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The option to wait in collective decisions and optimal majority rules ☆

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ABSTRACT

We consider a model in which voters over time receive more information about their preferences concerning an irreversible social decision. Voters can either implement the project in the first period, or they can postpone the decision to the second period. We analyze the effects of different majority rules. Individual first period voting behavior may become "less conservative" under supermajority rules, and it is even possible that a project is implemented in the first period under a supermajority rule that would not be implemented under simple majority rule.

We characterize the optimal majority rule, which is a supermajority rule. In contrast to individual investment problems, society may be better off if the option to postpone the decision did not exist. These results are qualitatively robust to natural generalizations of our model.

1. Introduction

In most political economy models, individuals know their preferences over candidates or social actions. In another branch of the literature, individuals know their fundamental preferences, but which action is best suited to implement them depends on an unknown state of the world. The main objective of this type of models is to analyze how individuals can aggregate dispersed information through strategic voting.[1]

In the present paper we focus on a third case that has received little attention so far: collective decisions under uncertainty when individuals discover their own preferences over time. In our model, individuals get additional information over time about their heterogeneous preferences regarding an investment project, and have to choose whether to implement it immediately, or delay the decision. In the latter case, they can either implement it after receiving additional information, or pass on it completely. While investment problems under uncertainty have been analyzed extensively for single decision makers,[2] we analyze such problems when the decision is made by a society through voting. Our main focus is twofold: firstly, we examine the effect of the majority rule on individual voting behavior and social decisions. We show that a higher majority rule makes individual voters in the first period more conservative towards projects whose expected payoffs in the future are low, and less conservative towards projects whose expected payoffs in the future are high. From an ex-ante point of view, this change of individual voting behavior is desirable and has the effect that the *optimal* majority rule is larger when society has the option to wait than when voters are forced to make a final up-or-down decision in the first period. In particular, we show in a symmetric setting, where simple majority rule is optimal without the option to wait, a supermajority rule becomes optimal with the option to wait.[3] Secondly, we show that society is often worse off (from an ex-ante point of view) if voters have the option to wait, rather than being forced to decide once and for all. This result holds even if society adopts the optimal majority rule in both cases.

Specifically, we consider the following dynamic social investment problem. In the first period, each voter knows his first period payoff, but his second period type is random. If the project is implemented in the first period, it is irreversible and payoffs to voters accrue in both periods according to their type realizations. Alternatively, if the project is not implemented in the first period, voters find out their respective second period types, and vote on whether to implement the project for the second period. We parameterize projects according

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[1] See, e.g., Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996), Feddersen and Pesendorfer (1998).

[2] See Dixit and Pindyck (1994) for a review of this literature.

[3] By a supermajority rule, we mean a voting rule that specifies that the status quo is only to be changed if a certain proportion of the electorate (greater than the 50%, the "simple majority") votes in favor of change.

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to the relative size of the gain of winners to the loss of losers. A "good" project is one where this ratio is large.

A possible advantage of delaying investment in the first period is that agents receive information about their payoffs in the next period: There is an "option value of waiting". We analyze how the type of majority rule influences the value of waiting, and thus, the voting behavior of individuals and the first period implementation decision. The expected second period payoff for a voter, if the project is delayed in the first period, may go in either direction as the majority rule changes: a higher majority rule may increase the risk that a "good" project with a positive expected value (i.e., one in which winners gain more than losers lose) is not implemented in the second period, thus diminishing the value of waiting and inducing voters to implement the project already in the first period. In contrast, a higher majority rule decreases the risk that a "bad" project is implemented in period 2, thus increasing the value of waiting.

A higher value of waiting makes voters more reluctant to implement the project already in the first period. Thus, a higher majority rule makes each voter more willing to agree to good projects, even if he is a loser today, and less willing to agree to bad projects, even if he is a winner today. There is also a second, direct, effect of a higher majority rule: more voters have to agree, making first-period implementation less likely. For bad projects, both effects go in the same direction, making implementation less likely for higher majority rules. In contrast, for good projects, the first effect may outweigh the second one, leading to more projects being implemented in period 1 under a higher majority rule.

On the normative side, we focus on an ex-ante point of view, that is, taking expectation over both voter type realizations and project types. We show that, relative to a situation where all decisions have to be made in the first period, the option to wait (weakly) increases the optimal majority rule in large electorates. Intuitively, higher majority rules have the advantage that, for socially bad projects, voters become more conservative and thus fewer of these projects are implemented, while for good projects, voters become more willing to implement in the first period. Moreover, since the best projects are already implemented in period 1, those projects that are reconsidered in period 2 form a negative selection from the set of all projects, and a higher majority rule is socially beneficial for these cases as well.[4]

We also characterize the ex-ante optimal supermajority rule explicitly under the additional assumptions that each voter has a 50 percent chance of being a high type, and that project types are uniformly distributed at the constitutional stage. The optimal supermajority rule in this case is approximately (i.e., up to integer constraints) between $7/11 \approx 63.6\%$ and $2/3$, for any number of voters.

It is also interesting to analyze the social ex-ante value of the option to wait. In unilateral investment problems, this value is always nonnegative, and often positive, as individuals may strictly benefit from postponing the decision. In contrast, a *society* may be better off if it is forced to invest either immediately or not at all, rather than having the option of postponing this decision. Indeed, we show that, from an ex-ante point of view (and with uniformly distributed project costs), this is the case even if society chooses the *optimal* majority rule for the case when waiting is possible.

Our results shed light on an important question in the endogenous determination of institutions: why do some organizations choose supermajority rules, and which features of decision problems influence this choice? Majority rules within organizations vary considerably, from simple majority to unanimity. Often, the choice of the majority rule that is to govern future decision making is a contentious issue itself, such as in the recent EU summit, which eventually adopted a supermajority rule. Most countries use supermajority rules for a change of the constitution, and, often implicitly, for "normal" legislation.[5] This paper contributes to the literature on the relative advantages of different majority rules by providing a new rationale for supermajority rules, which relies on voters' uncertainty over the consequences of project implementation, and the option value of waiting until new information is available. Thus, our model is most relevant for societies that frequently face decision problems with such characteristics.

Several previous papers have analyzed supermajority rules from an economic point of view. Buchanan and Tullock (1962) argue that, under a simple majority rule, a majority of people may implement socially bad projects because they can externalize a part of the associated cost to the losing minority, while under unanimity rule, only Pareto improving projects are implemented. However, Guttman (1998) shows that unanimity rule leads to a rejection of many projects that are not Pareto improvements, but nevertheless worthwhile from a reasonable social point of view. Assuming that the social goal is to minimize the sum of both types of mistakes, he shows that simple majority rule is optimal in a symmetric setting. The same result obtains in a symmetric setup in our model if voters have to make a once-and-for-all decision about the project in the first period. However, with the option to postpone a decision to the second period, we show that a supermajority rule is optimal.

Messner and Polborn (2004) analyze an overlapping generation model in which the median voter in the constitutional election decides on the majority rule that governs implementation decisions on possible projects in the future. For any project, these voters are simple one-time, up-or-down decisions. In contrast, in the present paper, our focus is on the timing of the implementation of reforms. Also, the electorate remains constant over time, thus removing the strategic incentive for the initial median voter to use supermajority rules to transfer power from future voters to his (more conservative) "average future self". Other rationales for supermajority rules include the problem of time inconsistency of optimal policies under simple majority rule (Gradstein, 1999; Dal Bo, 2006), the possibility of electoral cycles under simple majority rule (Caplin and Nalebuff, 1988), and protection against excessive redistribution (Aghion and Bolton, 2003).

Our model is most closely related to a small literature in which voters learn about their preferences over time. Compte and Jehiel (2008) develop an infinite-period search model in which the stopping decision is made by a committee, and proposals arrive exogenously and over time. The trade-off is that unanimity rule guarantees that only efficient projects are implemented, but it takes less time to reach an implementation decision under simple majority rule. If voters are sufficiently patient then higher majority rules imply that voters become more picky and average welfare increases. Albrecht et al. (2008) consider a simplified version of this framework in which all voters draw valuations from the same distribution and obtain results also for the case of intermediate and low patience levels. They show that the optimal majority rule is monotonically increasing in voters' discount rate and, if voters are sufficiently impatient, their expected equilibrium payoff increases with the size of the committee.

Both of these papers focus on the analysis of individual voting behavior and welfare under different (exogenous) majority rules in

[4] Even at the interim stage (i.e., in the first period when voters know the project type and their own first-period type), simple majority rule may be Pareto inefficient for some bad projects. This is the case if there is a simple majority of voters who approve immediate implementation under simple majority rule, but would prefer to postpone implementation, if the majority rule is changed to unanimity rule.

[5] For example, in parliamentary systems with a strong committee organization, a legislative proposal usually needs the support of *both* the respective committee and the house. In parliamentary systems with two chambers, certain legislative proposals need the support of both chambers. Tullock (1998), p. 216, estimates that legislative rules in the US for changing the status quo are "roughly equivalent to requiring a 60% majority in a single house elected by proportional representation".

a collective decision problem which is naturally framed as a stopping problem. While our setup differs somewhat, the main difference is in the questions we analyze. We are mostly interested in the optimal majority rule under the option to wait, and, in particular, how the optimal majority rule and voter welfare compare in the cases with and without the option to wait.

Strulovici (2010) analyzes a model in which a society has to choose in continuous time between a risky and a secure project. Ex-ante, all individuals are identical; over time, some individuals discover that they are winners and then receive a payoff forever after. The arrival rate is unknown, and voters continuously update their beliefs as long as the risky action is played. In contrast to our model, information arrives only as long as the risky action is played, and the project is reversible. Voters decide under simple majority rule or unanimity rule when to stop experimentation with the uncertain action. Strulovici finds that society always stops experimentation too early compared with a utilitarian optimum, and that unanimity rule may lead to more or less experimentation than simple majority rule.

Gersbach (1993b), in a framework generalizing Glazer (1989), considers a model where voters in period 1 choose between implementing an irreversible long run project that delivers benefits both in periods 1 and 2, and sticking with the status quo; in the latter case, they reconvene in period 2 and decide whether to implement a short-run project then. The voters' period 2 valuations are unknown in period 1. Our model shares the temporal setting of Gersbach (1993b), but differs in some aspects of the economy. In our model, there is an arbitrary number of voters whose second period valuations are iid draws from an arbitrary distribution. We also introduce a parameter that captures how much winners get relative to the loss of losers. This allows us to study how the type of project affects the value of the option to wait. For example, our more general setup allows us to show that Gersbach's example that the option value of waiting may be negative holds for a large class of investment problems, and even if the majority rule is chosen optimally. Most importantly, however, the focus of our analysis is different. While Gersbach assumes simple majority rule, we analyze the effect of different majority rules, both on voter behavior and on welfare, and characterize the optimal institutional response of a society that repeatedly faces such decision problems under uncertainty.

Another social learning paper in which new information arrives only as long as society is experimenting is Callander (2008). Citizens know how the status quo policy translates into outcomes, but the farther a policy is away from the status quo policy, the less certain is its consequences. Callander shows that an initial phase of experimentation and learning is eventually terminated if a policy achieves an outcome that is sufficiently close to the ideal outcome of the median voter.

Fernandez and Rodrik (1991) analyze a model of voting on reform projects that generate winners and losers. They show that a project that ex-post benefits the majority of the population need not be implemented, if the ex-ante expected benefit is negative for a majority of the population. If, instead, a majority of the population has positive ex-ante expected benefits, but ex-post, payoffs are negative for a majority, then a reform may be implemented initially, but would be reversed after payoff information becomes known. Thus, there is a bias in favor of the status quo. In contrast to Fernandez and Rodrik (1991), we analyze a setting in which reforms are not reversed, so that there is no status quo bias in our setup. Also, our focus is on comparing different majority rules and how they influence voting behavior and implementation decisions, while Fernandez and Rodrik (1991) only consider simple majority rule.

Glazer and Konrad (1993) consider a model of repeated collective decision making in which the outcome of the first period project may determine individual risk-averse voters' preference over projects in period 2. Since first period outcomes determine the voters' wealth in period 2, they will be more (less) concerned about the riskiness of the second period project if the project in period 1 leads to a loss (gain). This link implies that a collective decision about the two projects is effectively bundled. The authors show that bundling can lead to a negative decision in period 1 even if each voter individually would implement the project if he also had the control over the decision in period 2.

The paper proceeds as follows: the next section presents the model. Our main results follow in Section 3. We analyze the robustness of the model in Section 4. Specifically, we analyze what happens if projects that were implemented in the first period are (at least partially) reversible, and if society can use majority rules that vary between time periods. Section 5 concludes.

2. The model

2.1. Description

A group of N (odd) risk neutral individuals has to decide whether to undertake an investment project that creates costs and benefits (described in more detail below) for group members. The decision process has two stages. At the beginning of period 1, the group chooses between implementing the project right away and postponing the decision to the beginning of period 2. In the latter case, the group makes the final decision on whether to implement the project at the beginning of period 2. In both periods, the decision is governed by a *voting rule* indexed by m. The project is implemented if and only if at least m individuals approve. The majority rule may range from simple majority to unanimity, i.e. $m\in\{(N+1)/2,\ldots,N\}$.

In each period that the project is not implemented, all voters receive a net payoff normalized to 0. If the project is implemented in or before period t, then individual i receives a payoff of V_t^i-c in period t. We refer to V_t^i as voter i's *type* in period t. Types of different voters, as well as first period and second period types, are identically and independently distributed.[6] Specifically, we assume that V_t^i is equal to 1 with probability θ, and 0 otherwise. The *project type* $c\in(0,1)$ is the same for both periods, and can either be interpreted as the per-capita "cost" of the project (to be shared equally by all voters), or as a utility index that captures the size of the gains of those people who are better off than in the status quo, relative to the losses of those who would prefer the status quo.[7] For simplicity, we assume that individuals do not discount the future, so that they value future and current payoffs equally.

Before the period 1 election, individuals know (only) their period 1 type, and they learn their period 2 type before the period 2 election (if any). Each individual votes for the option that would provide him with the higher expected utility: in period 2, voter i votes for the project if and only if $V_2^i=1$. In period 1, voter i votes for the project if and only if he weakly prefers immediate implementation to the expected payoff from postponing (given that all voters behave in period 2 as described above). Formally, we use iterated elimination of weakly dominated strategies, a standard refinement in voting games.[8]

Eventually, we are interested in the optimal voting rule, chosen at an initial stage before type realizations for the project are known. At this time, all voters are identical and agree to choose the majority rule that maximizes their ex-ante expected payoffs. We can also interpret such a constitution normatively as the one that maximizes ex-ante utilitarian welfare.

[6] We relax these independence assumptions in Section 3 of the online appendix.

[7] Clearly, we could just specify the net payoff of each individual through one variable, but our approach allows us to use c to easily distinguish projects with a high expected average payoff (i.e., low c) from those with a low expected average payoff.

[8] This refinement, for example, eliminates (rather strange) Nash equilibria of the voting game in which everybody opposes investment, even if he would benefit from implementation.

Ex-ante payoffs, and thus the optimal majority rule, also depend on the project type c. Typically, however, it is not feasible to construct a constitution where the applicable majority rule depends on the type c of the project under consideration, since there would be verifiability problems, and such a rule would unavoidably lead to conflicts of interpretation. Thus, we focus on the majority rule that is optimal *in expectation*, when c is drawn from some distribution with cumulative distribution $F(c)$.

2.2. Discussion of modeling choices

The model is a relatively tractable framework for the analysis of intertemporal information arrival in social decision problems, and we discuss the robustness of the model to several extensions in Section 4. It is deliberately simple in some aspects. We restrict society to make a decision through voting and assume that project proposals cannot contain transfer payments between different voters. If, instead, types are observable and transfer payments are feasible, then, by the Coase theorem, any majority rule leads to implementation if and only if the project creates more benefits than costs. The assumption that transfer payments are not feasible is standard in most of the political economy literature and also appears to be quite realistic in many applications, for example because of informational constraints or legal provisions against vote buying.

Furthermore, the decision in the first period is restricted to the first-period implementation decision. For example, the first period electorate cannot choose to wait *and* commit the second period electorate to implement in the second period, or cannot choose to wait *and* forbid the second period electorate to consider implementation. While there are cases in which a majority of the first period electorate would like to take such measures, the assumption that today's electorate cannot commit a future electorate is both standard in the literature, and quite realistic for most democratic institutions, as such attempts would be very controversial (at least ex-post).

Our model has only two time periods. It is, in principle, not too difficult to extend this model to a setup with payoffs in infinitely many periods; however, a key assumption is that voters detect their preference for or against the project after some finite time, so that uncertainty is concentrated in early periods and later resolved.[9] In many applications, this appears realistic.

For example, consider the decision of an EU member state whether to join the (existing) Euro currency union. Initially, voters will be very uncertain about their personal costs and benefits of this potential project, both regarding common and idiosyncratic components of payoffs (e.g., for common payoffs, the effects on interest rates, or the risk of having to bail out other member states; for idiosyncratic components, people may be uncertain how often they will travel in the future to other countries that use the Euro, or they might not know how fast they can adjust to prices being denominated in a new currency that they are unaccustomed to). Over time, voters will learn some information about their payoffs, and, at least to some extent, they will learn even while their country remains outside the Euro. Also, if a country decides to join the Euro, but a majority of voters learns later on that they prefer to return to their national currency, that may not be feasible, so there is some element of irreversibility. (While we assume in the basic model that a decision to implement the project in period 1 is irreversible in period 2, we show in Section 4.1 that our results are robust when changing this assumption).

Many large reform proposals – such as, for example, whether to introduce private accounts in the pension system, tuition payments for state universities, or a cap-and-trade structure for greenhouses gases – share this structure that voters are initially uncertain about their payoffs from implementing the proposal, and learn more over time so that the extent of individual uncertainty diminishes over time. Our environment where voters are uncertain about their future payoff captures such a situation. The assumption that a voter's future type is completely uncorrelated with his present type is made for simplicity, but we show in the online appendix that our results are qualitatively robust as long as the correlation between a voter's first and second period type is not perfect.

In contrast, our model is not a good fit in situations where voters are fairly certain about their future preferences on the issue. For example, consider the issue of whether to introduce ethnic, racial or gender quotas for certain jobs. An individual who is a beneficiary of such a quota is likely to be in favor of it both today and tomorrow, and vice versa. There is not much uncertainty about the effects of the proposed policy on individual voters, and even if society can choose whether to introduce such a quota today or reconsider the proposal tomorrow, the problem for voters is essentially a static one.

Part of the information that voters receive may derive from implementation of the project in other jurisdictions, and the voters' ability to observe the experience of individuals there who are similar to themselves. The extent and sources of information are exogenous to our model.

3. Results

3.1. The benchmark case: no option to wait

We start with the benchmark case in which the period 1 decision about the project is final, i.e., a rejection cannot be reconsidered in period 2. Voter i's expected total payoff from immediate implementation is

$$U_I^i\left(V_1^i, c\right) = V_1^i + E\left(V_2^i\right) - 2c = V_1^i + \theta - 2c. \tag{1}$$

Each voter approves the project if and only if its net present value is nonnegative.[10] Thus, a voter with type $V_1^i = 1$ (a *high type*) votes in favor if and only if $1+\theta-2c \geq 0$, hence if $c \leq \frac{1+\theta}{2}$. Similarly, a *low type* voter ($V_1^i = 0$) votes in favor if and only if $E(V_2^i) - 2c = \theta - 2c \geq 0$, or $c \leq \theta/2$.

Thus, projects with type $c \leq \theta/2$ are unanimously approved, and those with $c > \frac{1+\theta}{2}$ are unanimously rejected. If, instead, $c \in \left(\frac{\theta}{2}, \frac{1+\theta}{2}\right]$ then the realization of types matters and a project is approved if and only if there are at least m high types. Consequently, the average or ex-ante expected payoff of voters is

$$\tilde{\pi}(m, N, c, \theta) \begin{cases} 2(\theta - c) & \text{if } c \leq \theta/2 \\ \sum_{k=m}^{N} \binom{N}{k} \theta^k (1-\theta)^{N-k} \left[\frac{k}{N} + \theta - 2c\right] & \text{if } c \in \left(\frac{\theta}{2}, \frac{1+\theta}{2}\right] \\ 0 & \text{if } c > (1+\theta)/2 \end{cases}. \tag{2}$$

To save on notation, we will usually suppress the arguments N and θ in $\tilde{\pi}$ and other functions when no confusion can arise (i.e., when we consider a situation in which N and θ are fixed).

Clearly, $\tilde{\pi}(m, c)$ is a piecewise linear function of c. Moreover, $\tilde{\pi}(m, c)$ jumps downward at $c = \theta/2$, and upward at $c = (1+\theta)/2$ for

[9] For example, we could generalize our model as follows. Once a project is implemented, it generates an infinite stream of payoffs for each voter (depending on the voter's type, as in our model, and discounted using a discount factor of δ). In the first period, voters know only their first period type. In the second period, they detect whether they are a high or low type for the remaining periods (or, more generally, the frequency with which they will be high types in the future). Thus, voting behavior from the second period on will be type dependent and thus, implementation either occurs in one of the first two periods, or not at all. As in our model, backwards induction can then be used to determine first period voting behavior.

[10] As a tie-breaking assumption, we assume that voters who are indifferent always approve the project. No results of our model qualitatively depend on this assumption.

all majority rules except unanimity rule. To see this, note that $\lim_{c\downarrow\theta/2}\tilde{\pi}(m,c)=\sum_{k=m}^{N}\binom{N}{k}\theta^k(1-\theta)^{N-k}\frac{k}{N}=\theta\sum_{k=m}^{N}\binom{N-1}{N-k}\theta^{k-1}(1-\theta)^{N-k}<\tilde{\pi}(m,\theta/2)=\theta$, and $\tilde{\pi}(m,\frac{1+\theta}{2})=\sum_{k=m}^{N}\binom{N}{k}\theta^k(1-\theta)^{N-k}[\frac{k}{N}-1]\leq 0$, while $\lim_{c\downarrow(1+\theta)/2}\tilde{\pi}(m,c)=0$. For unanimity rule, $\tilde{\pi}(N,\frac{1+\theta}{2})=0$, so that $\tilde{\pi}(N,\cdot)$ is discontinuous only at $c=\theta/2$, but not at $c=(1+\theta)/2$.

Intuitively, at $c=\theta/2$, high types strictly benefit from implementation, while low types are just indifferent. Hence, from an ex-ante perspective, voters strictly benefit if the project is implemented, which always occurs for $c\leq\theta/2$, while for $c>\theta/2$, implementation depends on the realization of preference types and is thus not guaranteed. Consequently, $\tilde{\pi}(m,c)$ drops at $c=\theta/2$. Similarly, for $c=(1+\theta)/2$, high types are just indifferent towards implementation, while low types strictly suffer. Thus, voters suffer from an ex-ante perspective if the project is implemented. For $c\in(\frac{\theta}{2},\frac{1+\theta}{2}]$, implementation depends on the realization of voter types, while it never occurs for $c>(1+\theta)/2$ (so that $\tilde{\pi}(m,c)=0$ if $c>(1+\theta)/2$). Fig. 1 shows the ex-ante payoff $\tilde{\pi}$ for the case $N=15$, $\theta=1/2$ and $m=8$ and $m=9$.

We now analyze the optimal voting rule for different levels of c. As mentioned above, when $c\leq\theta/2$ or $c>(1+\theta)/2$ then voting behavior is unanimous and independent of m. For $c\in(\theta/2,(1+\theta)/2]$, ex-ante utility as given by Eq. (2) is a probability-weighted sum of the terms in square brackets. The majority rule determines how many of these terms are included into the sum. Also, if the summand l is positive, then so is any summand $k>l$. Therefore, it is optimal to lower m as long as the additional term is positive, i.e. as long as $[m/N+\theta-2c]\geq 0$. Solving this inequality for m and taking into account the integer problem yields the optimal majority rule, $m^{**}=\lceil(2c-\theta)N\rceil$, provided that this is at least a simple majority. (We use $\lceil x\rceil$ to denote the smallest natural number at least as large as x.) Intuitively, lowering the majority requirement from $m+1$ to m is socially beneficial if and only if the average expected payoff is positive when there are *exactly* m high types (because this is the only circumstance in which the reduction of the majority rule matters for the outcome). Proposition 1 summarizes these observations.

Proposition 1. *Suppose that society can either implement the investment project in period 1, or not at all.*

1. *If $c\leq\theta/2$, or $c>(1+\theta)/2$, all majority rules yield the same expected payoff, i.e. $\tilde{\pi}(m,c)=\tilde{\pi}(m',c)$ for all $(N+1)/2\leq m,m'\leq N$.*
2. *For $c\in(\theta/2,(1+\theta)/2]$, the majority rule that maximizes the expected payoff is given by $m^*=max(\frac{N+1}{2},\lceil N(2c-\theta)\rceil)$.*

Note that, for c close to $(1+\theta)/2$, the unique optimal majority rule is unanimity rule. If $(2c-\theta)<1/2$, society is constrained by our assumption that m must be greater or equal to $(N+1)/2$. If, for some reason, society were able to use submajority rules in spite of their stability problems, then $\lceil N(2c-\theta)\rceil$ is always an optimal majority rule. This would not affect our qualitative results in the following.

Now consider the problem of choosing an optimal majority rule when the constitution cannot condition the majority rule on the project type c. From an ex-ante perspective, c is distributed according to some arbitrary distribution F. Again, we can focus on the interval $c\in(\theta/2,(1+\theta)/2]$, because this is the only set in which decisions are not unanimous so that the majority rule matters. Conditional on c being in this interval, the ex-ante expected utility is

$$\tilde{\Pi}(\cdot,N,\theta)=E\tilde{\pi}(m,N,c,\theta)=\sum_{k=m}^{N}\binom{N}{k}\theta^k(1-\theta)^{N-k}\left[\frac{k}{N}+\theta-2c^*\right], \quad (3)$$

where $c^*=E(c|c\in(\theta/2,(1+\theta)/2])$ is the expectation of c, conditional on c being in the middle interval. Following the same argument as above, the optimal majority rule is $m^*=\lceil N(2c^*-\theta)\rceil$ (if unconstrained).

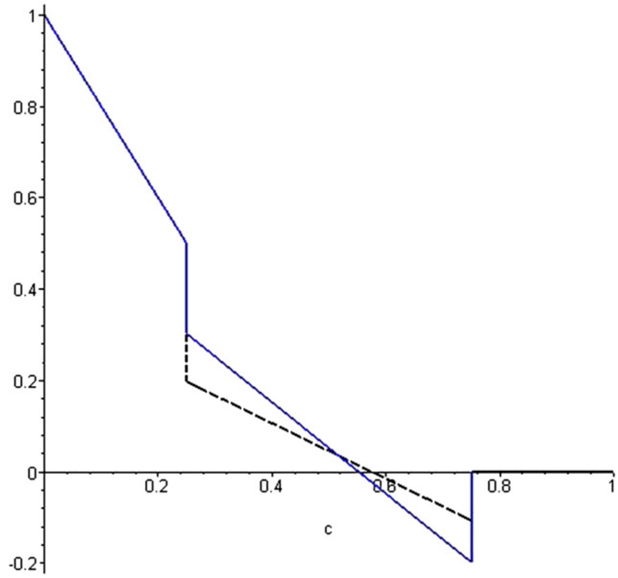

Fig. 1. The function $\tilde{\pi}$ for $N=15$, $\theta=1/2$ and $m=8$ (solid) and $m=9$ (dashed).

Proposition 2. *Suppose that society can either implement the investment project in period 1, or not at all. Furthermore, suppose that the constitution cannot condition the majority rule on c and that c is drawn from a distribution with cumulative distribution function F that satisfies $F((1+\theta)/2)-F(\theta/2)>0$.[11] Let $c^*=E(c|c\in[\theta/2,(1+\theta)/2))$. Then, $\tilde{\Pi}(\cdot,N,\theta)$ is maximized by*

$$m^*=max\{(N+1)/2,\lceil N(2c^*-\theta)\rceil\}.$$

It is interesting to consider the case where F is symmetric on $[\frac{\theta}{2},\frac{1+\theta}{2}]$ around the midpoint $\frac{1+2\theta}{4}$. This condition is, for example, satisfied when F is a uniform distribution. By symmetry, the expected implementation cost coincides with the midpoint of the interval, i.e. $c^*=\frac{1+2\theta}{4}$. Substituting this term in the above formula yields $m^*=\frac{N+1}{2}$. That is, for a symmetric distribution the ex-ante optimal majority rule is simple majority rule. This result is independent of the value of θ. To get an intuition, observe that the average payoff of a first period high type is $1+\theta-2c^*$, while the expected payoff of a first period low type is $\theta-2c^*$. If $c^*=\frac{1+2\theta}{4}$, then the expected payoffs of $\frac{1}{2}$ for a high type, and $-\frac{1}{2}$ for a low type are symmetric to each other. Simple majority rule maximizes the expected payoff in a situation where the payoffs of winners and losers are symmetric around 0. If $c^*<\frac{1+2\theta}{4}$, then winners gain more than losers lose, so that a social planner would like to encourage implementation even more; in this case, simple majority rule is the (constrained) optimal rule. Finally, if $c^*>\frac{1+2\theta}{4}$, then winners gain on average less than losers lose, so that a supermajority rule is optimal.

It is interesting to note that Proposition 2 implies that, if c is distributed uniformly, then simple majority rule is optimal, independent of the probability θ of being a winner. As explained above, while θ affects the interval in which individuals disagree with each other, conditional on c being in this interval, first period high and low types are, on average, symmetric. Thus, for example, even if θ is quite low, it is not optimal to change to a supermajority rule.

3.2. Individual voting behavior and the option to wait

We now analyze the effect of the option to wait with the implementation of the public project. Obviously, in period 2, player i votes in favor

[11] If $F((1+\theta)/2)=F(\theta/2)$, then all decisions are unanimous and thus, the majority rule never matters.

of the project if and only if $V_2^i=1$, and the project is implemented if and only if there are at least m high types. Let $I_2(m)$ denote the event that the project is implemented in period 2, given that the majority rule is m; and let $P(I_2(m))$ denote the probability of this event.

If the project is not implemented in period 1, then player i's expected payoff is $(E[V_2^i|I_2(m)]-c)P(I_2(m))$; we call this expression the *value of waiting*, U_W. To derive an explicit expression for U_W, we define the probability that there are exactly $m-1$ high types among the other $N-1$ voters as $p(m,N,\theta)=\binom{N-1}{m-1}\theta^{m-1}(1-\theta)^{N-m}$. We can think of p as the probability of voter i being pivotal under majority rule m. Let $q(m,N,\theta)=\sum_{l=m}^{N-1}\binom{N-1}{l}\theta^l(1-\theta)^{N-1-l}$ be the probability that there are m or more high types among the other $N-1$ voters. From voter i's point of view, q is the probability that the project is implemented through the votes of the other voters, independently of voter i's preference on the project. If voter i is pivotal, then a project is implemented if and only if i's payoff is positive; thus, in this case, the expected payoff is $E[max\{V_2^i-c,0\}]$. In contrast, if voter i is not pivotal and the project is implemented, the expected payoff for i is the unconditional expectation, $E(V_2^i)-c$. Summing up, the value of waiting is

$$\begin{aligned} U_W(c,m,N,\theta) &= p(m,N,\theta)E\left[\max\left\{V_2^i-c,0\right\}\right]+q(m,N,\theta)\left(E\left(V_2^i\right)-c\right) \\ &= \theta p(m,N,\theta)(1-c)+q(m,N,\theta)(\theta-c). \end{aligned} \tag{4}$$

Since voter i's payoff from implementing the project immediately is $U_I^i(V_1^i,c)=V_1^i+\theta-2c$, he will approve immediate implementation in period 1 if and only if

$$V_1^i+\theta-2c\geq\theta p(m,N,\theta)(1-c)+q(m,N,\theta)(\theta-c). \tag{5}$$

Note an important difference to the benchmark case without the option to wait: an individual voter's first period behavior as characterized by Eq. (5) depends on the majority rule m, because that rule determines the expected value of waiting.

If $c\leq\theta$, then both terms on the right-hand side of Eq. (5) are positive, so that the option to wait induces voters to behave more conservatively than in situations where the decision cannot be delayed. Moreover, $p(\cdot)$ is a decreasing function of m if $m\geq\theta(N-1)+1$, and $q(\cdot)$ is always a decreasing function of m. Thus, at least for all $\theta\leq 1/2$, individual voters behave more conservatively the lower is the majority rule m. In this case, the option to wait decreases the cost threshold below which a low voter type votes for a project, and this shift is the stronger, the lower the majority rule.[12]

If $c>\theta$, then the value of waiting is neither necessarily positive, nor is it necessarily decreasing in m. In contrast to private decisions, where the value of waiting is always positive, society sometimes implements projects that are not socially beneficial. If the right-hand side of Eq. (5) is negative, then it is possible that a high type voter votes for immediate implementation of an investment project even though his expected implementation payoff is negative. The reason for this seemingly strange behavior is that the voter's payoff from immediate implementation is at least better than his expected payoff if he forgoes immediate implementation and is then (perhaps) hit by implementation in the second period, when his type may be low. In this case, a higher majority rule may increase the value of waiting, as it increases the voters' protection in the next period against the implementation of a project that they oppose.

We now proceed to a more formal analysis of the value of waiting and its implications for individual voting behavior. Lemmas 1 and 2 are used repeatedly in the proofs of the following propositions, and are presented here in the text, because they are of independent interest and provide an intuition for the economic effects in our model.

Lemma 1 shows that a higher majority rule increases the probability of voter i being pivotal, relative to the probability that the project is implemented independent of voter i's preferences. This effect underlies a benefit of supermajority rules, since a voter always gets a nonnegative payoff if he is pivotal, but may receive a negative expected payoff if a project with high c is implemented independently of voter i's preferences. The proofs of all of the following results are in the Appendix A.

Lemma 1. *The ratio* $\frac{p(m,N,\theta)}{q(m,N,\theta)}$ *is increasing in m.*

Lemma 2 shows that the value of waiting $U_W(c,m,N,\theta)$*, defined in Eq. (4), is increasing in m if* $m<Nc$ *and decreasing in m if* $m>cN$*. Thus, for any* $c\in[0,1]$ *the value of waiting is single-peaked in m.*

Lemma 2. *If* $m<cN$*, then* $U_W(c,m,N,\theta)<U_W(c,m+1,N,\theta)$*. If* $m>cN$*, then* $U_W(c,m,N,\theta)>U_W(c,m+1,N,\theta)$*.*

If $c\leq 1/2$, the condition $m>cN$ is satisfied for all admissible majority rules. Intuitively, if $m/N>c$, the project is implemented in period 2 only if the per-capita benefit (i.e. the share of high types) exceeds the per-capita cost (c). A further increase of the majority rule then means that the project is not implemented in some cases where the project's average payoff is positive. Thus, the value of waiting in period 1 decreases. Conversely, the value of waiting increases in m if $m<cN$, as the project is implemented less often when the average payoff is negative.

Note that the single-peakedness of U_W in m implies that, if for some given c the value of waiting is negative for majority rule m^0, then the same must hold for any majority rule $m<m^0$ (i.e. $U_W(m^0,c)<0$ implies $U_W(m,c)<0$ for all $m<m^0$).

Proposition 3 below characterizes first-period voting behavior with the option to wait. For each majority rule m there are two cutoffs $\underline{c}$ and $\bar{c}$ such that low types vote for implementation if and only if $c\leq\underline{c}$ and high types vote for implementation if and only if $c\leq\bar{c}$. Again, there are three different regimes: if $c\leq\underline{c}$ or $c>\bar{c}$, all voters agree to implement or reject, respectively. If $c\in(\underline{c},\bar{c}]$, implementation depends on the number of first-period high types.

Proposition 3 also characterizes the range in which $\underline{c}$ and $\bar{c}$ lie, and how they change with m. Since $\underline{c}<1/2$, the value of waiting for c close to $\underline{c}$ is positive and decreases in m. A higher majority rule increases the willingness of low types to implement in period 1, as implementation in period 2 becomes less likely. Thus, $\underline{c}$ increases in m. In contrast, $\bar{c}>1/2$, and the value of waiting is non-monotonic in m in that region. For low majority rules, the value of waiting is negative for c close to $\bar{c}$, and increases with m. Thus, high types become more conservative as m increases, so that $\bar{c}$ decreases. In contrast, for high majority rules, the value of waiting is positive and decreases with a further increase in m, thus making high types less conservative, so that $\bar{c}$ increases in m for high levels of m. Thus, $\bar{c}$ is a U-shaped function of m.

Proposition 3. *For any majority rule m, there exist threshold values* $\underline{c}(m,N,\theta)$ *and* $\bar{c}(m,N,\theta)$*, with* $\underline{c}(m,N,\theta)<\bar{c}(m,N,\theta)$*, such that low types (high types) vote for first period implementation of a project if and only if* $c\leq\underline{c}(m,N,\theta)$ $(c\leq\bar{c}(m,N,\theta))$*.*

Moreover, $\underline{c}(\cdot,N,\theta)$ *is an increasing function and satisfies* $0\leq\underline{c}(\cdot,N,\theta)<\theta/2$*. In contrast,* $\bar{c}(\cdot,N,\theta)$ *is U-shaped, assumes its minimum for some* $m\in\{N/(2-\theta),\lceil(1+\theta)N/2\rceil\}$ *and satisfies* $1/(2-\theta)<\bar{c}(\cdot,N,\theta)<1$*. In addition,* $\bar{c}(N,N,\theta)<(1+\theta)/2$*.*

Proposition 3 provides a rather loose lower bound for the majority threshold that minimizes $\bar{c}$. In fact, we can show that $\bar{c}$ always assumes its minimum either at $\lceil(1+\theta)N/2\rceil$ or at $\lceil(1+\theta)N/2\rceil-1$.

[12] For $c\leq\theta$, Eq. (5) implies that high types always favor implementation, so that their behavior does not change relative to the case that waiting is not possible.

However, since the proof of this claim is considerably more cumbersome, we refrain from stating it formally.

3.3. Ex ante payoffs under different majority rules

Proposition 3 shows how the majority rule affects individual voting behavior: When c is low, individual voters behave more conservatively under lower majority rules, and, when c is high, individual voters behave more conservatively under higher majority rules. We now consider what this implies for voters' ex-ante payoffs and the optimal majority rule.

Denote a player's ex-ante payoff, that is, his expected payoff given m, c and θ, but before the player's type is known, by $\pi(m,N,c,\theta)$. Since we focus on the effect of m on payoffs, we will from now on suppress (when no confusion can arise) the variables N and θ as arguments of all functions in order to save on notation. For $c\leq\underline{c}(m)$, all voters vote for implementation in period 1, so that $\pi(m,c)=\tilde{\pi}(m,c)=2(\theta-c)$. If $c>\bar{c}(m)$, then all voters reject the project in period 1 and so $\pi(m,c)$ coincides with the value of waiting, $U_W(c,m)=q(m)(\theta-c)+\theta p(m)(1-c)$. Finally, if $c\in(\underline{c}(m),\bar{c}(m)]$, then the project is approved in period 1 if and only if there are at least m high types. After a first period rejection, which occurs with probability $[1-q(m)-\theta p(m)]$, the project may (in contrast to Section 3.1) still be implemented in period 2, so that $\pi(m,c)=\tilde{\pi}(m,c)+[1-q(m)-\theta p(m)]U_W(c,m)$. Rearranging terms and dropping the arguments from the functions q and p, we thus have

$$\pi(m,c)=\begin{cases}2(\theta-c) & \text{if } c\leq\underline{c}(m)\\ 2q(\theta-c)+\theta p(1+\theta-2c)+(1-q-\theta p)[q(\theta-c)+\theta p(1-c)] & \text{if } c\in(\underline{c}(m),\bar{c}(m)]\,.\\ q(\theta-c)+\theta p(1-c) & \text{if } c>\bar{c}(m).\end{cases} \tag{6}$$

Clearly, like $\tilde{\pi}(\cdot)$ in the benchmark case, $\pi(m,\cdot)$ is a piecewise linear function of c that exhibits a downward jump at $\underline{c}(m)$ for any m, and, unless $m=N$, an upward jump at $\bar{c}(m)$. Fig. 2 depicts the ex-ante payoff for $N=15$, $\theta=1/2$ and the cases $m=8$ and $m=9$.

We now turn to an analysis of the optimal majority rule for a given level of c, which may differ markedly from the benchmark case of Section 3.1. There are three different cases. First, if $c\leq\theta/2$, then each voter receives a positive ex-ante expected payoff from such a project, even if it is implemented independently of his own type. Thus, a rule that maximizes the probability of implementation is ex-ante optimal. If $c\leq\underline{c}(m)$, then the project is unanimously approved under rule m, and also under any larger majority rule $m'\geq m$ (as $\underline{c}(\cdot)$ is increasing in m). In particular, unanimity rule leads to implementation for the largest set of projects, $c\in[0,\underline{c}(N)]$. If, instead, $c\in(\underline{c}(N),\theta/2]$, then simple majority rule maximizes the implementation probability.

Second, if $c\in(\theta/2, min\ \bar{c}(\cdot)]$, then high types vote for and low types against first period implementation under any majority rule. For low values of c, the optimal rule is simple majority, because a high probability of first period implementation is socially optimal. In contrast, when c is relatively large, higher majority rules perform better.[13]

Third, if $c\geq(1+\theta)/2$, then rules for which $\bar{c}(m)>(1+\theta)/2$ cannot be optimal: At $c=\bar{c}(m)$, high types under majority rule m are indifferent between implementing immediately and waiting, and if $\bar{c}(m)>(1+\theta)/2$, then their implementation payoff is negative. Consequently, the ex-ante expected payoff is negative, because all three components of this weighted average (the implementation payoffs of high and low types, and the payoff from waiting) are negative for all $c\geq(1+\theta)/2$. In contrast, ex-ante expected payoffs are positive for all c under unanimity rule. Thus, for all $c\geq(1+\theta)/2$, the optimal m satisfies $\bar{c}(m)\leq(1+\theta)/2$. Hence, projects are unanimously rejected in the first period, so that the ex-ante expected payoff coincides with the value of waiting $U_W(c,m)$. By Lemma 2, the value of waiting increases in m if and only if $m<cN$. Thus, the majority rule that maximizes the value of waiting is given by $m^*(c)=\lceil cN\rceil$. In particular, $m^*((1+\theta)/2)=\lceil(1+\theta)N/2\rceil$. We show in the proof of Proposition 4 that this majority rule is also optimal for $c\in(\bar{c}(\lceil(1+\theta)N/2\rceil),(1+\theta)/2]$.

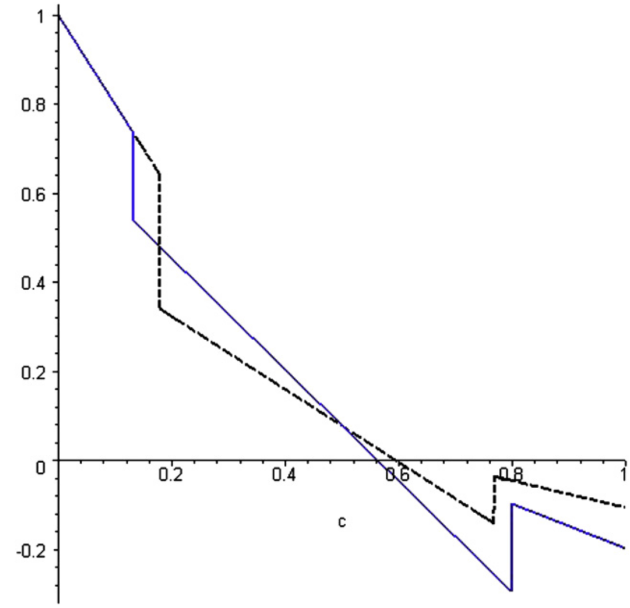

Fig. 2. Ex-ante payoffs $\pi(8,15,c,1/2)$ (solid curve) and $\pi(9,15,c,1/2)$ (dashed curve).

Proposition 4.

1. *For $c<\underline{c}(m)$ the ex ante expected payoff is maximized by any $m'\geq m$. In particular, unanimity rule is an optimal rule for all $c\leq\underline{c}(N)$ and strictly dominates simple majority for all $c\in(\underline{c}((N+1)/2),\underline{c}(N))$.*
2. *For $c>\bar{c}(\lceil(1+\theta)N/2\rceil)$ the (generically unique) optimal majority rule is given by $max\{\lceil(1+\theta)N/2\rceil,\lceil Nc\rceil\}$.*

It is also interesting that, for $c>(1+\theta)/2$, voters may be willing to increase a low majority rule even *after* the first period types are realized. That is, such rules are not only suboptimal in an ex-ante sense, but also ex-post. Consider, for example, a project with $c\in\left[(1+\theta)/2,\bar{c}\left(\frac{N+1}{2}\right)\right]$, and a society with a majority of high types. Under simple majority rule, the project is implemented in the first period by the support of high types. However, *all* voters (including high types) would be better off if society switched to unanimity rule, thereby killing the project in the first period. Thus, a change from simple majority rule to unanimity rule may be an ex-post Pareto improvement.[14]

3.4. Optimal majority rules

The option to wait produces effects both in favor and against low majority rules, so it is again natural to ask which of these effects dominates when the voting rule cannot be conditioned on the project type, i.e. when c is drawn from some distribution F. To gain tractability for the proofs, we focus – in this and the following section – on the case of uniformly distributed costs and types, i.e. $F(c)=c$ and $\theta=1/2$.

[13] An exact analytical characterization of the optimal rule is more cumbersome here than in the benchmark case, since π is a nonlinear function of p and q. Since there are no deeper insights to be gained, we refrain from doing so.

[14] The fact that high types may choose to implement a project with a negative expected return even for themselves is an example of what Bai and Lagunoff (2007) call the Faustian trade off in politics, where today's policy is determined by a desire to influence either the identity or the set of feasible choices of a future policy maker. By implementing immediately, today's pivotal voters make sure that tomorrow's “leaders” have no power to make a decision.

From the nature of our results (and continuity of solutions and value functions in the parameters), it is immediate that the results for $\theta=1/2$ are robust in a neighborhood of $\theta=1/2$ and in a (sup-norm-) neighborhood of the uniform distribution. A formal treatment of the general case ($\theta\in(0,1)$ and F general) is provided in the online appendix.

We denote the average ex-ante payoff of an individual voter under majority rule m by $\Pi(m,N)$. That is, $\Pi(m,N)=\int_0^1\pi(m,N,c,1/2)dc$. Remember that in the benchmark situation (without the option to wait) the voters' average ex ante payoff is maximized by simple majority. The following Proposition 5 shows that introducing the option to wait leads to a substantial increase in the optimal majority rule.

Proposition 5. *Suppose that c is ex-ante uniformly distributed on* $[0,1]$*, so that* $\Pi(m)=\int_0^1\pi(m,c)dc$.

i) $\Pi(m+1)-\Pi(m)<0$ *for all* $m\geq 2N/3$ *and*
ii) $\Pi(m+1)-\Pi(m)>0$ *for all* $(N+1)/2\leq m<7N/11$.

Moreover, $\Pi((N+1)/2)<\Pi(N)$.

While Proposition 5 does not determine the optimal majority rule exactly, it is clear from (i) that the optimal majority rule is at most $\lceil 2N/3\rceil$, i.e., the lowest majority rule that is higher than a two-thirds majority. From (ii), it follows that the optimal majority rule is a supermajority rule with $m/N\geq 7/11\approx 0.636$. In particular, if the number of voters N is large, then the optimal majority rule as a percentage of the electorate lies either within or arbitrarily close to the interval $[7/11, 2/3]$.

Interestingly, when the option to wait is introduced, simple majority not only loses its status as the optimal majority rule, but it actually becomes the worst majority rule. It is dominated even by unanimity (which is the worst of all supermajority rules that have $m\geq\lceil 2N/3\rceil$). Thus, loosely speaking, choosing a "too high" supermajority rule has a lower welfare cost than choosing a majority rule that is "too low".

While Proposition 5 holds for the uniform cost distribution, it is intuitive that the result is robust. For different cost distributions that are 'close' to a uniform distribution, the optimal majority rule would be close to the one characterized in Proposition 5, and thus a supermajority rule. For example, one can show that, for any density of the distribution that satisfies $1/4\leq f(c)\leq 2$ for all c, a supermajority rule is ex-ante better than a simple majority rule.[15]

3.4.1. Discussion

Supermajority rules are used in many international organizations like the European Union, and in most countries for a change of the constitution, and, often implicitly, for "normal" legislation. For example, in parliamentary systems with a strong committee organization, a legislative proposal usually needs the support of *both* the respective committee and the house. In parliamentary systems with two chambers, certain legislative proposals need the support of both chambers.

Our model provides a fundamentally new rationale for societies choosing supermajority rules, which relies on voters' uncertainty over the consequences of project implementation, and the option value of waiting until new information is available. Thus, our model is most relevant for societies that frequently face decision problems with such characteristics.

For example, one can argue that the European Union fits this description quite well. The most important decisions in the EU concern the admission of new members, transnational investment projects like the introduction of the Euro and the harmonization of industry regulations. Many of these projects are less "standard" (relative to the most important policy issues in the member states) and have uncertain payoff consequences for the member states. Consistent with this view, the European Council (the council of member state governments that makes the most significant decisions) uses a relatively high supermajority rule.

Also, most countries require a supermajority to change their constitution. Again, this area appears closer to the setting of this model than ordinary legislation issues: when the initial constitution is written, future needs are difficult to foresee and potential winners and losers are unclear, and even once a proposal arises, the consequences of changes for the distribution of gains and losses are not necessarily clear.

In contrast, most ordinary legislation in national legislatures concerns social or economic issues where preferences are more stable. As formally shown in the online appendix, the higher the correlation of voter types over time (and therefore, the less new information is expected to arise over time), the closer is the ex-ante optimal majority rule to simple majority rule. In this context, it is interesting to note that the European Union has recently adopted a new, lower supermajority rule for their decisions. With the expansion of their fields of responsibility, the EU appears to become more like a normal state in which standard decisions become more important, and thus the optimal supermajority rule decreases.

3.5. Does the option to wait increase the welfare of voters?

In settings with a single decision maker, the option to postpone a decision always weakly increases the decision maker's expected profit: the decision maker can still choose to go ahead and invest immediately, but sometimes he may strictly prefer to wait. While our setup here is similar, the answer to the title question is not obvious, as the option to wait influences a *game* between different voters, rather than the decision problem of a single decision maker.

Indeed, for some values of c, the option to wait hurts citizens from an ex-ante perspective.[16] For example, projects with $c>3/4$ are never implemented without the option to wait, as even the first period high types have a negative expected payoff from their implementation. In contrast, with the option to wait and simple majority rule, each project that was rejected in period 1 has a 50 percent chance of being accepted in period 2, and for most of these projects, the percentage of winners is smaller than c, making them socially undesirable. Moreover, $\bar{c}(m)>3/4$ for many majority rules, so that some projects that would definitely be rejected without the option to wait are actually implemented in period 1 with positive probability.

However, there are also project types for which the option of waiting increases expected social welfare. For instance, if $c=1/4+\varepsilon$, then a project may be rejected in period 1. Without the option to wait, such a rejection is final, while there is a second period chance for implementation with the option to wait, which is socially beneficial (for small ε).

Thus, there exist some cost levels for which ex-ante welfare increases, and others where welfare decreases with the option to wait. Again, it is interesting to see which effect dominates from an ex-ante perspective. Proposition 6 shows that the option to wait often harms voters in expectation.

Part 1 shows that, for a large range of low supermajority rules, as well as for simple majority rule, the option to wait lowers ex-ante payoffs, and only under very high majority rules, the option to wait is guaranteed to have a positive social value in terms of average ex-ante payoffs. In particular, for $N>5$, we show that the expected payoff without the option to wait and simple majority rule dominates the

[15] The (rather tedious) proof of this claim is available from the authors upon request. Also note that, while there may be even weaker assumptions under which supermajority rules are optimal, there are some distributions for which the result does not hold. For example, if $c=1/4$ with certainty, then simple majority rule is optimal for any N, with or without the option to wait.

[16] The earliest paper that has shown that the value of the option to wait may be negative is Gersbach (1993b), who constructs this result in an example with three players and correlated valuations in the second period. See also Gersbach (1993a).

expected payoff with the option to wait and the optimal supermajority rule.

Proposition 6.

1. *If $N>3$ and $m\leq\lfloor 3N/4\rfloor$ then $\tilde{\Pi}(m)>\Pi(m)$. If, instead, $m\geq 13N/16$ then $\tilde{\Pi}(m)<\Pi(m)$.*
2. *For $N>5$, $\max_m\tilde{\Pi}(m)>\max_m\Pi(m)$: the maximal average ex-ante payoff is strictly lower if voters have the option to wait.*

The statements in Proposition 6 starkly contrast with what we know about the value of waiting in individual decision problems, where an individual decision maker always benefits from the option to wait. For an intuition, consider a setting where N is large. Under simple majority rule without the option to wait, all projects with $c\leq 1/4$ are unanimously implemented, just as under the optimal supermajority rule. In addition, however, projects with $c\in(1/4,3/4]$ are implemented under simple majority rule if and only if a majority of voters has a high type, and this is, on average, better (from an ex-ante perspective) than not implementing any of these projects.[17] Again, nothing in this argument relies on c being drawn from a uniform distribution, and the result appears thus quite robust.

4. Robustness and extensions

In this section, we want to explore the robustness of the model when we relax some of our assumptions.

In Section 4.1, we investigate what happens when a project that was implemented in period 1 may be reversed in period 2. In Section 4.2, we analyze what would happen if it was possible to set different majority rules in the first and in the second period.

4.1. Partial reversibility

So far, we have assumed that a project that is implemented in period 1 cannot be terminated at the beginning of period 2. Of course, there are some collective investment projects that, under sufficiently favorable circumstances, can be reversed at a non-prohibitive cost. In what follows we show that the results of the basic model generalize – in a qualitative sense – to a setting where decisions are "partially reversible".

More specifically, we now assume that there is always an election at the beginning of period 2. As in the basic model, if the project was not implemented in period 1, then voters decide whether to implement it in period 2. In case that the project was implemented in the first period, the second period decision is whether or not the project is to be continued. It is natural to assume that m is the majority required to change the status quo, so to undo a project in period 2 requires (at least) m voters who want to terminate the project.

At the beginning of period 1 voters are uncertain about the benefits/costs of undoing a project. They only learn these benefits/costs at the beginning of period 2, before voting. In order to keep things as simple as possible we assume that there are only two possible outcomes. With probability γ, a decision to terminate the project yields a period 2 payoff of zero for all voters. That is, the investment decision is fully reversible in the sense that voters can fully recover the investment cost regarding the second period, c, and the termination of the project does not generate any further costs. Otherwise, with probability $1-\gamma$, the cost of terminating a project implemented in first period decision is so large that no voter can benefit from it. For instance, this is the case if voters cannot recover any investment costs by reversing the project. While in the first case, only high type voters benefit from a continuation of the project in the second case all voters prefer not to terminate the project.

[17] Clearly, this argument requires that N is large, but finite, because when we take N to infinity, then $lim\ \Pi(m,N)=lim\ \tilde{\Pi}(m,N)$.

Clearly, for $\gamma=0$, we are back to the setting of our basic model: a project that is started always remains in place. If, instead, $\gamma=1$ then a project that was implemented in period 1 can always by terminated in period 2, if voters choose to do so. In reality, most cases fall somewhere in between these extremes.

For notational and computational simplicity, we limit ourselves again to the case of $\theta=1/2$. Note first that the payoff from waiting, $U_W(c,m)$ is the same as in our basic model. The payoff from implementing the project in period 1 is given by

$$U_I(V_1,c,m)=V_1-c+(1-\gamma)[1/2-c] \\ +\gamma[p(1-c)/2+(1-p-q)(1/2-c)].$$

With probability $1-\gamma$, there is no opportunity to get out of the project, and thus, the period 2 payoff (given period 1 implementation) is the same as in the basic model. With probability γ, the circumstances are more favorable for terminating the project. From the perspective of an individual voter, he will be pivotal for the decision if there are exactly $m-1$ low types among the other $N-1$ voters (this happens with probability $p(m,N)$) and the project will be continued if and only if our voter's own second period type is high (probability 1/2, and payoff in this case is $1-c$). If there are less than m-1 low types (probability $1-p-q$), then the project is continued irrespective of the voter's type, and his expected payoff is $E[V_2]-c=1/2-c$. Finally, if there are at least m low types among the other players, which happens with probability $q(m,N)$, the project is terminated and the voter gets the default payoff zero.

Our first result characterizes the thresholds $c(m,N,\gamma)$ and $\bar{c}(m,N,\gamma)$ at which low and high types change their decisions, respectively. As the notation indicates, these thresholds now depend on γ.

Proposition 7.

i) *For each $\gamma\in[0,1)$, $\underline{c}(m,N,\gamma)$ is increasing in m.*

ii) *For each $\gamma\in[0,1)$, $\bar{c}(m,N,\gamma)$ is decreasing in m if $m/N\leq\underline{s}(\gamma)$ and it is increasing in m if $m/N\geq\bar{s}(\gamma)$, where $\underline{s}$ and $\bar{s}$ are defined by*

$$\underline{s}(\gamma)=\frac{11-3\gamma}{15-7\gamma} \quad \text{and} \quad \bar{s}(\gamma)=\frac{3-\gamma}{4-2\gamma}.$$

Both $\underline{s}$ and $\bar{s}$ are increasing and reach the value 1 as γ approaches 1/3.

iii) For each m, $\bar{c}(m,N,\cdot)$ is increasing in γ. $\underline{c}(m,N,\cdot)$ is decreasing in γ if $m\leq 2N/3$ and increasing in γ if $m/N>13/16$.

Parts i) and ii) of Proposition 7 tell us that the thresholds in this more general setting vary with m in much the same way as they do in the basic model. In order to see why this is so, consider first the threshold for low types. It is still true that increasing m lowers the value of waiting. Moreover, a higher m also increases the value of investing. Increasing m means that the investment decision is less likely to be reversed in period 2. For projects characterized by a low c a high probability of continuation is beneficial as, in expectation, every voter gets a high payoff. A decrease in the value of waiting, combined with an increase in the value of investing, implies an increase of the threshold up to which low type voters are willing to invest.

The intuition for the result that the threshold of high types is decreasing in m for low values of m is symmetric to the one described for the threshold of low types. Just as in the basic model, when m is already large, a further increase of m may lead to a decrease in the value of waiting. However, relative to the baseline model, the exact threshold where the value of waiting starts to decrease, changes, and consequently, also the majority rule where $\bar{c}$ reaches its minimum depends on γ. While $\bar{c}$ reaches its lowest value at $m=3N/4$ in the basic model, in the current setting, this minimum increases with γ and reaches 1 as γ approaches 1.

Part iii) of Proposition 7 states that $\bar{c}$ is increasing in γ, while $\underline{c}$ is decreasing in γ when m is low and increasing in γ when m is high. In order to understand this result, consider first $\bar{c}$, the threshold for the high types. An increase in γ implies a higher probability that the project will be terminated. Given that the costs are high this means that the expected value of investing in the first period increases for high types, and so they become more willing to invest: $\bar{c}$ increases.

When c is small, then an increase of γ has a detrimental effect on the value of investing. For low values of c, a termination of the project is socially beneficial only if most voters are low types. So, with low m, the project is terminated too often. Therefore, the value of investing is higher for low values of γ. On the other hand, for sufficiently high values of m, the project is not terminated often enough from a social point of view. In this case, increasing γ has a positive effect on the value of investing and so c increases in γ.

Just as in the basic model we can define the voters' ex-ante expected payoff.

$$\pi(m,c,\gamma)=\begin{cases} U_l(1/2,c,m) & \text{if } c\leq\underline{c}(m,\gamma)\\ \frac{p}{2}U_l(1,c,m)+qU_l(1/2,c,m)+(1-p/2-q)U_W(c,m) & \text{if } \underline{c}(m,\gamma)<c\leq\bar{c}(m,\gamma)\\ U_W(c,m) & \text{if } c>\bar{c}(m,\gamma). \end{cases}$$

There is one qualitative difference between the ex-ante payoff function for $\gamma=0$ in the basic model and the one here for $\gamma>0$. If $\gamma=0$, then the payoff generated by projects that are implemented for sure in period 1 (i.e. projects with $c\leq\underline{c}(m,N,\gamma)$) is independent of m (everyone votes in favor and the decision cannot be reversed in period 2). If, instead, $\gamma>0$ then even if a project is always implemented in period 1, it might be terminated at the beginning of period 2. Whether or not this happens depends on the prevailing majority rule m. If m is low, then the low cost projects in the range under consideration are terminated too often. Hence the ex ante expected payoff is decreasing in m as shown in the figure below. For sufficiently high values of m instead, even the very productive low-c projects are continued too often. In this case the ex ante payoff would increase if m were lowered. In the basic model ($\gamma=0$), the average ex-ante payoff is maximized for some majority rule between $m=7N/11$ and $m=2N/3$ (Fig. 3). As we have seen, the shape of the ex-ante payoffs remains essentially the same for positive γ. This suggests that the relative size of the gains and losses in ex ante payoffs from an increase in m should not vary much with γ either, and thus the optimal majority rule should not be too sensitive to γ. The following proposition confirms this intuition. It shows that for all $\gamma\leq 1/2$ the majority rule that maximizes average ex ante payoff lies between $7N/11$ and $3N/4$. Thus, the optimality of supermajority rules in this setting is very robust even if there is some possibility of terminating a project.

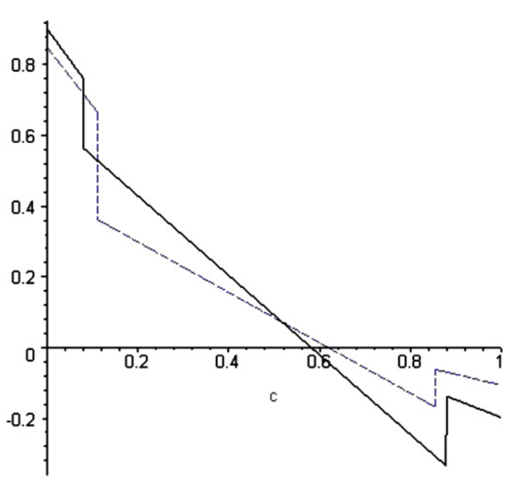

Fig. 3. Ex ante payoffs for $\gamma=0.5$, $N=15$, $m=8$ (solid) and $m=9$ (dashed).

Proposition 8. *If $\gamma\leq 1/2$ and $N\geq 31$, then $\Pi(m,\gamma)-\Pi(m+1,\gamma)<0$ for all $m<7N/11$. Moreover, $\Pi(m,\gamma)-\Pi(m+1,\gamma)>0$ for all $m>3N/4$.*

Proposition 8 shows that the optimal majority rule for the case $\gamma=0.5$ lies between $m=7N/11$ and $m=3N/4$. Remember that $7N/11$ is the lower bound on the optimal majority rule in the case of $\gamma=0$.

Proposition 8 only considers the case $\gamma\leq 0.5$ in the interest of brevity; showing that $7N/11$ is a lower bound on the optimal majority rule also when $\gamma>1/2$ would require very lengthy approximation arguments. However, as Fig. 4 shows, the optimal majority rule remains in between $7N/11$ and $3N/4$ also for $\gamma>1/2$. The figure depicts the average ex ante payoffs for four different majority rules ($N=51$, $m\in\{34,35,36,37\}$) as a function of γ. The slope of the curves decreases in the majority rule (i.e. the steepest one corresponds to $m=34$ and the flattest one to $m=37$).

Consistent with the results of our baseline model, when $\gamma=0$ then the optimal majority rule is the 2/3-rule ($m=34$). As shown in Proposition 8 the optimal majority rules for values of γ below 1/2 lie in the interval $7N/11$ and $3N/4$ ($m=34=2N/3$ for γ close to 0 and $m=35<3N/4$ for γ closer to 1/2). In addition, Fig. 4 shows that the bounds on the optimal majority rules stated in Proposition 8 apply also in the case $\gamma>1/2$, i.e. $2N/3<36,37<3N/4$.

The results of this section are very surprising: a high value of γ removes the technological links between periods (i.e., society can simply terminate projects that were implemented earlier, without incurring large expected termination cost, at least if γ is close to 1). Why does this not mean that we are getting closer and closer to a purely static setting in which we know from Guttman (1998) that – with a sufficiently symmetric setup, as we have assumed here – simple majority rule is optimal? Instead, society optimally uses supermajority rules that are even larger than in the basic model and that *introduce* institutional links between periods when there are fewer technological links (i.e., for $\gamma>0$).

It is easiest to see the intuition for this result when considering a society with many voters. Consider first simple majority rule. If γ is high, voters know that, whether or not society chooses to adopt the project in period 1, has almost no influence on whether the project will be implemented in period 2. All voters vote essentially myopically, and in each period, the project will be active if and only if there is a majority of high type voters. If N is larger, the percentage of high types in the population is almost always very close to 1/2, and so the expected payoff per voter if the project is active in a period is about $(1/2-c)$. Taking expectations over c in this symmetric setting, the expected average payoff is close to zero.

Consider now supermajority rules. In a large society, any supermajority rule means that the project status will remain unchanged in the second period: those projects that were implemented already in the first period will remain active, as it is very unlikely that there is a supermajority of low types that is sufficient to terminate the project, and those that were rejected in the first period will remain inactive, as it is very unlikely that there is a sufficient supermajority of high types in the second period. Thus, in the first period, a voter votes for implementation if and only if he would rather like to see the project implemented in both periods than inactive in both periods. Since a first period low type has an expected payoff over both periods of (about) $1/2-2c$, those projects with $c\leq\underline{c}\approx 1/4$ are unanimously approved, and lead to an expected payoff per voter of about $E(1/2-c|c\leq 1/4)$. In contrast, all projects that generate disagreement between high types and low types in the first period

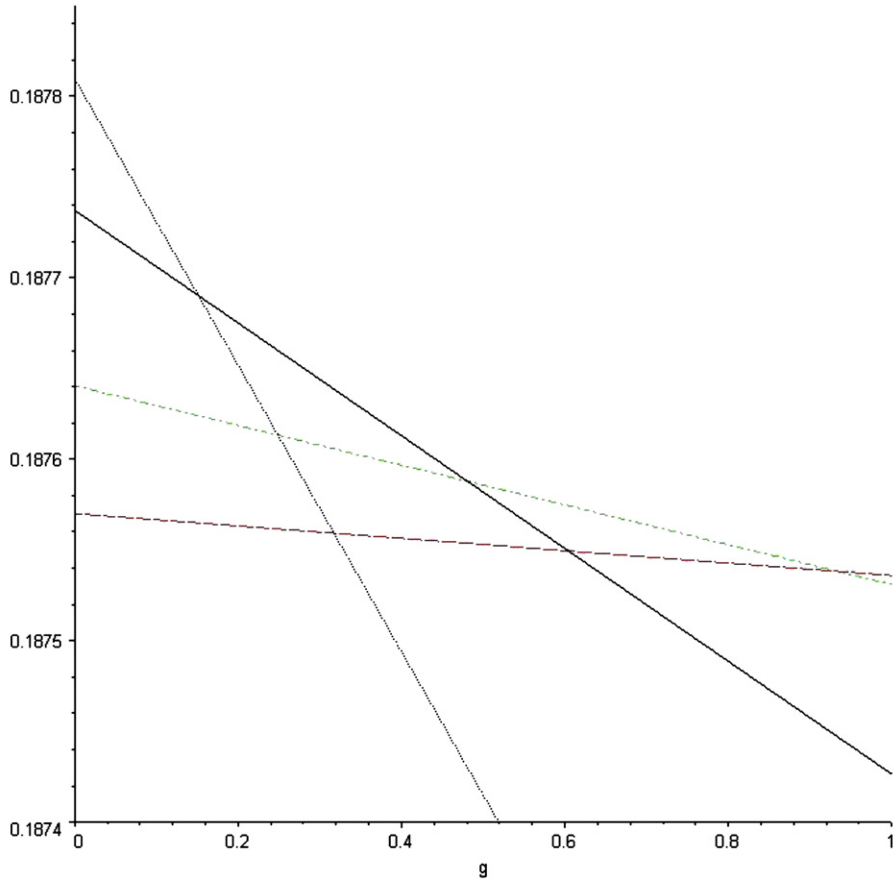

Fig. 4. Average ex-ante payoffs as function of γ; $N=51$, $m=34$ (gray, steepest), $m=35$ (black), $m=36$ (green) and $m=37$ (red, flattest).

are very unlikely to receive the supermajority of support needed for implementation.[18]

In summary, under simple majority rule, the only social benefit generated derives from the fact that the percentage of high types is a little bit higher than the percentage of low types in all periods in which the project is implemented (in periods in which the relation between high types and low types is reversed, the project will be inactive). However, the project type c does not affect whether or not a project is implemented in a particular period. In contrast, supermajority rules do worse on selecting projects that benefit a majority of voters (because sometimes a majority, but not the required supermajority are high types), but they do substantially better in terms of selecting projects in which the winners' gains outweigh the losers' losses.

4.2. Time dependent majority rules

In this section we allow for majority rules to vary across periods. As argued earlier, in most real world situations, time-varying majority rules are not feasible because it would be impossible to determine objectively whether a supposedly new project is really new so that the decision should be taken according to the first period rule, or if it is just an old project that is presented as a new one. Still, in some cases, time-varying majority rules may be feasible, and also, from a theoretical point of view, it is interesting to see what would happen if time-varying rules were feasible.

To keep things as simple as possible, we continue assume that $\theta=1/2$. Let m_t be the majority rule in period $t=1,2$. Clearly, first period voting behavior in this model depends only on m_2, since the value of postponing the decision is a function of m_2, but not of m_1. More specifically, low type voters vote for the project if and only if $c\leq\underline{c}(m_2)$, and high type voters approve the project if and only if $c\leq\bar{c}(m_2)$. The functions $\underline{c}$ and $\bar{c}$ here are the same threshold functions as in the basic model. While m_1, is irrelevant for voting behavior, it still determines the outcome of the first period election. In particular, the payoff that a voter expects to get before he learns his first period type is (but given c)

$$\pi(m_1, m_2, c) = \begin{cases} 2(\theta-c) & \text{if } c\leq\underline{c}(m_2) \\ q_1(\theta-c)+p_1\theta(1-c)+(1-\theta p_1-q_1)[q_2(\theta-c)+p_2\theta(1-c)] & \text{if } \underline{c}(m_2)\leq c\leq\bar{c}(m_2) \\ q_2(\theta-c)+p_2\theta(1-c) & \text{if } \bar{c}(m_2)<c, \end{cases}$$

where q_t and p_t, $t=1,2$, are respectively the probability that in period t the project will be approved without the voter's consent and the probability that the voter will be pivotal in period t. This function is piecewise linear with two jumps at $\underline{c}(m_2)$ and $\bar{c}(m_2)$, just like in the case of a constant voting rule.

In the previous sections we have argued that the reason why the ex ante optimal majority rule is close to a two-third majority, is that for smaller majority rules voters' behavior is too much driven by the value of waiting. Low type voters are willing to reject good projects today because they see a good chance that the project will be implemented in the next period when they might also benefit from it. High type voters instead are willing to implement projects with a negative expected return because they know that there is a good chance that in the second period the project may be implemented anyway. Thus, it is better for them to grab at least the current period gains. Increasing the majority rule lowers the value of postponing the decision in

[18] The ex-ante expected per-capita payoff under supermajority rules in a large society is about $E(1/2-c|c\leq1/4)\times(1/4)$times$2=6/32=0.1875$ (where the second term is equal to the probability that $c\leq1/4$, and the "2" takes into account that the payoff accrues over 2 periods). This is very close to the payoff values in Fig. 4.

absolute terms and thus has the benefit that low type voters become more willing to accept good projects and high type players become more willing to reject bad projects. Of course, these benefits of a higher majority rule come with the cost that, in both periods, a larger consensus is needed to implement projects that would be beneficial on average.

Allowing for time dependent majority rules changes this basic trade off. A decrease of the (absolute value) of the value of waiting can be obtained without modifying m_1. Thus, we should expect that it is optimal to keep the period 1 majority rule low, while the optimal period 2 majority rule should be at least as large as the optimal rule in the baseline model. Proposition 9 confirms this intuition. The statement of the proposition assumes that $N>60$, which allows for a significantly shorter proof. However, the result holds for all N, and a proof covering this case is available from the authors upon request.

Proposition 9. *Suppose that c is uniformly distributed so that* $\Pi(m_1, m_2)=\int_0^1 \pi(m_1,m_2,c)dc$, *and assume* $N>60$. *Let* $(m_1{}^*,m_2{}^*)$ *maximize* $\Pi(m_1,m_2)$. *Then* $m_1{}^*\leq N(1+p(m_2{}^*,N))/2$ *and* $m_2{}^*\geq 7N/11$.

The next proposition shows that having the option to wait is no longer detrimental for society if it adopts the optimal pair of majority rules (m_1^*,m_2^*). Intuitively, this can be seen as follows. If the decision cannot be delayed then the optimal majority rule is simple majority. Now compare the ex-ante payoff when the decision is taken once and for all with simple majority with the voters' ex ante payoff under simple majority in period 1 and unanimity in period 2, $(m_1,m_2)=((N+1)/2,N)$. In the previous sections we have seen that $\underline{c}(N)\approx 1/4$ and $\bar{c}(N)\approx 3/4$, where $c=1/4$ and $c=3/4$ are the thresholds at which low and high type voters change their behavior when the decision cannot be delayed. This means that first period voting behavior is essentially the same as in the case that there is only one decision. If the first period decision is positive, then voters get the same payoff under both regimes. But while voters get nothing in case of a negative decision when the decision is once and for all, there is still a (small) chance that all voters agree to implement the project in period 2 under unanimity rule. So, for every $c>1/4$ the ex ante payoff is slightly higher in the case where the decision can be delayed. Of course, since $((N+1)/2,N)$ is a feasible pair of majority rules, the same must hold a fortiori for the optimal pair of majority rules $(m_1{}^*,m_2{}^*)$.

Proposition 10. *Suppose that c is uniformly distributed. Then* $\Pi(m_1^*, m_2^*) > \tilde{\Pi}((N+1)/2)$.

5. Conclusion

We analyze a model in which voters have to choose whether to implement a project immediately, or postpone a decision till the second period and vote then. Our main focus is the effect of the majority rule on individual voting behavior and social decisions in this framework.

We show that the option to wait makes voters in the first period more conservative towards projects that have a positive expected second period value, and more inclined to implement projects that have a negative expected second period value. To counteract these socially undesirable tendencies when society has the option to wait, the optimal majority rule increases relative to the case where postponing is not possible. For example, for a uniform project type distribution, the optimal majority rule changes from simple majority without the option to wait to a supermajority rule that is between $7/11\approx 63.6$ percent, and about 2/3.

Surprisingly, the case for a supermajority rule remains intact and even is strengthened if society can, in principle, repeal a first-period decision to implement a project. In this case, society reacts to increased technological flexibility (i.e., ability to repeal an implemented project) by making repealing the project more difficult by requiring a supermajority to do this. The fundamental reason for this result is that supermajority rules lead to a considerably better selection of implemented projects than simple majority rule.

In the online appendix, we also show that the benefits of supermajority rules are further strengthened if individual voters' preferences are correlated with each other. In contrast, correlation between first and second period valuations reduces the size of the optimal majority rule. Perfect correlation re-establishes simple majority rule as the optimal rule; however, even if correlation is high, but not perfect, the optimal majority rule remains a supermajority rule.

Another important result was that the option of waiting, which is always positive for individual decision problems, can be negative for our social decision problem. Indeed, we show that this is the case when the project cost is uniformly distributed from an ex-ante perspective, even if society adopts the optimal majority rule in the case that they have the option to wait. This indicates that societies may have some demand for commitment brought about by inflexible rules that force a vote at an inflexible moment in time.

One direction in which future research can expand on our model framework is as follows. In our model, individuals only choose how to vote. In some instances, individuals may also be able to adapt to the policy enacted and thereby influence the distribution of their payoff in the second period. This may be important, for example, in issues where the project is some sort of environmental regulation, say, increasing the private cost of some polluting activity. Adaptation, such as buying a smaller car or isolating one's home, may make compliance less costly over time, but the enacted policy (as well as the expectation of which regulation will be in force in the next period) will affect the optimal extent to which individuals adapt.

Appendix A

In this appendix, we will often suppress the argument m of functions (e.g., we write p for $p(m)$), and generally denote functions evaluated at $m+1$ by primes (e.g., $p'\equiv p(m+1)$).

Lemma A1.

$$\frac{p'}{p}=\frac{\theta}{1-\theta}\frac{(N-m)}{m}.$$

Proof. Using the definition of p, $\frac{p'}{p}=\frac{\binom{N-1}{m}\theta^m(1-\theta)^{N-1-m}}{\binom{N-1}{m-1}\theta^{m-1}(1-\theta)^{N-m}}=\frac{\theta}{1-\theta}\frac{N-m}{m}$. □

Lemma A2. For all N and all $m\geq\theta N$, $\frac{q(m,N,\theta)}{p(m,N,\theta)}\leq\frac{\theta(N-m)}{m-\theta N}$.

Proof. Let $s=m/N$. For $s\geq\theta$, we have

$$\begin{aligned}\frac{q(m,N,\theta)}{p(m,N,\theta)}&=\sum_{l=m+1}^{N}\frac{p(l,N,\theta)}{p(m,N,\theta)}=\sum_{l=m}^{N-1}\frac{\binom{N-1}{l}\theta^l(1-\theta)^{N-1-l}}{\binom{N-1}{m-1}\theta^{m-1}(1-\theta)^{N-m}}\\&=\sum_{l=m}^{N-1}\left[\left(\frac{\theta}{1-\theta}\right)^{l-m+1}\prod_{k=m}^{l}\frac{N-k}{k}\right]\leq\sum_{l=m}^{N-1}\left(\frac{(1-s)\theta}{s(1-\theta)}\right)^{l-m+1}\\&\leq\frac{(1-s)\theta}{s(1-\theta)}\sum_{l=0}^{\infty}\left(\frac{(1-s)\theta}{s(1-\theta)}\right)^{l}=\frac{(1-s)\theta}{s-\theta}=\frac{\theta(N-m)}{m-\theta N}.\quad\square\end{aligned}$$

Proof of Lemma 1. Observe that $\frac{p'}{q'}\geq\frac{p}{q}$ if and only if

$$\frac{p'}{p}\geq\frac{q'}{q}=\frac{q'}{q'+p'}=\frac{1}{1+\frac{p'}{q'}}. \tag{7}$$

Using Lemma A1, and $\frac{p'}{q'} \geq \frac{(m+1)-\theta N}{\theta(N-(m+1)}$ by Lemma A2, a sufficient condition for Eq. (7) to hold is

$$\frac{\theta}{1-\theta}\frac{(N-m)}{m} \geq \frac{1}{\frac{\theta(N-(m+1))}{\theta(N-(m+1))}+\frac{(m+1)-\theta N}{\theta(N-(m+1))}} = \frac{\theta(N-(m+1))}{(1-\theta)(m+1)}$$

which is always true. □

Proof of Lemma 2. The difference between the value of waiting at m' and at m,

$$U_W\left(c,m',N,\theta\right)-U_W(c,m,N,\theta) = \theta(1-c)\left(p'-p\right)+(c-\theta)p', \tag{8}$$

is positive if and only if

$$\frac{p}{p'} \geq \frac{c}{1-c}\frac{1-\theta}{\theta}. \tag{9}$$

Using Lemma A1, Eq. (9) holds if and only if

$$\frac{1-\theta}{\theta}\frac{m}{N-m} \leq \frac{c}{1-c}\frac{1-\theta}{\theta} \Leftrightarrow \frac{m}{N} \leq c. \quad \square$$

Proof of Proposition 3. The cutoff structure follows immediately from the fact that $\frac{\partial U_I}{\partial c} = -2 < \frac{\partial U_W}{\partial c} = -\theta p - q$. Moreover, $U_I(1,c,\theta) > U_I(0,c,\theta)$ implies that $c < \bar{c}$.

We now show that $c < \theta/2$. Since $\theta/2$ is the value of $\underline{c}$ in the benchmark case (without the option to wait), $c < \theta/2$ if and only if the value of waiting at this threshold is strictly positive. We have

$$U_W(\theta/2,m,N,\theta) = \theta p(m,N,\theta)(1-\theta/2)+q(m,N,\theta)(\theta-\theta/2) > 0, \tag{10}$$

for all θ. Thus, low types become more conservative with the option to wait.

We next show that $\underline{c}$ is an increasing function in m. Since $m \geq (N+1)/2$, this claim follows immediately from the Lemma 2: For all $m \geq (N+1)/2$ the value of waiting is decreasing in m for all $c < 1/2$. Thus, $\underline{c} < \theta/2 < 1/2$ means that $\underline{c}$ must be increasing in m if $m \geq (N+1)/2$.

We now turn our attention to the threshold of high types, $\bar{c}(\cdot,N,\theta)$. Solving $U_I(c,1,\theta) = U_W(c,m,N,\theta)$ for c yields

$$\bar{c}(m,N,\theta) = \frac{1+\theta(1-p(m,N,\theta)-q(m,N,\theta))}{2-\theta p(m,N,\theta)-q(m,N,\theta)}. \tag{11}$$

The claim that $\bar{c}(N,N,\theta) < (1+\theta)/2$ follows immediately from the fact that at $c = (1+\theta)/2$ the value of implementing the project immediately is just zero, while the value of waiting is $p(N,N,\theta)\theta(1-\theta)/2 > 0$.

Since

$$\begin{aligned} &U_I(1,1/(2-\theta),\theta)-U_W(1/(2-\theta),m,N,\theta)\\ &= 1+\theta-\frac{2}{2-\theta}-\theta p\left[1-\frac{1}{2-\theta}\right]-q\left[\theta-\frac{1}{2-\theta}\right] = \frac{1-\theta}{2-\theta}[\theta(1-p)+(1-\theta)q] > 0, \end{aligned} \tag{12}$$

it follows that $\bar{c}(\cdot,N,\theta) > 1/(2-\theta)$.

By Lemma 2, $\bar{c}(m) \geq \bar{c}(m')$ if and only if $\bar{c}(m) \geq m/N$. Using Eq. (11), we obtain

$$\frac{1+\theta(1-p-q)}{2-\theta p-q} \geq \frac{m}{N} \Leftrightarrow (1+\theta)N-2m-[p\theta(N-m)-q(m-\theta N)] \geq 0. \tag{13}$$

Lemma A2 implies that the term in square brackets is positive for all $m \geq \theta N$. This means that condition (Eq. (13)) can never hold for $m \geq (1+\theta)N/2$. Hence, $\bar{c}$ is increasing at least from $m = \lceil(1+\theta)N/2\rceil$ onwards.

Finally, since $2 > \theta p + q$, the left hand side of Eq. (13) is decreasing in m, which implies that Eq. (13) changes sign at most once. □

Lemma A3. $Np(\lceil 3N/4\rceil,N,1/2) \leq 4$ for all N.

Proof. Given that N is an odd number we have that either $3N+1$ is divisible by 4 (for $N = 5,9,13,\ldots$) or $3(N+1)/4$ is an integer (for $N = 3,7,11,\ldots$). In the first case we have that $\lceil 3N/4\rceil = (3N+1)/4$, while in the latter case we have $\lceil 3N/4\rceil = 3(N+1)/4$. Notice also that in either case we have $\lceil 3(N+4)/4\rceil - \lceil 3N/4\rceil = 3$.

Let $f(N) = Np(\lceil 3N/4\rceil,N,1/2)$. In what follows we will show that $f(N+4)-f(N) < 0$. This is sufficient for proving our statement since it implies that $\max_N f(N) = \max\{f(3),f(5)\} = \max\left\{3\binom{2}{2}/2^2, 5\binom{4}{3}/2^4\right\} = 5/4 < 4$.

Letting $m = \lceil 3N/4\rceil$ the increment $f(N+4)-f(N)$ is given by

$$\begin{aligned} &\frac{N+4}{2^{N+3}}\binom{N+3}{m+2}-\frac{N}{2^{N-1}}\binom{N-1}{m-1}\\ &= \frac{N}{2^{N-1}}\binom{N-1}{m-1}\left[\frac{(N+4)(N+3)(N+2)(N+1)}{16(N+1-m)(m+2)(m+1)m}-1\right]. \end{aligned}$$

Observe that $(N+j+2) < 4(m+j)$ for $j = 0,1,2$ and that $N+1 \leq 4(N+1-m)$. Thus the first term in the square brackets is a product of four numbers which are smaller than one, and thus the term in square brackets is negative. □

Proof of Proposition 4. The main arguments are in the text. It remains to be shown that $m^*(c) = \lceil(1+\theta)N/2\rceil$ for all $c \in (\bar{c}(\lceil(1+\theta)N/2\rceil), (1+\theta)/2]$. The ex-ante payoff under majority rule m is a weighted average of the values of immediate implementation for low and high types and the value of waiting associated with rule m (where the weights depend on the behavior of players). Thus, it is sufficient to show that for all c in the specified range we have $U_W(c,\lceil(1+\theta)N/2\rceil) > \max(U_W(c,m),U_I(V_1,c))$ for all m and all V_1. The definition of $\bar{c}(\lceil(1+\theta)N/2\rceil)$ implies immediately that $U_W(c,\lceil(1+\theta)N/2\rceil) > U_I(V_1,c)$ for all $c > \bar{c}(\lceil(1+\theta)N/2\rceil)$ and V_1. For $m > \lceil(1+\theta)N/2\rceil$, $U_W(c,\lceil(1+\theta)N/2\rceil) > U_W(c,m)$ follows immediately by Lemma 2. Furthermore, if for any majority rule $m < \lceil(1+\theta)N/2\rceil$ we had $U_W(c,\lceil(1+\theta)N/2\rceil) < U_W(c,m)$ for some $c \in (\bar{c}(\lceil(1+\theta)N/2\rceil),(1+\theta)/2]$, then this inequality would also hold for any lower c, and in particular for $\bar{c}(\lceil(1+\theta)N/2\rceil)$. But this implies $\bar{c}(m) < \bar{c}(\lceil 3N/4\rceil)$, a contradiction to the fact that $\bar{c}(\cdot)$ is minimized at $m = \lceil(1+\theta)N/2\rceil$. □

Proof of Proposition 5. We have to show that the difference $\Pi(m') - \Pi(m)$ is negative whenever $m \geq 2N/3$. It is a matter of tedious but straightforward algebraic manipulations that this difference is equal to the ratio[19]

$$\frac{2pqp'+8qp-11p-8pp'-q^2p-p\left(p'\right)^2-16qp'+21p'-2q\left(p'\right)^2+4\left(p'\right)^2+3q^2p'}{4(4-2q+p')(4-2q-p)}. \tag{14}$$

The denominator of this expression is clearly positive. Thus the sign of the difference in average ex-ante payoffs coincides with the sign of the numerator of this expression. Denote this numerator by $d(p,q,p')$.

[19] The following expression is obtained by using $q' = q-p'$.

We first show that for any p and q, d is monotonically increasing in p'. We have

$$\frac{\partial d\left(p,q,p'\right)}{\partial p'} = 2pq - 8p - 2pp' - 16q + 21 - 4qp' + 8p' + 3q^2$$
$$\geq -8p - 2pp' - 16q + 21 - 4qp' \geq 13 - 2pp' - 4qp' > 0,$$

where the second inequality sign in this expression follows from the fact that $q + p/2 \leq 1/2$.

Since $p' = (N-m)p/m$, we have that $p' \leq p/2$ for all $m \geq 2N/3$, and thus

$$d\left(p,q,p'\right) \leq \max_{p' \leq p/2} d\left(p,q,p'\right) = d(p,q,p/2)$$
$$= p\left[\frac{pq}{2} - \frac{1}{2} - 3p + \frac{q^2}{2} - \frac{p^2}{4}\right] < \frac{p}{2}\left[pq - 1 + q^2\right].$$

Given that $p,q \leq 1/2$ we thus have that $d(p,q,p') < 0$ whenever $m \geq 2N/3$. This proves the first part.

Since $p' = (N-m)p/m = \frac{1-s}{s}p$ for $s = m/N$, we have $p' \geq ap$ for $m \leq sN$, where $a = (1-s)/s$. Monotonicity of d in p' therefore implies that

$$d\left(p,q,p'\right) \geq \min_{p' \geq ap} d\left(p,q,p'\right) = d(p,q,ap)$$
$$= \{[a(4-p)-8]ap + [2pa(1-a) + (3a-1)q - 8(2a-1)]q + 21a - 11\}p.$$

Now observe that the sign of $\partial d(p,q,ap)/\partial q$ coincides with the sign of $2pa(1-a) + (6a-2)q - 8(2a-1)$. Since

$$2pa(1-a) + (6a-2)q - 8(2a-1) < 8 - a(16 - 2(2q+p))$$
$$+ 2q(1-a) \leq 8 - 14a$$

it follows that whenever $a \geq 8/14 = 4/7$ then $\min_{p' \geq ap} d(p,q,p')$ is decreasing in q. Using the fact that $q \leq (1-p)/2$ we thus have that for all $1 \geq a \geq 4/7$

$$d\left(p,q,p'\right) \geq \min_{q \leq (1-p)/2}\left\{\min_{p' \geq ap} d\left(p,q,p'\right)\right\} = d(p,(1-p)/2,ap)$$
$$= \frac{p}{4}\left(55a - 29 - 2ap - ap^2 - 14p + 12a^2p - p^2\right) =: D(p,a).$$

$D(p,a)$ is clearly increasing in a. In the case of simple majority we have $a = (N-1)/(N+1)$, which is increasing in N. Thus under simple majority we have that $a \geq 2/3$ if $N \geq 5$ (for $N = 3$ all majority rules satisfy $m \geq 2N/3$). Since $D(p,2/3) = p(23 - 30p - 5p^2)/12 > 0$ for all $p \in (0,1/2)$ we can therefore conclude that at simple majority the increment of Π is positive.

The preceding observations allow us to restrict our attention in the remainder of the proof to supermajority rules $m > (N+1)/2$. Since for all such rules we have $1/2 \geq p(m-1)/2 + q(m-1)$ and $p(m) \geq p(m-1)/2$, the identity $q(m-1) = p(m) + q(m)$ implies $3p(m)/2 + q(m) \leq 1/2$ or equivalently $q \leq (1-3p)/2$. Exploiting this fact we can thus claim that if $m > (N+1)/2$ then we have for all $a \in (4/7, 1)$ that

$$d\left(p,q,p'\right) \geq \min_{q \leq (1-3p)/2}\left\{\min_{p' \geq ap} d\left(p,q,p'\right)\right\} = d(p,(1-3p)/2,ap)$$
$$= \frac{p}{4}\left(50ap + 15p^2a - 29 - 42p - 9p^2 + 8a^2p^2 + 55a + 12a^2p\right) =: \hat{D}(p,a).$$

Since $\hat{D}(p,a)$ is strictly increasing in a it follows that for all $a \geq 4/7$ we have

$$d\left(p,q,p'\right) \geq \hat{D}(p,4/7) = \frac{p\left(119 - 466p + 107p^2\right)}{196}.$$

It is straightforward to see that this expression is strictly positive for all $p \in (0,1/4]$. Since $p(m)$ is decreasing in m for every N and also $p((N+3)/2)$ decreases with N, it follows that for $2N/3 > m > (N+1)/2$ we must have $p(m) \leq p(7,11) = 105/520 < 1/4$ (notice that only for $N \geq 11$ there are majority rules in the specified range). Hence, for all p which may arise for $2N/3 > m > (N+1)/2$ we know that $\hat{D}(p,a) > 0$, whenever $a \geq 4/7$. The condition $a = (1-s)/s \geq 4/7$ in turn is equivalent to $s = m/N \leq 7/11 \approx 0.636$. Thus we can conclude that $d(p,q,p') > 0$ for all $(N+1)/2 < m < 7N/11$. This proves statement ii).

Finally, using $p((N+1)/2)/2 + q((N+1)/2) = 1/2$ and $q(N) = 0$ it is straightforward to show that

$$\Pi(N) - \Pi((N+1)/2) = \frac{16 - 27p + 10p^2 + p^3}{48(4-p)},$$

which is strictly positive for all $p < 1$. □

Proof of Proposition 6. For the proof of the first statement, we drop the arguments from the functions p and q (like in earlier proofs). Calculating the difference between $\Pi(m)$ and $\tilde{\Pi}(m)$ gives

$$\Pi(m) - \tilde{\Pi}(m) = \frac{2p + 3 + q^2 - 4q}{4(4-2q-p)} - \frac{3+2p}{16} = \frac{3p + 4q^2 - 10q + 4qp + 2p^2}{16(4-2q-p)}. \quad (15)$$

The denominator of this expression is clearly positive and thus the sign of the difference is determined by the numerator. Denote this numerator by $d(p,q)$.

We first show that $d(p,q)$ is negative for $m = (N+1)/2$. Remember that, in this case, we have $p = 1 - 2q$ and thus

$$d = 3p + 4q^2 - 10q + 4qp + 2p^2 = 5 - 20q + 4q^2.$$

This expression is negative if $q((N+1)/2, N) \leq 5/2 - \sqrt{5} \approx 0.26$, which is satisfied for all $N > 5$.

Next consider any supermajority m such that $(N+1)/2 < m \leq \lfloor 3N/4 \rfloor$. For any such majority rule we have that $m \leq N-2$. Therefore, it follows that

$$q(m) \geq p(m+1) + p(m+2) = p(m+1)\left(1 + \frac{N-m-1}{m+1}\right)$$
$$= p(m)\frac{N-m}{m}\frac{N}{m+1}.$$

Notice that the last term in this expression is decreasing in m. Thus we may write $p \leq (q\lfloor 3N/4 \rfloor \lceil 3N/4 \rceil)/((N - \lfloor 3N/4 \rfloor)N)$. Using Lemma A3, it can be shown that the right-hand side of this last inequality is smaller than $(12/5)q$.[20]

Next observe that the fact that m is a supermajority rule implies that $1/2 \geq p(m-1)/2 + p(m) + q(m) \geq 3p(m)/2 + q(m)$. Combining this observation with the preceding one, we obtain $p \leq min\{(1-2q)/3, 12q/5\}$, or equivalently,

$$p \leq \begin{cases} 12q/5 & \text{if } q \leq 5/46 \\ (1-2q)/3 & \text{if } q > 5/46. \end{cases}$$

Notice that $d(12q/5, q) = -(14/5 - 628q/25)q < 0$ for all $q \leq 5/46$ and $d((1-2q)/3, q) = (11 - 104q + 20q^2)/9 < 0$ for all $5/46 < q \leq 1/2$. Since $d(p,q) \leq d(min\{(1-2q)/3, 12q/5\}, q)$, this proves the first claim.

As for the second part of number 2, observe that a sufficient condition for the numerator of Eq. (15) to be positive is $3p > 10q$. From Lemma A2, we have $\frac{q}{p} \leq \frac{N-m}{2m-N}$ so that for all $m \geq \frac{13}{16}N$, the numerator of Eq. (15) is positive.

[20] If $N = 7, 11, 15\ldots$, then $12/5 - (\lfloor 3N/4 \rfloor \lceil 3N/4 \rceil)/((N - \lfloor 3N/4 \rfloor)N) = (N + (N-1)/2 + 8)/10N > 0$.

For number 2, note that Proposition 5 implies that the optimal majority rule with the option of waiting is lower or equal to $\lceil 2N/3 \rceil$, which is lower or equal to $\lfloor 3N/4 \rfloor$ for all $N>5$. By the second statement of Proposition 6, for all such rules, the expected ex-ante payoff is higher without the option to wait. □

Proof of Proposition 7. Calculating the thresholds we obtain

$$\underline{c}(m,N,\gamma)=\frac{1-p-(1+\gamma)q}{4-(p+2q)(1+\gamma)} \quad \text{and} \quad \bar{c}(m,N,\gamma)=\frac{3-p-(1+\gamma)q}{4-(p+2q)(1+\gamma)}.$$

It is a matter of straightforward algebra to show that the difference $\underline{c}(m+1,N,\gamma)-\underline{c}(m,N,\gamma)$ is proportional to the following expression.

$$\underline{\Delta}(\gamma)=2p-2\gamma p'+p'^2\left(1-\gamma^2\right)+(1-\gamma)\left(p-p'\right)\left[1-q'(1+\gamma)\right],$$

where $p=p(m,N)$, $p'=p(m+1,N)$ and $q'=q(m+1,N)$. Since $q'<1/2$ and $1+\gamma\leq 2$ the third term of this expression is positive. Also $p>\gamma p'$ and so $\underline{\Delta}>0$.

The difference between $\bar{c}(m+1,N,\gamma)-\bar{c}(m,N,\gamma)$ is proportional to

$$\bar{\Delta}(\gamma)=-2\gamma p-2p'+p'^2\left(1-\gamma^2\right)+(1-\gamma)\left(p-p'\right)\left[1-q'(1+\gamma)\right].$$

Using $p/p'=m/(N-m)=s/(1-s)$ the condition $\bar{\Delta}\leq 0$ can be rewritten as follows

$$\frac{3-4s}{1-s}+\gamma\frac{4s-1}{1-s}\geq\left(1-\gamma^2\right)\left[p'-\frac{2s-1}{1-s}q'\right]. \tag{16}$$

By Lemma A2 we know that $q(2s-1)/(1-s)\leq p$. Together with Lemma 1 this implies that the rhs of this inequality is positive. On the other hand, $p'\leq 1/4$ for all m,N, implies that the rhs of Eq. (16) is no larger than $(1-\gamma^2)/4$. Using these bounds it is straightforward to check that Eq. (16) must hold if

$$s\leq\frac{11-4\gamma+\gamma^2}{15-16\gamma+\gamma^2}.$$

Moreover, Eq. (16) must be violated if

$$s\geq\frac{3-\gamma}{4(1-\gamma)}.$$

Taking the derivative of $\bar{c}$ with respect to γ delivers an expression which is proportional to $3p-p^2+2q(1-p)$. Clearly, this expression is positive.

The derivative of c is proportional to

$$p(1-p)-2q(1+p). \tag{17}$$

Since $q\geq p'$ this expression is smaller than $p-2p'$. But since $p'/p=(1-s)/s$ it follows that $p-2p'\leq 0$ if $s\leq 2/3$.

On the other hand, since q is smaller than $(1-s)p/(2s-1)$ (Lemma A2) Eq. (17) must be positive if

$$1-p-(1+p)\frac{1-s}{2s-1}\geq 0$$

or equivalently,

$$4s>3-p.$$

For $p<1/4$ then s needs to be larger than 13/16 in order for this inequality to hold. But $p<1/4$ is always satisfied if $m>2N/3$. □

Proof of Proposition 8. Calculating the difference $\Pi(m+1,\gamma)-\Pi(m,\gamma)$ delivers a polynomial in p,p',q,q' and γ. Using $q'=q-p'$ and $p'=p(1-s)/s$ we can eliminate p' and q' (at the cost of introducing $s=m/N$). After eliminating from the resulting expression all common (positive) factors we obtain a polynomial whose terms we collect in two groups. The first group contains all terms that do not contain p as a factor. The second group is given by the following polynomial in q.

$$\begin{aligned}2s^2\Big\{&42-64s+\gamma(26-32s)+q\left[-37+64s-\gamma(106-160s)-\gamma^2(37-48s)\right]\\&+q^2\left[14-24s+\gamma(154-96s)+\gamma^2(124-112s)+\gamma^3(18-24s)\right]\\&+q^3\left[-3+4s-\gamma(8-16s)-\gamma^2(18-32s)-\gamma^3(16-24s)-\gamma^4(3-4s)\right]\Big\}\end{aligned} \tag{18}$$

Consider first the expression in the first line (including the term $2s^2$ outside the curly brackets). This expression is linear in q. Hence, it assumes its minimum in q at one of the possible extreme values of q (0 or 1/2). If $q=0$, we are left with

$$2s^2(42-64s+\gamma(26-32s)).$$

It is straightforward to show that this expression is concave in s. Thus, it assumes its minimum at one of the extreme points of the interval [1/2,7/11]. The value at $s=7/11$ is smaller than the value at $s=1/2$ and equal to $(14/11)(14+62\gamma)/11$. Hence, we can conclude that conditional on $q=0$ the expression in the first line must be larger than $(14/11)^2$.

As for the case $q=1/2$, consider first only the expression within the curly brackets. The sum of those terms is equal to

$$\begin{aligned}&42-64s+\gamma(26-32s)+\left[-37+64s-\gamma(106-160s)-\gamma^2(37-48s)\right]/2\\&=\left[47-64s-\gamma(54-96s)-\gamma^2(37-48s)\right]/2.\end{aligned}$$

This expression is decreasing in s for all $\gamma\leq 1/2$. If we therefore set $s=7/11$ we obtain an expression that is increasing in γ. After substituting $\gamma=0$ we are left with $47-64(7/11)=69/11$. Thus, conditional on $q=1$ (and taking into account the factor $2s^*$) the term in the first line must be larger than 69/22. Since this number is larger than $(14/11)^2$ we continue to work with $(14/11)^2$ as lower bound for the expression in the first line.

Next consider the terms in the second and third line (neglecting for the moment the factor $2s^*$). Clearly, the expression in the second line is decreasing in s and the third line is increasing in s. Since $q\leq 0.5$ it is easy to see that the sum of the two lines must be decreasing in s. So we can set $s=7/11$ which yields

$$q^2\left[-14+1022\gamma+580\gamma^2+30\gamma^3+q\left(-5+24\gamma+26\gamma^2-8\gamma^3-5\gamma^4\right)\right]/11.$$

Both polynomials in γ that appear in this expression are increasing in γ for $\gamma\in[0,1/2]$. Setting $\gamma=0$ we are left with

$$q^2(-14-5q)/11\geq-33/88,$$

where we have used the fact that $q\leq 1/2$. Taking into account the factor $2s^2$ we get as lower bound for the terms in the last two lines $-21/44$.

Adding this to $(14/11)^2$ yields 553/484 as a lower bound for the overall expression.

The group of terms that contain p are

$$\begin{aligned}
&ps\Big\{\Big[7-19s+12s^2\Big]q^2\gamma^4+\Big[-28+76s-48s^2+q\Big(48s^2-72s+24\Big)\Big]q\gamma^3\\
&+\Big[48s^2-77s+29-3/4+q\Big(-128s^2+208s-72\Big)+q^2\Big(73s^2-106s+34\Big)\Big]\gamma^2\\
&+\Big[50-130s+64s^2-7+q\Big(-76+252s-168s^2\Big)+q^2\Big(24-72s+48s^2\Big)\Big]\gamma\\
&+37-149s+112s^2+q\Big(-32+104s-72s^2\Big)+q^2\Big(7-19s+12s^2\Big)\Big\}\\
&+p^2\Big\{\Big[-2+10s-14s^2+6s^3\Big]q\gamma^4+\Big[4-20s+28s^2-12s^3\\
&+q\Big(-6+32s-42s^2+12s^3\Big)\Big]\gamma^3+\Big[4-20s+28s^2-12s^3\\
&+q\Big(-8+44s-68s^2+24s^3\Big]\gamma^2+\Big[8-52s+88s^2-36s^3\\
&+q\Big(-6+32s-54s^2+24s^3\Big)\Big]\gamma+4-24s+44s^2-24s^3+\Big\}\\
&+p^3\Big\{\Big[-1+3s-3s^2-s^3\Big]\gamma^4+\Big[-3+10s-7s^2\Big]\gamma^3+\Big[-4+14s-12s^2+2s^3\Big]\gamma^2\\
&+\Big[-3+10s-11s^2+4s^3\Big]\gamma-1+3s-3s^2+s^3\Big\}.
\end{aligned} \tag{19}$$

Substituting all polynomials in s in this expression with their minimum over the range $s\in[1/2,7/11]$ we obtain an expression that is decreasing in γ and q. Finally substituting $q=\gamma=0.5$ leaves us with the following polynomial in p

$$-7.51p-1.45p^2-0.3125p^3.$$

It is straightforward to verify that the absolute value of this polynomial is smaller than 553/484 for all $p\leq 0.1478$. Finally, $p((N+1)/2,N)<0.1478$ for all $N\geq 31$.

We now show that for $m\geq 3N/4$ the difference $\Pi(m+1,\gamma)-\Pi(m,\gamma)$ is negative. In order to do so, take again Eq. (18) and replace all terms that contain s with the maximum that they assume for $s\in[3/4,1]$. Doing so yields an expression that is increasing in q. Now remember that $q\leq p(1-s)/(2s-1)$ (Lemma 1) and $p\leq 4/N$ for all $s\geq 3/4$ (Lemma A3). Thus, since $N\geq 31$ it follows that $q\leq 0.065$. Substituting this value we end up with a polynomial that is increasing in γ. Substituting $\gamma=0.5$ we obtain a value of about -0.9. Thus, for $s\geq 3/4$ (Eq. (18)) is bounded above by -0.9. Now notice that since $p<4/31$ it follows that Eq. (19) must be very small too. In fact, it is not difficult to show that Eq. (19) cannot exceed 0.9. □

Proof of Proposition 9. In order to show that $m_2^*\geq 7N/11$ we show that $\Pi(m_1,m_2+1)-\Pi(m_1,m_2)$ is positive as long as $s=m_2/N<7/11$. We write p_2 and p_2' for $p(m_2,N)$ and $p(m_2+1,N)$, respectively. A similar notation is applied to other variables as well. Remember that for all $m_2<2N/3$ we have $\bar{c}(m_2)>\bar{c}(m_2+1)$. Since $\pi(m_1,m_2+1,c)-\pi(m_1,m_2,c)\geq 0$ for all $\underline{c}'\in[0,c]\cup[\bar{c}',\bar{c}]$, it follows that

$$\begin{aligned}
&\Pi(m_1,m_2+1)-\Pi(m_1,m_2)\\
&\geq\int_{\underline{c}'}^{\bar{c}'}[\pi(m_1,m_2+1,c)-\pi(m_1,m_2,c)]dc\\
&\quad+\int_{\bar{c}}^{1}[\pi(m_1,m_2+1,c)-\pi(m_1,m_2,c)]dc\\
&=(1-q_1-p_1/2)\int_{\underline{c}'}^{\bar{c}'}\Big[(q_2-q_2)(1/2-c)+\Big(p_2'-p_2\Big)(1-c)\Big]dc\\
&\quad+\int_{\bar{c}}^{1}\Big[\Big(q_2'-q_2\Big)(1/2-c)+\Big(p_2'-p_2\Big)(1-c)\Big]dc.
\end{aligned}$$

We use $q_2'=q_2-p_2$ and $p_2'=p_2(1-s)/s$, (where $s=m_2/N$) in order to eliminate q_2' and p_2' from all expressions in the integral. It is then a matter of straightforward but tedious algebra to show that this integral is proportional to the following expression

$$\begin{aligned}
&p_2^2\Big[7-2p_2-(10-2p_2)q_2+3q_2^2\Big]+2sp_2\Big[28-19p_2+3p_2^2\\
&+\Big(54-28p_2+3p_2^2\Big)q_2+(32+9p_2)q_2^2-6q_2^3\Big]\\
&+s^2\Big[4\Big(44-92q_2+71q_2^2-24q_2^3+3q^4\Big)-4p_2\Big(54-91q_2+50q_2^2-9q_2^3\Big)\\
&+p_2^2\Big(75-92q_2+27q_2^2\Big)-p_2^3(8-6q_2)\Big]\\
&+s^3\Big[4\Big(-64+128q_2-96q_2^2+32q_2^3-4q_2^4\Big)\\
&+4p_2\Big(48-72q_2+36q_2^2-6q_2^3\Big)-4p_2^2\Big(12-12q_2+3q_2^2\Big)+p_2^3(4-2q_2)\Big].
\end{aligned}$$

It is not difficult to see that the expression in the first row of this sum is positive. So we only have to show that the sum of the second and third row is positive too. We first argue that the third row is negative so that the sum of the last two lines is decreasing in s. The sum of the terms that are pre-multiplied by p_2^2 and p_2^3 is certainly negative. So consider the sum of the first expression in the last row. Since $q_2\leq(1-p_2)/2$ we have that $128q_2\leq 64-64p_2$ and thus

$$\begin{aligned}
&\Big(-64+128q_2-96q_2^2+32q_2^3-4q_2^4\Big)+p_2\Big(48-72q_2+36q_2^2-6q_2^3\Big)\\
&\leq\Big(-64p_2-96q_2^2+32q_2^3-4q_2^4\Big)+p_2\Big(48-72q_2+36q_2^2-6q_2^3\Big)<0.
\end{aligned}$$

Next we will show that if $s=3/5$ then the sum

$$\begin{aligned}
D=&\Big[4\Big(44-92q_2+71q_2^2-24q_2^3+3q^4\Big)-4p_2\Big(54-91q_2+50q_2^2-9q_2^3\Big)\\
&+p_2^2\Big(75-92q_2+27q_2^2\Big)-p_2^3(8-6q_2)\Big]\\
&+s\Big[4\Big(-64+128q_2-96q_2^2+32q_2^3-4q_2^4\Big)+4p_2\Big(48-72q_2+36q_2^2-6q_2^3\Big)\\
&-4p_2^2\Big(12-12q_2+3q_2^2\Big)+p_2^3(4-2q_2)\Big]
\end{aligned}$$

is decreasing in q_2. The second derivative of D with respect to q_2 (evaluated at $s=7/11$) is proportional to

$$436-480q_2+120q_2^2-1192p_2+684p_2q_2+213p_2^2.$$

Under the assumption that $N>60$ p_2 is small enough such that this expression is positive. But if D is convex in q_2 then D must be decreasing for all q_2 if it is decreasing for the largest possible value $q_2\leq(1-p)/2$. Calculating the first derivative of D with respect to q_2 and evaluating it at $q_2=(1-p)/2$ yields an expression proportional to

$$-138+741p_2+297p_2^2,$$

which is negative if $N>60$ (so that p_2 is small).

Evaluating D at $(p,q,s)=(p,(1-p)/2,3/5)$ we obtain

$$63-414p_2-405p_2^2$$

which is positive for p_2 small enough (i.e. $N>60$). This tells us that Π is increasing at least up to $m_2=3N/5$. Finally, if $m_2>3N/5$, then $q\leq 2p$. Evaluating D at $(p,q,s)=(p,(1-p)/2,3/5)$ yields

$$144-1960p_2+6209p_2^2-7460p_2^3+3100p_2^4,$$

which is again positive for p_2 small enough (i.e. $N>60$).

Next we show that $m_1^*=(N+1)/2$. First observe that for a given m_2, $\pi(m_1,m_2,c)$ does not vary with m_1 if either $c\leq\underline{c}(m_2)$ or $c>\bar{c}(m_2)$. Thus, writing p_1' and q_1' for $p(m_1+1,N)$ and $q(m_1+1,N)$, respectively, we have

$$\begin{aligned}
&\Pi(m_1,m_2)-\Pi(m_1+1,m_2)\\
&=\int_{\underline{c}(m_2)}^{\bar{c}(m_2)}(q_1-q_1')(1/2-c)+(p_1-p_1)(1-c)/2-(q_1+p_1/2-q_1-p_1/2)U_W(m_2,c)dc\\
&=\int_{\underline{c}(m_2)}^{\bar{c}(m_2)}\frac{p_1+p_1'}{2}[1-c-U_W(m_2,c)]-\frac{p_1'}{2}dc\\
&=\frac{p_1}{2m_1}\int_{\underline{c}(m_2)}^{\bar{c}(m_2)}N[1-c-q_2(1/2-c)-p_2(1-c)]-(N-m_1)dc\\
&=\frac{p_1}{2m_1}\left\{[m_1-N(q_2+p_2)/2](\bar{c}-\underline{c})-N[1-q_2-p_2/2]\frac{\bar{c}^2-\underline{c}^2}{2}\right\}\\
&=\frac{p_1(\bar{c}-\underline{c})}{4m_1N}\left[\frac{2m_1}{N}-q_2-p_2-[1-q_2-p_2/2](\bar{c}+\underline{c})\right]
\end{aligned}$$

where we have exploited that $q_1'+p_1'=q_1$ (second line) and $p'_1=p_1(N-m_1)/m_1$ (third line).

Next observe that if $m_2=(N+1)/2$, then $p_2/2+q_2=1/2$ and thus

$$q_2+p_2+[1-q_2-p_2/2](\bar{c}+\underline{c})=\frac{p_2}{2}+\frac{1}{2}+\frac{3-p_2}{6}=1+\frac{p_2}{3}.$$

So if $m_2=(N+1)/2$, and $m_1\geq N(1+p_2)/2$ then

$$\frac{2m_1}{N}-q_2-p_2-[1-q_2-p_2/2](\bar{c}+\underline{c})\geq 1+p_2-(1+p_2/3)=2p_2/3>0.$$

In order to conclude the proof we need to show that the expression $q_2-p_2-[1-q_2-p_2/2](\bar{c}+\underline{c})$ is decreasing in m_2 (so that if Π is decreasing in m_1 when $m_2=(N+1)/2$, then it must also be decreasing in m_1 for larger values of m_2). So write q_2' and p_2' for $q_2(m_2+1,N)$ and $p_2(m_2+1,N)$, respectively. Then,

$$\begin{aligned}
&q_2-q_2'+p_2-p_2'+[1-q_2-p_2/2](\bar{c}+\underline{c})-\left[1-q_2'-p_2'/2\right]\left(\bar{c}'+\underline{c}'\right)\\
&=p_2+[1-q_2-p_2/2]\frac{4-2p_2-2q_2}{4-p_2-2q_2}-\left[1-q_2+p_2'/2\right]\frac{4-2q_2}{4+p_2-2q_2}\\
&=p_2+[1-q_2]\left[\frac{p_2'}{b}-\frac{p_2}{a}\right]-\frac{p_2}{2}\left[1-\frac{p_2}{a}\right]-\frac{p_2'}{2}\left[1-\frac{p_2'}{b}\right]\\
&=\frac{p_2-p_2'}{2}-(1-q_2)\left[\frac{p_2}{a}-\frac{p_2'}{b}\right]+\frac{p_2^2}{2a}+\frac{p_2'^2}{2b},
\end{aligned}$$

where $a=4-p_2-2q_2$ and $b=4+p_2-2q_2$. Since $3\leq a<b<4$, $1-q_2<1$ and $p_2=p_2(N-m_2)/m_2$ it follows that

$$\begin{aligned}
&\frac{p_2-p_2'}{2}-(1-q_2)\left[\frac{p_2}{a}-\frac{p_2'}{b}\right]+\frac{p_2^2}{2a}+\frac{p_2'^2}{2b}\\
&>\frac{p_2-p_2'}{2}-\frac{bp_2-ap'_2}{ab}+\frac{p_2^2}{2a}+\frac{p_2'^2}{2b}\\
&=\frac{p_2-p_2'}{2}-\frac{4\left(p_2-p_2'\right)-2q_2\left(p_2-p_2'\right)+2p_2p_2'}{ab}+\frac{p_2^2}{2a}+\frac{p_2'^2}{2b}\\
&>\frac{p_2-p_2'}{18}-\frac{2p_2p_2'-p_2^2-p_2'^2}{ab}>0.\quad\square
\end{aligned}$$

Proof of Proposition 10. We only have to show that $\tilde{\Pi}(N+1)/2)<\Pi((N+1)/2,N)$. Calculating the difference in $\tilde{\Pi}((N+1)/2)-\Pi((N+1)/2,N)$ yields an expression that is proportional to

$$\begin{aligned}
&-2240-156p(N,N)+117p(N,N)^2+1952p((N+1)/2,N)\\
&\quad-488p(N,N)p((N+1)/2,N).
\end{aligned}$$

Since $p(N,N)\leq 1/4$ for all N, it follows that the sum of the three terms that involve $p(N,N)$ must be negative. Moreover, since $p((N+1)/2,N)\leq 1/2$ for all N, we also have that

$$-2240+1952p((N+1)/2,N)<0$$

and so we are done. □

Appendix B. Supplementary data

Supplementary data to this article can be found online at doi:10.1016/j.jpubeco.2012.01.002.

References

Aghion, P., Bolton, P., 2003. Incomplete social contracts. Journal of the European Economic Association 1 (1), 38–67.

Albrecht, J., Anderson, A., Vroman, S., 2008. Search by Committee. Georgetown University.

Austen-Smith, D., Banks, J., 1996. Information aggregation, rationality, and the condorcet jury theorem. American Political Science Review 90, 34–45.

Bai, J., Lagunoff, R., 2007. On the Faustian Dynamics of Policy and Political Power. Georgetown University.

Buchanan, J., Tullock, G., 1962. The Calculus of Consent. University of Michigan Press.

Callander, S., 2008. Can Good Policies be Found? Repeated Elections, Learning, and Policy Dynamics. Northwestern University, Kellogg.

Caplin, A., Nalebuff, B., 1988. On 64%-majority rule. Econometrica 56, 787–814.

Compte, O., Jehiel, P., 2008. Bargaining and Majority Rules: A Collective Search Perspective. University College London.

Dal Bo, E., 2006. Committees with supermajority voting yield commitment with flexibility. Journal of Public Economics 90, 573–599.

Dixit, A., Pindyck, R., 1994. Investment Under Uncertainty. Princeton University Press.

Feddersen, T., Pesendorfer, W., 1996. The swing voter's curse. The American Economic Review 86, 408–424.

Feddersen, T., Pesendorfer, W., 1998. Convicting the innocent: the inferiority of unanimous jury verdicts under strategic voting. American Political Science Review 92, 23–35.

Fernandez, R., Rodrik, D., 1991. Resistance to reform: status quo bias in the presence of individual-specific uncertainty. The American Economic Review 81, 1146–1155.

Gersbach, H., 1993a. Environmental preservation and majority decisions. Land Economics 69 (2), 147–155.

Gersbach, H., 1993b. Politics and the choice of durability: comment. The American Economic Review 83 (3), 670–673.

Glazer, A., 1989. Politics and the choice of durability. The American Economic Review 79 (5), 1207–1213.

Glazer, A., Konrad, K.A., 1993. The evaluation of risky projects by voters. Journal of Public Economics 52, 377–390.

Gradstein, M., 1999. Optimal taxation and fiscal constitution. Journal of Public Economics 72, 471–485.

Guttman, J., 1998. Unanimity and majority rule: the calculus of consent reconsidered. European Journal of Political Economy 14, 189–207.

Messner, M., Polborn, M.K., 2004. Voting on majority rules. The Review of Economic Studies 71 (1), 115–132.

Strulovici, B., 2010. Learning while voting: determinants of collective experimentation. Econometrica 78 (3), 933–971.

Tullock, G., 1998. Reply to Guttman. European Journal of Political Economy 14, 215–218.