Exam 1 Fall 2011 Driskill Econ 263

1 Part I: True/false. Mark each answer with a clear "T" of "F." Make sure you write distinctively enough that there is no ambiguity about what is a "T" or an "F".

One (1) point for each correct answer. 20 total points for this section.

1. Consider the following sequence over time of the pound sterling price of whiskey in the United Kingdom:

Date	1900	1930	1960	1990
Bottle of whiskey	.18	.71	1.95	8.80

   The most plausible explanations for this pattern would be

   (a) _____ that a change in tastes ocurred that increased people's consumption of whiskey.(F)

   (b) _____ that the increase in world income that has ocurred has increased demand for whiskey.(F)

   (c) _____ that there has been a reduction in the world supply of silicon, the raw material needed to make the glass for bottles.(F)

   (d) _____ that there has been an increase in the British money supply.(T)

2. U.S. cotton production

   (a) _____ is the recipient of subsidies paid for by U.S. taxpayers.(T)

   (b) _____ benefitted from Eli Whitney's invention because easily-cleaned cotton only grew near the seacoast .(T)

   (c) _____ owed its dominant world position in the pre-civil war South in part to the lack of property rights in other cotton-growing countries .(T)

   (d) _____ is dependent today on the arrival of a "hard freeze" which allows cotton to be picked.(F)

3. In most economic models encountered in undergraduate textbooks, the number of endogenous variables is two (2) because:

   _____Economist's have been unable to develop more complicated models.(F)

   _____Economists have attempted to make these models "graphically friendly" for expository and pedagogical purposes.(F)

_____Most "real world" situations can be adequately analyzed by models having at most two (2) endogenous variables.(F)

_____Economists have frequently used advanced techniques to prove that more complicated models addressing the same problem as does the two-variable model do not lead to different qualitative conclusions.(T)

4. In this class

   _____there is both a regularly scheduled and alternate last exam.(F)

   _____you are free to not take a particular exam and double the weight of the next exam on your grade.(F)

   _____you will be expected to use calculus.(T)

   _____Professor Driskill will give a make-up last exam if the regularly scheduled exam interferes with your flight home.(F)

5. The Ice Trust melted away because

   _____unusually cold weather created an exceptional ice harvest. (F)

   _____of the aggressive trust-busting strategy of President Teddy Roosevelt.(F)

   _____because of an increase in the number of new suppliers of ice in response to the high prices created by the formation of the trust.(T)

   _____ of all of the above.(F)

2 Part II: Budget constraint questions. 30 points total

Each month, Andy gets the following endowments:

$$\overline{C}_A = 5, \overline{T}_A = 3.$$

1. 5 points. If the price of coffee is 7 units of currency per unit of coffee, and the price of tea is 5 units of currency per unit of tea, what is Andy's income/month measured in currency?

   (a) A:

$$\overbrace{5}^{\overline{C}_A} \times \overbrace{7}^{P_C} + \overbrace{3}^{\overline{T}_A} \times \overbrace{5}^{P_T} = 50.$$

2. 5 points. What is his income/month measured in units of coffee? In units of tea?

   (a) A:

$$\frac{50}{7} = 49\frac{1}{7} = 7.1429;$$
$$\frac{50}{5} = 10.$$

3. 5 points. What is the relative price of coffee? What are the units of this price?

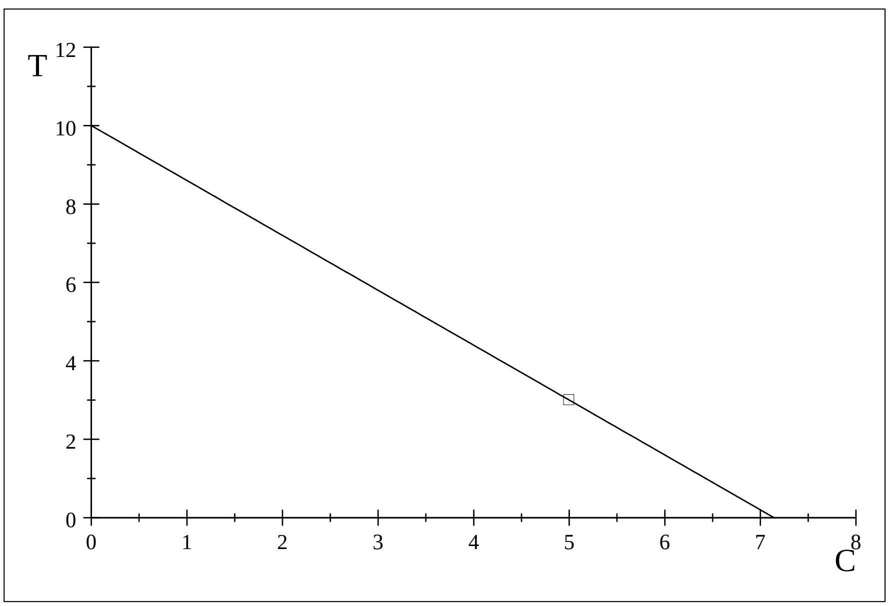
   (a) A:
   $$\frac{P_C}{P_T} = \frac{7}{5} = 1.4$$
   The units are units of tea/unit of coffee.

4. 5 points. Write his budget constraint in standard slope-intercept form with consumption of tea/month on the left-hand-side of the equality sign.

   (a) A:
   $$T_A = 10 - 1.4C_A.$$

5. 5 points. With tea on the vertical axis and coffee on the horizontal, draw a schematic diagram of his budget constraint, making sure you identify all relevant features, i.e., slope, intercepts, and endowment point.

   (a) A:$y = 10 - 1.4x$. Coffee-intercept is 7.1429., endowment point is $(5, 3)$



6. 5 points. What would happen to this schematic diagram if both P_C and P_T were to double? Triple? Be cut in half?

   A: nothing. Both slope and intercept remain unchanged.

3 Part III: Autarkic Equilibrium and Trade. 100 points total

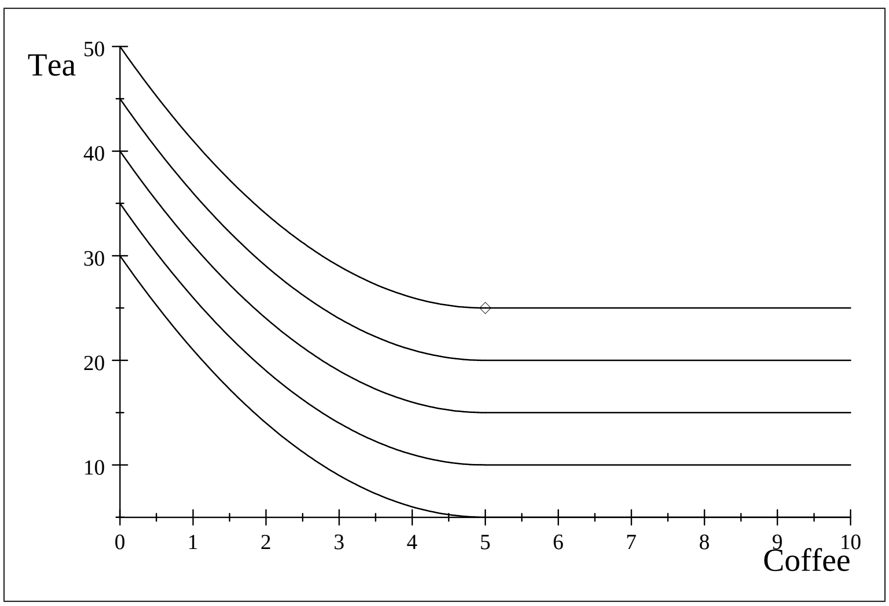
Consider the following POW economy. There is an English camp in which Andy and Bob reside. Each month they each receive a Red Cross endowment of 5 units of coffee and 25 units of tea. That is,

$$\begin{aligned} \overline{C}_A &= \overline{C}_B = 5; \\ \overline{T}_A &= \overline{T}_B = 25. \end{aligned}$$

Andy has preferences represented by the following utility function:

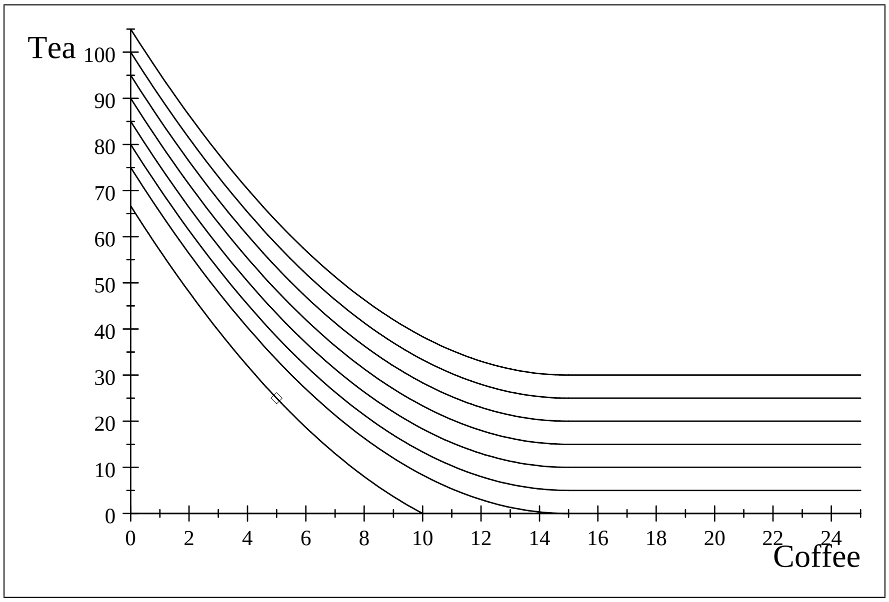
$$U_A = \begin{cases} T_A + 10C_A - (C_A)^2 & if \quad 0 \leq C_A \leq 5 \\ T_A + 25 & if \quad C_A > 5 \end{cases}$$

His indifference curve map looks like this:



Bob's preferences can be represented by the utility function:

$$U_B = \begin{cases} T_B + 10C_B - (\frac{1}{3})(C_B)^2 & if \quad 0 \leq C_B \leq 15 \\ T_B & if \quad C_B > 15 \end{cases}$$



1. 20 points. Calculate the equilibrium values of the endogenous variables p_a, C_A, C_B, T_A, and T_B. That is, your answers should be of the form: $p_a =$ ___; $\widehat{C}_A =$ ___; $\widehat{C}_B =$ ___; $\widehat{T}_A =$ ___; $\widehat{T}_B =$ ___.

   Answer: First derive the two demand curves. Start by finding the slopes of the indifference curves:

$$\begin{aligned} T_A &= U_A - 10C_A + (C_A)^2; \\ \frac{dT_A}{dC_A} &= -10 + 2C_A. \end{aligned}$$

   Or

$$\begin{aligned} dU_A &= dT_A + [10 - 2C_A]dC_A = 0; \\ dU_A &= 0 \rightarrow \frac{dT_A}{dC_A} = -10 + 2C_A. \end{aligned}$$

   So,

$$-\frac{dT_A}{dC_A} = 10 - 2C_A = p.$$

   Or,

$$C_A^d = -\frac{1}{2}p + 5.$$

   Likewise,

$$dU_B = dT_B + [10 - \frac{2}{3}C_B]dC_B = 0.$$

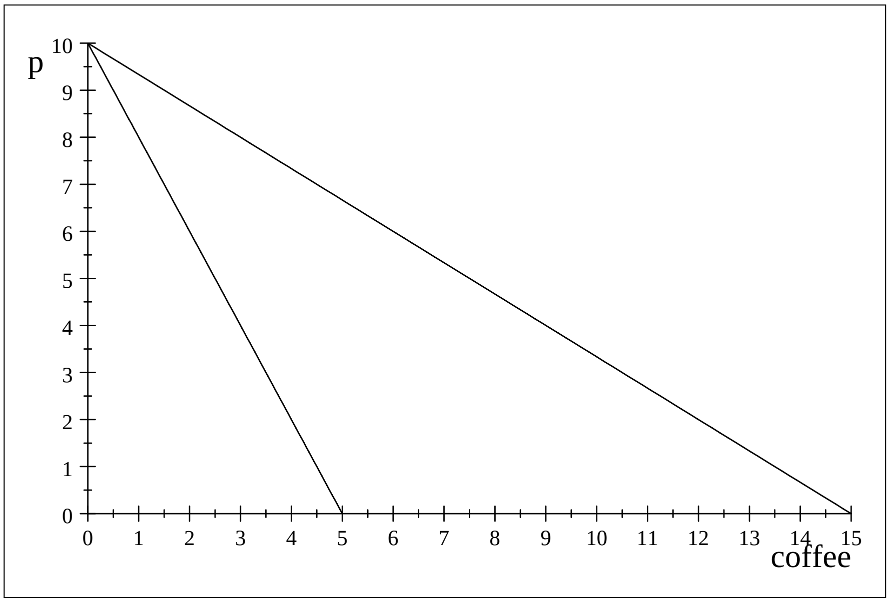
   So,

$$-\frac{dT_B}{dC_B} = 10 - \frac{2}{3}C_B = p.$$

Or,

$$C_B^d = -\frac{3}{2}p + 15.$$

The individual inverse demand curves look like this:



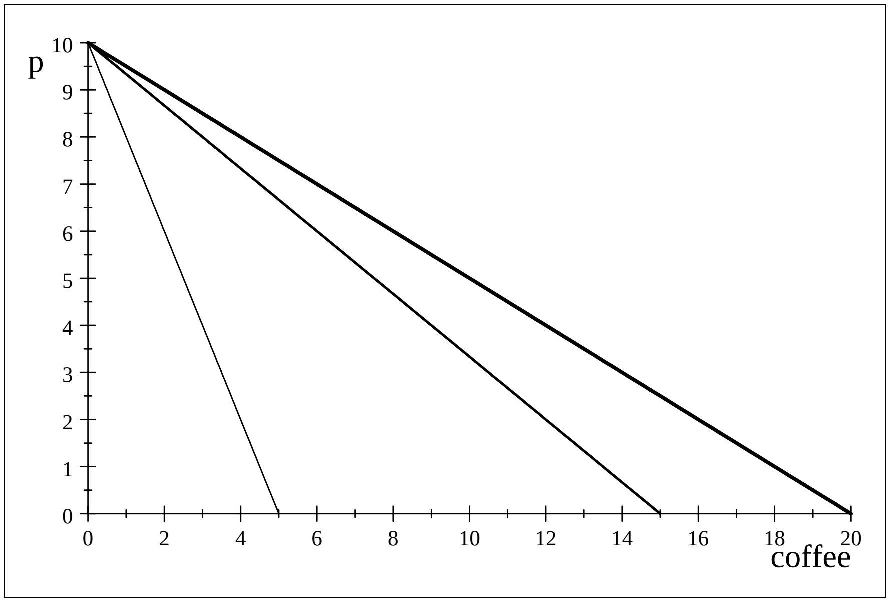
2. Market, or aggregate, demand is thus

$$C_A^d + C_B^d \equiv C^d = 20 - 2p.$$

In inverse form this is

$$p = 10 - \frac{1}{2}C^d$$

This inverse demand curve looks like this (with the individual demand curves depicted as well)



Excess supply is thus

$$ES = \overbrace{10}^{\overline{C}_A + \overline{C}_B} - (20 - 2p) = -10 + 2p$$

Supply is $5 + 5 = 10$, so equilibrium implies

$$\widehat{p}_a = 5$$

Subbing back into the demand curves yields

$$\widehat{C}_A = 2.5; \ \widehat{C}_B = 7.5$$

To calculate tea consumption, use the budget constraints:

$$\overbrace{12.5}^{p\widehat{C}_A} + T_A = \overbrace{25}^{p\overline{C}_A} + \overbrace{25}^{\overline{T}_A} = 50.$$

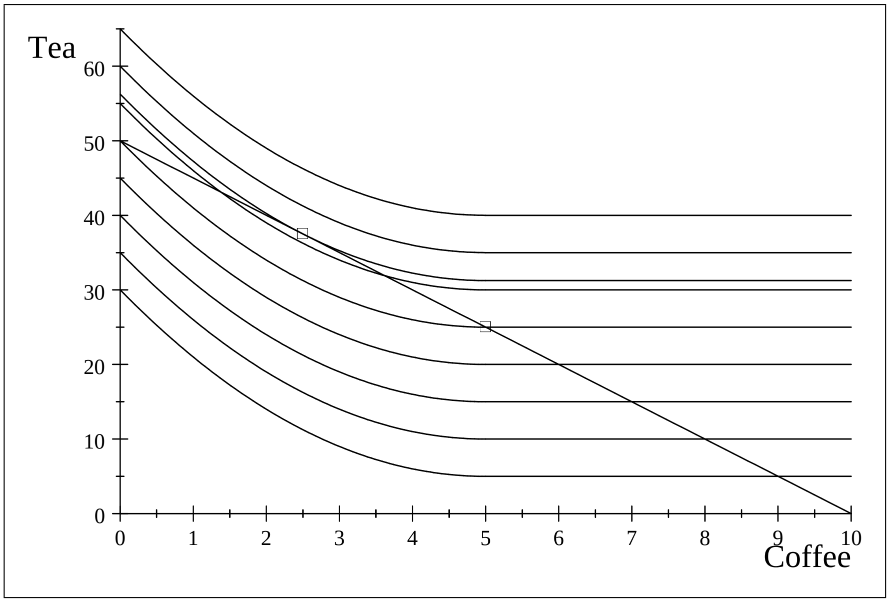
So,

$$\widehat{T}_A = 37.5$$

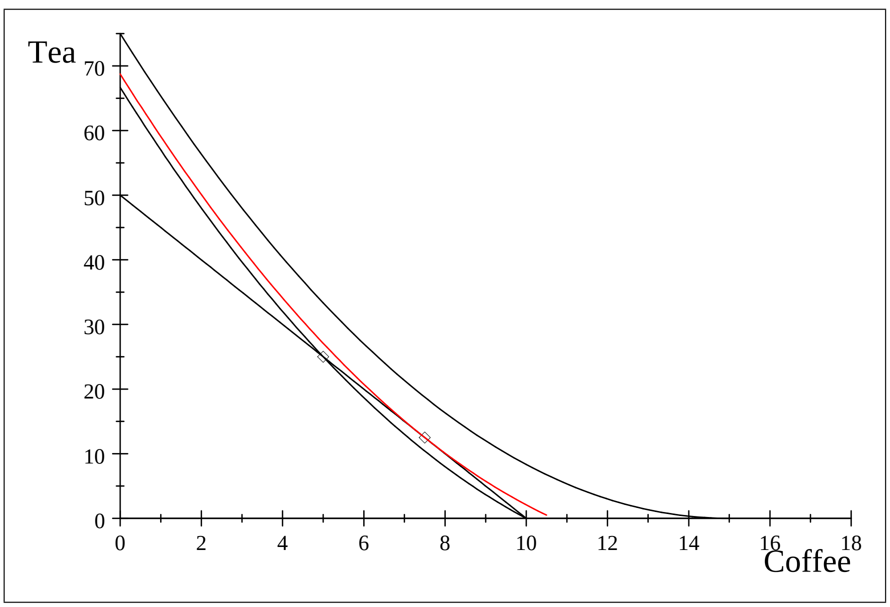
Because total tea consumption must equal total supply of 50 (in autarky),

$$\widehat{T}_B = 12.5$$

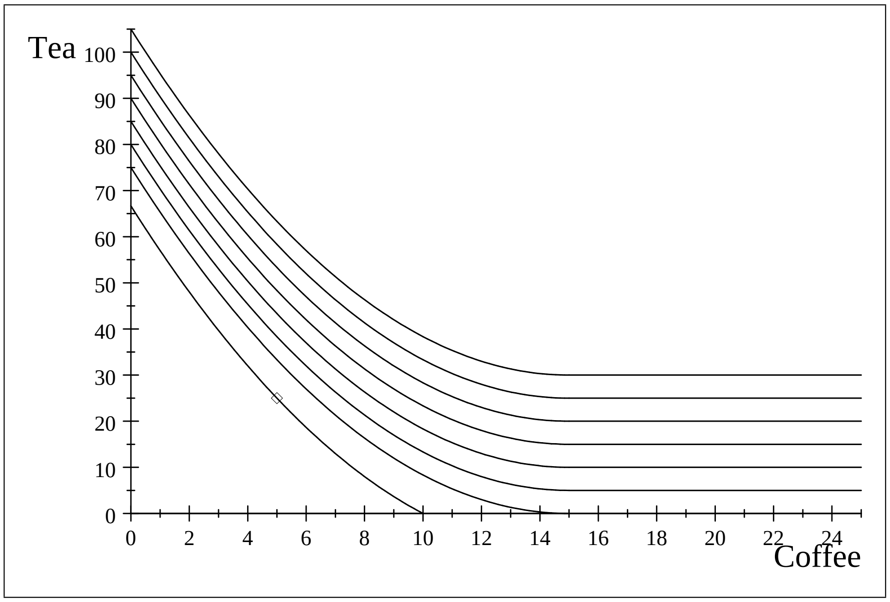
The depiction of Andy in autarkic equilibrium is



For Bob:



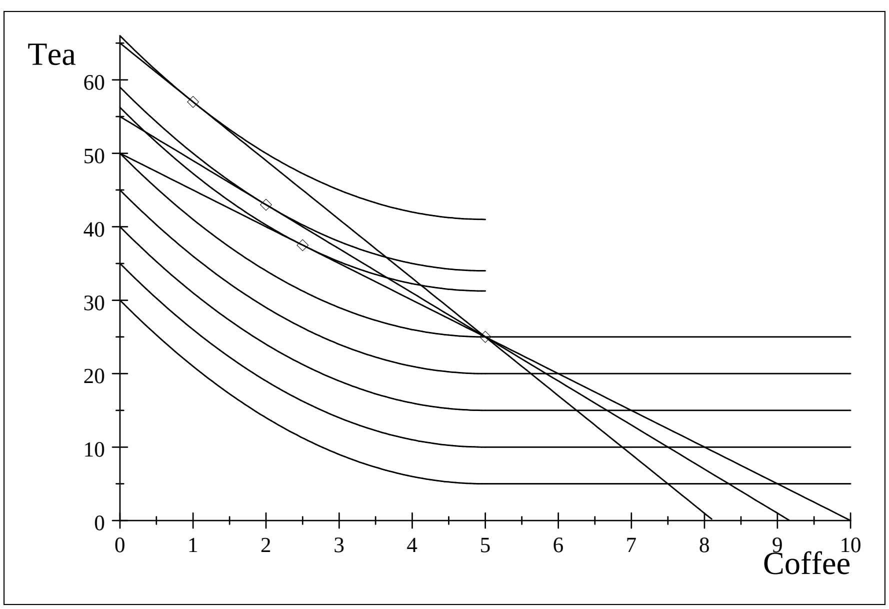
For contrast, here again is a more complete picture of Bob's preferences.



4 A more complete graphical depiction (just Alex)

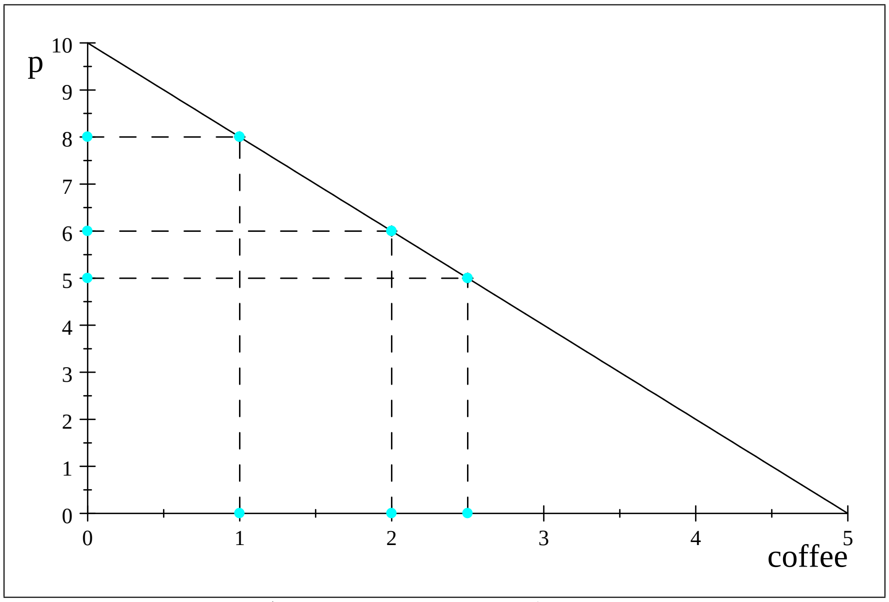
4.1 Preferences, endowments, BC's, MPP's

Endowment point: $(\overbrace{5}^{\overline{C}_A}, \overbrace{25}^{\overline{T}_A})$; MPP's: $p = 5$, $MPP : (\overbrace{2.5}^{C_A}, \overbrace{40}^{T_A})$; $p = 6$, $MPP : (2, 50)$; $p = 8$, $MPP :\ (1, 57)$



$$p = 5,\ 6, 8;\ C = 2.5, 2, 1$$

4.2 Andy's inverse demand curve



Alex's inverse demand function

5 Trade

1. 20 points. Now consider the French camp, in which reside 5 POW's who each receive each month a Red Cross endowment of 1 unit of coffee and 10 units of tea. Each of their preferences is represented by the following utility function:

$$U_f = \begin{cases} T_f + 12.5C_f - (\frac{5}{4})(C_f)^2 & if \quad 0 \leq C_f \leq 5 \\ T_f + 31.25 & if \quad C_f > 5 \end{cases}$$

Calculate the autarkic equilibrium values p_a^*, $\widehat{C}_f$, and $\widehat{T}_f$ of this camp, denoting the French relative price of coffee by p^*.

(a) Answer: by the usual methods, the individual French POW demand curve is found as:

$$p^* = -\frac{dT_f}{dC_f} = 12.5 - 2.5C_f.$$

Or,

$$C_f^d = -\frac{2}{5}p^* + 5$$

Market demand, denoted by C_F^d, is thus

$$C_F^d = 5C_f^d = 25 - 2p^*$$

Excess demand is thus

$$ED^* = 25 - \overbrace{5}^{5\bar{c}_f} - 2p^* = 20 - 2p^*$$

Hence, the autarkic price is just $\widehat{p}^* = 10$. Because all individuals are identical, they all end up consuming their endowment in autarky:

$$\widehat{C}_f = 1,\ \widehat{T}_f = 10.$$

2. Now imagine there exists an arbitrageur who behaves as if he were a perfect competitor, i.e., he is a price-taker. He incurs costs from transporting coffee from one camp to another. These costs, measured in units of tea, are a function of the amount of coffee he transports. Denote the amount of coffee transported by A. The cost function is given by

$$C(A) = A + \frac{1}{2}A^2$$

Excess supply and excess demand functions for the above English and French camps of questions one (1) and two (2) are, respectively:

$$\begin{aligned} ES &= -10 + 2p; \\ ED^* &= 20 - 2p^*. \end{aligned}$$

Calculate the equilibrium free-trade prices p and p^*, the equilibrium quantity of coffee exported from the British POW camp, and arbitrage industry profits.

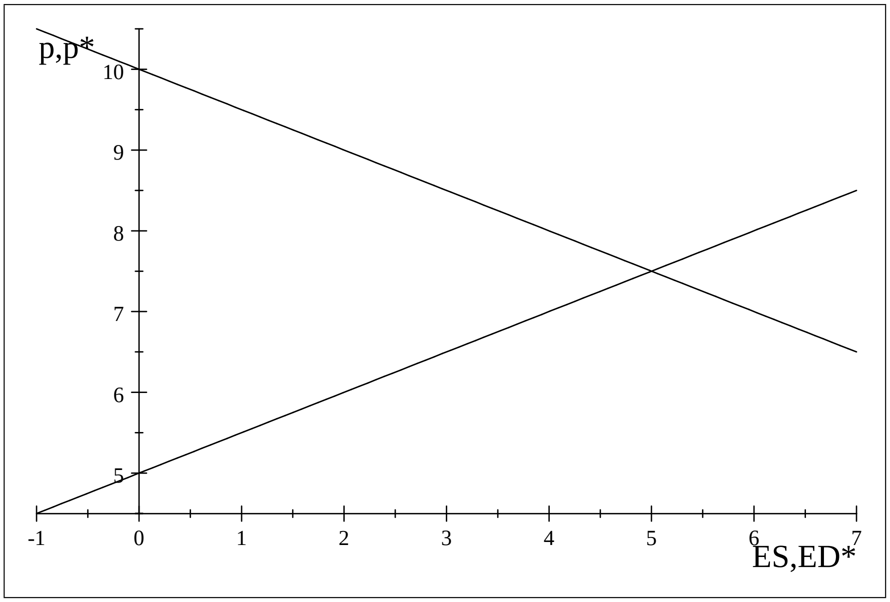
Answer: The three equilibrium conditions are

$$\begin{aligned} ES &= A;\ ED^* = A; \\ p^* - p &= MC(A) \end{aligned}$$

First solve for the inverse demand functions:

$$\begin{aligned} p^* &= -\frac{1}{2}ED^* + 10; \\ p &= \frac{1}{2}ES + 5. \end{aligned}$$

Here is the picture



Now, using $ES = A$ and $ED^* = A$, we have

$$\begin{aligned} p^* - p &= -\frac{1}{2}\overbrace{A}^{ED^*} + 10 \\ &\quad - \left(\frac{1}{2}\overbrace{A}^{ES} + 5\right) \\ &= -A + 5 \end{aligned}$$

Now,

$$MC(A) = 1 + A.$$

So, the third equilibrium condition tells us that

$$\overbrace{-A + 5}^{p^* - p} = \overbrace{1 + A}^{MC(A)}.$$

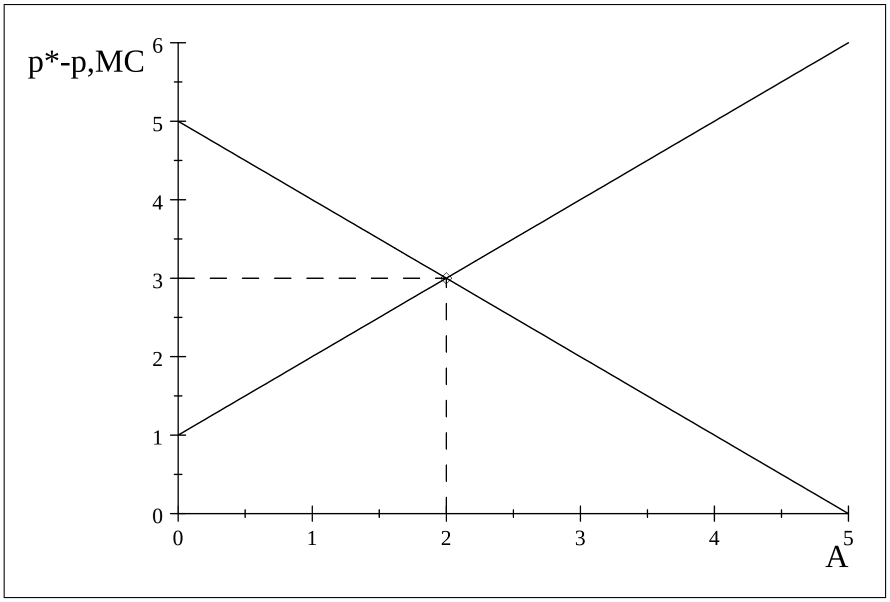
Solving for $A:$

$$2A = 4, \;\; \widehat{A} = 2.$$

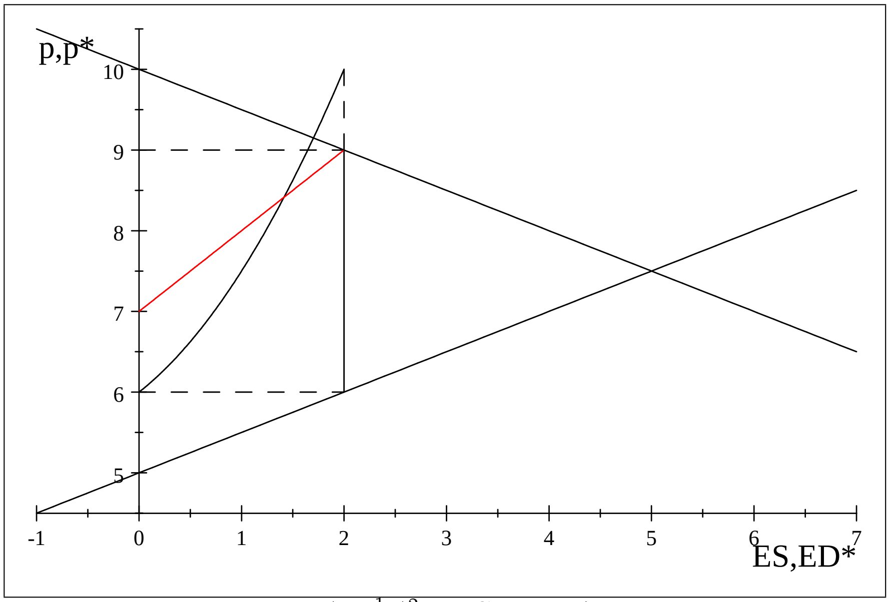
Substituting this value of A into the inverse ES and ED^* equations yields

$$\begin{aligned} p^* &= -\frac{1}{2} \times 2 + 10 = 9; \\ p &= \frac{1}{2} \times 2 + 5 = 6. \end{aligned}$$

Here is the picture



Now, knowing $p^* - p$ and A, we can depict the solution in our ES-ED* graph:



$\chi = A + \frac{1}{2}A^2, \; MC = 1 + A$

Alternatively, start with the third equilibrium condition, $p^* - p = MC(A) =$

$1+A$, which implies

$$p^* - p - 1 = A.$$

Then the first two equilibrium condition equations can be written as 2 two equations in the two unknowns p and p^* :

$$\begin{aligned}
\overbrace{-10+2p}^{ES} &= \overbrace{p^* - p - 1}^{A}; \\
\overbrace{20-2p^*}^{ED^*} &= \overbrace{p^* - p - 1}^{A}.
\end{aligned}$$

In canonical form:

$$\begin{aligned}
p^* - 3p &= -9; \\
3p^* - p &= 21
\end{aligned}$$

which solves as

$$\begin{aligned}
\widehat{p} &= 6; \\
\widehat{p^*} &= 9.
\end{aligned}$$

Now, with these values,

$$\widetilde{\widetilde{A}} = 2$$

Hence, coffee exports, which must equal coffee imports, are 2. For profits:

$$\begin{aligned}
\pi_A &= (p^* - p) \times A - A - \frac{1}{2}A^2 \\
&= 3 \times 2 - 2 - 2 = 2.
\end{aligned}$$

3. 20 points. How much coffee and tea does each British prisoner consume in this new equilibrium?

   (a) Answer: At $\widetilde{p} = 6$,

$$\begin{aligned}
\widetilde{C}_A &= 5 - \frac{1}{2}\overbrace{6}^{\widetilde{p}} = 2; \\
\widetilde{C}_B &= 15 - 9 = 6.
\end{aligned}$$

   To calculate tea, we use the budget constraints and get

$$\begin{aligned}
\widetilde{T}_A &= 43 \\
\widetilde{T}_B &= 19
\end{aligned}$$

4. 20 points. Who is better off in this new equilibrium? Who is worse off? How do you know?

answer: Only the British "coffee-lover" (Bob) is worse off; every one else is better off. For the French, we do not need to compute relative utility levels, because in autarky they each consume their autarkic bundle. Free trade "rotates" each of their budget constraint lines through this autarkic consumption point, thus "cutting the autarkic indifference curve." A diagram would help the exposition here.$U_A = T_A + 10C_A - (C_A)^2$

(a) $37.5 + 10(2.5) - (2.5)^2 = 56.25 = U_A(autarky)$
$43 + 10(2) - (2)^2 = 59.0 = U_A(FT)$
$U_B = T_B + 10C_B - \frac{1}{3}(C_B)^2$
$12.5 + 10(7.5) - \frac{1}{3}(7.5)^2 = 68.75 = U_B(autarky)$
$19 + 10(6) - \frac{1}{3}(6)^2 = 67.0 = U_B(FT)$

5. Now consider the parametric case for transport technology:

$$\chi(A) = c_0 A + \frac{c}{2}A^2$$

with associated marginal cost function

$$MC(A) = c_0 + cA.$$

The excess demand and excess supply functions remain in inverse form as

$$\begin{aligned} p^* &= -\frac{1}{2}ED^* + 10; \\ p &= \frac{1}{2}ES + 5. \end{aligned}$$

Find the parametric solution for this model, that is, find solution (or equivalently reduced-form) equations of the form

$$\widehat{p} = f(c_0, c);\ \widehat{p}^* = g(c_0, c);\ \widehat{A} = h(c_0, c);\ \widehat{\pi}_A = l(c_0, c).$$

Of course, a full solution would include quantities consumed by everyone and associated levels of well-being-but that is too much computation.

Answer: As before,

$$p^* - p = -A + 5.$$

But now

$$p^* - p = c_0 + cA$$

So

$$\begin{aligned} -A + 5 &= c_0 + cA; \\ A(1 + c) &= 5 - c_0; \\ A &= \frac{5 - c_0}{1 + c}. \end{aligned}$$

Hence

$$\begin{aligned}
p^* &= -\frac{1}{2}\left(\frac{5-c_0}{1+c}\right)+10; \\
p &= \frac{1}{2}\left(\frac{5-c_0}{1+c}\right)+5; \\
p^*-p &= -\left(\frac{5-c_0}{1+c}\right)+5 \\
&= \frac{-5+c_0+5+5c}{1+c} \\
&= \frac{5c+c_0}{1+c}; \frac{d(p^*-p)}{dc} = \frac{5+5c-5c-c_0}{(1+c)^2} = \frac{5-c_0}{(1+c)^2}. \\
\pi_A &= \left(\frac{5c+c_0}{1+c}\right)\left(\frac{5-c_0}{1+c}\right) - c_0\left(\frac{5-c_0}{1+c}\right) - \frac{c}{2}\left(\frac{5-c_0}{1+c}\right)^2.
\end{aligned}$$

One thing the parametric approach lets us see clearly is that we must have $c_0 \leq 5$ for trade to take place. Also, as either c_0 or c gets smaller (lower transport costs), p^* gets smaller and p gets bigger. As for profits, consider what happens as $c \to 0$:

$$\pi_A = \left(\frac{c_0}{1}\right)\left(\frac{5-c_0}{1}\right) - c_0\left(\frac{5-c_0}{1}\right) = 0.$$

5.1 Potential Pareto Improvement: Alex

Autarky: $p_a = 6,\ MPP_a:\ (2.5, 37.5);$

Free trade: $p_{FT} = 8;\ Endowment\ is\ MPP_a;\ MPP_{comp}\ is\ (1, 50)$