tighten the nut on each foot.
3. One of your carts contains a plunger. The plunger can be cocked by pressing down and lifting up.

[*] *Aresto Momentum!*
[†] Angular momentum (which you will study later) never transforms into momentum, and momentum never transforms into angular momentum.

Pressing the top pin releases the plunger.



4. Magnets are embedded on the ends of the carts. This can cause the carts to be attracted or repelled. Place the carts on the track and play with them to see which ends attract and which repel. Note that there are no magnets on the end with the plunger.

Exercise 2: Inelastic Collisions

Data Acquisition

5. Depress and cock the plunger. (For now, this is just to keep it out of the way.)

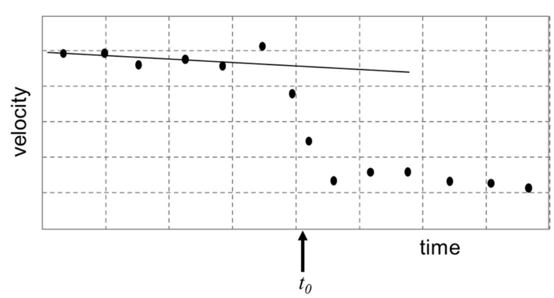
6. Orient the carts on the track so they are attracted to each other. Place a mass bar in one of the carts.
7. Run the Capstone program Momentum. Start recording.
8. With one cart stationary, have the second cart collide into it. The carts should stick together.

Caution: Do not let the carts collide with the detectors!

At this point, you could just get some points before and after the collision and see how the momenta compare. But, if you look closely at the data, you will notice several problems.

- There is some noise in the data, but with a single point, you cannot estimate your uncertainty.
- Your carts are not a truly isolated system. Friction is steadily slowing the carts both before and after the collision.
- You only really care about the moments just before and just after the collision. However, you don't actually have good velocity data around the time of the collision.

You can compensate for this by fitting the good data with a straight line. Then, extrapolate to find the velocity at the moment of the collision.



9. Find the initial velocity (including the uncertainty) at the moment just before the collision. (For the stationary cart, you can assume the initial velocity is exactly zero.)
10. Repeat to find the final velocity. Record your results on the tables below.

Data Analysis

11. Given your measured values for the initial velocity, calculate what the final velocity (and the associated uncertainty) should be if momentum is conserved. Is this calculated value consistent with the measured velocity? (Show your work on a separate sheet.) Calculate the kinetic energy before and after the collision. What fraction of the initial kinetic energy is lost in the collision?

Explosion

12. Orient the carts so they are attached and the plunger is in the middle. Place a mass bar in one of the carts.
13. Give the connected carts an initial velocity
14. While the carts are moving, use the flat of a ruler to apply a quick blow to the release pin. Acquire the velocity data and repeat the analysis.

Exercise 3: Elastic Collisions

15. Orient the carts so they repel each other. Remove any mass bars.
16. With one cart stationary, gently[*] collide the second cart into it. Acquire and analyze the data as above.
17. Add a mass bar to the stationary cart, then repeat the above experiment.
18. Within the resolution of the experiment, are momentum and kinetic energy conserved?

[*] Don't push so hard that the carts physically touch. Let the magnets do the pushing.

Discussion

19. If the momentum does not appear to be conserved within the uncertainty of you measurements, what could explain this? Be specific.

20. The position sensors work by assuming that the speed of sound is 344 m/s and measuring the time for the echo to bounce off the target. Doubtless, this speed is off by a bit. How would this effect your observation of the conservation of momentum or kinetic energy? Explain.

Collision

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

Calculated final velocity = Fractional change of kinetic energy =

Explosion

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

Calculated final velocities = Fractional change of kinetic energy =

Bounce 1

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

Bounce 2

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

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Pre-Lab Preparation Sheet for Lab 4: Force, Mass, and Acceleration

(Due at the Beginning of Lab)

Directions:

Read over the lab and then answer the following questions.

1. Consider the experimental configuration shown in Figure 1. Starting from Newton's 2nd law, show that the acceleration of the cart is on the string is given by:

$$a = \frac{mg}{m+M}.$$

2. In the limit where $M >> m$, what is the acceleration?

3. In the limit where $M << m$, what is the acceleration?

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Lab 4: Force, Mass, and Acceleration

"Well, the Force is what gives a Jedi his power . . . . It surrounds us and penetrates us. It binds the galaxy together."

-- Obi-Wan Kenobi on Newton's 2^{nd} Law

Equipment

Motion sensor

Low-friction cart

Torpedo level

Foam crash pad

Force sensor

2.2 meter track

Low –friction pulley

0.02, 0.05, 0.10, and 0.20 kg hooked masses

Note: The acceleration due to gravity varies with location. Here at Vanderbilt, this acceleration has been measured as

$$g = (9.7943 \pm 0.0032)\frac{m}{s^2}$$

Use this value throughout the semester.

Introduction

Newton's 2^{nd} Law is the most important concept you will learn in this class:

$$\vec{F}_{net} = m\vec{a}$$

If you know the **mass** and **net force** on an object, you know the **acceleration** of the object.

If you know the **acceleration** of an object and its **initial velocity** and its **initial position**, you know the complete **trajectory** of the object.

Consider the problem illustrated below: a frictionless wheeled cart is pulled by string attached to a falling mass.

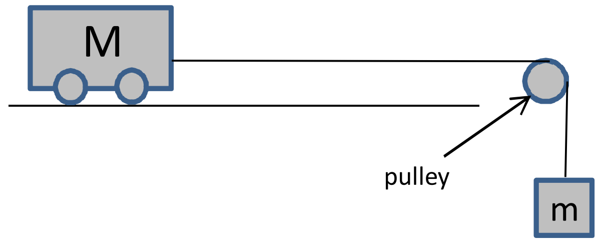

Fig. 1

It can be shown that the acceleration a is given by

$$a = \frac{mg}{M + m} \tag{1}$$

Of course, that is the theory. In the real world, things can get messy. Welcome to experimental physics.

Exercise 1: Data Acquisition

1. Label the individual forces on the diagram above.
2. Mass the combined cart and force sensor with the electronic scale

$M =$ ____

> **Note:** Before each measurement with the scale, you should *tare* the scale. Empty the scale, then press the Z or TARE button found on the panel. This resets the zero point of the scale.

You will be using a set of hooked weights for you falling masses. These have nominal values of 0.020 kg, 0.050 kg, 0.100 kg, 0.200 kg.

3. Measure their precise masses with the digital scale and record the results and associated uncertainties on Table 1 below.

You will use the PASCO Force Sensor to measure the tension in the string. Note the sign convention for the direction of the force on the hook.

Assemble the cart, force sensor, string, and falling mass. The falling mass will be one of four hooked weights: 0.020 kg, 0.050 kg, 0.100 kg, 0.200 kg. **Verify that a crash pad is positioned underneath the falling mass.**

4. Run the Capstone program Velocity&Force.

Note: Before each measurement with the force sensor, you must *tare* the force sensor. Remove any force from the hook, then press the TARE button found on the side of the sensor. This will ensure that when the force is zero, the device returns zero.

5. Holding cart stationary, measure the static tension T_{static} on the string and the corresponding uncertainty. Record the result in Table 1 below. Briefly explain how you determined the uncertainty below.

Note: Do not assume that every digit which Capstone reports is significant.

Table 1: Measured values

m	T_{static}	$T_{dynamic}$	a

6. **Dedicate one of you members to catching the cart before it crashes into the pulley.** Please, do not turn this into a projectile motion lab!
7. Start recording, then release the cart. Record the dynamic tension on the string $T_{dynamic}$, the acceleration a, and the associated uncertainties. Justify your determination of each of these uncertainties below.

Exercise 2: Data Analysis

8. From the pre-lab, what is the acceleration in the limit of $m << M$? Is this confirmed in your observations? Using Excel, plot your data in way to support your argument.

While you have measured the acceleration above, it can also be calculated from Equ. 1, or from the measured tension of the string and the cart mass.

9. Fill in the table below. Include the associated uncertainties

Table 2: Calculated accelerations

$\frac{mg}{M+m}$	$\frac{T_{dynamic}}{M}$

10. Are the measurements in Table 2 consistent with each other? Explain.

11. Are the measurements of $\frac{mg}{M+m}$ consistent with the measured accelerations a in Table 1? Explain.

12. Thus far, we have been blithely ignoring friction of the cart. Given that friction is present, how would that effect the acceleration of the cart?

13. Would the addition of a constant friction force resolve any discrepancy between the observed acceleration and $\frac{mg}{M+m}$? Explain.

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Pre-Lab Preparation Sheet for Lab 5:
Friction

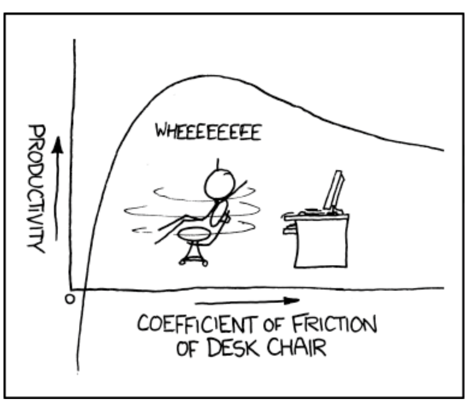
(Due at the Beginning of Lab)

Directions:

Read over the lab and then answer the following questions.

1. What are the units for the coefficients of kinetic and static friction?

2. Write a detailed procedure to measure the coefficient of kinetic friction for a block of wood sliding along a track which would utilize the equipment you are familiar with from previous labs. How would you estimate the uncertainty?



xkcd.com

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Lab 5: Static and Kinetic Friction

Q: How does a physicist milk a cow?

A: First, let's assume a spherical cow . . . .

--old physics joke (Physicists find this hilarious.[*])

Equipment

- 2.2 meter track
- Motion sensor
- Wooden block
- Force sensor
- Mass set
- Masonite board

Introduction

Friction is incredibly complicated. A full discipline of science (tribology) is devoted to it. Its basic origins remains a subject of high level research. Nevertheless, in many cases, a very simple model (a so-called *spherical cow*) describes the behavior of friction with surprising accuracy.

If we have two solid materials in contact, we can define two types of friction:

Static friction: the force that keeps two surfaces from slipping. The force of static friction is parallel to the surface. Its magnitude is whatever is required to prevent slippage. Its maximum magnitude is the coefficient of static friction m_s times the normal force *n*.

$$0 \le F_s \le \mu_s n \tag{2}$$

Kinetic friction: a constant force between two slipping surfaces. The force is parallel to the surface, and its direction is opposite the motion. The magnitude is

$$F_k = \mu_k n \tag{3}$$

This model is purely empirical; there is no underlying explanation or principle. The coefficients must be measured on a case by case basis. But this model happens to work well enough to be useful. Note that this model makes several assumptions:

- Friction is binary: static or kinetic, nothing else.
- Kinetic friction is independent of both contact area and velocity.

[*] Sad, I know.

- While $\mu_s \geq \mu_k$, the coefficients are otherwise unrelated.

Table 1 provides a short list of examples.

Table 1: Friction coefficients

Material 1	*Material 2*	μ_s	μ_k
glass	glass	0.9 – 1.0	0.4
glass	nickel	0.78	0.56
rubber	concrete	1.0	0.8
Teflon™	Teflon™	0.04	0.04

Experiments

1. Along with your lab partners, develop a procedure to measure the coefficient of static friction and corresponding uncertainty for the wooden block and the Masonite board. Clearly describe your procedure and present your results.

2. Along with your lab partners, develop a procedure to measure the coefficient of kinetic friction and corresponding uncertainty for the wooden block and the Masonite board. Also, justify your answers to the following questions:
   a. Is f_k independent of the velocity?
   b. Is f_k independent of the surface area?

   Clearly describe your procedure and present your results.

A question to ponder until the next lab:

Why do race cars have wide tires?

Name________________________ Section _______ Date_____________

Pre-Lab Preparation Sheet for Lab 7: Scaling and the Properties of Elastic Materials

(Due at the Beginning of Lab)

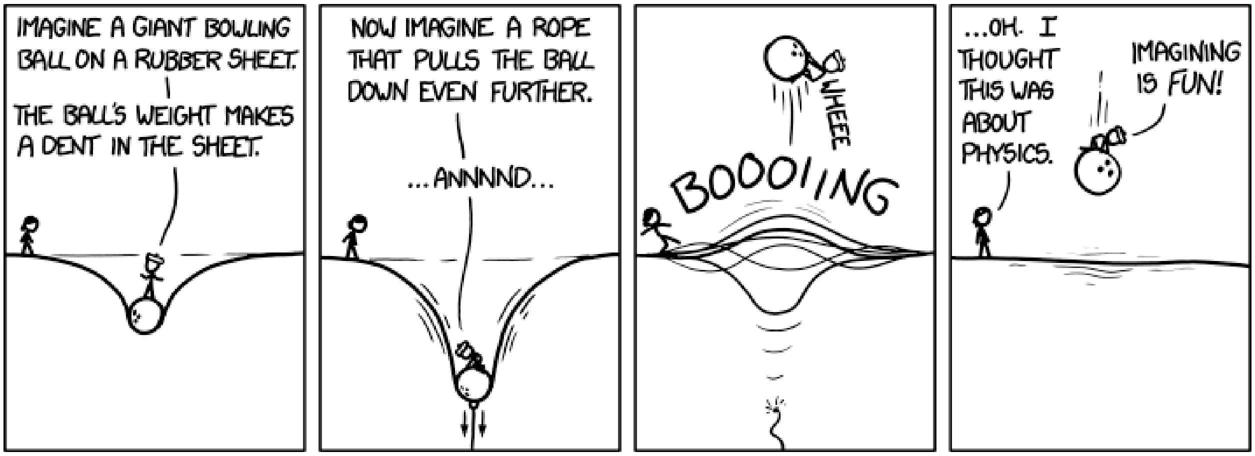
Directions:

Read over the lab and then answer the following questions. You will need to search the web or other resources to answer some of these questions.

1. Suppose you had two springs (A and B) identical in every way but that spring A was 3x longer than spring B. How would their spring constants compare?

2. Suppose you had two flat rubber bands (A and B) identical in every way but band A was 3x the width of band B. How would their spring constants compare?

3. Suppose you had two rubber bands (A and B) identical in every way but band A was 3x the width and 3x the thickness of band B. How would their spring constants compare?


IMAGINE A GIANT BOWLING BALL ON A RUBBER SHEET.
THE BALL'S WEIGHT MAKES A DENT IN THE SHEET.
NOW IMAGINE A ROPE THAT PULLS THE BALL DOWN EVEN FURTHER.
...ANNNND...
BOOOIING
WHEEE
...OH. I THOUGHT THIS WAS ABOUT PHYSICS.
IMAGINING IS FUN!

xkcd.com

Name ____________________ Date __________ Partners ________________

TA ________________ Section _______ ________________

Lab 7: Scaling and the Properties of Elastic Materials

To see a world in a grain of sand

And a heaven in a wild flower,

Hold infinity in the palm of your hand,

And eternity in an hour.

--William Blake

GOALS AND OBJECTIVES

To develop a general appreciation of the effects of scale on the properties of materials.
To determine the domain of applicability of Hooke's Law for particular objects.
To recognize the difference between material elastic properties and object elastic properties.
To predict the elastic properties of an object of arbitrary scale.
To observe and characterize hysteresis during the extension and contraction of elastomers.
To link this hysteresis to an expanded concept of conservation of energy.

EQUIPMENT

2-meter measuring stick
Rubber strips and bands
SONAR reflector
Motion sensor
Right-angle clamp and rod for mounting the force sensor
Safety goggles
Clamp, mount and 1.5-m long rod
Clips
Hanging masses
Force sensor

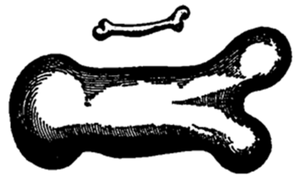
Galilean Scaling

In his final book, *Discourses and Mathematical Demonstrations Relating to Two New Sciences*[*], Galileo described the mathematics of scaling. Consider two dogs, Bruno and Pip. Bruno is exactly 3× the size (in every direction: height, depth, and width) as Pip, but is otherwise identical. Clearly, Bruno will be heavier. Since mass is proportional to volume, Bruno's weight will be 3^3 or 27× the weight of pip.

Of course, Bruno will also be stronger, but how much stronger? The strength of a bone is proportional to the area of the bone's cross section. Thus he will only be 3^2 or 9× stronger than Pip. *Pound for pound, Pip is much stronger than Bruno!* This is the problem of *scaling*: an apparently simple change in size can lead

[*] A remarkably boring title, but considering what he went through after publishing *The Starry Messenger*, that can be forgiven.

to surprising changes in an object's other properties.



An illustration from Galileo's book.

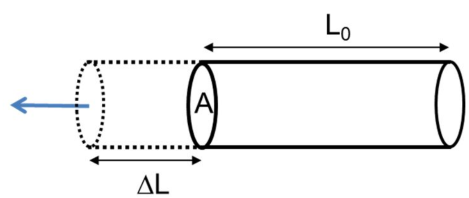
To make up for this scaling problem, the bones of large animals must be proportionally much thicker than the bones of small animals. King Kong could never exist. He would collapse under his own weight. Likewise, ants are famously strong for their size, able to lift objects many times heavier than themselves. This is no great mystery but a straightforward consequence of scaling.

Rubber Bands

Different systems scale in different ways. In this lab, you will study the scaling properties of rubber bands. (And, you will learn a few other things about rubber as well.)

For a wide range of elastic materials, the relationship between force and deformation is well described by Hooke's Law:

$$F_s = -k\Delta L \tag{1.4}$$



However, this is not a *Law* as much as *Spherical Cow*: an approximation that is *unreasonably effective*[*] over a limited range of deformation.[†] Within this limited range, one can characterize an object's elastic properties by its spring constant *k*.

Exercise 1: Inventory

Use the calipers to carefully measure the width and thickness of each rubber band. Take care not to compress the bands as you measure them. You want the relaxed dimensions.

[*] . . . as Eugene Wigner would say.
[†] Stretch it too much and Murphy's Law takes over.

	Diameter (m)	cross-sectional area (m^2)
Small		
Large		

Exercise 2: Scaling the Spring Constant with length

Use the large cord for the following.

1. Placed the two hose clamps on the cord separated by 1.20 m.
2. Hang one clamp from the force sensor. Hang the sonar reflector from the other clamp.
3. Start the program Spring Constant. Then, gently pull the cord ~2 cm.

Stop acquiring data before releasing the cord.

4. Repeat two more times.
5. Find the average spring constant.

measured k (1.20 m) = ______________________________

6. Given your pre-lab prediction for how the spring constant should scale with length, estimate the spring constants for 0.60 m and 0.40 m cords.
7. Now, measure these spring constants as you did for the 1.20 m cord.

	estimated	measured	% diff
k (0.60 m)			
k (0.40 m)			

Strictly speaking, a spring constant only conveys information about a particular spring or cord. A parameter that was independent of length would be more generally applicable.

8. Rewrite the spring constant as a function of the length of the cord.

$k(L_0)$ =

Exercise 3: Scaling in all dimensions

Now, suppose you had a double cord (two cords side by side)

9. Estimate the spring constant for two parallel cords 0.60 m long. Then measure it.

	estimate	measured	% diff
k_{double} (0.60 m)			

10. How should the spring constant scale with the diameter of the cord? Explain.

11. Write an expression for the spring constant of a cord with arbitrary diameter and length.

12. Estimate the spring constants for the small cord with lengths of 0.60 m and 0.30 m.

	estimate	measured	% diff
k_{small} (0.60 m)			
k_{small} (0.30 m)			

Exercise 4: Wild extrapolation

13. How should the spring constant of a cord scale with uniform changes in its over-all size (length, width, and thickness)?

14. An idiot decides to jump from the New River Gorge Bridge in West Virginia (267 m above the river) using a bungee cord made of the same material you have been using. He plans to use a 200 m cord which stretch to a maximum of 240 m. His mass is 70 kg

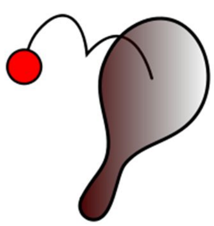
    a. Using conservation of energy, calculate the spring constant of this cord.

    b. What should the diameter of the cord be?

c. We have neglected the weight of the cord itself. Is this reasonable? (Do the math.)

Exercise 5: Professor, I Shrank My Lab Partner!

Imagine your lab partner is playing with a paddle ball.* Out of nowhere, Rick Moranis† zaps her with his shrink ray making her and the toy 100 times smaller (in all three dimensions). Setting aside her psychological trauma, how does this change of scale affect her game-play? Discuss this with your partners, and then your TA.



* For those of you unfamiliar with toys from the mid-20th century, it consists of a rubber ball attached to a paddle by a thin rubber band. You then whack the ball repeatedly with the paddle, sort of like playing Ping-Pong with yourself. In the pre-internet age, this was regarded as fun.

† You know, the nerdy father from *Honey, I Shrunk the Kids*. (Yeah, I'm old.)

Exercise 6: Hysteresis

Thus far, we have only dealt with the stretching behavior of the cord. You may have noticed that the relaxation behavior is a little different. Now, let’s look at this in detail.

15. Assemble the clips on the large cord so that its effective length is 1.20 m.
16. While acquiring data, both stretch and relax the band. Do this several times to get a good idea of how the force curve during stretching compares to the force curve during relaxation. This behavior is called Hysteresis.
17. Sketch the curves in the graph below. Label each axis and which segments of the curve corresponds to stretching and relaxation.

18. How does the work done stretching the cord compare to the work released during relaxation?

19. Applying the principle of conservation of energy (aka the First Law of Thermodynamics), explain your answer above.

What you've observed here is a concrete example of a phenomenon known as *hysteresis*.

Name_________________________ Section_______Date_____________

Pre-Lab Preparation Sheet for Lab 8: Torque and Rotational Inertia

(Due at the Beginning of Lab)

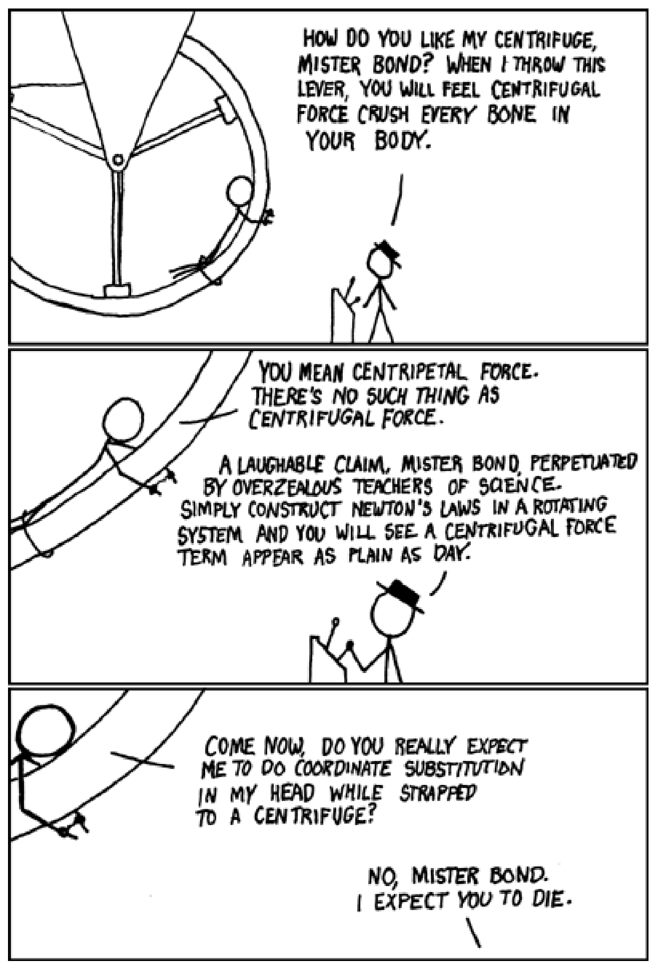
Read over the lab and Appendix B, and then answer the following questions.

1. What is Newton's 2nd law in terms of rotational motion?

2. To account for friction, you must solve for two unknown parameters. What are they?

3. A car is traveling North. In which direction are the wheels turning?

4. How is the *z*-axis of a Right Handed coordinate system related to the *z*-axis of a Left Handed coordinate system?



Name ____________________ Date __________ Partners_____________
TA ________________ Section ______ _____________

Lab 8: Torque and Rotational Inertia

". . . You make the rocking world go round"
--B. May *et al., Jazz,* 2 (1978).

Objectives

- To relate an objects rotational inertia to its geometry
- To understand Newton's laws in terms of rotational motion
- To observe the conservation of energy in terms of rotational kinetic energy

Equipment

- PASCO Rotational Motion Sensor with aluminum holder
- Calipers
- thread
- electronic balance
- Flat aluminum disk
- 10 g mass
- Table clamp, 2 rods, and right angle clamp
- Super pulley

DISCUSSION:

Angular velocity $\boldsymbol{\omega}$ and angular momentum $\boldsymbol{L}=I\boldsymbol{\omega}$ are vectors. The direction of these vectors is given by the right hand rule. The equations describing Newton's 2nd law for rotational motion are fully analogous to the equations for linear motion. A net torque on an object will change its angular velocity $\boldsymbol{\omega}$:

$$\vec{\tau} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} \qquad (1)$$

where I is the rotational inertia*, and the angular acceleration $\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$. We may also write the 2nd Law to relate the net torque to change angular momentum $\boldsymbol{L}$.

$$\vec{\tau} = \frac{d\vec{L}}{dt} \qquad (8.2)$$

Thus, the direction of the applied torque is always in the same direction as the resulting angular acceleration and the change in the angular momentum.

---

* Also known as the moment of inertia

If multiple objects are attached together, the total rotational inertia is the sum of the individual moments of inertia.

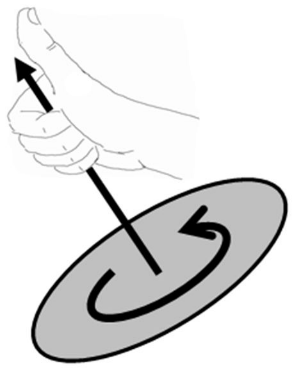


Fig 1

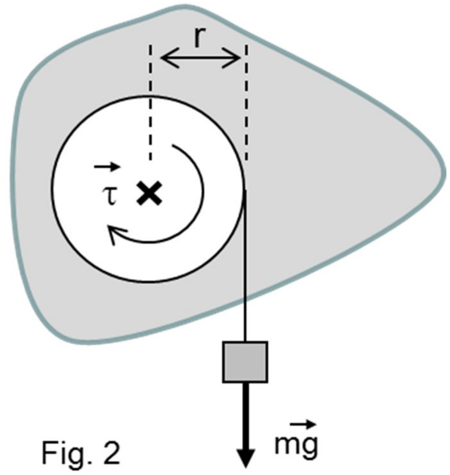


Fig. 2

Consider Fig. 2. An arbitrary object (in this case, the lump sketched above is attached to a pulley. A torque is applied to the object by a hanging weight from the pulley. The net torque on the system is

$$\begin{aligned} \vec{\tau} &= I\vec{\alpha} \\ \vec{r}\times(m\vec{g}) &= \left(I_{object} + I_{weight}\right)\vec{\alpha} \\ rmg &= \left(I_{object} + mr^2\right)\alpha \end{aligned} \tag{8.3}$$

Note that *the hanging weight must be included in the sum* even though it is moving in a straight line!

There is another complication. Since OSHA does not permit us to perform undergraduate laboratories in frictionless vacuums, we should also include the torque of friction in the calculation as well.

$$rmg + \tau_f = \left(I_{object} + mr^2\right)\alpha \tag{8.4}$$

Exercise 1: Rotational inertia of the flat disk

From theory, we know that the rotational inertia of a flat disk is

$$I_{disk} = \frac{1}{2}MR^2 \tag{8.5}$$

Measure the radius and mass of the disk and calculate its rotational inertia.

I_{disk} = ____________________________

Exercise 2: Rotational inertia of the PASCO rotational motion sensor

First, we will measure the rotational inertia of the motion device.

a. Open the file Labfile/Disk Cylinder Inertia L8.A2-1.ds

b. Measure the radius of the largest pulley on the PASCO rotational motion device.

c. Wind the string attached to the 10g mass around that pulley until the mass is suspended about 1m above the floor, but make sure the string falls freely from the pulley before the mass hits the floor.

d. Start data acquisition and let the mass drop.

e. On the angular velocity plot, you should notice two distinct slopes. These correspond to the angular accelerations of the device before and after the weight fell away. Using the slope tool measure these angular accelerations. Note the sign.

α_1= ____________________________ α_2 = _________________________________

f. Place the disk on the device and repeat the above experiment to measure the angular accelerations before and after the weight falls free.

α_3= ____________________________ α_4 = _________________________________

Print out these plot. Annotate this plot to indicate the start of the fall and the release of the weight. Also include the acceleration during each segment of the plot.
For a moment, let's ignore friction. Using equation(8.3), calculate the following moments of inertia.

The PASCO device by itself, I_{PASCO} = _______________________________________

The combination of the PASCO device and the disk, I_{total} = ______________________

Taking the difference, find the rotational inertia of just the disk, and the percent difference between this value and the value found in Exercise 1.

I_{disk} = ____________________________ % diff = ____________________________

Exercise 3: Accounting for friction

Now, let's face reality and deal with friction. As the mass is accelerating the pulley system, the physics is described by the expression using friction

$$rmg + \tau_f = \left(I + mr^2\right)\alpha_1 \tag{8.6}$$

After the mass falls free, this simplifies to

$$\tau_f = I\alpha_2 \tag{8.7}$$

With these two equation, we have two unknowns: I and τ_f. Do the algebra (on a separate sheet) to find expressions for I and τ_f in terms of known values. Write them below.

Now, again calculate the following moments of inertia while accounting for friction:

The PASCO device by itself, I_{PASCO} = ______________________________

The combination of the PASCO device and the disk, I_{total} = ______________________

Taking the difference, find the rotational inertia of just the disk, and the percent difference between this value and the value found in Exercise 1.

I_{disk} = ____________________________ % diff = ____________________________

Exercise 4: Conservation of energy

Before the mass is released, the initial energy of the system E_i is entirely found in the gravitational potential energy of the 10 g mass.

$$E_i = mgh \tag{8.8}$$

Here, h is the distance the weight travels before falling free from the device, *not the initial height above the floor*. This is related to the rotation of the device by

$$h = r\theta \tag{8.9}$$

where r is the radius of the pulley.

The energy just after the weight falls free E_f is the rotational kinetic energy of the system

$$\begin{aligned} E_f &= \frac{1}{2} I\omega^2 \\ &= \frac{1}{2}\left(I_{PASCO} + I_{disk} + mr^2\right)\omega^2 \end{aligned} \tag{8.10}$$

Applying conservation of energy

$$\begin{aligned} E_i - W_f &= E_f \\ mgh - \tau\theta &= \frac{1}{2}\left(I_{PASCO} + I_{disk} + mr^2\right)\omega^2 \end{aligned} \tag{8.11}$$

where W_f is the work done by the torque of friction. Calculate both the left and right sides of Equation (8.11) and find the percent difference between them.

$E_i - W_f$ = ________________________

% diff = _____________________________

E_f = ______________________________

Exercise 5: Changing the axis of rotation

The rotational inertia of a thin disk rotating about its edge is

$$I_{edge} = \frac{1}{4}MR^2 \tag{8.12}$$

g. From your measurements of the mass and radius, calculate the rotational inertia for the disk when rotating on its edge.

h. Using the same method of Exercise 3, measure the rotational inertia in this orientation.

i. Find the percent difference between these values. Record them below.

Review

j. Throughout this lab, what specific assumptions did you make about the apparatus (such as about the mass, friction, *etc*.)

k. In accounting for the torque of friction, you assumed that the torque is constant. How can you show that this is a valid assumption?

l. In Exercise 4, why did you not account for the linear kinetic energy ($\frac{1}{2}mv^2$) of the falling weight?

m. Suppose the mass of the string were not negligible. How would that affect the data?

Name___________________________ Section _______ Date_____________

Pre-Lab Preparation Sheet for Lab 9: Fluids, Pressure, and Buoyancy

(Due at the Beginning of Lab)

Directions:

Read over the lab and then answer the following questions.

1. Relative to water, what is the relative density $\left(\dfrac{\rho}{\rho_{water}}\right)$ of the following materials:

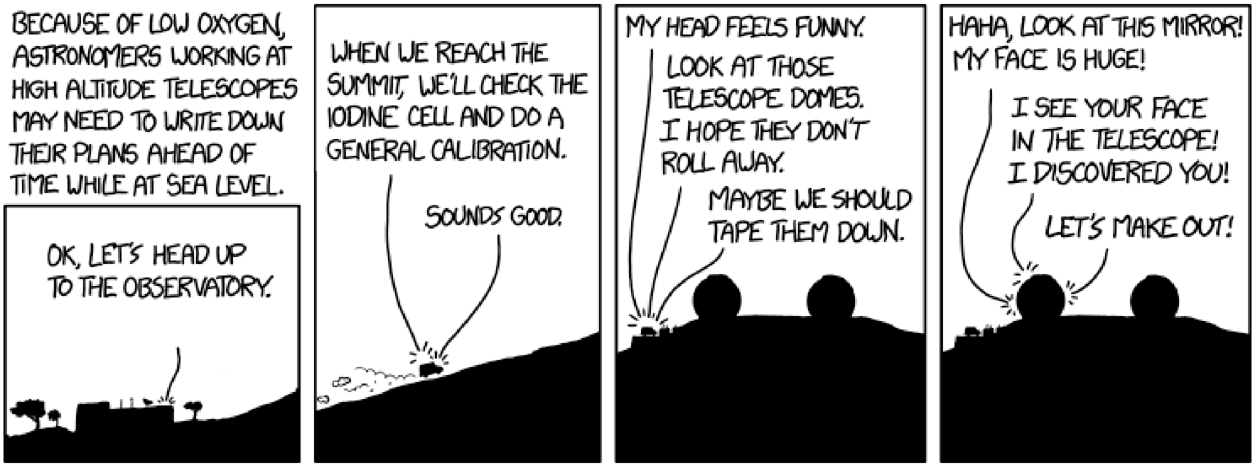
    a. air (sea level):

    b. seawater:

    c. mineral oil:

    d. iron:

2. Starting with Archimedes' Principle, derive equation (7) for a submerged object.


BECAUSE OF LOW OXYGEN, ASTRONOMERS WORKING AT HIGH ALTITUDE TELESCOPES MAY NEED TO WRITE DOWN THEIR PLANS AHEAD OF TIME WHILE AT SEA LEVEL.
OK, LET'S HEAD UP TO THE OBSERVATORY.
WHEN WE REACH THE SUMMIT, WE'LL CHECK THE IODINE CELL AND DO A GENERAL CALIBRATION.
SOUNDS GOOD.
MY HEAD FEELS FUNNY.
LOOK AT THOSE TELESCOPE DOMES. I HOPE THEY DON'T ROLL AWAY.
MAYBE WE SHOULD TAPE THEM DOWN.
HAHA, LOOK AT THIS MIRROR! MY FACE IS HUGE!
I SEE YOUR FACE IN THE TELESCOPE! I DISCOVERED YOU!
LET'S MAKE OUT!
xkcd.com

Name ____________________ Date __________ Partners ________________

TA ________________ Section _______ ________________

Lab 9: Fluids, Pressure, and Buoyancy

"εὕρηκα!"

--Archimedes

Equipment

Force sensor (port A)

Motion sensor (ports 1&2)

Rotation sensor (ports 3&4)

Hooked masses of 50g, 200g, and 500g

2 PASCO harmonics springs

Mass hanger with two 50 g masses and one 100 g mass

Two loops of cord: ~170 cm and ~110 cm lengths

Pressure

Pressure describes the distribution of force over a surface and is defined as force per unit area.

$$P = \frac{F}{A} \tag{1}$$

The basic unit of pressure in SI units is the Pascal

$$1\ \mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2} \tag{2}$$

For perspective, standard atmospheric pressure at sea level (1 atm) is about 100 kPa:

1 atm = 101300 Pa.*

Pressure may be measured in one of two ways:

- ***Gauge pressure*** measures the difference in pressure between two fluids. For example, a tire gauge measures the difference in pressure between the air inside a tire and the air outside.
- ***Absolute pressure*** measures the pressure a single fluid exerts on a surface, or equivalently the difference in pressure between a particular fluid and a vacuum.

In this lab, we will only measure the gauge pressure relative to the ambient air pressure.

* Also, 1 atm = 14.65 psi (pounds per square inch).

CAUTION: The sensor **must remain dry**. Do not let water reach the sensor!

Part I: Pressure and depth

What is the water pressure at a given depth? Imagine a cylinder of water stretching from the surface to a depth *d*. (See Fig. 1.)

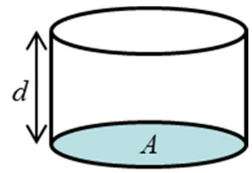


Fig. 1

First, let's define density ρ (in particular volumetric density)* as mass per unit volume:

$$\rho = \frac{m}{V} \tag{3}$$

Therefore, this volume of water will have a weight

$$\begin{aligned} mg &= \left(\rho_{H_2O} V\right) g \\ &= \left(\rho_{H_2O} Ad\right) g \end{aligned} \tag{4}$$

where ρ_{H2O} is the density of water: about 998 kg/m^3. Thus, the gauge pressure (the difference between the pressure at depth *d* and the pressure at the surface) is

$$\begin{aligned} P &= \frac{F}{A} \\ &= \frac{\rho_{H_2O} Adg}{A} \\ &= \rho_{H_2O} gd \end{aligned} \tag{5}$$

This expression holds for any fluid where the density is constant.

The *absolute pressure* at a given depth will be the gauge pressure plus the air pressure at the surface.

$$P_{absolute} = P_{air} + \rho gd \tag{6}$$

1. At what depth does the pressure increase by 1 atmosphere?

Exercise A: Inventory

2. With the graduated cylinder, measure the density of water and the associated uncertainty.

* NOTE: ρ is the Greek letter *rho*. It is **NOT p**!!! Do not call it p!

$\rho_{H2O} =$ ____

3. Using water displacement, measure the volume and associated uncertainty of the aluminum bar. Then, measure the mass, density, and the associated uncertainties of the aluminum bar.

$V_{Al} +$ ____

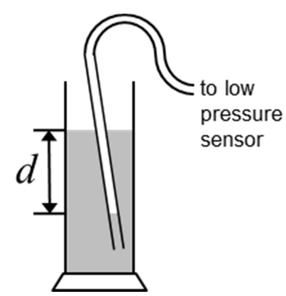
$m_{Al} =$ ____

$\rho_{Al} =$ ____

Exercise B: Fluid Pressure

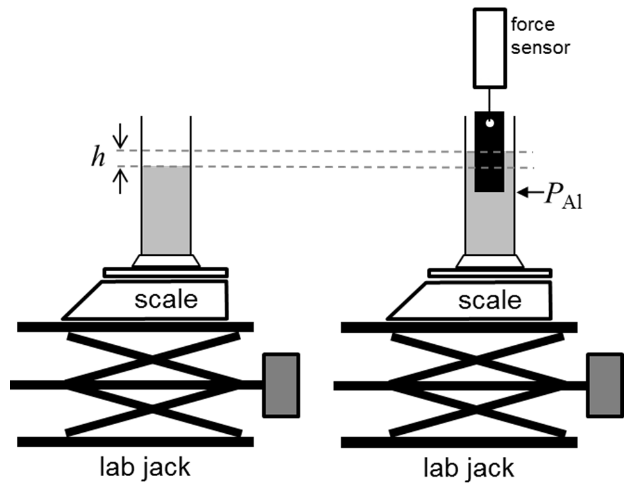
4. Calibrate the Low Pressure Sensor
    a. Verify that the end of the hose is exposed to the ambient air.
    b. Click Calibration  on the left side of the screen.
    c. Select One Standard (1 point offset)
    d. With the Standard Value = 0, set the current value to the Standard Value.

5. Lower the end of the hose into the water. The water level in the hose is the level at which the water pressure is in equilibrium with the air pressure in the hose. Measure the pressure at the following depths:



d (m)	$P_{measured}$(Pa)	ρgd	% difference
0.05			
0.10			
0.15			

Exercise C: Buoyancy -- qualitative observations



6. Place the scale on the lab jack; then place the graduated cylinder on the scale.
7. Tare the scale.
8. Note the water level in the cylinder.
9. By raising the lab jack, slowly lower the aluminum block into the water. Note the following:

   a. How does the apparent weight (as measured by the force sensor) change as the aluminum bar is lowered?

   b. How does the reading on the scale change as the bar is lowered?

   c. How does the sum of the force sensor and scale readings change as the bar is lowered?

d. Given the uncertainties of the force sensor and scale, what is the most accurate way to measure the apparent weight of the bar in water?

Exercise D: Buoyancy – quantitative observations

10. Lower the block about 2/3 the way into the water. Measure the following

apparent weight of the Al block in the water = ______________________________________

change in water level h = ____

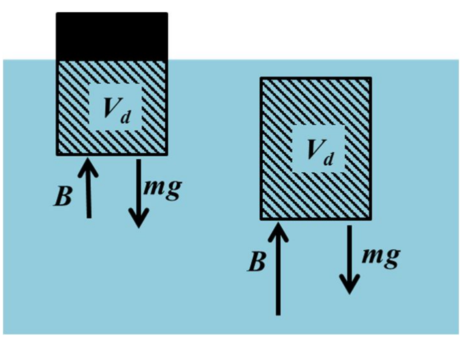
water pressure at the bottom of the Al block P_{Al} = __________________________________

11. Draw a free body diagram for the Al block. From the diagram, how should P_{Al} relate to the apparent weight of the Al block in the water? Then, verify this with your measurements.

12. How should change in water level relate to the change in . . .
    a. . . . the pressure at the bottom of the cylinder?
    b. . . . the force on the scale?

13. How does P_{Al} relate to the apparent weight of the Al block?

14. The Buoyant force is the net force of a fluid on an object. Archimedes said that *the buoyant force is equal to the weight of the displaced fluid.* Algebraically, show that this is true.



Exercise D: Density

Using Archimedes' Principle, it can be shown that the density of a completely submerged object is given by

$$\rho = \rho_{H_2O}\left(1 - \frac{\textit{apparent weight in water}}{\textit{actual weight}}\right)^{-1} \tag{7}$$

15. Applying this rule, precisely measure the density of aluminum and the corresponding uncertainty.

16. How does the uncertainty of this measurement compare with your earlier measurement of the density? Is it better or worse? Why?

Often, it is useful to measure a quantity relative to another quantity. This can eliminate unnecessary uncertainty.

17. Applying Equ. (7), measure the ration of the density of aluminum to the density of water $\left(\frac{\rho_{Al}}{\rho_{H_2 0}}\right)$ and note the corresponding uncertainty.

18. You will find a short rod of a white metal on your table. Carefully measure its density, then (with some assistance from the internet) identify its composition. (Hint: It is not an alloy but made of a single element.) Briefly, explain your reasoning.

Name__________________________ Section _______ Date_____________

Pre-Lab Preparation Sheet for Lab 10: Harmonic Motion

(Due at the Beginning of Lab)

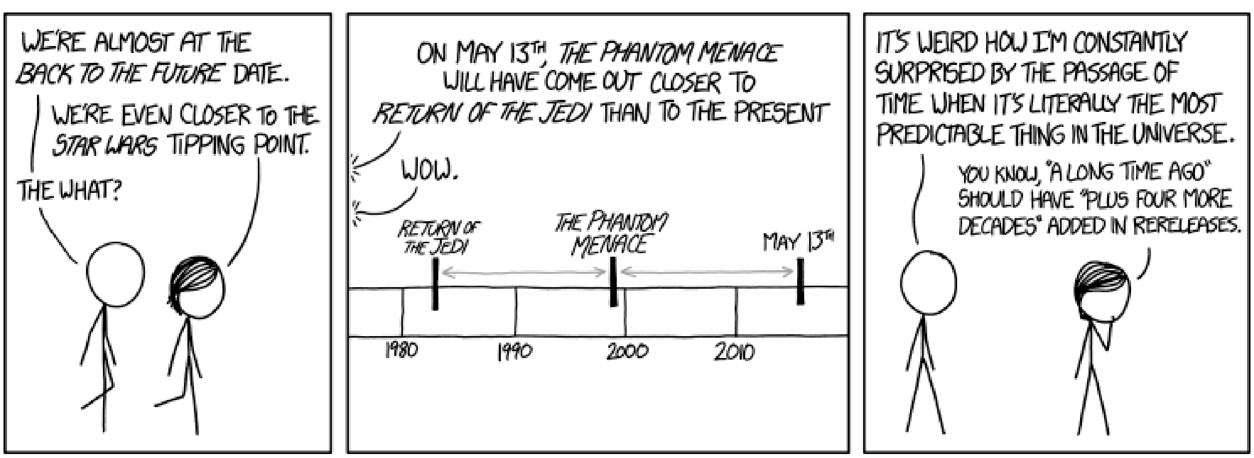
Directions: Read Appendix A. Then, read over the lab and then answer the following questions.

1. What type of force **F**(x) will produce Harmonic Motion?

2. What type of force **F**(x) will produce Simple Harmonic Motion?

See other side

3. Demonstrate that Equ. (11.5) is a solution to Equ. (11.4). That is, find the 2nd derivative of Equ. (11.5); then verify that Equ. (11.4) remains valid.



xkcd.com A little old, but still relevant. (And deeply depressing.)

Name ____________________ Date __________ Partners ________________

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Lab 10: Harmonic Motion

"He wrested the world's whereabouts from the stars, and locked the secret in a pocket watch."

--Dava Sobel on John Harrison

Objective

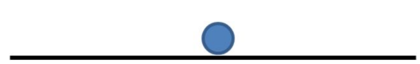
- To recognize the force and energy patterns that result in harmonic oscillation and simple harmonic oscillation
- To recognize when oscillatory motion can be approximated as simple harmonic oscillation
- To understand the relationship between simple harmonic oscillators and time-keeping

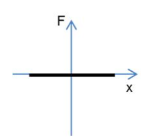
Equipment

Force sensor (port A)	2 PASCO harmonics springs
Motion sensor (ports 1&2)	Mass hanger with two 50 g masses and one 100 g mass
Rotation sensor (ports 3&4)	Two loops of cord: ~170 cm and ~110 cm lengths
Hooked masses of 50g, 200g, and 500g	

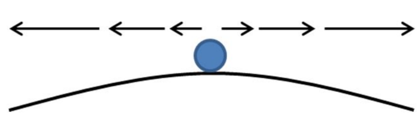
An object is in equilibrium when there is no net force on it. We define three basic types of equilibrium. Consider a ball on a track.

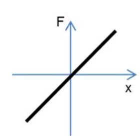
If the track is flat, there is no net force on the ball. If the ball received a bump it would roll in the direction of the bump at a constant speed. This is *neutral equilibrium*.



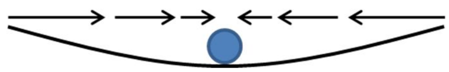


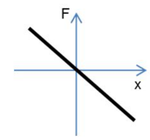
If the ball is on the peak of a hill, again there is no net force on it. But, if it receives a bump to the left (or right) it experiences a repelling force which further accelerates the ball to the left (or right) away from its equilibrium position. This is *unstable equilibrium*.





If the ball is at the bottom of a trough, again there is no net force on it. However, if it receives a bump to the left, it experiences a restoring force to the right returning the ball to its equilibrium position. The ball will then oscillate back and forth about its equilibrium position.* This is *stable equilibrium*.





Oscillation about a stable equilibrium point is the focus of this lab.

Equilibrium and Potential Energy

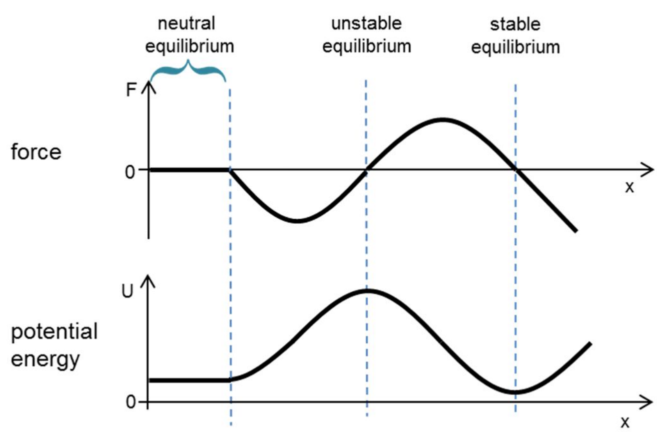
When dealing with conservative forces, we can also understand equilibrium in terms of potential energy U(x).

$$U(\vec{x}) \equiv -\int_0^x F(a) \bullet da \tag{11.1}$$

$$F(x) = -\frac{dU(x)}{dx} \tag{11.2}$$

In this perspective:

- Neutral equilibrium: regions where the potential energy is constant
- Unstable equilibrium: points where the potential energy is at a local maximum
- Stable equilibrium: points where the potential energy is at a local minimum



* . . . unless there is excessive damping.

Simple Harmonic Oscillators and Springs

The simplest type of oscillation arises with a simple spring:

$$F = -kx \tag{11.3}$$

Applying some basic algebra . . .

$$ma = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \tag{11.4}$$

The solution to this differential equation is

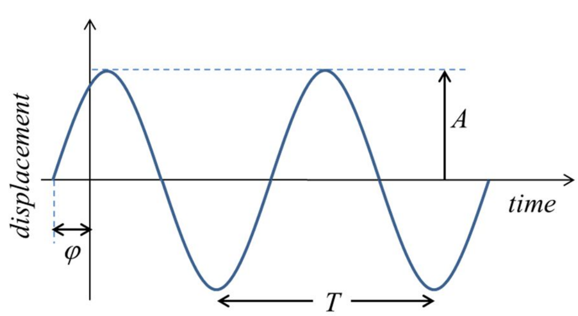
$$\begin{aligned} x(t) &= A\sin\left(\sqrt{\frac{k}{m}}\,t + \varphi\right) \\ &= A\sin\left(\omega\,t + \varphi\right) \end{aligned} \tag{11.5}$$

Here,

- A is the amplitude of the oscillation
- $\omega = \sqrt{\dfrac{k}{m}}$ is the angular frequency (in radians/sec)
- φ is the phase shift

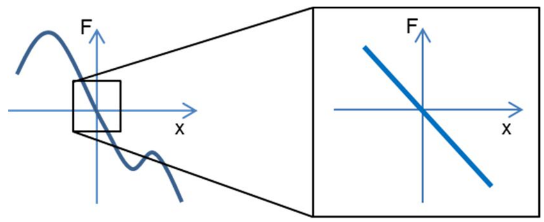
We can relate ω to the frequency f and period T by

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{11.6}$$



This is called a *Simple Harmonic Oscillator*.

In general, a restoring force can be quite complicated. A restoring force need only be positive when x is negative and *vice versa*. One would expect the resulting oscillation to be very complicated as well. However, ***if the oscillations are small enough***, the force close the equilibrium point can be usually approximated as linear* [$F(x) = -kx$] and the observed motion can be approximated by a sinusoid [$A \sin(\omega t + \varphi)$].† Note the diagram below.



In short, **if you can write the force as $F(x) = -kx$‡, you have a simple harmonic oscillator**. Then, you *immediately* know is motion is described by Equ. (11.5). *You don't actually have to do any math!!*

In practice, we see such oscillators *all the time*: a tree swaying in the breeze, a boat rocking on a lake, a quartz crystal vibrating in a watch. And once you recognize a system as approximately simple harmonic, you can quickly explain a lot about it.

For a Simple Harmonic Oscillator:

1. How does the oscillating frequency scale with mass?

2. How does the oscillating frequency scale with the spring constant?

3. How does the oscillating frequency scale with the amplitude of the motion?

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* There are oscillators which cannot be approximated by a linear function. These usually involve sharp accelerations (*i.e.*, delta functions). For example, a bouncing ball.

† This is equivalent to approximating the force function with the first term of a Taylor series expansion:

$$\begin{aligned} F(x) &= c_1 x + c_2 x^2 + c_3 x^3 + \dots \\ &\approx c_1 x \\ &\approx -k x \end{aligned}$$

‡ Note that x does not have to be a distance. It could be an angle or an electronic voltage. The solution is still an oscillator.

Exercise 1: Springs

1. Run the program LabFile/A Labs/Simple Harmonic Motion. Select the first tab, Force vs. Displacement.
2. You have two springs. Select one, and generate a force vs. distance curve. **Stretch the spring no more than 0.2 m.**
3. Record the spring constant on Tables 1 and 2 below? Is this close to an ideal spring?

4. Repeat with the 2^{nd} spring.
5. Record the spring constant on Table 3 below? Is this close to an ideal spring?

6. Select the 2^{nd} tab, Spring Oscillator. Suspend the hanger from a single spring so it has a total mass of 100 g. Position it over the motion sensor. Gently pull it down 2 centimeters, then release.
7. Record its motion for about 15 cycles. Then, fit this data to an appropriate function.
8. Repeat for displacements of approximately 4 and 8 cm, and fill in Table 1 below. (Get the exact amplitude from the fit.) Does the frequency scale with amplitude?

Table 1: 1st spring, 100 g mass, k = ______________________________

A (m)	$\omega_{expected}$ (rad/s)	$\omega_{measured}$ (rad/s)	% diff

9. Measure the oscillating frequency for a variety of masses, but this time keeping the amplitude roughly constant. Fill in Table 2 below.

Table 2: 1st spring, k = ________________________________

m (kg)	A (m)	$\omega_{expected}$ (rad/s)	$\omega_{measured}$ (rad/s)	% diff
0.050				
0.100				
0.150				

10. Attach the 2nd spring. Keeping the amplitude roughly constant, fill in Table 3 below.

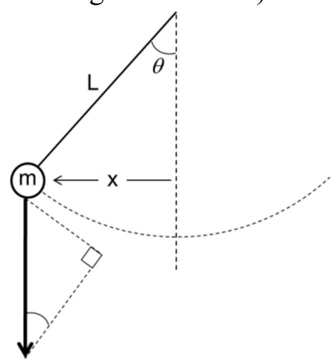
Table 3: 2nd spring, k = ___________________________________

m (kg)	A (m)	$\omega_{expected}$ (rad/s)	$\omega_{measured}$ (rad/s)	% diff
0.050				
0.100				
0.150				

Exercise 2: Pendulums

Consider the pendulum illustrated below.

1. Consider the free body diagram for the bob. Calculate the restoring force (that is, the force tangent to the arc) on the bob. Would this force produce simple harmonic motion?



2. Re-calculate the restoring force in the limit of small angles.

$$\sin\theta \approx \theta \approx \frac{x}{L} \tag{11.7}$$

In this limit, what is the effective spring constant for the pendulum?

3. Write an expression for the angular frequency of the pendulum in terms of the length of the pendulum and the mass of the bob. How should the frequency of the pendulum scale with the mass of the bob? With the length of the pendulum?

4. Using your short string, hang the 200 g mass from the rotation sensor to form a pendulum. Select the 3rd tab, Pendulum. Measure the angular frequency for amplitudes of approximately 0.1, 0.2, 0.4, and 0.8 radians. Fill in the table below. Measure the pendulum length from the axis to the middle of the mass. Keep the pendulum's motion in the plane of the rotation sensor.

Note: Here at Vanderbilt, $g = (9.7943 \pm 0.0032)\ \mathrm{m/s^2}$

CAUTION: Don't hit anyone with the pendulum!
Don't step into a swinging pendulum!

Table 4: Pendulum: 200 g bob, L = ________________________

A	$\omega_{measured}$	$\omega_{expected}$	% diff

5. By looping each string twice, you have four different pendulum lengths available. Using the 200 g mass, measure the angular frequency for pendulums of four different lengths. Keep the amplitudes small, less than 0.1 radians.

Table 5: Pendulum: 200 g bob

L	A	$\omega_{measured}$	$\omega_{expected}$	% diff

6. With your short string, find the angular frequency of the pendulum for three different masses. Fill in the table below. Do you see any trend to the frequency data? If so, explain.

Table 6: Pendulum: L = ________________________

m	A	$\omega_{measured}$
50 g		
200 g		
500 g		

Clocks

Clocks work by counting the number of times a repeating phenomenon happens, such as the rising of the sun, the swing of a pendulum, or the vibration of a quartz crystal. The frequency of a *good* clock is nearly constant and immune to small disturbances of its motion.

Question 1: A particular clock is accurate to within one minute per week. What is its percent uncertainty?

Question 2: Why are simple harmonic oscillators good clocks?

Question 3: Why does a grandfather clock use a long pendulum?

Name___________________________ Section _______ Date_____________

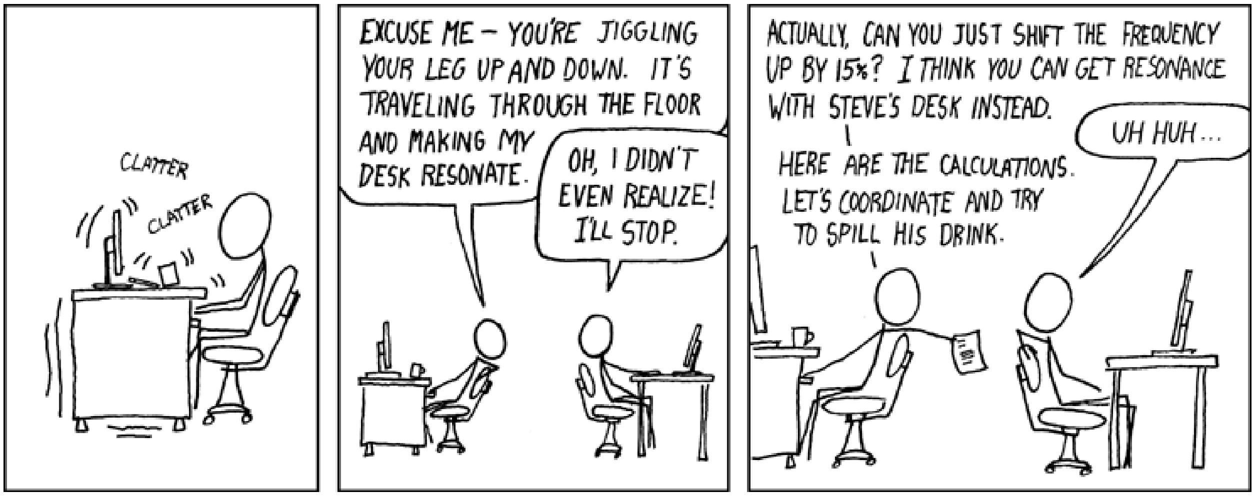
Pre-Lab Preparation Sheet for Lab 11:
Standing Waves and Resonance

(Due at the Beginning of Lab)

1. On a standard 88 key piano, Middle C has a frequency of 261.626 Hz. What is the wave length in air?

2. What are the wavelengths of the highest (4186 Hz) and lowest (27.50 Hz) keys?

3. The notes A4 (440.000 Hz) and D4 (293.665 Hz) form a perfect fifth: a frequency ratio of 3:2. Find three harmonic frequencies which these notes share? (Or very nearly share.)


CLATTER
CLATTER
EXCUSE ME – YOU'RE JIGGLING YOUR LEG UP AND DOWN. IT'S TRAVELING THROUGH THE FLOOR AND MAKING MY DESK RESONATE.
OH, I DIDN'T EVEN REALIZE! I'LL STOP.
ACTUALLY, CAN YOU JUST SHIFT THE FREQUENCY UP BY 15%? I THINK YOU CAN GET RESONANCE WITH STEVE'S DESK INSTEAD.
HERE ARE THE CALCULATIONS. LET'S COORDINATE AND TRY TO SPILL HIS DRINK.
UH HUH...

xkcd.com

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Lab 11: Standing Waves and Resonance

There is geometry in the humming of the strings; there is music in the spacing of the spheres.

--Pythagoras

Objectives

- Understand the relationship between harmonic oscillators and waves
- Understand how boundary conditions lead to resonance

Equipment

- Function generator with mason string. 90 g for tension
- Oscillator
- Precision mass set
- Large slotted mass, 4 kg total
- Steel wire
- Microphone
- Spectrum Lab
- Super pulley + small pulley
- 3 table clamps
- Risers for sound board / equipment box
- Bridge

Introduction: The harmonic oscillator vs. the unbounded wave

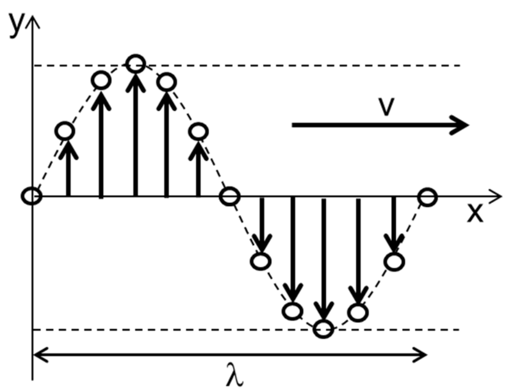
The equation for a simple harmonic oscillator describes the motion of a single rigid object.

$$y(t) = A\cos(\phi - \omega t) \tag{12.1}$$

Here, ω is determined by the mass and spring constant of the oscillator:

$$\omega = \sqrt{\frac{k}{m}} \tag{12.2}$$

A wave (for example, waves on a string) can be regarded as a collection of coupled simple harmonic oscillators, where each oscillator is a single segment of the string. However, each segment of the string is slightly out of phase with its neighbor. Thus, the phase shift ϕ of each oscillator is a function of x.



$$\begin{aligned} y(x,t) &= A\cos(kx - \omega t) \\ &= A\cos\left(\frac{2\pi}{\lambda}x - 2\pi ft\right) \\ &= A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right) \\ &= A\cos\left(\frac{2\pi}{\lambda}\left[x - vt\right]\right) \end{aligned} \tag{12.3}$$

Simulation 1: Run the file Waves 1.html.

The speed of the wave v is determined by the tension T and the linear mass density μ of the string.

$$v = \sqrt{\frac{T}{\mu}} \tag{12.4}$$

The speed is related to (*but* ***not*** *determined by*) the wavelength λ and frequency f:

$$v = \lambda f \tag{12.5}$$

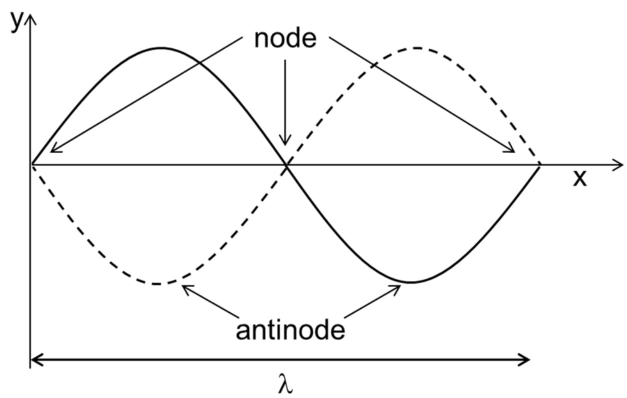
In other words, if you increase the wavelength you will lower the frequency, but the speed remains unchanged.

Interference and standing waves

If two waves of the same frequency are simultaneously traveling in opposite directions, they well interfere with each other to create a **standing wave**.

Simulation 2: Run the file Waves 2.html.

Note how the wave shape remains, but the wave does not appear to be going anywhere. Instead, a pattern of **nodes** (where the string does not move) and **antinodes** (where the string exhibits maximum motion) appear.



Exercise 1: Real strings and boundaries

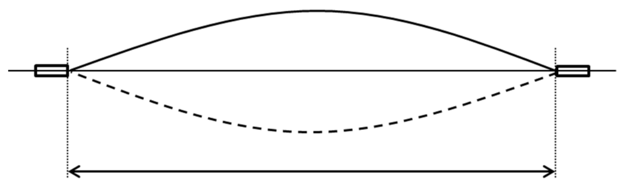
Equation (12.3) nicely describes a wave traveling along an infinitely long string which is magically suspended in space. Any frequency/wavelength may be produced on the string

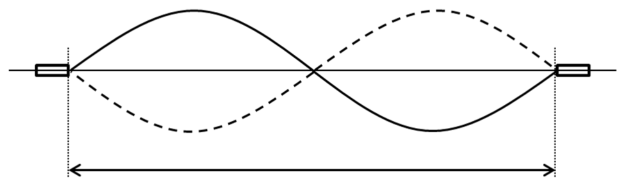
However, real strings have ends and are held in place by clamps, bridges, nuts, and whatnot. These create **boundary conditions:** points where we constrain the position of the string to a particular value. Consider a string of length L clamped at opposite ends of its length.

a. Since waves reflect off the clamps, waves will be traveling in both directions on the string. This will create a standing wave.
b. Since the ends cannot move, only standing waves with nodes separated by a distance L can exist on the string.

Hence, *the allowed wavelengths are quantized**: only those waves which match the boundary conditions can exist on the string. The first two solutions are illustrated below, but other solutions also exist. Each solution is called a **resonance**.

* No, this is not quantum mechanics. But it is very analogous to the quantization of energy levels in quantum mechanical systems.





1. In the blank space below, sketch the next two solutions, and record each wavelength λ_n in terms of L.
2. Then, write down an expression for the wavelength λ for all possible solutions.

The longest possible wavelength λ_1 is called the **fundamental** or the **first harmonic**.

Half of this wavelength $\lambda_1/2$ is the **second harmonic**.

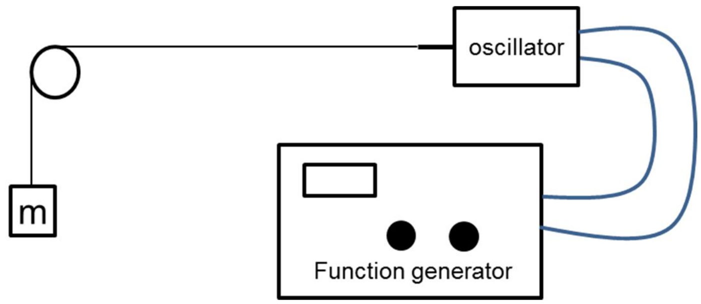
One third of the fundamental wavelength $\lambda_1/3$ is called the **third harmonic**.

Et cetera.*

3. Label your sketches with the corresponding harmonic.

* For some systems (such as certain organ pipes) not all harmonics are allowed.

Exercise 2: Driven waves on a string



1. Measure the linear mass density (mass / length) of the of the mason's twine.

μ = ____

2. Apply 90 g to tension the string. Calculate the velocity of waves on the string.

v = ____

3. Turn on the oscillator and adjust the frequency of the function generator to about 5 Hz. Then, steadily increase the frequency until the string resonates with a maximum amplitude in its fundamental mode. Adjust the amplitude knob as needed.

f_1 = ____

4. Describe the boundary conditions for the wave on this string. (Note: this string is not clamped at each end.)

5. Noting the positions of the nodes, measure the wavelength of the fundamental λ_1.

6. Knowing the frequency and wavelength, calculate the speed of waves on the string. Then, calculate the percent difference between this value and the wave-speed calculated earlier.

$v =$ ____

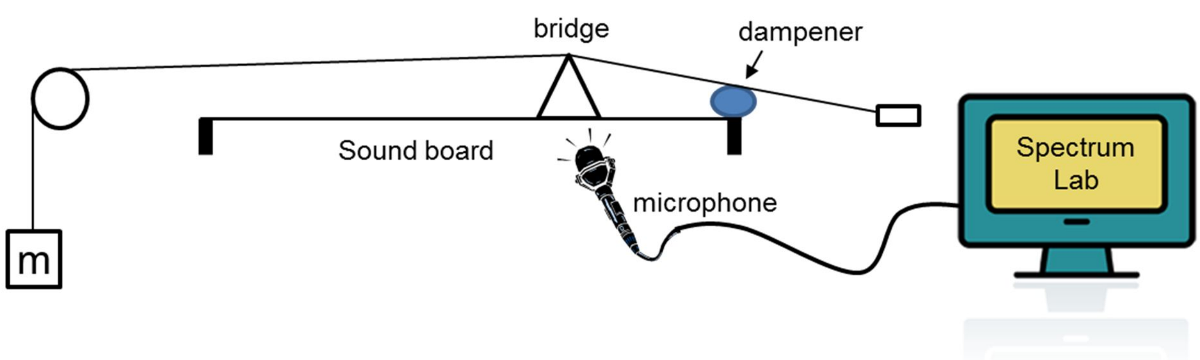
% diff = ____________________________________

7. Estimate the frequencies of the next three harmonics and enter the values in the table below. Then with the function generator, experimentally measure the frequency of each harmonic.

	Estimated frequency	Measured frequency
f_1	XXXXXXXXXXXXXXXXXXXXX	
f_2		
f_3		
f_4		

Exercise 3: The one string Tennessee music box.*

In the previous exercise, you actively drove the string at particular frequencies to see which frequencies would resonate. In this exercise, you will randomly dump energy into the wire (*i.e.*, pluck the wire) and see what frequencies naturally appear.



1. Study the steel wire and describe the boundary conditions of its motion.

* Because three strings is just way too complicated.

2. With the wire sample provided, measure the linear mass density μ of the wire. Also, measure the vibrating length L of the wire.

$\mu =$ ________________________________

$L =$ ________________________________

3. Using a hanger mass of 4 kg to tension the wire, calculate the speed of waves on the wire.

$v =$ ________________________________

4. Calculate the frequency of the first four harmonics and fill in the table below.
5. Place the microphone underneath the sound board. Then, run the program Spectrum Lab (found under the Start menu).
6. Pluck the wire and measure the frequencies of the first four harmonics. (You can use the mouse to identify the frequency components on the screen.)

Tension with 4 kg on hanger

	Calculated frequency	Measured frequency
f_1		
f_2		
f_3		
f_4		

7. Reduce the tension on the string by a factor of two. Recalculate the frequencies and fill in the table below.

8. Using Spectrum Lab, measure the new value of the frequencies on the string.

Reduced tension

	Calculated frequency	Measured frequency
f_1		
f_2		
f_3		
f_4		

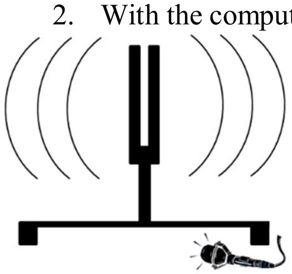
Exercise 4: Selecting harmonics

1. Locate the middle of the string.
2. Pluck the string; then gently touch the string in the middle. You should notice a distinct increase in the pitch.
3. Which harmonics are suppressed? Explain why.

4. Where should you place your finger to dampen to dampen the first two harmonics, but retain the third? Try it.

Exercise 4: Harmonic patterns

1. Select a tuning fork within your vocal range. Gently strike the fork with the rubber mallet and place the bottom of the fork against the sound board.
2. With the computer, note the pattern of harmonics displayed on the screen.



3. From your group, select the best opera singer.*
4. Holding the microphone, with a steady clear voice sing "*Ahhhhhhhhhhh*". Try to match the tone of the tuning fork. Note the pattern of harmonics displayed on the screen.
5. Using the same pitch as before, sing "*Eeeeeeeeeeeeeeee*".
6. Briefly describe how the harmonic patterns differ.

* . . ., or whoever is wearing a Viking helmet.

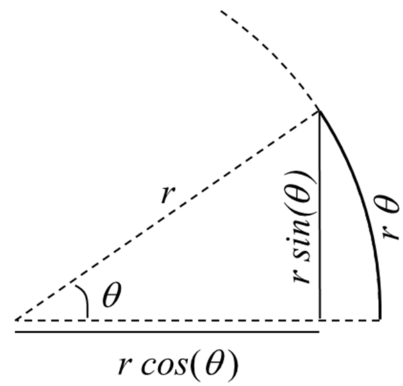
7. Hiss into the microphone. "*Sssssssssssss"* How does this frequency pattern compare the vowel sounds above? Why is it not possible to sing with consonants?

Appendix A: The Small Angle Approximation

Often, you will find yourself dealing with small angles. In such cases, the following approximations can make your calculations a lot easier.

If an angle θ is measured in radians, the arc length of the corresponding circle segment s is given by

$$s = r\theta \ .$$



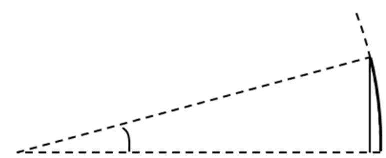
By definition

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \ .$$

Inspecting the diagram, it is obvious that for small angles

$$r\sin(\theta) \approx r\theta$$

$$\boxed{\sin(\theta) \approx \theta} \ .$$



It is also obvious that for small angles

$$r\cos(\theta) \approx r$$

$$\boxed{\cos(\theta) \approx 1}$$

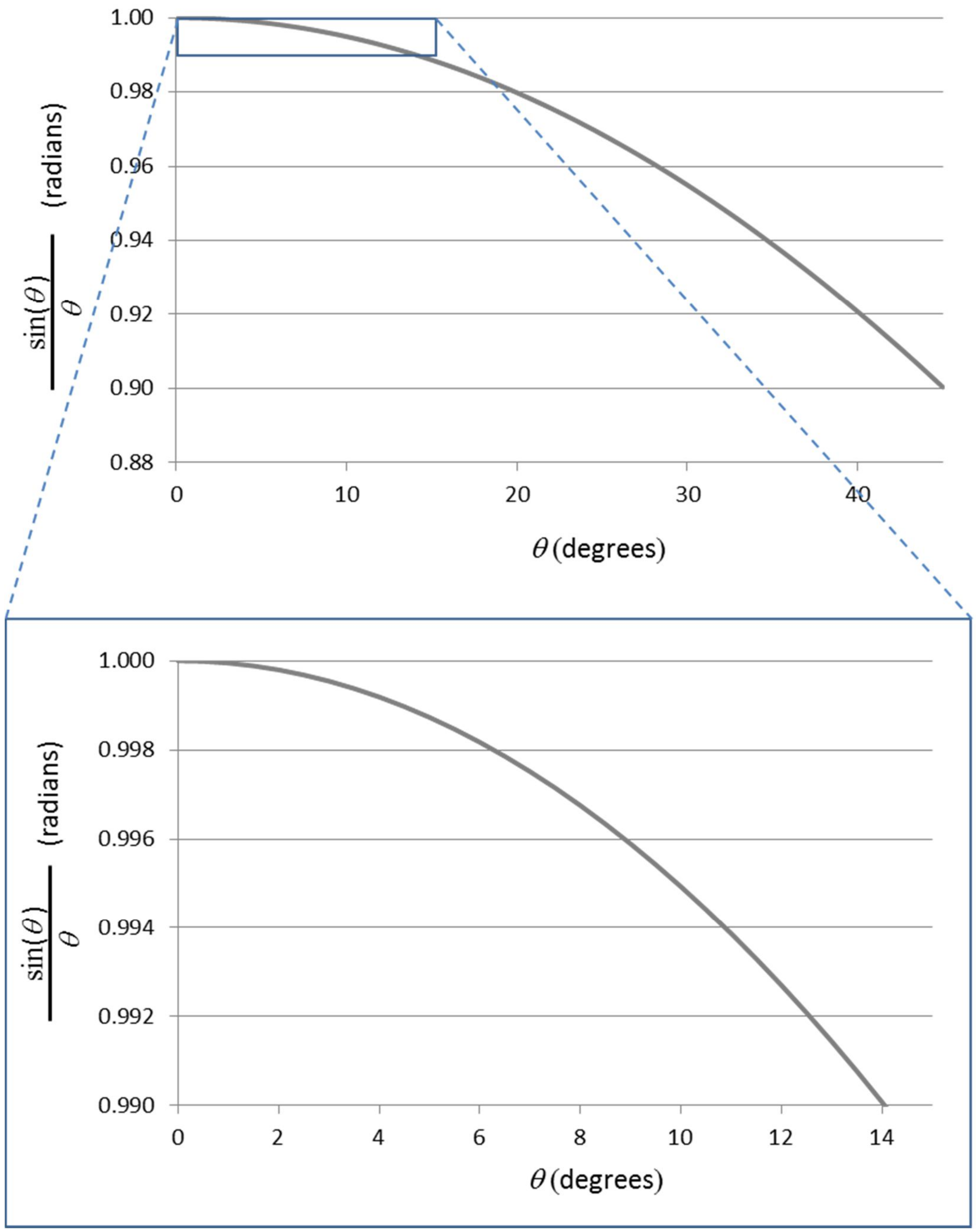
Thus,

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\boxed{\begin{aligned} \tan(\theta) &\approx \sin(\theta) \\ &\approx \theta \end{aligned}}$$

So, for what angles does this approximation work? Well, it depends on how accurate you need to be. The graph below is a useful guide. The closer $\dfrac{\sin(\theta)}{\theta}$ is to 1, the better the approximation.

Note: The angles on the x-axis are in degrees. The angles on the y-axis are calculated in radians.

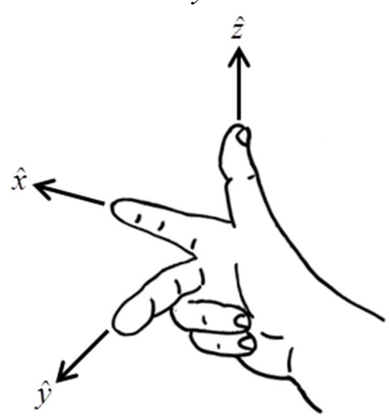


Appendix B: The Right Hand Rule and Right Handed Coordinates

Many of the equations used in this lab and the associated class require that you use a *Right Handed Coordinate System*, particularly those equations involving cross products. Using the wrong coordinates can lead to considerable confusion and grief; so, **get in the habit of drawing your coordinate axes correctly!**

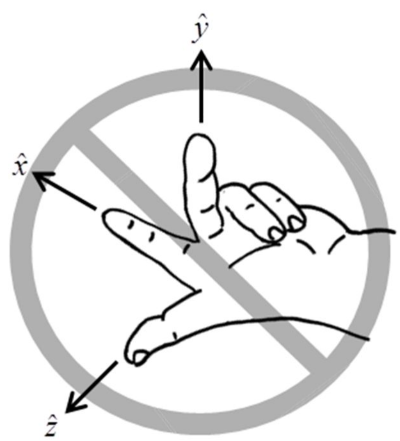
Consider the illustration of a Right Handed Coordinate axes below. The unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ correspond to the index finger, middle finger, and thumb respectively. Here

$$\hat{x} \times \hat{y} = \hat{z}$$



Right Handed Coordinates

Below is an example of a Left Handed Coordinate system where $\hat{x} \times \hat{y} = -\hat{z}$, and *that's wrong*!!



Left Handed Coordinates

Appendix C: Uncertainties with Dependent and Independent Measurements

Suppose you had measured three different rods:

$x = (2.34 \pm 0.03)$ m $\quad$ $y = (5.43 \pm 0.05)$ m $\quad$ $z = (8.31 \pm 0.02)$ m

What would be the length of these three rods placed end to end?

In Lab 1, you were instructed to combine the uncertainties by simply adding the absolute uncertainties.

$$\begin{aligned} L &= x + y + z \\ &= (2.34 + 5.43 + 8.31)\text{ m} \pm (0.03 + 0.05 + 0.02)\text{ m} \\ &= (16.08 \pm 0.10)\text{ m} \end{aligned}$$

In fact, this is an upper limit of the uncertainty. In this particular case, we can actually do a bit better. The logic goes like this:

If you make several independent measurements subject to random uncertainties, we expect some measurements will be a little bit high and some to be a little bit low. These variations should partially cancel out.

I'll spare you the ugly details of the math, but it boils down to this:

When adding several independent measurements

$$Q = q_1 + q_2 + \ldots + q_n$$

the corresponding absolute uncertainties are added in quadrature.*

$$\Delta Q = \sqrt{\Delta q_1^{\,2} + \Delta q_2^{\,2} + \ldots + \Delta q_n^{\,2}}$$

Thus, in the example above we have

$$\begin{aligned} L &= x + y + z \\ &= (2.34 + 5.43 + 8.31)\text{ m} \pm \left(\sqrt{0.03^2 + 0.05^2 + 0.02^2}\right)\text{ m} \\ &= (16.08 \pm 0.06)\text{ m} \end{aligned}$$

* That is, added like the Pythagorean Theorem.

Note that for this to apply, we must be adding independent measurements. Consider for a moment a different problem:

> *You are given a rod of length* $r = (1.54 \pm 0.04)$m, *What would be the length of 3 similar rods placed end to end?*

Here, **we only have a measurement for a single rod**; so, there is no longer a partial canceling out of uncertainties. In this case, the best we can do is to simply add the uncertainties:

$$\begin{aligned} 3r &= (1.54 + 1.54 + 1.54)\,\text{m} \pm (0.04 + 0.04 + 0.04)\,\text{m} \\ &= 3\,(1.54 \pm 0.04)\,\text{m} \\ &= (4.62 \pm 0.12)\,\text{m} \end{aligned}$$

What about when multiplying (or dividing) measurements? The method is similar:

When multiplying (or dividing) several independent measurements

$$Q = q_1 \times q_2 \times \ldots \times q_n$$

the corresponding relative uncertainties are added in quadrature.

$$\Delta Q = \sqrt{\left(\frac{\Delta q_1}{q_1}\right)^2 + \left(\frac{\Delta q_2}{q_2}\right)^2 + \ldots + \left(\frac{\Delta q_n}{q_n}\right)^2}$$

So, if I were to take the measurements in the first example and multiply them together (say, to find a volume), the result would be

$$\begin{aligned} V &= x \times y \times z \\ &= (2.34 \times 5.43 \times 8.31)\ \text{m}^3\ \pm \left(\sqrt{\left(\frac{0.03}{2.34}\right)^2 + \left(\frac{0.05}{5.43}\right)^2 + \left(\frac{0.02}{8.31}\right)^2}\right) 100\% \\ &= 16.08\ \text{m}^3 \pm (0.016)100\% \\ &= 16.08\ \text{m}^3 \pm 1.6\% \end{aligned}$$

But, suppose I had a measured only one rod and imagined constructing a cube 3 rods long by 2 rods tall and 1 rod deep. What would be the volume of this structure?

Now with only one actual measurement

$$r = (1.54 \pm 0.04)\text{m}$$

the relative uncertainties are simply added:

$$\begin{aligned} 3r \times 2r \times r &= \left(6 \times 1.54^3\right)\text{m}^3 \pm \left(\frac{3\times 0.04}{3\times 1.54} + \frac{2\times 0.04}{2\times 1.54} + \frac{0.04}{1.54}\right)\times 100\% \\ &= 21.91\,\text{m}^3 \pm \left(0.08\right)\times 100\% \\ &= 21.91\,\text{m}^3 \pm 8\% \end{aligned}$$

Moral of the story:

Independent measurements: add uncertainties in quadrature.

Non-independent measurements: simply add the uncertainties.