Linking distinct markets via arbitrage

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1 Learning objectives

1. Understand the concepts of **excess demand** and **excess supply** functions.

2. Understand why **arbitrageurs** transport goods from one location to another, and why the marginal cost of transport just equals the price difference between two locations for the profit-maximizing amount of transport.

3. Understand why how the emergence of trade between distinct locations lowers the autarkic equilibrium relative price in one location and raises it the other.

4. Understand why the location with the lower autarkic equilibrium relative price for some commodity exports that good, and why the location with the higher autarkic equilibrium relative price imports the good.

5. Recall why the value of an autarkic equilibrium relative price is determined by the interplay of tastes and resources.

6. Understand the determinants of transport costs.

7. Understand why transport costs can be so high that some goods are not traded between locations.

8. Understand why distance affects the volume of trade between different locations, and how this relationship is expressed in the **gravity equation**.

9. Understand and graphically depict how trade can create winners and losers within a country.

10. Understand and graphically depict how a hypothetical costless feasible redistribution of endowments among the members of a location would make them all better off under free trade than under autarky.

11. Understand and use the Edgeworth Box to show that many such **potential Pareto improvements** exist for the endowment economy, even with non-identical endowments for individuals.

12. Understand why the existence of such **potential Pareto improvements** that arise from a change from autarky to free trade inform economists' thinking on trade policy.

2 Introduction

As noted, one of the key economic lessons provided by Radford's POW camp is that it presents a real-world example of a change from autarky to free trade. We now model the economic activity that links otherwise separate markets, and analyze the implications of such *market integration*.

2.1 "The Bottle Return"

In the long-running and popular television sitcom *Seinfeld*, one episode ("The bottle return") illustrated how and under what conditions trade links distinct markets. In the episode, Neuman, a postal worker, notices that 5 cents is paid on each bottle returned to a vender in New York, but 10 cents if returned in Michigan. He remarks upon this to his friend Kramer, and the dialogue from that point goes something like this:

Neuman: "Wait a minute, you get 5 cents here, 10 cents there, we can round up bottles here, run them out to Michigan ... "

Kramer: "It doesn't work. You overload your inventories and blow your margins on gasoline. I've tried it every which way, I couldn't crunch the numbers... "

Neuman: "We could load up an 18-wheeler..."

Kramer: "Too much overhead; there's permits, way-stations, ..."

Neuman: "So we could put the bottles in a u-haul, go small..."

Kramer: "You're way out of your league..."

In the vocabulary of economists, Neuman and Kramer were thinking about whether they could make money as *arbitrageurs,* purchasing bottles in New York

argitrageur: someone who simultaneously buys and sells the same good in order to profit from price discrepancies.

at a relatively cheap price and transporting them to Michigan to sell them at a higher price. Kramer pointed out to Neuman that this economic activity of transport involves using resources the cost of which would outweigh the profits to be made from the price discrepancy.

The solution to the cost of arbitrage hit upon by Neuman later in the episode was to use extra space in a mail truck going from New York to Michigan on Mother's day. The scheme, of course, doesn't go well, and the trip involves various bumps and interruptions.

The episode does in fact point to the answer to the question: How and under what conditions does trade link distinct markets? When there are geographically distinct markets for the same good, profit opportunities may exist for entrepreneurs who "buy cheap (5 cents per bottle)" in one location, and "sell dear (10 cents per bottle) in another location. If the resources used in the transport are sufficiently cheap, money can be made.

What might we expect to be the result of such arbitrage? If the prices of the good in each location are sensitive to supply and demand, the price in the originally cheap location should rise in response to the demand by the arbitrageurs, and the price in the originally expensive location should fall in response to the supply from the arbitrageurs. Thus, the prices in the two geographically distinct areas are brought closer together, with the higher price falling and the lower price rising.

When the costs of arbitrage are sufficiently low so that the prices in the two geographically distinct markets are essentially the same, we say that the two markets are *integrated* (or in equivalent language, *unified*). We model such a situation by assuming that the two markets are actually one, and total or market demands and supplies are the sums of individual demands and supplies from both areas. Thus, the model assumes there is one price that clears the market, a price that is the same in each location.

As *The Bottle Return* emphasizes, this limiting case of perfect arbitrage in which prices are equalized in different locations is a poor model for many commodities and especially for many services. Haircuts, for example, are a service for which there is unlikely to be any arbitrage: arbitrageurs would have to transport either the barber or the customer, and the price differential for haircuts across locations is unlikely to cover the cost of a plane ticket. Despite the obvious existence of such non-traded goods, the abstraction of a unified market turns out to a useful simplification that helps illuminate many trade issues.

Brain teaser: Of course if you can afford to transport your barber with you where ever you go, at least for you the price of a haircut is $ 400 everywhere. Is this an example of one market or a segmented market?

In this section, we build a *model* of arbitrage that formalizes the decision processes alluded to in "the bottle return." The model allows us to see clearly under how transportation technology determines whether or not markets are linked by arbitrage, and under what conditions we might expect such linked distinct geographic markets to be approximated as an integrated market.

To do this, we expand the model of one local market within which one good is traded for another by positing geographically distinct markets for the same good and existence of profit-maximizing entrepreneurs who, under certain conditions, link these distinct markets by buying cheap in one market and selling dear in the other. When two previously autarkic economies are linked by such arbitrageurs, we say there exists a *trading equilibrium* for the two economies.

What are the effects of linking two previously autarkic economies via trade? We show that the behavior of arbitrageurs closes the gap between the two autarkic equilibrium prices of the two economies. That is, the two trading equilibrium prices will be closer to each other than they were in autarky. This means that the trading equilibrium price is higher than the autarkic equilibrium price in the economy in which the autarkic price was lowest, and lower in the economy in which the autarkic equilibrium price was highest.

To further help fix ideas, think about the actual situation (as opposed to the hypothetical situation described by "The bottle return") described by R.A. Radford in his article "The Economic Organization of a P.O.W. Camp". In that camp, there were various distinct compounds for the different nationalities of POW's: one for the British, one for the Americans, one for the French, and so on. The British were initially confined to their own compound. In this isolated environment, markets sprang up for exchange of the various goods supplied to the soldiers from the Red Cross. Radford's description of these markets within the British POW compound suggests that they behaved very much like our simple model of a perfectly competitive market, with price determined by the intersection of demand and supply curves. The model of such an autarkic economy was the subject of the previous chapter.

As recounted by Radford, after some time the English, for the price of a few cigarettes paid to the guards, were allowed to send a few people to visit the other compounds. These emissaries found that coffee was "relatively cheap among the tea-drinking English" and yet commanded a "fancy price" in terms of other goods in the French compound. In response to this discrepancy in price between the English and French compounds, the POW soldiers who were allowed to move between compounds engaged in arbitrage: they bought coffee in the English camp, transported it to the French camp, exchanged it there for other goods at the "fancy price," and then brought these other goods back to the British camp, earning a profit in terms of these other goods. This link between two distinct locations is what we model here.

As usual, we develop the model with the use of sub-models, and we depict and illustrate the parts and workings of the model with both equations and graphs. To make the exposition as clear as possible, we develop the key parts of the model by using a very general specification, and then illustrate this general case with examples using both specific functional forms with arbitrary parameter values and specific functional forms with specific numerical values for parameters. This makes for a lot of equations but also makes for a more complete understanding of the model.

The sub-models are demand and supply functions of non-mobile residents of the two distinct economies–the British and French POW camps – and a sub-model of the behavior of arbitrageurs. The sub-model of the non-mobile residents is just the preceding model of an autarkic economy. We recast this model slightly by introducing the concepts of **excess demand** and **excess supply** . This allows us to diagrammatically solve the model of simultaneous equilibrium in two trading economies. Arbitrageurs, as noted, are those individuals who purchase a good in one market and transport it to another market in order to make a profit from selling it there at a higher price.

In the demand and supply sub-models, camp residents are assumed to take the relative price of coffee as exogenous, and to choose their optimal consumption of coffee and tea based on this price. Arbitrageurs also take as exogenous the relative price of coffee in both the British camp and the French camp, and choose how much coffee and tea to transport between the two camps. For the model as a whole, the endogenous variables are the relative prices of coffee in the two markets, the quantities of coffee and tea consumed by each resident in each camp, and the amounts of coffee and tea transported from one camp to another, i.e., the exports of coffee from one economy, which are the imports of coffee by the other economy, and the imports of tea into the coffee-exporting country, which are the exports of tea from the coffee-importing country.

Anticipating our next topic, we note that for purposes of understanding what economists generally mean by "gains from trade," the most important implication of the above analysis is that the equilibrium relative price of coffee in each camp differs from their autarkic values once the economies are linked by arbitrage.

3 The model of trade between the French and English

3.1 Demand and supply functions of the non-mobile residents

The first element or sub-model of this model is a specification, for each market, of demand and supply functions of the non-mobile residents of each market. The second element is a specification of the demand and supply of goods by the arbitrageurs. The final element is a specification of market equilibrium in each locality.

Let us make the simplifying assumption that there are two types of people: arbitrageurs and non-arbitrageurs. This simplifies the analysis by allowing us to ignore how the profits of the arbitrageurs might feed back into the demands for goods for consumption purposes. It is an **innocuous** assumption: relaxing this assumption would not affect any of our key conclusions. To make things concrete, assume the non-arbitrageurs are the residents of the POW camps, i.e., British and French POW's, all of whom are confined to their respective compounds. Arbitrageurs are assumed to come from outside the compounds, and can be thought of, for example, as the prison guards.

Consider the market for, say, coffee vis-a-vis tea. In the home market – which we will arbitrarily assign as a label to the English POW camp – demand and supply by non-arbitrageurs are assumed to have the usual properties, as in equations (14.ii) and (15) of the previous chapter:

$$C_E^d = F(p);\ C_E^s = \overline{C}_E.$$

where p is the relative price of coffee, i.e., the number of units of tea it takes to purchase one unit of coffee, and the subscript E indicates these are the *market* demand and supply functions for the English camp. The "overbar" on $\overline{C}_E$ just emphasizes that aggregate English supply is exogenous (the sum of the individual English POW's endowments).

For an autarkic single-market analysis, the above demand-supply model is well-suited to graphical representation. Both functions can be represented on the same diagram, with price on the vertical axis and quantity on the horizontal. The intersection of the two curves depicts the equilibrium price and the equilibrium quantity. Answers to the canonical question we ask of our models–what happens to the values of endogenous variables when the value of an exogenous variable changes–are readily depicted as the new intersection of the demand and supply curves when an exogenous variable such as endowment of coffee changes and shifts one of the curves.

3.2 Excess demand and excess supply functions of non-mobile participants

When we introduce, as we will, an interaction between two economies, the graphical depiction of the model becomes more difficult to display. However, by collapsing some if the information contained within the demand and supply functions, we can continue to use graphical methods to help understand and depict the model.

To this end, let us introduce two new concepts: an excess supply function and an excess demand function. We will use an excess supply function to collapse information about the English market, and an excess demand function to collapse information about the French camp.

Excess supply is simply the difference in quantity supplied and quantity demanded for any given price. An excess supply function, then, is the function that relates these quantities to various permissible prices. It is formed by subtracting the demand function from the supply function. For the excess supply function in the English camp we have:

$$ES_E^C = \overline{C}_E^s - C_E^d \equiv h(\overset{+}{p}). \tag{1}$$

where we define the function as $h(p)$. The "+" sign above the argument p of the function indicates that this composite function is an upward–sloping function of the relative price of coffee. The superscript C indicates that this is the excess supply function for *coffee*.

Such an inverse excess supply function is depicted as a thick line in Figure 1, along with the standard inverse supply and demand functions depicted by thin lines. The dotted horizontal line connects the intersection of the regular

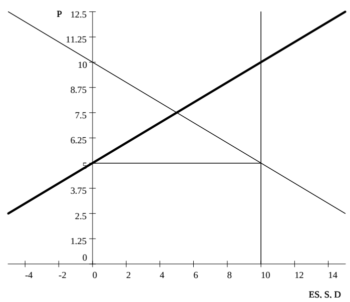


Figure 1: Inverse D and S and inverse ES

demand and supply functions with the vertical intercept of the excess supply function, emphasizing that this vertical intercept occurs at the value of p at which demand equals supply. Note that our vertical supply curve (i.e. our assumption that supply is an exogenous endowment) means that the slope of the inverse excess supply curve is equal to the slope of the inverse demand curve. In more general cases, the inverse excess supply curve slope will depend on both demand and supply reactions to different hypothetical market prices. Note that an excess supply function can take on negative values.

Now consider the French POW camp and the related concept of an excess demand function. An excess demand function is just the difference between demand and supply. Denote the relative price of coffee in the French camp (the foreign country) by a "*" superscript. Demand and supply in the French camp are given by

$$C_F^d = F^*(p^*);\ C_F^s = \overline{C}_F.$$

Now form (in a similar fashion to the excess supply function) the excess demand function by subtracting the supply function from the demand function:

$$ED_F^C = F^*(p^*) - \overline{C}_F^s \equiv h^*(\bar{p}^*). \tag{2}$$

Again, the relationship between the normal demand and supply curves and the excess demand curve is shown in Figure 2. Note, as with the excess supply function graph, that a dashed horizontal line connects the intersection of demand and supply to the vertical intercept of the excess demand function. Also as with the excess supply function, the excess demand function can take on negative values.

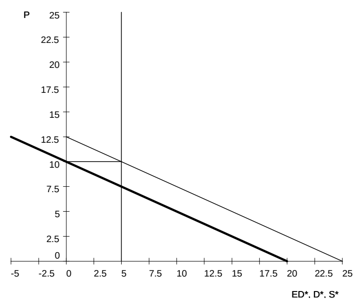


Figure 2: Inverse excess demand

3.3 Market Equilibrium in Isolation

In geographic isolation, the equilibrium price in the English camp, is determined by the equilibrium condition that the quantity demanded by the English POWs equals the quantity of their endowment. Formally, in terms of excess supply function in the English camp, we have:

$$ES_E^C(p) = 0.$$

This equation may be solved for a market-clearing relative price, which we will denote p_a, where the subscript "a" alerts us this is the English *autarkic* equilibrium price.

For the French, the equilibrium price is also determined by the intersection of French demand and the French endowment. The market clearing price is the one that makes excess demand equal to zero in the French camp:

$$ED_F^C(p^*) = 0\ .$$

This equation may be solved for a market-clearing relative price, which we will denote p_a^*, where the subscript "a" alerts us this is the French *autarkic* equilibrium price.

As we will see, a necessary condition for arbitrage to occur is that the autarkic equilibrium prices differ across the two camps. A sufficient condition for arbitrage to occur is that the difference in price is large enough to compensate for the cost of arbitrage. In the context of our POW model, differences in autarkic relative prices reflect the interplay of demand and the supply in each location separately. That is, different autarkic relative prices arise when, the values of p

at which $ES_E^C(p) = 0$ is different from the value of p^* at which $ED_F^C(p^*) = 0$. For the case in which all prisoners get the same Red Cross packet, regardless of whether they are English or French, supply would be the same in each autarkic camp. In this case, then, different autarkic equilibrium prices across the two camps arise only because of differences in demand. For a case in which supply in each camp is different, different autarkic equilibrium prices could arise even with identical market demand curves in each autarkic economy. The general point here is that different autarkic equilibrium prices is another way of saying that the intersection of the supply and demand curves in each market occurs at a different price. It could also be the case that the supply and demand curves intersect in the same place, in which case there are no gains from trade and no profits from arbitrage (even if arbitrage costs are zero). This case is sort of pathological and yet provides a benchmark.

Inquiring minds want to know: A parametric example

It may help if we illustrate this new concept with specific linear functional forms. Assume the English market demand function $F(p)$ is approximated by the linear function:

$$C_E^d = a_0 - ap;\ a_0 > 0,\ \alpha > 0.$$

The composite excess supply function $h(p)$ would thus be

$$ES_E^C \equiv h(p) = \overline{C}_E - a_0 + \alpha p.$$

For notational ease, we now define new parameters as a composite of others and rewrite the excess supply equation as follows:

$$ES_E^C = \alpha_0 + \alpha p;\ \alpha_0 \equiv \overline{C}_E - a_0.$$

To simplify notation further, we use the illustrative parameter values

$$\alpha_0 = -10; \alpha = 2,$$

so that

$$ES_E^C = -10 + 2p$$

As usual, we can express the excess supply function in inverse form:

$$p = \frac{ES_E^C}{\alpha} - \frac{\alpha_0}{\alpha}.$$

or, with our specific numerical values,

$$p = \frac{ES_E^C}{2} + 5.$$

For the French, consider the following linear specifications:

$$C_F^d = a_0^* - \alpha^* p^*;\ \overline{C}_F^s = b_0^*;\ a_0^* - b_0^* > 0;\ \alpha^* > 0; a_0^* > 0; b_0^* > 0$$

As with the English, let us be even more concrete by introducing hypothetical numerical values for these parameters:

$$a_0^* = 25; b_0^* = 5; a^* = 2.$$

The demand function may now be expressed as:

$$C_F^d = 25 - 2p^*;\ \overline{C}_F^s = 5.$$

Illustrating with our parametric specific linear functional form, we have

$$ED_F^C = a_0^* - b_0^* - a^* p^*$$

Again for notational ease, we define new parameters as a composite of others and rewrite the French excess demand equation as follows:

$$ED_F^C = \beta_0 - \beta p^*;\ \beta_0 \equiv a_0^* - b_0^*,\ \beta \equiv a^*.$$

In terms of our illustrative numerical parameters, we have:

$$\beta_0 = 20; \beta = 2$$

3.4 Demand and Supply by Arbitrageurs

We now introduce to arbitrageurs the model, individuals who seek to profit from buying coffee in the low price location and transport and sell it in the high price location. We continue with our example in which the autarkic relative price of coffee in the British POW compound is low relative the autarkic relative price of coffee in the French POW compound. Consider a guard who is able to move between these camps buying coffee in the British compound and transporting it to the French compound, where he sells it. The per unit revenue from such an endeavor (measured in units of tea) would be given by the difference between the French relative price of coffee and the British: $p_a^* - p_a$ (recall this was differential was 5 in our example).

We want to consider how the cost of arbitrage affects the level of market integration. Therefore, we suppose that the guard may choose to either work an additional hour on guard duty or spend the time off-duty conducting arbitrage (in this setting guards we be found to systematically stay at the office longer than their logged hours). We assume that on each trip between the camps the guard can carry only one kilo of coffee. Thus a guard makes net revenue of $p^* - p$ units of tea. A guard thus has two competing influences on his well-being that he must balance when deciding upon how much arbitrage to undertake. On the one hand, the more trips between the camps he makes, the more profit he makes. On the other hand, the more trips he makes, the less leisure time he has and the more energy he expends. Note that the number of trips undertaken per day is equal to the guard's *demand* for coffee per day in the English market (because we assume the guard can only take one unit of coffee per trip). Because all units of coffee bought in the English camp are transported and sold in the French camp, the quantity demanded by arbitrageurs in the English camp is just equal to the quantity supplied by arbitrageurs in the French camp.

The solution to this balancing act is known as the solution to the *labor/leisure trade-off model*. The guard likes both leisure and the tea he earns from the arbitrage process, which requires his labor. We could look at this sub-model and its solution in more detail. If we did, such a sub-model would, as usual, involve a specification of tastes and a specification of a budget constraint. The specification of tastes would be represented by a family of indifference curves in the tea-leisure time plane. The budget constraint would describe all those pairs of leisure time and tea from which a guard could choose, given the amount of time he is endowed with and given the rate at which he can exchange leisure for tea, i.e., given the relative price of leisure. In contrast to the model in which the two goods were coffee and tea, for which the relative price was simply the market-determined relative price of coffee, the relative price of leisure in this model must be constructed from the relative prices of coffee in each compound, and the *technology of transport*.

In such a sub-model, the variables and parameters assumed exogenous to the guards would be the technology of transport, namely how long it takes to make a trip between markets and the energy expended is so doing, the amount of coffee and tea that can be transported per trip, and the market prices in each market, p and p^*. The endogenous variables would be the choices of how much tea and leisure the guard wants to consume. Knowing the most-preferred choice of leisure time, we could infer the resulting amount of time spent in transport, and thus the amount of coffee transported from one compound to another. This would be the variable of most interest to us because it would allow us to specify demand and supply in the larger complete model of the linked English and French POW economies.

Rather than work through the details of such a model, we simply note that the standard assumption in economics is that the *marginal cost* (measured in units of tea) of transporting coffee increases as the amount of coffee transported by guards increases. The concept of marginal cost is the *extra* cost associated with transporting one more "small-as-possible," i.e., infinitesimal, unit of coffee. If we denote the amount of coffee transported per day as A and marginal cost as MC, the assumption of increasing marginal cost implies a marginal cost *function* in which larger values of A are associated with larger values of marginal cost, A. If we denote this function by the lower-case letters mc, we would express this functional relationship as follows:

$$MC = mc(\overset{+}{A}). \tag{3}$$

As usual, the "plus" sign over the argument A tells us that higher values of A are associated with higher values of MC.

The profit-maximizing rule To understand how guards make the "best" choice of how much coffee to transport each day, we ask the question: what is the extra additional, or *marginal,* net revenue associated with transporting one more unit of coffee? As noted, this is just $p^* - p$. If guards are making the best choice of how much coffee to transport each day, they are choosing that amount such that the marginal net revenue, $p^* - p$, is just equal to the marginal cost. That is, the optimal choice of A must satisfy the equation that makes marginal net revenue equal to marginal cost:

$$p^* - p = mc(\overset{+}{A}). \tag{4a}$$

If this were not true, the guards could always do better. For example, if they were transporting an amount such that the marginal cost were greater than the marginal revenue, they would be losing money. To improve upon things, they would transport less, still forgoing the net revenue $p^* - p$, but reducing cost by more (because the marginal cost is declining in the amount transported), thus reducing the loss. If they were transporting an amount such that marginal cost was less than marginal revenue, they could increase net revenue by transporting one more unit. Thus the profit-maximizing amount of coffee transported per day must be the amount that equates marginal cost to marginal net revenue.

We can depict the profit-maximizing choice of A in a diagram on which the vertical axis measures both marginal cost and $p^* - p$ and the horizontal axis measures A. On such a graph, an upward-sloping curve would depict

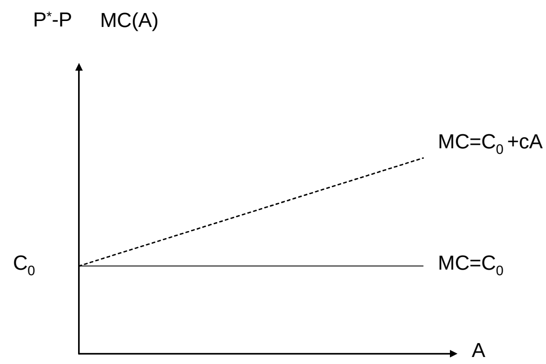


Figure 3: Marginal cost as a function of A

the positive relationship between marginal cost and A, the amount of coffee transported per day. For any conceivable value of $p^* - p$ we can then "read off" the corresponding optimal choice of A by extending a horizontal line from the particular value of $p^* - p$ to where it intersects the upward-sloping marginal cost curve. This is illustrated in Figure 3 with two examples. One example illustrates the case of constant marginal cost, with marginal cost always equal to the value C_0, regardless of the quantity of A. This marginal cost function is a horizontal line of height C_0.

The other example is drawn for increasing marginal cost: as A takes on higher values, marginal cost increases. This example is represented by the upward-sloping line $MC = C_0 + cA$, where c is a positive parameter.

3.5 Market Equilibrium

In the home country, market equilibrium is found by equating the excess supply of coffee by non-arbitrageurs to the demand for coffee by arbitrageurs:

$$\overbrace{h(p)}^{ES_E^C} = A. \tag{5.i}$$

As usual, to make our equations consistent with our graphs, we express this in inverse form as

$$p = h^{-1}(\overset{+}{A}). \tag{5.ii}$$

This just says that, *in equilibrium*, the graph of p as a function of the amount of coffee demanded by arbitrageurs, A, (which, by virtue of our considering a condition of equilibrium, just equals English excess supply of coffee), is an upward-sloping curve.

In the foreign country, equilibrium is found by equating excess demand by non-arbitrageurs to the excess supply of the arbitrageurs:

$$\overbrace{h^*(p^*)}^{ED_F^C} = A. \tag{6.i}$$

Again, we express this in terms of the inverse excess demand function in preparation for graphical analysis:

$$p^* = (h^*)^{-1}(\overset{-}{A}). \tag{6.ii}$$

This just says that, *in equilibrium*, the graph of p^* as a function of A is a downward-sloping curve.

3.6 Solving the model

A solution to this model is, as always, a determination of values of the endogenous variables as functions of the exogenous components of the model. That is, we seek the relationship between values of p, p^*, and A and the exogenous components of the model, namely the preferences of the various individuals, their endowments, and the costs of arbitrage.

To determine these relationships, we note that, in equilibrium, we have a relationship between $p^* - p$ and A that is found by subtracting $h^{-1}(A)$ from $(h^*)^{-1}(A)$:

$$p^* - p = (h^*)^{-1}(\overset{-}{A}) - h^{-1}(\overset{+}{A}) \equiv H^{-1}(\overset{-}{A}). \tag{7a}$$

This says that *in equilibrium*, the difference between the foreign price p^* and the home-country price p is a decreasing function of A. This relationship is depicted in Figure 4, where it is superimposed on the two examples of marginal cost curves from Figure 3.

Figure 4 thus depicts arbitrageurs' profit-maximizing choice of A as a function of $p^* - p$, and the downward-sloping relationship between $p^* - p$ and A that must hold in equilibrium because of the requirement that the excess supply of coffee from the home country must equal the excess demand by the foreign country. The intersection of these two curves thus determines the *equilibrium* amount of arbitrage and the *equilibrium* gap between the foreign and domestic price of coffee. In the Figure, we depict the equilibrium if there were constant marginal cost as A_{eq} and the equilibrium if there were increasing marginal cost as A'_{eq}.

What, though, determines the individual equilibrium values $\widehat{p}^*$ and $\widehat{p}$, along with the equilibrium gap between them? To determine these individual equilibrium prices, note that, once we have determined the equilibrium value of

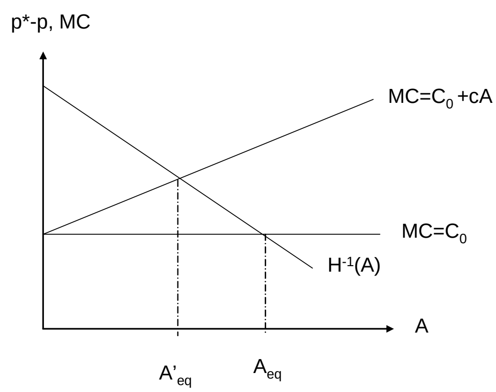


Figure 4: Equilibrium of A and $p^* - p$

the quantity of coffee bought and sold by arbitrageurs, which we denote as $\widehat{A}$, then we can determine $\widehat{p}$ and $\widehat{p}^*$ by substituting this value of A for ES_E^C into $h^{-1}(ES_E^C)$ and for ED_F^C into $(h^*)^{-1}(ED_F^C)$, respectively.

This can also be depicted in a diagram that superimposes $h^{-1}(ES_E^C)$ onto $(h^*)^{-1}(ED_F^C)$, with both p and p^* measured on the vertical axis, and ES_E^C, ED_F^C, and A measured on the horizontal axis. In equilibrium, we know that $ES_E^C = ED_F^C$. Hence, any potential equilibrium must involve points on $h^{-1}(ES_E^C)$ and $(h^*)^{-1}(ED_F^C)$, i.e., values of p and p^*, respectively, that lie on the same vertical line segment, drawn at some identical value for both ES_E^C and ED_F^C. The difference between these values of p and p^* must, in equilibrium, just equal the marginal cost associated with the value of A that equals the value of ES_E^C (and, of course, ED_F^C). Thus, we can imagine "searching" for an equilibrium by looking at all those vertical segments between $h^{-1}(ES_E^C)$ and $(h^*)^{-1}(ED_F^C)$ and choosing the one for which the difference between p^* and p is just equal to the marginal cost associated with the distance of that segment from the origin, i.e., the marginal cost associated with that particular value of A. This is illustrated in Figure 5.

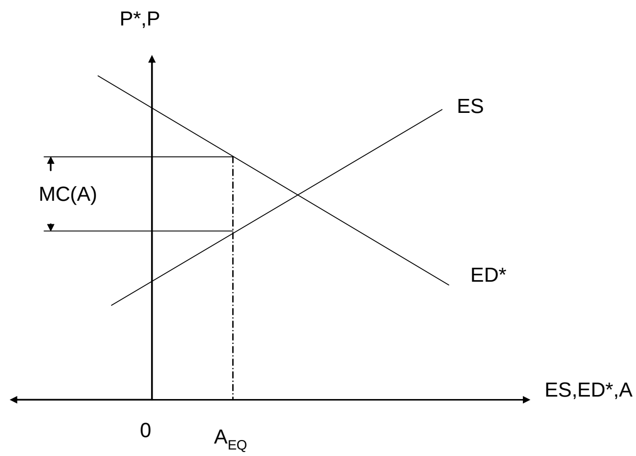


Figure 5: Equilibrium of A, p, and p^*

4 Implications for the pattern of trade

Note that the equilibrium quantity of A is the amount of coffee *exported* from the English POW camp and the quantity *imported* into the French POW camp. Further note that the coffee-exporting camp is the one that has the lower autarkic equilibrium price, and that the coffee-importing camp is the one that has the higher autarkic equilibrium price. This illustrates the general, generic explanation of the *pattern of trade*: locations which have the lower autarkic equilibrium relative price for some good export that good, and locations that have the higher autarkic equilibrium relative price for some good import that good.

This result naturally leads us to a more fundamental question: why do autarkic equilibrium relative prices differ? The foregoing development of the autarkic model tells us the answer: differences that lead to an intersection of autarkic supply and demand curves at different relative prices. But what causes demand and supply curves to intersect at different relative prices? As noted earlier, the answer is either differences in tastes or differences in endowments. In more general models that we develop later, in which supply is no longer just an exogenous endowment but a result of production, we will see that differences in resources and technology lead to differences in supply. In general, we thus summarize the causes of differences in autarkic equilibrium relative prices as differences in *tastes, resources,* and *technology.*

4.1 The effect on equilibrium prices

Relative to the autarkic price, trade between these two economies raises the free-trade relative price of the good in the exporting country and lowers the relative price of the good in the importing country. This again is a general result that extends beyond our simple endowment economy model. Later, we analyze how these changes in relative prices effect the various individuals in the two economies.

4.2 The possibility of non-traded goods and the role of distance

So far, we have analyzed a case in which the marginal cost of arbitrage is always small enough that some trade takes place. Must this be so?

The short answer to this question is: no. The marginal cost curves depicted in Figures 3 through 5 could be so high in the $(A, p^* - p)$ plane that their vertical intercepts are above the vertical intercept of the $H^{-1}(A)$ function. What this configuration of curves means is that the cost of arbitrage is higher than the benefit. Arbitrage in any amount is not profitable. Thus, the two markets remain in autarky, with separate market clearing price.

For many items that consumers purchase this is the appropriate description of reality: Services, such as haircuts, for example, tend not to enter into trade flows. Arbitrage in the haircut market is effectively a decision of a consumer to move from one location to another (assuming we maintain the assumption of endowments of haircut suppliers are exogenously fixed as we have been to this point). Given it is costly to travel to a distant location for a haircut, most individuals stick to a nearby by outlet. Such a case is illustrated in Figure 6.

Our model, for reasons of expositional clarity, only considers two goods. In reality, many goods might or might not be traded between distinct geographical areas, and whether or not they are depends on both the size of gap between autarkic equilibrium relative prices and on the cost of transport. The historical development of trade among regions is in large part due to reductions in transport costs.

One reason marginal transport costs for even the first unit of some potentially traded good might be greater than the gap between autarkic equilibrium relative prices is distance: the farther two geographical regions are apart, the greater the marginal cost of shipment. This observation leads us to a prediction about the patterns of trade: the greater the distance between two nations, the less trade will take place, ceterus paribus. For any given commodity, the costs of transport is higher the greater the distance they must be shipped. Hence, a smaller quantity of that commodity will be traded between nations that are far apart than between nations that are closer. And furthermore, more commodities will fall into the "non-traded" category between countries relatively far apart than for countries relatively close. Thus, the total amount of trade between countries relatively far apart will be less than between countries relatively close to each other.

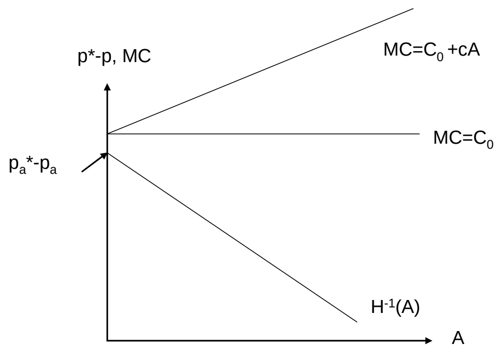


Figure 6: The non-traded case

This ceterus paribus prediction has been tested and is known as the **gravity model** of international trade. The ceterus paribus assumption is that the size of the trading countries is held constant. The robust finding of a large empirical literature is that the volume of trade between two countries is directly proportional to the product of their sizes and inversely proportional to the distance between them. Symbolically, if we denote the size of two economies E and F by the symbols GDP_E and GDP_F, respectively, and the distance between them as d_{EF}, then we can express the relationship between the volume of trade between the two countries, denoted by V_{EF}, as follows:

$$V_{EF} = \theta \times \frac{GDP_E \times GDP_F}{d_{EF}}$$

where θ is a factor of proportionality. This is known as the gravity model because of its similarity to the prediction that the force of gravity between two bodies is directly proportional to the product of their masses and inversely proportional to the distance between them.

5 Two useful special cases

With the nuts and bolts of the analysis of how distinct locations, e.g., different countries, are linked by trade, we now point out two "simplifying assumptions" that economists frequently invoke for mostly pedagogical purposes.

5.1 The limiting case of "zero transport costs"

To exposit our theoretical trade models most clearly, we frequently make an assumption of **zero transport costs**. This assumption can be thought of as an assumption that marginal cost of transport is zero. The implication of this assumption is that in equilibrium $p^* = p$.

What is perplexing about this limiting case to many non-economists is the following conundrum: with zero transport costs, there are zero costs, but also zero profits, but there are still arbitrageurs carrying out their business, buying and selling coffee and tea and transporting these commodities between the locations. The question arises: why bother, if there are no profits? This is less of a puzzle if one views this as a limiting argument: profits are never really zero, but are "close" to zero and the incentives for the arbitrageurs remain.[1]

5.2 The limiting case of a "small"country

Imagine that the French POW camp was much larger than the English camp. What would happen in such a circumstance if the two camps were linked by arbitrage? The coffee transported from the English camp would only constitute a relatively small increase in supply in the French camp. Hence, relative to autarky, we would expect only a small change in the French price arising from the change to free trade.

We frequently formalize this situation as a limiting case in which the foreign country is "infinitely" larger than the home country, and call this the **small-country** assumption for the home country. Formally, this is modeled by assuming a **perfectly elastic** foreign excess demand function for the good imported into the foreign country and a perfectly elastic foreign excess supply function for the good exported from the foreign country. This in turn implies that the small country faces fixed, exogenous relative prices in a trading environment. It should be apparent to you that the fixed, exogenous relative price would be the autarkic relative price in the large country. In practical terms, what matters is how large the small open economies excess demand (or supply) is for the good relative to the excess supply (or demand) in the large market. Nigeria is smaller in the international market for manufacturers than in the global oil market.

6 Introducing tariffs and quotas

6.1 Tariffs

What happens if the French camp imposes a tariff on any coffee coming into the camp in amount t units of tea per unit of coffee? An arbitrageur thus receives $p^* - t$ units of tea for every unit of coffee sold. Their arbitrage schedules are

[1] The alert student might notice that the same conundrum holds for the case of constant marginal cost. The same justification applies: one shold think of this a limiting case, or an approximation, in which profits are just arbitrarily close to zero.

thus functions of $p^* - t - p$ instead of $p^* - p$. Thus, the profit-maximizing condition for an arbitrageur becomes, equation (4), becomes

$$p^* - t - p = mc(\overset{+}{A}). \tag{4b}$$

With a slight rearrangement, this is

$$p^* - p = mc(\overset{+}{A}) + t.$$

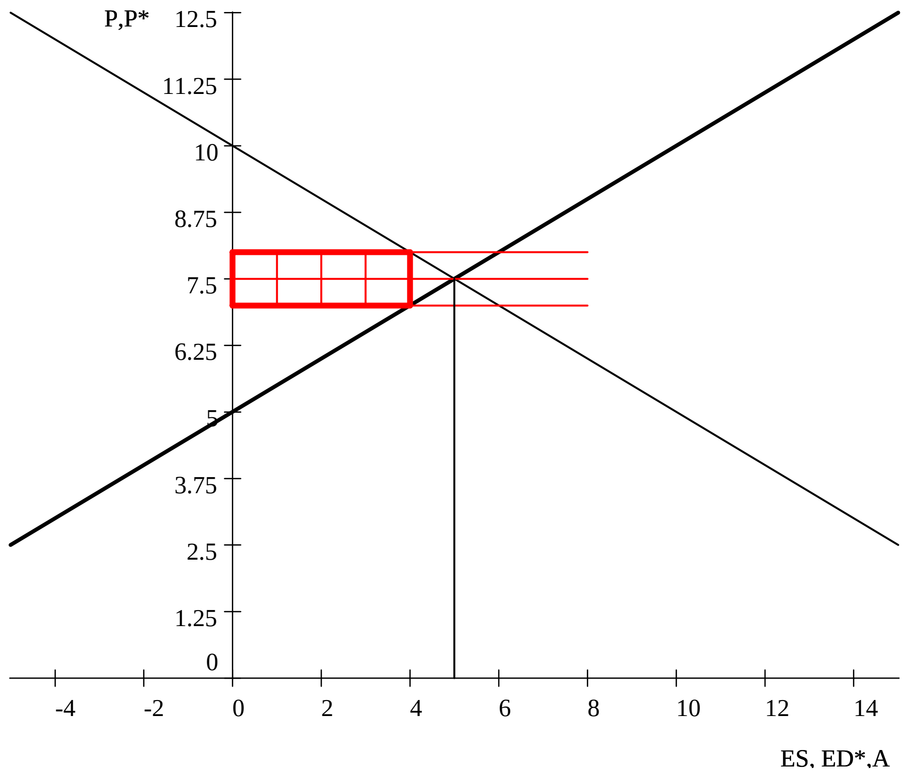
In equilibrium, this means that equation (7) becomes

$$p^* - p = (h^*)^{-1}(\overset{-}{A}) - h^{-1}(\overset{+}{A}) \equiv H^{-1}(\overset{-}{A}) + t. \tag{7b}$$

Consider the simple limiting case of zero transport costs. We can illustrate the effect of a tariff by using a diagram similar to that of Figure 5, in which we superimpose $h^{-1}(ES_E^C)$ onto $(h^*)^{-1}(ED_F^C)$, with both p and p^* measured on the vertical axis, and ES_E^C, ED_F^C, and A measured on the horizontal axis. As noted,in equilibrium we know that $ES_E^C = ED_F^C$, and any potential equilibrium must involve points on $h^{-1}(ES_E^C)$ and $(h^*)^{-1}(ED_F^C)$, i.e., values of p and p^*, respectively, that lie on the same vertical line segment, drawn at some identical value for both ES_E^C and ED_F^C. With a tariff, the difference between these values of p and p^* must, in equilibrium, just equal the marginal cost associated with the value of A that equals the value of ES_E^C (and, of course, ED_F^C), *plus* the value of the tariff, t. Thus, as in the non-tariff case, we can imagine "searching" for an equilibrium by looking at all those vertical segments between $h^{-1}(ES_E^C)$ and $(h^*)^{-1}(ED_F^C)$ and choosing the one for which the difference between p^* and p is just equal to the marginal cost associated with the distance of that segment from the origin, i.e., the marginal cost associated with that particular value of A, plus t.

As depicted in Figure 7, in which marginal costs are constant and equal to zero (0), the graphical depiction of equilibrium without a tariff implies that $p = p^* = 7.5$. The quantity of arbitrage undertaken in this limiting case is five (5) units of tea. Now, imposition of a tariff of t requires that, in equilibrium, $p^* - p = t$. Thus, we can "search" for values of A which makes the value of $(h^*)^{-1}$ minus the value of h^{-1} just equal t. In the figure, a tariff of $t = 1$ is depicted, and equilibrium occurs with $A = 4$, $p = 7$, and $p^* = 8$.

The **tariff revenue** collected by the French government is equal to the tariff per unit, t, times the quantity of imports. In Figure 7, this is represented by the area in the rectangle delimited by the thick red lines.



The effect of a tariff

To summarize, the implications of an imposition of a tariff are:

1. The price in the tariff-imposing (importing) country goes up relative to free trade;
2. The price in the foreign country goes down;
3. The tariff-imposing government collects tariff revenue.

6.2 Quotas

Note that a quota limits the amount of an item that can be imported. The effect of an appropriately sized quota is thus exactly like that of a tariff in its effects on prices in the two countries. That is, there exists a quota that replicates the effects of a tariff on the two prices. The difference is that a quota does not generate any revenue for the government.

Other differences between tariffs and quotas exist in more complicated settings, such as in situations involving uncertainty about supply and demand, but the major difference is the lack of government revenues associated with a quota.

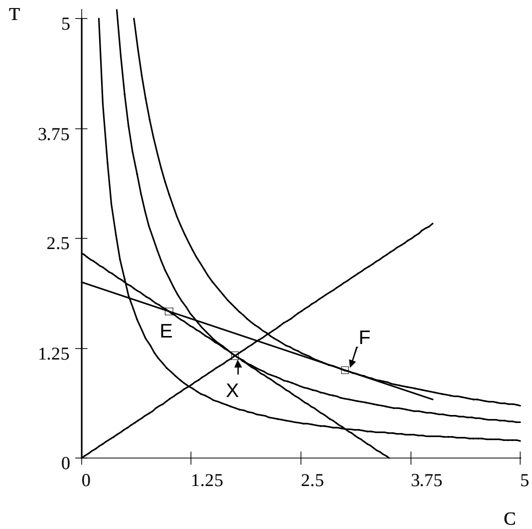


Figure 7: Imposition of a tariff

6.3 Optimal tariffs and tariff wars

6.3.1 The small-country case

For a small country, teh excess supply function it faces is flat. Is there a tariff that can make satisfy the compensation principle? The answer is no: the optimal tariff (in the sense of being a potential Pareto improvement) is zero for a small country.

To see this, consider the simplest possible case in which our country has one consumer, Baptiste. Without a tariff, Baptiste has a budget constraint with slope equal to the negative of the exogenous world price (denoted $\overline{p}$, where the overbar reminds us that this price is fixed) that goes through his endowment point. The most-preferred coffee-tea pair lies on this budget constraint at the point at which an indifference curve is tangent to this budget constraint. At this point, our French POW imports coffee and exports tea.

Consider an imposition of a tariff t^* by the French government. Because of the small-country assumption, this raises the French relative price of coffee to $\overline{p} + t^*$. For Baptiste, this rotates his budget constraint clockwise in teh coffee-tea plane around his endowment point. By itself, this would make him worse off than he would be with free trade. This is illustrated in Figure 8, where point E represents teh endowment point, point F represents the free trade most-preferred point, and point X represents teh most-preferred point with the tariff imposed.

But the French government collects tariff revenue. If this is rebated to Baptiste, could he be made better off than he was under free trade? The rebate would shift up Baptiste's budget constraint.

How much does Baptiste's budget constraint shift up? Let us write down his budget constraint *in equilibrium*:

$$\widehat{T} = \overline{T} + (\overline{p} + t^*)\,\overline{C} - (\overline{p} + t^*)\,\widehat{C} + S,$$

where S is the "subsidy" which is equal to the tariff revenue rebated by the government, and the "hat" over C and T indicate that these are equilibrium values, chosen so that his margianl rate of substitution at these values equals $-(\overline{p} + t^*)$. But this tariff revenue is:

$$S = t^*(\widehat{C} - \overline{C}).$$

Substituting this back into Baptiste's budget constraint, we see that his equilibrium choice $(\widehat{C}, \widehat{T})$ must lie on the straight line through his endowment but with slope $-\overline{p}$:

$$\widehat{T} = \overline{T} + (\overline{p}) \cdot \overline{C} - (\overline{p})\,\widehat{\cdot C}.$$

Baptiste cannot be made better off. With the tariff imposed, Baptiste chooses a most-preferred pair for which the marginal rate of substitution is greater than the marginal rate under free trade. This must be true after he gets the subsidy as well. His optimal choice of coffee and tea with a balanced budget tariff-with-subsidy scheme lies along the same budget constraint as he faced with free trade, but it cannot be at the free-trade point.

This is depicted in Figure 9. Baptiste's new optimal consumption point is depicted as point R. Note how the budget constraint that determines his margianl rate of substitution goes through R, but with the steeper slope created by the tariff.

6.3.2 The large-country case

For a country large enough that the excess supply function it faces is not flat, the imposition of a tariff *reduces* the world price of its import. In this case, can imposition of a tariff improve the well-being of its citizens?

The general answer to this question is: yes. To give a heuristic understanding of the logic behind this result, we will discuss a special case in which the excess supply functions facing each country are perfectly inelastic, and in which the demands for the imported good of each country are unaffected by income. These assumptions lead to a model that highlights the key features behind the general result that a large country can impose an **optimal tariff**.

In Figure 10, titled "No-income-effects preferences," we depict an indifference curve map for which the demand for coffee will be unaffected by income. In the Figure, we have superimposed a series of parallel downward-sloping budget

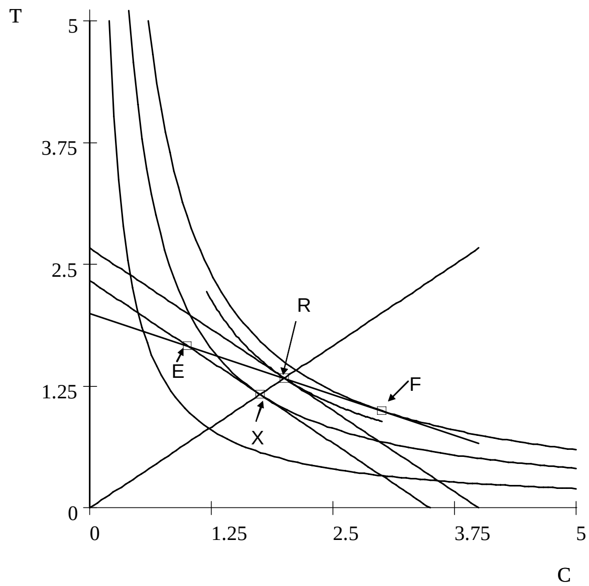


Figure 8: Baptiste's choice with subsidy

constraints, each of which is tangent to an indifference curve at the same value of coffee. This illustrates that, at any given price, increases in income which shift up in parallel fashion the budget constraint leave unchanged the quantity of coffee consumed: all of the increase in income is spent on tea.

In Figure 11, we illustrate that this indifference map can also lead to a perfectly inelastic excess supply of tea. In the Figure, the endowment point is depicted as $(\overline{C}^* = 0, \overline{T}^* = 1.5)$. As the relative price of coffee falls, the point of tangency between each of the budget constraints and an indifference curve is always with $T^* = 1$. These points are connected by the dotted line.

Consider our usual case in which the French import coffee. As usual, we assume their inverse excess demand function is downward-sloping. But in contrast to the standard case, we now assume the English excess supply function is perfectly inelastic. What happens if the French impose a specific tariff of t^* units of tea per unit of coffee imported? Because of the assumed inelastic supply of imports, the effect is simply to lower the English relative price p: the French price $p^*$is fixed at the intersection of excess demand and excess supply, but the difference between the English price and the French price must be equal to t^*. Thus the effect of the tariff is to reduce the English price by the entire amount of the tariff. The quantity imported remains unaffected because of the assumed perfectly inelastic supply of imports. The only other effect is that the French government collects tariff revenue equal to the tariff t^* times the quantity of imports.

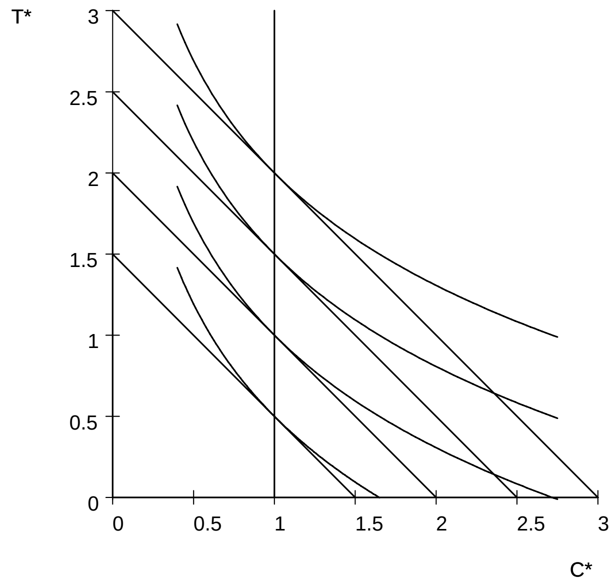


Figure 9: No-Income-effects preferences

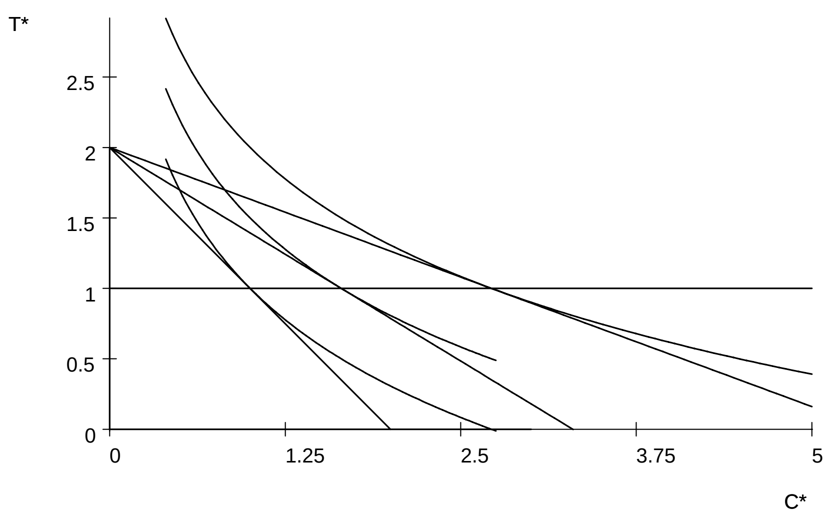


Figure 10: Constant tea excess supply

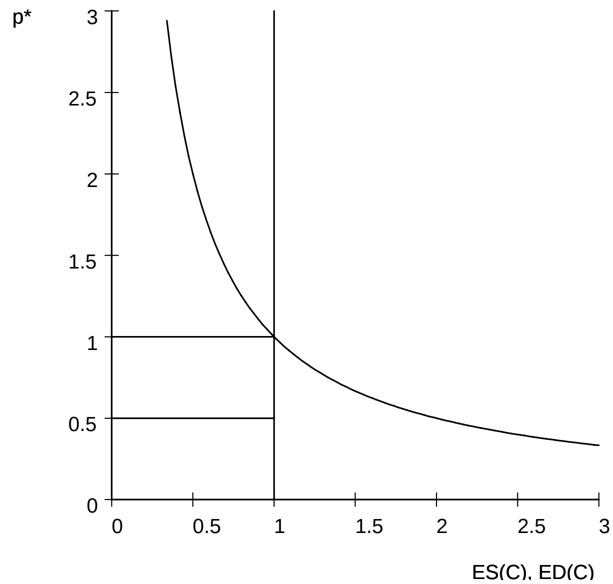


Figure 11:

Are the French better off? If the French government distributes the tariff revenue to the French people, these individuals can now spend this extra income. By assumption, we assume they only spend it on tea, so that their inverse excess demand function for coffee is unaffected. Hence, after the tariff, each French resident consumes the same amount of coffee as before (again, an implication of our assumption of perfectly inelastic supply)and more tea. Each French resident is better off.

This is illustrated in Figure 12, where the French price is one (1), the tariff is one-half (1/2), and the English price is thus one-half (1/2). The tariff revenue is indicated by the area in the rectangle of height one-half (1/2) and width one (1), which in the figure is the amount of excess supply.

What about the English? As the French tariff gets bigger and bigger, the price received by the English gets smaller and smaller. They export the same quantity (by assumption), but get less and less tea in exchange. Thus,they consume the same amount of coffee, but consume less and less tea. They are unambiguously worse off.

We can also display these effects in a diagram which depicts the excess demand for tea by the British and excess supply of tea by the French. Again we assume an inelastic supply by the exporter, the French in the case of tea. In Figure 11, the relative price of *tea* $(1/p)$ is measured on the vertical axis and the excess supply of and excess demand for tea are measured along the horizontal axis. The solid vertical line depicts French excess supply with no tariff. The intersection of this excess supply with the English inverse excess demand

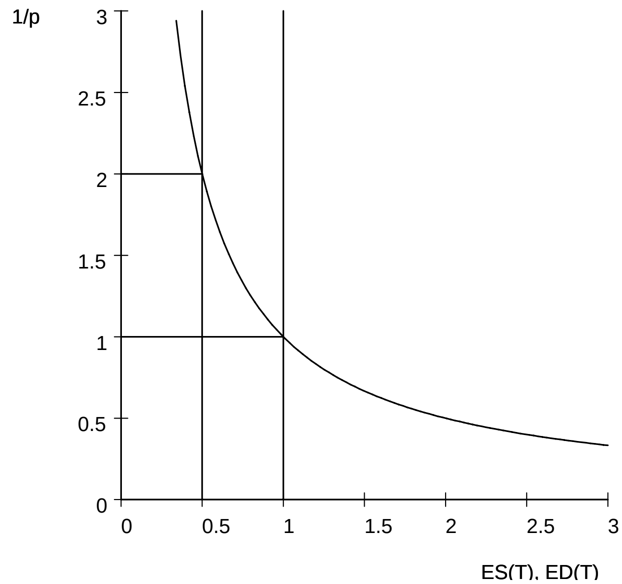


Figure 12: The tea market with supply shift

function determines the equilibrium relative price of tea. Of course, because of Walras' Law, we know that this depiction conveys no different information than what is found in the depiction of equilibrium in the market for coffee.

But we can depict how the imposition of a tariff by the French changes the quantity of tea supplied. Because the French consume more tea because of the collection and distribution of the tariff revenue, they now supply less to the English. That is, the inverse excess supply function shifts back toward the origin. This is depicted by the dotted vertical line in the Figure 13. The intersection of this dotted line with the inverse excess demand curve depicts the new higher relative price of tea.

Can the English afford themselves of the same welfare-increasing strategy as the French? Assume again that, along with the perfectly elastic export supply of tea from the French, increases in income for the English lead to unchanged demand for tea. From the English perspective, they have a downward-sloping inverse excess demand curve for tea that intersects the perfectly-inelastic excess supply of tea at some price $1/p$. By the same logic as we used with the French, we see that imposition of a tariff t for the English reduces the relative price of tea that the French receive, and generates tariff revenue for the English that when distributed to the English residents makes them better off.

Note, though, that by imposing their own tariff, the English make the French less well off than they would have been without an English tariff. In fact, we might think of the English as imposing their own tariff as *retaliation* in response to the initial French tariff. And after the English retaliate, we might think that the French would install another round of tariff increases, followed by further English retaliation, and so on and so forth. Thus we see the possibility of a **tariff war**.

Apart from the possibility of a tariff war, we can ask another question about tariffs: taking as given another country's tariff (say the English), what would be the **optimal tariff** rate from the perspective of a home country (the French)? The preceding description of the effects of the tariff tells us that the optimal rate is the one that generates the most revenue: regardless of the tariff rate, the same quantity of coffee is imported and consumed by the French, so more tariff revenue always leads to increased tea consumption but no decrease in coffee consumption. Which rate generates the maximum revenue? In this case, this optimal tariff is as close as possible to p^*, as can be seen from Figure 10. We say "as close as possible" because if the tariff equals p^*, imports would be zero, and there would be no imports, and no tariff revenue. A tariff that eliminates all imports is known as a **prohibitive tariff.** In our model, such a tariff puts the French economy back in its autarkic state.

What about for the English? From the English perspective, they have a downward-sloping inverse excess demand curve for tea that intersects the perfectly-inelastic excess supply of tea at some price $1/p$. By the same logic as we used with the French, we see that the optimal tariff t for the English is as close as possible to $1/p$.

These results suggest that large countries have an incentive to manipulate their term of trade by imposing tariffs. They also suggest that this motivation might lead to higher and higher tariffs, either through evolving tariff wars or through imposition by each country of its optimal tariff rate. In these strategic settings, the attempts by each country to make itself better off at the expense of its neighbor leads to a situation where they are both worse off and close to an autarkic situation. Later we will see how these results have led some economists to argue that the formation of international organizations such as the World Trade Organization was and is an attempt to eliminate these policy dilemmas.

7 A Recap of arbitrage, with interpretation and implications

First note that this analysis of arbitrage lets us explain a few features of the world economy. First, not all goods are traded. Haircuts, as noted, are seldom traded, because the transportation cost is simply too high: you either have to travel to a foreign country to get a haircut, or you have to pay to bring a hairdresser to you from abroad. Many goods fit this "non-traded" category, and the reason is simply that transport costs are so high as to make arbitrage unprofitable.

This also suggests that one reason previously autarkic countries and regions ended up with similar prices is that technical innovation in transport costs fell enough that arbitrage became possible. Large price disparities between the Great Lakes region of the United States and New York City disappeared after construction of the Erie Canal dramatically lowered transport costs. Many bulky appliances such as hot water heaters used to be produced in many counties within a state until rail and road transport became cost effective, leading many states to now have no production facilities for these appliances. Advances in telecommunications have now made many services such as medical transcription, services that were recently supplied locally in the United States, now imported from India and other distant locations.

Finally, the implication of this model of arbitrage is that international trade changes relative prices vis a vis autarky. This has important implications for determining who wins and who loses in a move from autarky to free trade. The implications for individuals of these changes in relative prices are what we take up next.

8 Effects of international trade on individual well-being

For the purpose of evaluating what happens to an individual's well-being when arbitrage links two previously autarkic economies, the most important implication of the arbitrage link is that the equilibrium relative price in the trading equilibrium is different from the autarkic equilibrium relative price. This has implications for the well-being of all individuals in both economies. In this section, we first demonstrate that it is possible for there to be "winners" and "losers" within each economy. We then consider the "thought experiment" that economists use to show that there exists a hypothetical, costless, feasible redistribution of the endowments among the individuals within each economy such that, with this redistribution, all individuals would be better off under free trade than under autarky. Finally, we consider how this result informs economists' views on trade policy.

8.1 Winners and losers

Consider the effect on Andy and Bob of the increase in the relative price of coffee that occurs when arbitrageurs link their camp with the French camp. In Figure 14, Andy and Bob's autarkic equilibrium positions are depicted, with the autarkic budget constraint depicted as the black line through the endowment point $(1, 1)$ and the autarkic consumption bundles depicted as points on this budget constraint through which an indifference curve passes that is tangent to the budget constraint. Andy, the coffee-lover (relative to tea and relative to Andy) consumes more coffee than his endowment and less tea. His autarkic equilibrium indifference curve is depicted as the blue curve. Bob, the tea-lover (relative to coffee and relative to Andy), consumes more tea than his endowment and less coffee. His autarkic equilibrium indifference curve is depicted as the black curve.

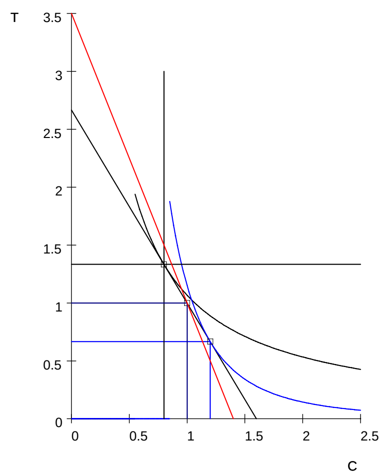


Figure 13: Andy wins, Bob loses

In a trading equilibrium, Andy and Bob continue to have the same endowment point, but their (coincident) budget constraints are now rotated in a clockwise fashion through the endowment point $(1, 1)$, reflecting the higher relative price of coffee that now prevails. This new budget constraint is depicted as a red line. As drawn, it is clear that Bob is better off and Andy worse off. For Bob, the new trading equilibrium budget line cuts his autarkic equilibrium indifference curve; he can now consume pairs of coffee and tea that have more of both than did his autarkic consumption point. That is, there are points on his new budget constraint that lie to the northeast of his autarkic equilibrium endowment point. For Andy, free trade has rotated his budget constraint so that it lies everywhere below his autarkic equilibrium indifference curve: he is worse off.

This result for our model economy is illustrative of what frequently happens in actual experience when trade barriers, either man-made or technological, are dismantled: not everyone is made better off. For example, the reduction of textile import restraints clearly hurts some people in the U.S. What criterion, then, are economists using when they pronounce such trade liberalizations as "good for the country as a whole?"

8.2 A compensation criterion

To understand how economists think about this problem, we first have to appreciate the conceptual possibility of a mechanism whereby we can imagine everyone in our model economy being made better off by a change from autarky to free trade. To understand this, imagine the following scenario in the English POW camp.

Suppose, following a disbursement of the Red Cross packages to Andy and Bob, Andy and Bob exchange between themselves, as would be their usual custom. But now, just after having made their exchanges but before they have actually consumed their coffee and tea, they are unexpectedly offered an opportunity to exchange with an arbitrageur at a higher relative price of coffee than that which they agreed upon in autarky for their initial exchanges with each other. What would their opportunities look like now?

First consider what their budget constraints would now look like. For both of them, their actual quantities of coffee and tea available for trade are the amounts that they had planned on consuming before this new opportunity arose. That is, their "endowment point" is now their optimal autarkic consumption point. Andy's budget constraint now runs through his autarkic consumption point, but has a steeper slope than did his budget constraint in autarky. A similar situation applies to Bob: his budget constraint also passed through his autarkic consumption point and also has the same steeper slope. We depict their old autarkic equilibrium points and their new ones in Figure 8. Because their new budget constraints have been rotated through a point on their old budget constraints at which the indifference curve through that point was just tangent to the old budget constraint, this new budget constraint "cuts" that budget constraint. By moving in a northwest direction along their new budget constraints from their autarkic consumption points, Andy and Bob can attain more-preferred points of coffee and tea, i.e., points that lie on a higher indifference curve. This is illustrated in Figure 15.

The existence of a hypothetical reallocation of endowments among the members of the POW could make them all better off under free trade than under autarky is sometimes described by saying: the gains to the winners from free trade are sufficiently large that they could compensate the losers and still be better off. A change in economic circumstances that satisfies this criterion is said to satisfy the "Hicks-Kaldor" compensation criterion,so named because of the two economists who first developed the concept. It is also described as having **potential Pareto improvements**.

8.3 How the compensation criterion informs thinking about policy

There are a number of ways of thinking about the implications of this result for actual trade policy. First, the simplicity of the two-person "country," i.e., the

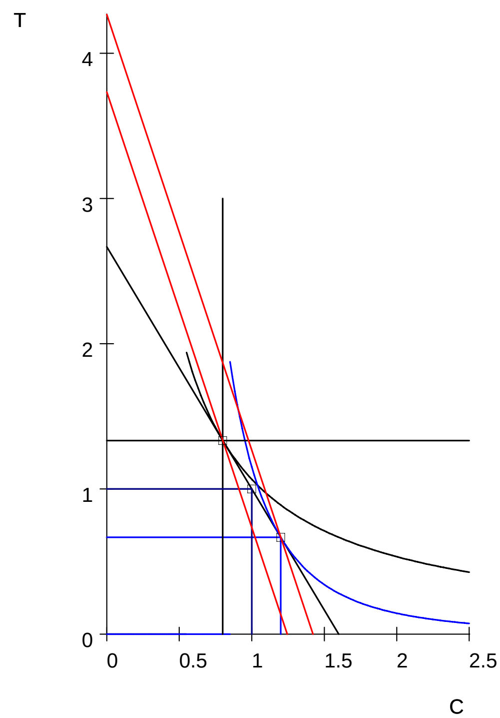


Figure 14: Depiction of compensation

British POW camp, which helps us see clearly the possibility of a rearrangement of endowments that would permit both Andy and Bob to be better off, also might obscure the practical difficulties of implementing such a compensation scheme in a more realistic setting.

This line of thought leads us to consider how this purely hypothetical possibility informs economists' thinking about real-world trade policy. Perhaps the following quotation best represents why economists use this theoretical result to inform their thinking about free trade and other market-based policy prescriptions:

> "*Heuristic theorem:* Most technical changes or policy choices directly help some people and hurt others. For some changes, it is possible for the winners to buy off the losers so that everyone could conceivably end up better off than in the prior status quo. Suppose that no such compensatory bribes or side payments are made, but assume that we are dealing with numerous inventions and policy decisions that are quasi-independent. Even if for each single change it is hard to know in advance who will be helped and who will be hurt, in the absence of known "bias" in the whole sequence of changes, there is some vague presumption that a hazy version of the law of large numbers will obtain: so as the number of quasi-independent events becomes larger and larger, the chances improve that any random person will be on balance benefitted by a social compact that lets events take place that push out society's utility possibility frontier, even though any one of the events may push some people along the new frontier in a direction less favorable than the status quo."
>
> from Samuelson, P.A., (1981), Bergsonian welfare functions, in *Economic welfare and the Economics of Soviet Socialism*, ed. Rosefield. Cambridge: Cambridge University Press. (p. 227)

Note that Samuelson described this as a "heuristic" theorem: a guide to thinking, rather than a logical inference from a well-defined set of premises. We emphasize this to make clear where the line of demarcation should be drawn between what economists know as implications of their models and what they believe on the basis of further inferences based on less-precise forms of knowledge such as imperfect observation and informed guesses.

Note also that the implication of belief in the heuristic theorem is that the "gains from trade" must be viewed in some sort of "long view" context: at any moment in time, some people are hurt and others helped, but only over time might we expect these effects to average out with a positive net gain for any randomly chosen individual.

We should also re-emphasize that this theoretical result about a hypothetical redistribution of resources does not imply that an actual policy of redistribution will necessarily lead to everyone being better off. If such redistributions use up resources, our theory no longer assures us that everyone could be made better off.

Finally, note that the assumptions that underlie this heuristic theorem alert us to situations in which there might be disagreements even among economists about the gains to a society from pursuing a policy of free trade. For example, the rapid decrease in telecommunications costs has led some economists to believe that there will be a massive change in the "pattern of trade," with tens of millions of services that are now produced domestically in the United States being provided from abroad. Such a massive dislocation doesn't fit neatly with the idea of "quasi-independent" events to which the law of large numbers would apply.

Nonetheless, we emphasize this result because this example of the possibility of a hypothetical redistribution that makes everyone better off is representative of a general result that is implied by more complicated models of perfect competition, models that include production, many goods, many people, and many countries. It remains the foundation of economists' arguments in favor of free trade.

9 Summary

When economies in distinct geographical areas have different autarkic equilibrium relative prices, the possibility exists that arbitrage will link them. Whether and to what extent this happens depends on the costs of arbitrage activities. Typically arbitrage costs are associated with 'transportation costs.' If transport costs are low enough that arbitrage does take place, the country with the lower autarkic relative price of some commodity will export that commodity, and the country with the higher autarkic relative price will import that commodity. In comparison to the autarkic equilibrium, the effect of this trade will be to increase the relative price in the exporting country and decrease the price in the importing country. As a useful simplifying assumption, we sometimes model transport costs as zero, so that the "law of one price" is assumed to hold: relative prices are the same in both locations.

Tariffs can be thought of as an addition to transport costs: they drive an additional wedge between the domestic import price and the foreign export price. The effect is thus to raise the import price and lower the export price. In addition, the tariff-imposing nation collects tariff revenue. Quotas mimic the effects of tariffs, except that they don't generate revenue for the government.

Relative to autarky, the changes in relative prices in the two distinct locations have effects on the well-being of individuals within each location: in general, it may be that there are both "winners" and "losers" within each location.

What, then, can be said about whether such trade is "good for the nation?" After all, a nation is composed of individuals, and if not every individual is made better off by a change in economic circumstance, what could it mean for "the nation" to be better off? What we show in this model, and what we will show is a result that continues to hold in more general and complicated models that include production, is that there exists a hypothetical redistribution of income that makes everyone better off under free trade than under autarky. Why this informs economists' thinking on trade policy issues requires a longer argument, one that we will continue to develop throughout the text.

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