The autarkic endowment economy

Crucini-Driskill

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1 Learning Objectives

1. Understand how economists model the exchange of goods and services among individuals in a market during a period of time, e.g., a month.

2. Understand how economists model individual consumer **preferences**; that is, how individuals make pair-wise comparisons of one basket of goods and services vis à vis another, and what it means to say they are **indifferent** between the two baskets?

3. Understand why preferences can be represented as families of concave-to-the origin, downward-sloping, non-intersecting indifference curves provided preferences satisfy the axioms of: (1) completeness, (2) non-satiation, (3) transitivity, and (4) diminishing marginal rate of substitutability.

4. Understand that a marginal rate of substitution function systematically relates the slope of an indifference curve to the point on the curve at which the slope is evaluated.

5. Understand the concept a rank-ordering of indifference curves, and understand the contrast between a rank-ordering (or, in equivalent language, an ordinal ranking) and a cardinal ranking.

6. Understand what economists mean by a **budget constraint**, and become comfortable with the two-dimensional graphical representation of a budget constraint.

7. Understand how the solution of the sub-model of the individual consumer describes a systematic relationship (known as an individual's general equilibrium demand curve or general equilibrium demand function) between a consumer's optimal choice of a consumption bundle, i.e., a collection of goods and/or services, and the exogenous (to the consumer) variables relative prices and endowment bundle.

8. Understand that market demand and market supply functions are summations of individual demand and individual supply functions, respectively.

9. Understand how the solution to the endowment economy model describes a systematic relationship between endogenous variables (namely relative prices and quantities consumed by each individual) and each individual's rank-ordering of the consumption bundle they consume, and the exogenous variables of endowments.

10. Understand how the **autarkic equilibrium relative price** that is part of the solution of the autarkic endowment model is determined by the interplay of tastes and resources.

11. Understand the basic model of money demand, money supply, and price-level determination.

2 Key concepts

1. Budget constraints.

2. Endowments.

3. Relative prices.

4. Nominal and real income.

5. Preference relationships.

6. Indifference curves and indifference maps (or indifference families).

7. Individual demand curves.

8. Market demand curves and market supply curves.

9. Market equilibrium.

10. An equilibrium (or solution) for the model as a whole.

3 Introduction

> "The propensity to truck, barter, and exchange one thing for another . . . is common to all men, and to be found in no other race of animals. . . . ... Nobody ever saw a dog make a fair and deliberate exchange of one bone for another with another dog" - Adam Smith, Wealth of Nations.

As children, we would trick-or-treat with siblings on Halloween and when we returned home, we did what everyone must: engage in trade. One of us preferred hard candy relative to tootsie rolls and the two items were exchanged at some ratio, analogous to what trade economists refer to as the terms of trade. Inevitably, older siblings tended to get more in exchange for less than younger ones, something economists refer to as a favorable terms of trade. In most markets such bargaining power arises due to scarcity of supply as opposed to physical intimidation, but not always (think of the drug trade or 'sweat shops').

Halloween is just one example of exchange that takes place as a natural extension of human interaction. Children trade Halloween candy, baseball cards, beanie babies, Magic cards, seashells collected on the shore and more. Teenagers trade music on Ipods, and clothes and tips to interesting internet addresses. Adults tend to trade more in organized markets, selling their labor in exchange for money and exchanging money for goods and services. Good neighbors often exchange clothing with parents of three-year-olds giving clothing of their now four-year-olds to friends across the street with younger children. The terms of trade here appear most favorable. Friendships and relationships rarely endure overtly skewed terms of trade, and eventually some goods or services flow in the other direction.

An intriguing historical example of trade that we will use repeatedly is the exchange of Red Cross rations among prisoners of war and their guards. The "propensity to truck, barter, and exchange" was one of the two key behavioral assumptions about people that Adam Smith, the founder of modern economics, used to form his model of the " economic person." His other key behavioral assumption, as noted earlier, was that people pursue their self-interest.

These examples exemplify situations in which people trade: (1) Equal quantities of goods and different tastes, as in the hard-for-soft candy Halloween exchange; (2) unequal quantities of goods and identical tastes, as might happen if twins decided to focus their trick-or-treating efforts on different sides of the street and then trade to equalize the spoils of their efforts. If neither tastes nor quantities of goods differ across people, there will be no basis for trade.

exemplify: to show or illustrate by example.

The converse is not true, though. If both tastes and quantities available differ, people may, or may not, trade. One of the tasks of this chapter will be to provide the framework for understanding how the *interplay* of tastes and resources, i.e., quantities of goods available, determines whether or not trade takes place, either between individuals within or between nations.

In this chapter we develop the theoretical foundations of exchange without worrying at all about where the supply of goods come from or why individuals receive different looking or different sized allocations. This is not a statement about the origins of supply being less important than the origins of demand. It is nothing more than breaking a complex problem down into its component parts to make it manageable. Consider your first soccer practice, say, at age 5. The good coach doesn't play a DVD of World Cup soccer and ask: "Any questions?" He starts with rudimentary elements, called drills, develops players skills and crafts a game-plan. Economists and economic models are like sports coaches and game plans. Like game plans, economic models sometimes disappoint when confronted with real-world data (a game). Yet, without game plans the sport is poorly understood and executed and unpleasant to watch.

allocation: the money or goods that consumers receive and have at their disposal to used in the process of exchange.

We also introduce the rudiments of a monetary economy as well. As noted in earlier chapters, economists usually dichotomize their analyses into "real" and "monetary." The real part assumes that money works so efficiently as a medium of exchange that we can model the world as if people simply engaged in costless barter. This analysis of a real economy constitutes the first part of the chapter. The key variables in this analysis are quantities of commodities consumed and traded, the **relative prices** at which these exchanges take place, and the levels of well-being of the individuals involved in this economy.

We graft onto this real model an extremely simple model of the demand and supply of money. This allows us to discuss the determination of **nominal prices**, which includes the nominal exchange rate. This provides the minimum background necessary to organize thought about the international economy.

4 The real model

4.1 Overview

The best real-world example we have found in which the supply of goods is outside the control of individuals is based on the vivid account of R.A. Radford, who was interred during World War Two in a series of German prisoner of war (POW) camps. The model that we build here is an economic formalization of Radford's account of trade in the camp.

In these POW camps the bulk of goods provided to the prisoners came from packages distributed by the Red Cross. As is true today, these packages are provided free of charge, one to a prisoner. Because prisoners, by and large, were not allowed to work in any fashion to supplement these rations they constituted the prisoners' entire endowment of resources. Each prisoner received exactly the same package, containing such things as packets of coffee, tea, cigarettes, tins of meat, and various other necessities. This setting corresponds almost perfectly to what economists' model as an endowment economy. In an endowment economy, supply is exogenous. Economists use the term exogenous to describe variables and events outside the control of individual decision-makers. The Red Cross packages are exogenous from the point of view of individual prisoners. The packages are endogenous with respect to the Red Cross, it's donors, and the commandant. From this example it should be clear that from the point of

endowment: a good or service given to someone.

view of a particular decision-maker a variable may always be classified as either endogenous or exogenous. The subtle part is that what is exogenous to one actor in the model may be endogenous to others. Variables that are exogenous to all decision makers in the economy are things like the weather. These distinctions are important in economics and in life; they take a lot of practice to master. As we progress through various models you will get the drill down perfectly.

If each prisoner received the same rations and consumed them, the POW camp would not offer any interesting economic insights. This is not what happened. As Radford describes, extensive trade sprang up between individuals and eventually took place in organized exchanges with posted prices and with use of cigarettes as a unit of account and medium of exchange. We will not dwell on the special role of cigarettes as the currency of the POW camp until later in the chapter. If individuals voluntarily engage in trade, there must be gains from it for both parties. Our focus will be on what exchanges took place, at what prices and for what reasons. Much of economics will be conveyed through this real-world example and our mathematical formalization of it.

An even more beneficial aspect of the circumstances facing the prisoners from the standpoint of international economists, is that prisoners were isolated in different compounds based on their nationality: French soldiers in one compound and English soldiers in another. This offers a virtual microcosm of trade among nations because initially the two compounds were not permitted to trade and latter the commandant decided to change the rules and permit unrestricted trade. Thus the two compounds went from **autarky** – complete economic and physical isolation – to free trade. Because barriers to trade have varied enormously across time and countries, a comparison of autarky and free trade lays the groundwork for our exploration of natural and official barriers to exchange.

autarky: here we refer to the absence of economic interaction between a country and all others; an alternative definition is economic self-sufficiency, which seems to suggest there are no gains from trade.

4.2 The importance of understanding autarkic equilibrium

Our analysis of a model of a POW camp will proceed in two steps. First we will model an autarkic economy that corresponds to the initial condition described by Radford in which different nationalities were kept segregated from each other. Because the autarkic economies will be similar in structure, we will focus on the English compound in our exposition. The elements of this model will be: (1) a description of the institutional features of a market economy; (2) a model of consumer demand for goods found in the Red Cross package; (3) a very simple model of supply; and (4) and a complete solution of the model.

We will assume there are at least two classes of individuals in this autarkic economy, distinguished by their tastes. Within each class, though, all individuals are identical. For this case of two different classes of people, the analysis *can* be interpreted as modeling trade between two regions within an autarkic country when the people within each region are identical in terms of tastes and incomes but tastes differ across regions.

The rationale for introducing taste differences is twofold. First, without taste differences there would be no incentive to trade given everyone receives the same package. Second, it is important to demonstrate that even with heterogeneity in the population and uneven benefits from trade, *all* parties *could* benefit from trade. This provides an important understanding that helps illuminate why a common idea promulgated by many non-economists, namely that all trade, whether among many people or only between two individuals, benefits one party only at the expense of another, is false.

Second, we will analyze how two previously autarkic economies are linked together by trade when the barriers to trade - in this case the policies of the German camp commandant - are removed. This analysis will introduce the key concept of arbitrage and will describe how, when the interplay of tastes and endowments leads to different autarkic prices in the two compounds, trade can occur between the previously-isolated compounds.

This last point is why it is important to understand the determinants of equilibrium prices in an autarkic economy before modeling trade between economies: if autarkic equilibrium prices don't differ, there will be no trade. A fundamental understanding of the *pattern of trade* between nations requires first an understanding of why autarkic prices differ.

4.3 Model components

Like in most economic analysis, our strategy breaks the model into "sub-models:" smaller components of the complete model. The basic components of the complete autarkic model are sub-models of consumers, which are in turn used to construct a sub-model of market demand curves–the "demand" side–and sub-models of the supply side.[1] These sub-models are then connected by the assumption of market equilibrium, that requires quantity demanded to equal quantity supplied.

4.4 The consumer

Suppose your cousin from Europe visits you in the United States. When your cousin arrives at the airport, her luggage is lost. And the first thing on your list of things to do becomes shopping. The questions you would ask your cousin in preparation for that shopping trip form the essential building blocks of an economist's formal model of consumer behavior. You might begin by asking what items were in the luggage that she would like to replace. Implicitly the contents of the luggage represents her preferences or given a reasonable sized-suitcase, a sub-set of her preferred bundle. The next question would be how much they have to spend to replace the lost items. Maybe you want to throw in a few dollars under the circumstances. Economists model consumption choice like a successful shopping trip: one in which the most-preferred bundle is chosen subject to the available budget. We begin with the budget constraint. It is the easiest part of the formal model to understand if not always easy to stay within.

[1] Quotations around "demand" alert us that the demand side is, in more general models, connected to the supply side by virtue of peoples' incomes being dependent on what is produced and sold and at what price these outputs are sold.

4.4.1 The Budget Constraint

We begin with a budget constrained by the flow of income during the current period of expenditure. We postpone discussion of borrowing and lending or uncertainty about the availability of income over the course of the period until Part II of the book. To be concrete, suppose you are given a fixed dollar budget at the start of the semester (beyond the required tuition and room-and-board) and you want to plan out your expenditures over the course of the semester. How will you allocate the budget across the many thousands of items that market economies produce each year? For our visiting cousin, the problem is easier. She announces a budget constraint, and assuming she sticks to it, she shops (with your help) for the best bundle available that satisfies that budget given market prices.

The budget constraint is not the same thing as a budget plan. A budget plan is the result of preferences confronting the budget constraint. The budget constraint is on the one hand the dollar amount of resources available (the easy part) and on the other hand, the set of all feasible combinations of goods and services that fit within that budget given market prices. The set of all feasible combinations of choices is easiest to convey with a single good. If my income is \$ 100 and the price of the good is \$ 10, I can purchase any amount from 0 units to 10 units. A more interesting problem involves two goods since this gives rise to a tension across acquisition of the goods. If I purchase more of one good I must purchase less of another. The budget constraint describes this trade-offs in algebraic terms.

Because we are going to fully develop our POW camp model from real-world observations, the two obvious goods to engage in the analysis are tea and coffee. Why? Because the English prefer tea to coffee and much of what develops in trade will hinge on either preferences for or production of goods. We symbolize the quantity of tea per unit of time that hypothetically could be purchased by individual i as T_i, and the quantity of coffee per unit of time that hypothetically could be purchased by individual i as $C_{i.}$

We emphasize that these variables are measures of quantities *per unit of time*. For example, in Radford's POW camp, Red Cross deliveries might have taken place at one-month intervals, and our measurement of coffee and tea would thus be something like "6 tins of tea per month" and "5 pounds of coffee per month." Clearly, 6 tins of tea per month is not the same as 6 tins of tea per week, which would in fact amount to approximately 24 tins per month. Our units of measurement must account for this. More important, though, we emphasize the time dimension because consumption of commodities is a recurrent activity: it is repeated time period after time period. In the more complicated models of later chapters in which people can purchase long-lived assets, this distinction becomes crucial.

Because the prices most familiar to people are prices expresses as how much of a particular currency it takes to purchase a unit of some good, we will start our analysis using these **nominal** or, equivalently, **currency** prices. While the actual currency used in the POW camps was cigarettes, a commodity itself, we will use the more familiar dollars when dealing with currency prices.

The currency prices of tea and coffee are denoted P_T and P_C, respectively. They represent how many units of a currency, dollars for our example, that are exchanged for one unit of tea and coffee, respectively. This nomenclature is familiar to everyone.

Finally, we symbolize the income of this individual, measured in currency, as Y_i. Like with T_i and C_i, this quantity should be thought of as relative to a fixed interval of time, that is, a decision period such as a month. Because we are effectively considering one period in our analysis we will not make this explicit again.

The budget constraint is the mathematical statement that expenditure on tea plus expenditure on coffee, at market prices, must be less than or equal to income:

$$P_T T_i + P_C C_i \leq Y_i$$

Since we are abstracting from borrowing or lending and we assume people do not become satiated with consumption before their income is exhausted (we formalize this property of preferences below), it is innocuous to assume that each individual will always spend their entire income. Mathematically, this means the budget constraint will hold with exact equality:

$$P_T T_i + P_C C_i = Y_i \tag{1}$$

Of the things symbolized here, we consider P_T and P_C to be exogenous *to the individual*, that is, to be variables whose values the individual assumes are unaffected by his or her own decisions about how much tea and coffee to buy. In our POW camp Y_i is also exogenous *to the individual* as it will represent the **market value** of a Red Cross packet, and the packet is not the subject of choice for an individual. This reduces our list of endogenous variables to T_i and C_i. Note also, that only one of these choices is independent. That is, if I chose how much tea to consume, given the market prices of tea and coffee and my nominal income, I have effectively also chosen the amount of coffee to consume.

Graphically, we can represent all pairs of $(C_i,\ T_i)$ that satisfy the above budget constraint for any given prices P_T and P_C and income Y_i as a straight line in a two-dimensional graph. To understand how this is done, first rearrange the equation so that it is in standard slope-intercept form, with T_i on the left-hand-side[2]:

$$T_i = \frac{Y_i}{P_T} - \frac{P_C}{P_T} C_i \tag{2}$$

For particular values of $P_{T,}$ P_C, and Y_i, the graph of this equation has vertical intercept equal to $\frac{Y_i}{P_T}$, horizontal intercept equal to $\frac{Y_i}{P_C}$, and slope equal to $(-\frac{P_C}{P_T})$. All the pairs of T_i and C_i on this negatively-sloped line exactly exhaust the budget. Points below the budget line leave some money unspent. The points

[2] We could have put C_i on the left-hand-side instead of T_i. There is no significance to which variable we plot on the vertical axis and which on the horizontal. We will be consistent through the text to avoid confusion.

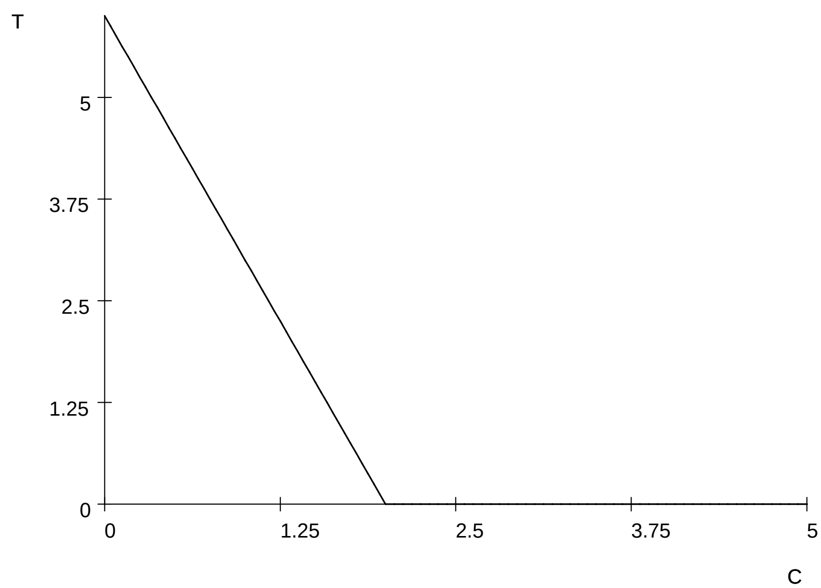

Figure 1: Budget constraint

on the line and below the line are **feasible** consumption points. However, we focus on points on the budget line because any point below the budget line involves unused resources and would never be chosen.

An example of this equation is presented in Figure 1, where we have set $P_C = \$6$, $P_T = \$2$, and $Y_i = \$12$. Hence, in this graph the vertical intercept equals $\frac{Y_i}{P_T} = 6$ units of tea/unit of time and the slope equals $-\frac{P_C}{P_T} = -3$ units of tea/unit of coffee.

When written in standard slope-intercept form, both the intercepts and the slopes represent important economic concepts. Now, P_T represents how many dollars it takes to purchase one unit of tea, and P_C represents how many dollars it takes to purchase one unit of coffee. The quantity $\frac{Y_i}{P_T}$ thus measures income in units of tea. What $\frac{Y_i}{P_T}$ tells us is the maximum amount of tea this individual's nominal income could purchase at nominal price P_T. We refer to this as individual i's **real income** *measured in units of tea.* By the same reasoning, $\frac{Y_i}{P_C}$ measures individual i's **real income** *measured in units of coffee.* The modifier "real" is used to indicate that the income is measured in terms of a "real" commodity, not in terms of a currency.

The distinction between real and nominal variables is one of the most important you will encounter in all of economics. It is important because modern currencies are pieces of paper and coins that have almost no intrinsic value. That is, their value arises because they serve as a medium of exchange. While money does have alternative uses, as when coins are thrown into fountains or pressed into medallions, or when dollar bills are rolled up and used as straws,

Concept check: Suppose an individual earns \$150/week, and music downloads cost \$1.50/unit. What is this individual's real income measured in units of music downloads?

folded as origami, or used as wallpaper, these uses are dwarfed in importance by its use as a medium of exchange. Because of this feature, the demand for money must be treated differently than the demand for other commodities. We do not take up this aspect of money in this chapter, because it turns out not to be relevant for our model.[3]

Now consider the absolute value of the slope coefficient: $\frac{P_C}{P_T}$. The units of P_C and P_T are dollars per unit of coffee and dollars per unit of tea, respectively. Thus the units of $\frac{P_C}{P_T}$ are units of tea per unit of coffee. For example, if P_C were \$6/unit of coffee and P_T were \$2/unit of tea, then $\frac{P_C}{P_T} = 3$ units of tea/unit of coffee.

$\frac{P_C}{P_T}$ is known as the **relative** price of coffee in terms of tea. It tells us the rate at which an individual *can* trade-off tea for coffee along his or her budget constraint. For the above example, this rate would be three (3) units of tea for every unit of coffee.

An alternative way of thinking about a relative price is to contemplate spending a fixed amount of dollars (your nominal income) on coffee and tea. To buy more coffee, you necessarily must buy less tea. For example, if, as above, P_C = \$6/unit of coffee, P_T = \$2/unit of tea, and Y_i = \$12.00/unit of time, various combinations of coffee and tea you could purchase could be displayed in the following chart:

coffee	0	1	2
tea	6	3	0
total expenditure	\$12	\$12	\$12

This chart shows that if you contemplate an initial purchase of no coffee and 6 units of tea, then to move from this initial point to a purchase of one unit of coffee, you must forgo 3 units of tea. Likewise, starting from a point of 1 unit of coffee and 3 units of tea, if you purchased one more unit of coffee, you would have to forego another 3 units of tea.

We emphasize real incomes and relative prices as opposed to nominal income and nominal prices because these real variables are the endogenous variables of our soon-to-be-developed full-blown general equilibrium models. This focus on real, as opposed to nominal, variables also reflects the understanding that what ultimately matters for consumers is quantities of goods and services consumed, not the number of dollar bills in their wallets. Those dollars are only a means to an end.

Incomes in the Endowment Economy What is an individual's income in a POW camp? In the United States, the sources of an individual's income includes wage payments, capital gains and dividends on financial assets, gains from the sale of physical assets such as real estate and commercial property, transfer payments and gifts. In the POW camp, no one works and no one has access to financial assets or property located outside the physical boundaries of

[3] You may recall from Chapter 2 that economists frequently break their analyses of economic phenomenon into "real" and "monetary" parts. This model is a "real" model and is independent of monetary phenomenon.

the camp. What the POWs have to consume or trade is their Red Cross packages. These are the real world equivalent of what economists call endowment income. Since every prisoner in the camp has the same package, everyone has the same income. The key things we would like to understand are: what determines the value of the package, what determines what individuals will trade, and how individuals are differentially affected by the opportunity to trade.

For the POW camp example, we can think of POW's receiving their packets of endowments of tea and coffee from the Red Cross, and then taking these packets with them to a central marketplace. We will denote such a pair of endowments as the ordered pair $(\overline{C}_i,\overline{T}_i)$, and call it an **endowment point**. At the central marketplace, an auctioneer is assumed to call out nominal prices in an attempt to find a price that clears the market. As the auctioneer calls out these nominal prices, the individual calculates the value of his or her endowment at these prices. This value is the individual's nominal income. Symbolically, we thus express this nominal income as:

$$Y_i = P_T\overline{T}_i + P_C\overline{C}_i \tag{3}$$

where again, $\overline{T}_i$ and $\overline{C}_i$ represent the exogenously given quantities of tea and coffee that individual i receives from the Red Cross. We can also measure income in units of coffee as :

$$\frac{Y_i}{P_C} = \overline{C}_i - \frac{P_T}{P_C}\overline{T}_i \tag{4}$$

or in units of tea as

$$\frac{Y_i}{P_T} = \overline{T}_i - \frac{P_C}{P_T}\overline{C}_i \tag{5}$$

Manipulation of the Endowment Budget Constraint Diagram The budget constraint for individual i in an endowment economy is thus expressed symbolically in standard slope-intercept form as:

$$T_i = \{\overline{T}_i + \frac{P_C}{P_T}\overline{C}_i\} - (\frac{P_C}{P_T})C_i \tag{6}$$

The graph of this budget constraint has slope $(-\frac{P_C}{P_T})$ and intercept $\{\overline{T}_i + \frac{P_C}{P_T}\overline{C}_i\}$. The slope is equal to minus the value of the relative price of coffee, and the intercept is the value of real income measured in units of coffee.

A key feature of this line is that it goes through the endowment point $(\overline{C}_i, \overline{T}_i)$. This is most easily seen if we rearrange the budget constraint in the following fashion:

$$T_i - \overline{T}_i = (\frac{P_C}{P_T})[\overline{C}_i - C_i]$$

Clearly, when $T_i = \overline{T}_i$ and $C_i = \overline{C}_i$, the equation is satisfied.

In preparation for development of the full-blown model of consumer behavior, consider how the graph of the budget constraint changes in response to changes in the variables that we will treat as exogenous to the individual consumer.

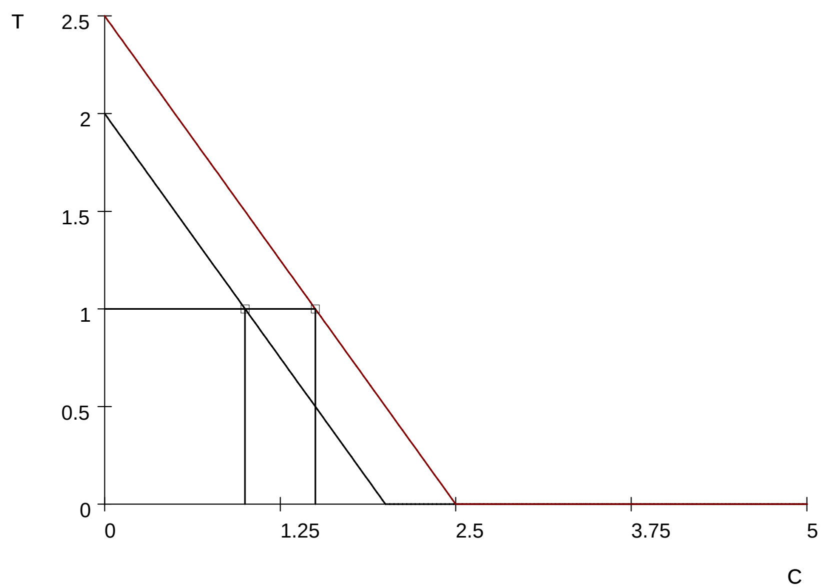

Figure 2: Two budget constraints: $\overline{C}_i = \overline{T}_i = 1; \overline{C}_i = 1.5, \overline{T}_i = 1$

First consider a change in endowment with the relative price of coffee $\frac{P_C}{P_T}$ assumed unchanged. This means that the slope of the budget constraint is unchanged. Hence, an increase in endowments simply "shifts" the budget constraint in a parallel fashion. That is, the new budget constraint associated with the different level of endowments but with unchanged relative prices has different intercepts but the same slope as the initial budget constraint. Two budget constraints that have different endowment points are depicted in Figure 2, where the budget constraint that goes through the endowment point $(1.5, 1)$ is parallel to but farther from each axis than the one that goes through the point $(1, 1)$.

Now consider an increase in the relative price of coffee, $\frac{P_C}{P_T}$, but with the endowment point unchanged. Because the budget constraint must go through the endowment point, the line that depicts the constraint "rotates" around the endowment point as relative prices change. In Figure 3, the effect on the budget constraint depiction of an increase in the relative price of coffee is shown by having the constraint with the lower initial relative price drawn with a flatter slope while the constraint with the higher relative price has a steeper slope. We have denoted the two relative prices "P_1" and "P_2", where $P_1 < P_2$. We will usually denote a relative price as p, written in lower case and written with no subscript used to denote the commodity associated with a particular nominal price. In the Figure, the subscripts "one" and "two" simply denote that we are talking about two different **relative** prices, *not* the nominal price for good

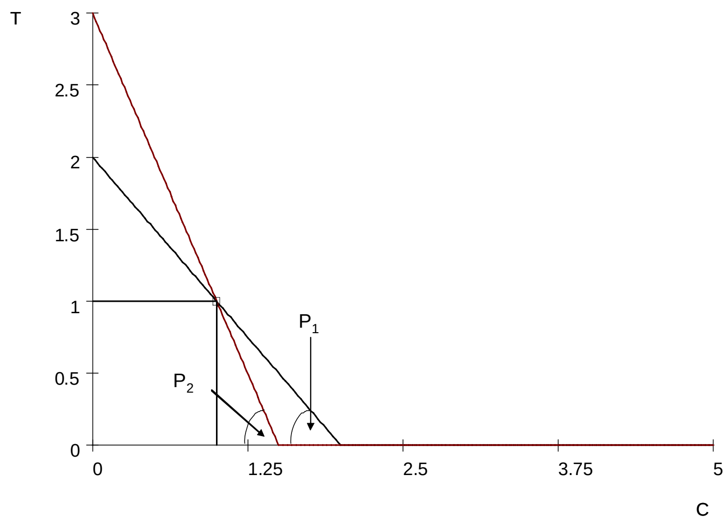

Figure 3: Two budget constraints: $\overline{C}_i = \overline{T}_i = 1; P_1 = 1; P_2 = 2$

"one" and good "two."

4.4.2 Preferences

First we will consider how economists think about preferences. All of us probably have intuitive notions of what it means for someone to prefer one thing to another, and these notions indeed are part of economists' model of preferences. To make progress, though, in analyzing how these preferences affect the workings and characteristics of a system with interactions between a variety of individuals, economists have had to develop a more explicit and better articulated model of peoples' preferences. This model can be easily criticized as having assumptions that appear violated by many real-world circumstances. Nonetheless, the model has been extremely useful in making predictions about behavior that have been verified in the real world again and again. Furthermore, the model embodies one of the epistemic virtues of science: it has opened up fruitful approaches to more thorny problems such as the influence of group norms, of advertising, and of habit on consumer choices.[4]

Our development of this approach will be brief, and thus limited to the graph-friendly two-good case. We simply note that the approach applies to as many goods as one can imagine.

[4] The epistemic virtues of science that help us evaluate the usefulness of theories include not just prediction, but also fertility in the sense of success in opening up new areas of research.

Preference Relations The basic idea behind the economists' model of preferences is that individuals are capable of making a particular type of comparison between *pairs* of bundles of goods and services. That is, when one of Radford's POW's was confronted with two distinct bundles of things, such as bundle A consisting of, for example, three units of tea, three units of coffee, four units of cigarettes, and one unit of chocolate, and bundle B consisting of three units of chocolate, three units of cigarettes, four units of coffee, and two units of tea, he was assumed to be able to say one of three things: whether he would *prefer* to consume bundle A rather than bundle B, or whether he would *prefer* to consume bundle B rather than bundle A, or whether he would be *indifferent* between consuming bundle A or consuming bundle B. When someone is indifferent between two bundles, we say that for such an individual each bundle is "at least as good as" the other bundle.

The reason we say individuals can make "a particular type" of comparison is that the comparison is not about *how much* more or less an individual likes one bundle vis-a-vis another, but only *whether* they like one bundle more or less than another. Such a qualitative as opposed to quantitative comparison is known as a *rank-order* or, in equivalent language, an *ordinal* comparison.

Note that each bundle consists of as many individual things as are conceivable for a person to consume. In a modern economy, this can be quite a large number of things. As noted, for expositional purposes, we will almost always restrict our attention to the artificial case of two commodities. This lets us present our models as diagrams. As we have noted, for most people diagrams in conjunction with the equations they represent are more readily understood than a purely mathematical treatment. Little is lost from a more general treatment by this restriction to "picture-friendly" examples.

The diagrammatic representations of our model of consumer preferences will be presented on a graph on which one axis will measure units of tea per unit of time and on which the other axis will measure units of coffee per unit of time. As with our analysis of budget constraints, It doesn't matter on which axis-the vertical or the horizontal-we measure which variable, but, to be reader-friendly, we will always be consistent and put tea per unit of time on the vertical axis (the y-axis or axis of *ordinates* in the standard terminology of analytic geometry) and coffee per unit of time on the horizontal axis (the x-axis or axis of *abscissas* in the terminology of analytic geometry). Because there are no negative quantities of tea and coffee, the diagram will be the northwest quadrant (quadrant I) of the coffee-tea Cartesian plane. This is indicated in Figure 4 as the striped area.

Each point in the plane represents a bundle of coffee and tea about which an individual has preferences vis-a-vis other bundles. We will denote any such point by the ordered pair (C_i, T_i), where the first element of the pair represents the contemplated amount of coffee per unit of time and the second element represents the contemplated amount of tea per unit of time. The subscript identifies the quantities as being associated with a particular individual.

The set of all pair-wise comparisons over all possible consumption bundles for any individual is known as his or her *preference relation.* There are two features of this concept that need emphasis as they are somewhat counter -

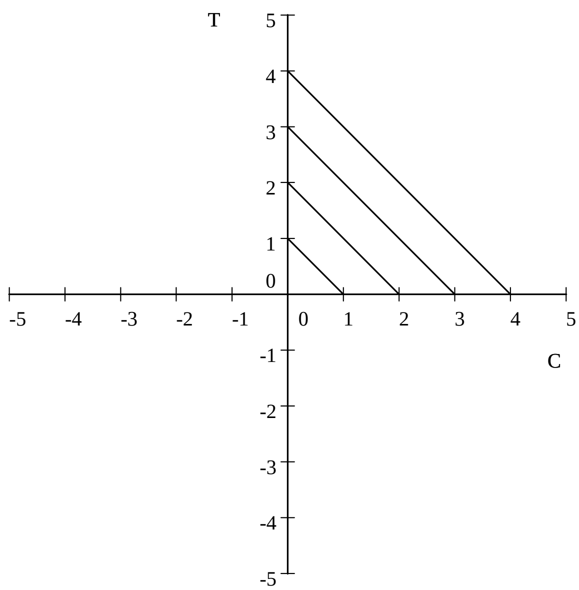

Figure 4: The coffee-tea plane

intuitive compared to our non-technical ideas about what it may mean to prefer one thing to another.

First, this model treats preference relations as exogenous, that is, as specified from "outside the model." This implies that the above pair-wise comparisons are not influenced by what other people think, or by what other people consume, or by advertising, or by the prices of the various goods, or by one's own income. Of course, this assumption clearly ignores important aspects of peoples' real-world experiences, and is thus incapable of helping us understand some interesting questions. It has, though, proven to be an enormously fruitful abstraction, allowing economists to understand an enormous range of consumer-choice phenomena.

Second, in an important sense, these comparisons are about hypothetical situations: it asks the question: *If* income were no object, which of these two bundles would you prefer? It also assumes that preferences are *independent* of prices. The conceptual separation of tastes from prices and incomes is what gives this model so much of its predictive power.

And once more, one other feature of this concept also must be emphasized: the comparisons between pairs don't tell us *how much* more or less an individual likes one bundle vis-a-vis another. It simply tells us whether one bundle is better, worse, or the same in terms of desirability vis-a-vis another bundle.

Axioms of Choice and Indifference Curves With this notion in mind of what it means for someone to have preferences, i.e., that one can make pair-wise comparisons between bundles of commodities, we can list the four assumptions ("axioms" in the more formal language of mathematics) that form the economists' model of individual preferences. With only these four assumptions, derived from introspection and observation and viewed by most people as quite plausible, economists have built a model of consumer choice that has both made empirical predictions born out by many observations and that has allowed them to tackle such thorny problems as "the gains from trade."

The first assumption is that an individual can make such a pair-wise comparison between *all* conceivable bundles of commodities he or she may consume. The technical term that describes this assumption is that such a preference relation is **complete**.

The second assumption is that, in comparison to some particular bundle–call it the "initial" bundle–a commodity bundle that has more of all of the commodities than does the initial bundle is always preferred to the initial bundle. We might call this the **more is better** assumption. A more-impressive sounding title is the axiom of **non-satiation**.

The third assumption is that a person's pair-wise comparisons exhibit a certain type of rationality called **transitivity**. This means that for any three bundles, say A, B, and C, if A is at least as good as or preferred to B and B is at least as good as or preferred to C, then A is at least as good as or preferred to C. For example, imagine our POW camp in which the only available goods are coffee and tea. Now imagine a prisoner who prefers a bundle with one unit of coffee and one unit of tea (call this bundle A) to a bundle with 1.1 units of coffee and 0.80 units of tea (call this bundle B). This prisoner also prefers bundle B (1.1 units of coffee and 0.80 units of tea) to a bundle with 1.2 units of coffee and 0.75 units of tea (call this bundle C). For this prisoner's preference relation to be transitive, it must be that he prefers bundle A to bundle C.

Finally, we have the axiom of **diminishing marginal substitutability**. This means that, starting with any hypothetical bundle of goods that we may want to consider, if we imagine taking away successive units of one of the commodities in the bundle and adding more of one other commodity in the bundle *so as to keep the person indifferent between these successive bundles*, then it takes more and more of the second commodity for each successive reduction by one unit of the first commodity.

When we abstract from indivisibilities, these four assumptions imply that we can graphically depict, i.e., visually represent, a preference relation by a family of non-intersecting, downward-sloping continuous curves known as an **indifference map**. The individual curves that make up this family are known as **indifference curves**. Each curve represents the collection of all those coffee - tea pairs among which the individual in question is indifferent. Other members of the family are different indifference curves passing through combinations of goods that are strictly more or less preferred to combinations found on other indifference curves: family members farther away from the origin than some other member, i.e., a "reference" member, have combinations of goods that are more preferred to any of those on the reference curve, while family members closer to the origin than the reference member have combinations of goods that

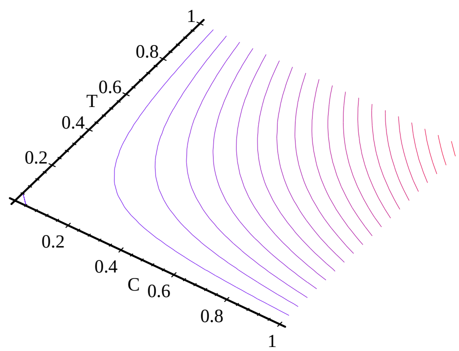

Figure 5: An indifference map

are less preferred than any of those on the reference curve. Furthermore, such indifference curves not only slope down, but the slope of the curve gets flatter and flatter, i.e., less in absolute value, as we move farther along the indifference curve in the direction of increasing quantity of coffee. A curve with such properties is described as *convex* to the origin. A convex indifference curve is one that is "bowed into" the origin. In Figure 5 we depict some members of such a family as viewed from above the $C-T$ Cartesian plane.

What is sometimes difficult to grasp is that we can always "draw," i.e., fit, more non-intersecting indifference curves between any two we have depicted. This reflects both a feature of analytic geometry - the abstract "point" in the Cartesian plane takes up no space - and the axiom of completeness: an individual is assumed to be able to compare every possible pair of bundles of commodities, which are represented by every possible point in the positive quadrant of the coffee-tea plane. Again, the abstraction of measuring variables as real numbers is a convenience and is not representative of economic reality. What we have done is abstract from the essential discreteness of most items that people purchase. People don't buy fractions of newspapers, or Coca-Colas, for example. This abstraction allows us to use the powerful techniques of analytic geometry in the development and the solutions of our models, and is another example of what economists call an "innocuous" assumption.

Want to know more? See Appendix A.

This abstraction has one other important implication: when, as we frequently do, discuss the thought experiment of changing a particular variable by "the smallest possible unit," we really mean that we are changing it by an infinitesimally small amount. This is because there is no "smallest possible amount" when things are measured along the real number line: no matter how small a number you pick, there are always more numbers between that number and zero.

The marginal rate of substitution *function* One last point needs emphasis. The assumption of a diminishing marginal rate of substitution makes use of the *concept* of the marginal rate of substitution: it is the slope of an indifference curve, and it tells us about the willingness of an individual to exchange one thing for another. It is important to remember that the slope of an indifference curve is a *number*, which means it is a variable. This number in general depends on the point of the indifference curve at which the slope is being measured. Consequently, we can always form the ordered triplet in which the first element is the number that is the marginal rate of substitution and the second and third elements are the values of C_i and T_i at that point on the indifference curve at which we are measuring the slope. The set of all such ordered triplets for all permissible values of C_i and T_i is a three-variable function. We could write this in general functional notation as

$$MRS_{T_i C_i} = mrs_i(C_i, T_i) \tag{7}$$

where we name the function mrs_i.[5] It is important to realize that the particular value of the marginal rate of substitution along an indifference curve is something determined by the choice of particular values of C_i and T_i.

Inquiring minds want to know: For the mathematically minded

For example, in the Appendix we use the indifference curve defined as all those pairs of $C_i$and T_i that satisfy the equation $T_i = \frac{1}{(C_i)^3}$. In that example, the marginal rate of substitution function is

$$mrs_i(C_i, T_i) = \frac{3T_i}{(C_i)^4}.$$

For calculus users, this is found by taking the total derivative of the implicit function $1 = (T_i)(C_i)^3$ and solving for $\frac{\partial T_i}{\partial C_i}$.

How should we usefully think about the marginal rate of substitution, and what is most important about this concept? First, we can heuristically think of the marginal rate of substitution at some point as the rate at which someone is willing to exchange an amount of one thing for "the smallest possible unit" of another. Second, what is useful about the concept of the marginal rate of substitution is its interpretation as the *willingness* as opposed to the *capability* of an individual to make an exchange of one thing for another. We can infer that this is a willing exchange from the fact that the hypothetical exchange leaves him or her on the same indifference curve.

Some key points about indifference curves and their slopes For the future development of our model of consumer equilibrium and for the future

[5] It may seem odd to name a function by three letters instead of one as in the more usual notation such as $y = f(x)$, where the function name is f, but the use of mrs is designed to be a mnemonic device. Furthermore, in this model as well as in most economic models, we will eventually have too many functions for them all to have one - letter names!

development of “gains from trade,” a few features of indifference curve slopes are important. First, the tangent line at some point on a smooth indifference curve - whose slope is defined as the slope of the indifference curve at that point - has a key feature: *it just touches that indifference curve at that point, and touches it at no other point.* This means it does not “cut” the indifference curve: all points on the tangent line except for the point that touches the indifference curve are below the indifference curve, i.e., the vertical coordinate of any point on the line at some value of the horizontal coordinate variable is always less than the vertical coordinate of a point on the indifference curve evaluated at the same value of the horizontal coordinate variable.

This also means that any line through a point on an indifference curve other than the tangent line would “cut” the indifference curve: it would intersect the indifference curve at least one and perhaps two distinct points, and there would be points on such a line both above and below the indifference curve.

What does it mean for different people to have different preferences? How do we capture the idea that two people have different preferences? The key idea here is that when we describe someone’s preferences, we can only really say how much of one thing a person is willing to trade off for some amount of another good. That is, intensity of preference can only be thought of as a relative concept. To say that Andy prefers coffee to tea is meaningless. What we can say is: for any possible bundle of coffee and tea, Andy’s preference relation tells how much tea from his initial amount he would be willing to exchange for a small amount of extra coffee. That is, we can say what his marginal rate of substitution is at that initial point.

When we want to compare two individuals, then, we want to compare their marginal rates of substitution. For example, if at some point, say $(2, 1)$, Andy’s marginal rate of substitution of tea for coffee were 3 (three), we would say: at $(2, 1)$, Andy would be willing to exchange tea for coffee at the rate of three (infinitesimal) units of tea for one (infinitesimal) unit of coffee. Now, suppose Bob’s marginal rate of substitution at $(2, 1)$ were $\frac{1}{3}$ (one-third). Then we would say that Bob would be willing to exchange tea for coffee at the rate of $\frac{1}{3}$ ("smallest possible units" or "infinitesimal") units of tea for 1(infinitesimal) unit of coffee.

The question we can now ask is: which one of these people likes coffee relative to tea more so than the other person: Andy or Bob? To stay on his indifference curve, Andy is willing to give up three units of tea for one unit of coffee. Bob, on the other hand, is only willing to give up one-third unit of tea for one unit of coffee to stay on his indifference curve. Clearly, to maintain the same level of satisfaction, Andy is willing to give up more tea for one more unit of coffee than is Bob. The natural way to classify these two people is with Andy as the “coffee-lover” and Bob as the “tea-lover.” The distinguishing characteristic that leads to this classification is the relative slopes of each individual’s indifference curve vis a vis the other. Andy, the coffee-lover, has a steeper indifference curve at the point of comparison than does Bob (where coffee is on the horizontal axis and tea on the vertical axis).

Inherent in this classification is that if Andy is the coffee-lover relative to Bob, then Bob is the tea-lover relative to Andy. Also, there is no way to say Andy likes coffee better than Bob in any absolute sense: *there is no way to measure "how much" better off Andy would be than Bob if they both had an extra unit of coffee, and we make no attempt to do so.* All we do is measure the relative trade-offs of coffee for tea each individual would be willing to make so as to keep each of them on the indifference curve that went through their initial point of comparison.

Section recap We have now constructed the first piece of the sub-model of an individual consumer's demand. Individuals are assumed to have a preference relation that allows them to rank-order all conceivable bundles of commodities. Any bundles that share the same rank-order number are members of set that we call an indifference curve. Indifference curves slope down, are convex to the origin, and do not intersect each other. Furthermore, through every commodity bundle runs an indifference curve. All of these indifference curves constitute an indifference map.

4.4.3 Consumer's optimal choice

The preceding sections have depicted an individual's preferences by means of an indifference map in the coffee-tea plane, and have depicted this individual's feasible purchases of coffee and tea by the straight-line budget constraint in the coffee-tea plane. We now have to describe the conditions that must hold if this person is in fact choosing the most-preferred bundle of coffee and tea from among those on her or his budget constraint. When these conditions are met, we say that the consumer is in equilibrium, i.e., is at a "rest point" from which she or he would not move unless there is a change in some exogenous feature of the consumer sub-model.

Describing and depicting a consumer optimum In Figure 6, we overlay a representative budget constraint on a POW's indifference curve map. As we look along his budget constraint, clearly his most-preferred choice is the bundle at which his budget constraint "just touches", i.e., is tangent to, an indifference curve. At any other point, the budget constraint "cuts" an indifference curve. Hence, by moving along his budget constraint away from such a point, he could hit a higher indifference curve. Only when he is at a point on his budget constraint that is tangent to an indifference curve does any movement away from such a point put him on a lower indifference curve. Thus, of all the pairs of coffee and tea this POW *could* consume (for a particular relative price and particular endowment), the most-preferred pair is the one at which the budget constraint just touches, or, in more formal language, is tangent to, an indifference curve.

As noted, we denote such a point a *consumer equilibrium*: unless there is an exogenous change of some sort such as a change in the relative price of coffee

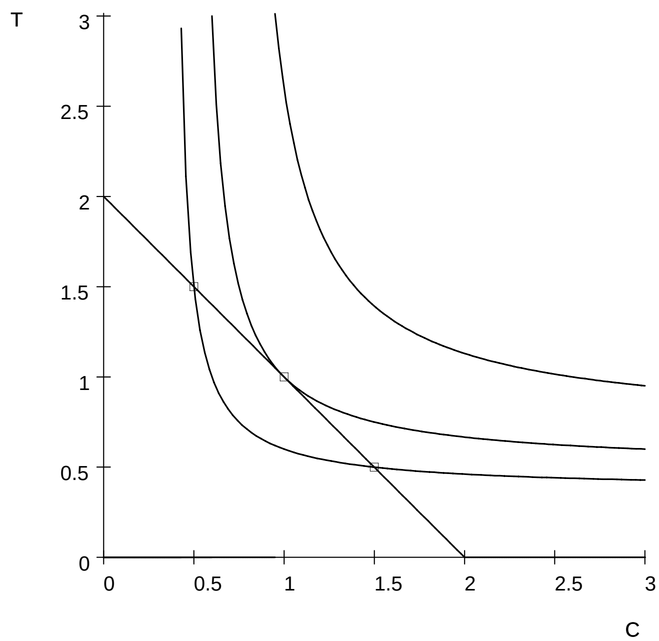

Figure 6: Consumer equilibrium

(which would rotate the budget constraint around the endowment point) or a change in endowments (which would shift out the budget constraint in a parallel fashion), the consumer will continue to choose the same most-preferred pair.

Another way of thinking about why this point of tangency between budget constraint and indifference curve is most preferred is to remember the interpretations we give to the slopes of these two curves. The slope of a budget constraint tells an individual the rate at which he or she *can* trade tea for coffee in the marketplace. The slope of an indifference curve describes an individual's *willingness* to trade tea for coffee so as to remain indifferent between remaining at one point or moving away from that point. If an individual is not at a point on his or her budget constraint where these two things are equal, a small movement along the budget constraint will make such an individual better off.

The description of consumer equilibrium is thus a pair of equations: a **tangency condition** that expresses the consumer's choice of a consumption bundle–a value of coffee and a value of tea in our examples–that makes his or her marginal rate of substitution just equal to the (exogenous to the consumer) relative price; and the budget constraint, another relationship between p, the relative price, and the endogenous or choice variables of the consumer. The budget constraint is symbolically represented by equation (9), which we replicate here with the notation $p = \frac{P_C}{P_T}$:

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i. \tag{9}$$

The tangency condition is symbolically represented as:

$$p = mrs_i(C_i, T_i); \tag{10}$$

Equation (10) describes a collection of points, namely coffee-tea pairs that lie on an indifference curve at a point where the slope of the indifference curve equals p. This collection of points is one curve in the (C_i, T_i) plane. Examples of such curves are depicted as straight-line rays from the origin in Figure 7[6]. The flatter ray is associated with a lower value of p. These curves need not be straight lines.

[6]For preferences that satisfy the four axioms of choice such a curve could have a variety of shapes, but the presumption usually made by economists is that the curve is, if not non - negatively sloped, at least steeper than $-p$. Our figure displays curves that are straight lines through the origin, but should not mislead anyone into thinking that such curves *must* be straight lines.

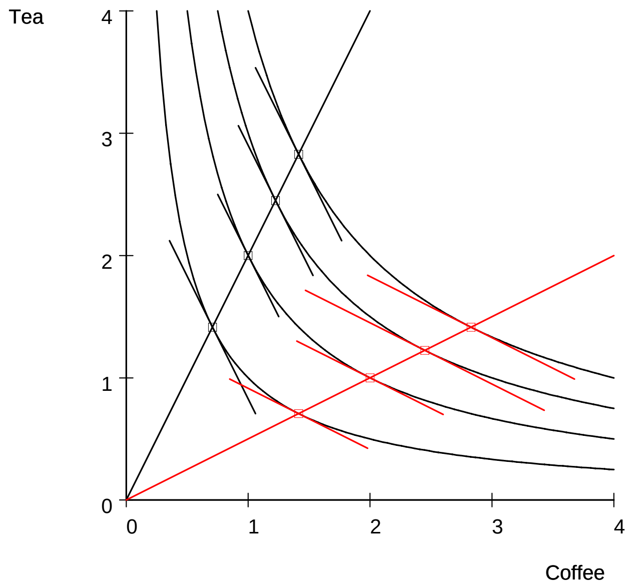

Figure 7: $p = mrs(C, T)$

Inquiring minds want to know: For the mathematically minded

The two tangency condition curves depicted in Figure 9 are in fact the following two **parametric example**s of the functions described by $p = mrs_i(C_i, T_i)$:

$$\overbrace{2}^{p} = \overbrace{(C_i)^{-\left(\frac{1}{2}\right)} (T_i)^{\left(\frac{1}{2}\right)}}^{mrs_i}$$

$$\overbrace{\frac{1}{2}}^{p} = \overbrace{(C_i)^{-\left(\frac{1}{2}\right)} (T_i)^{\left(\frac{1}{2}\right)}}^{mrs_i}$$

Note that each of these equations can be rewritten with T_i "solved out" so as to make clear they are represented by straight lines through the origin in the tea-coffee plane:

$$T_i = \overbrace{4}^{p^2} C_i$$

$$T_i = \overbrace{\frac{1}{4}}^{p^2} C_i.$$

The budget constraint, as noted, describes another collection of points in the coffee-tea plane. The intersection of these two collections of points is the

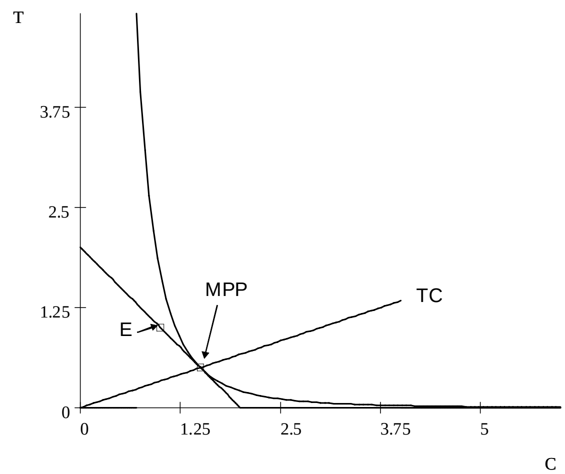

Figure 8: Consumer equilibrium again

unique point of consumer equilibrium. Such an intersection is depicted in Figure 8, along with the associated indifference curve that is tangent to the budget constraint at this most-preferred point. The collection of points that constitute the tangency condition is labeled "TC", and an endowment point is depicted as well, labeled "E." The most-preferred point is labeled "MPP."

Associated with every equilibrium pair is a value for another endogenous variable: the number of the indifference curve of which an equilibrium pair is a member, i.e., the rank-order, according to the individual's preference relation, of the equilibrium pair among all imaginable consumption pairs.

Solution of the sub-model. We can now describe and depict the solution of the sub-model of the individual consumer. That is, we can characterize the relationship between the exogenous (to the individual) variables $p, \overline{T}_i$, and $\overline{C}_i$, and the endogenous variables C_i^d, T_i^d, and U_i^d. We put the superscript "d" on C_i , T_i , and U_i to identify and emphasize that these are equilibrium values for the submodel of consumer demand.

Our solution strategy will be to use the conditions of consumer equilibrium to characterize the relation between the equilibrium values of C_i and T_i and the exogenous variables, and then to determine the associated value of U_i. This strategy exploits what is known as the **recursive structure** of our sub-model: part of the model - the part concerned with the solution of C_i and T_i- can be solved independently of the part concerned with the solution of U_i. That is, we can solve for the equilibrium pair (C_i^d, T_i^d) associated with various values of the exogenous variable p without reference to U_i. Then, we can consult the individual's preference relation and rank-order his or preferences for these various equilibrium outcomes This recursive structure, a feature of many models, is something that we will exploit frequently.

In symbolic terms, the solution of this sub-model is a triplet of **reduced form** functions: one function for each endogenous variable. Each member of the triplet is a multivariate function that tells us the value of the endogenous variable associated with the each three-tuple $(p, \overline{T}_i, \overline{C}_i)$:

$$C_i^d = f(p, \overline{T}_i, \overline{C}_i) \tag{11}$$

$$T_i^d = g(p, \overline{T}_i, \overline{C}_i) \tag{12}$$

$$U_i^d = h(p, \overline{T}_i, \overline{C}_i) \tag{13}$$

Our focus here will be on the relationship between p and the endogenous variables (rather than between the exogenous variables $\overline{T}_i$ and $\overline{C}_i$ and the endogenous variables). This relationship is more important for our purposes of elaborating how general equilibrium models work and of demonstrating the economist's concept of gains from trade.

Equation (11) is known as the individual's **general equilibrium demand curve** or, equivalently, general equilibrium demand function, for coffee.[7] It answers the question: ceterus paribus, at such-and-such a relative price of coffee p,what is the equilibrium quantity per unit of time of coffee chosen by this individual? Some of you are familiar with this concept from introductory economics courses. Equation (12) also answers the same type of question: ceterus paribus, at such-and-such a price p, what is the equilibrium quantity of tea per unit of time chosen by this individual?[8]

Equation (13) is known as the **indirect utility function**. We include it here mostly to reinforce the distinction between different types of logical relationships between variables. As noted, this equation is **not** a structural equation: it is a reduced form. Conceptually, it simply allows us to keep track of the rank-orderings of the various optimal choices of a consumer at various prices. It also emphasizes that a consumer's well-being, as represented by order-preserving utility function numbers, depends on the price that actually prevails in the marketplace.

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[7] An ordinary demand curve is used in partial - equilibrium models and expresses the quantity demanded as a function of relative price and real income separately. In general equilibrium models such as we use here, real income is a function of relative price.

[8] By convention, though, this is not referred to as the demand curve for tea, because the argument in the function is the relative price of coffee. We could, though, transform equation (12) into a demand curve for tea by associating the value of tea chosen for any price p with the reciprocal of p, $\frac{1}{p}$. That is, wherever in the function we would find p, we would replace it with $1/\frac{1}{/p}$. The units of $\frac{1}{p}$ are units of coffee per unit of tea, and is denoted as the relative price of tea. This transformation would then relate the quantity of tea chosen to the relative price of tea, and we would call this transformed function a demand curve. Both functions yield the same information, though.

Our task is to *characterize* these reduced-form relationships. This characterization is most easily made with the aid of diagrams.

Depicting the consumer sub-model solution: demand functions and order-preserving indifference-curve numbers. We have already started construction of a demand function in the preceding discussion of consumer equilibrium. In that section, we depicted in Figure 9 a point of consumer equilibrium: the most-preferred pair of coffee and tea chosen for a particular relative prices. Thus, with such a point we have started to map the relationship between the exogenous (to the individual consumer) variable p and the endogenous variables C_i and T_i.

Now imagine that we contemplate varying p over all possible values between infinity (∞) and one (1), and identify the most-preferred pair (C_i, T_i) for each value of p. As p varies in this fashion, we can imagine the budget constraint rotating around the endowment point $(\overline{C}_i, \overline{T}_i)$ in a counterclockwise manner, starting from a vertical position ($p = \infty$) and ending in a horizontal position ($p = 0$). For each value of p, we could identify the optimal pair, denoted as $(C_i^d(p), T_i^d(p))$ as that one point at which the budget constraint is just tangent to an indifference curve. On a graph with p on one axis and C_i on the other axis, we could then plot the pairs (p, C_i^d) for every value of p. This set of ordered pairs would constitute individual $i's$ demand function for coffee.

Let us construct in more detail individual demand curves for two POW's, Andy and Bob. Two family members of the indifference maps for Andy and Bob are depicted in Figure 9, with Andy's indifference curves depicted as the steeper-sloped lines (IA_1 and IA_2) and Bob's depicted as the flatter (IB_1 and IB_2). The point to note here is that Andy and Bob have different preferences: Andy is relatively a "coffee-lover" while Bob is relatively a "tea-lover." We also plot the endowment point (EP), which is depicted as the same for both Andy and Bob, and the member of Andy's indifference curve family (IA_1) and the member of Bob's indifference curve family (IB_1) which go through this point. Finally, we plot two budget constraints, each associated with a different value of p, with $p_2 > p_1$, and the associated most-preferred bundles for both POW's (MPPA_1,MPPA_2,MPPB_1,MPPB_2), which are the points of tangency.[9]

Note that from looking at the diagram we can see that the most-preferred pair chosen by Andy when $p = p_2$ is on a lower indifference curve than the most-preferred pair chosen for $p = p_1$. For Bob, things are reversed: his most-preferred pair when $p = p_2$ is on a higher indifference curve than his most-preferred pair when $p = p_1$. While the points of consumer equilibrium reflect Andy's and Bob's best choices *for a given* p, their well-being is obviously affected by the particular value of p that occurs. In this case, because Bob is the coffee-lover relative to Andy, the outcome that we describe in which Bob is better off with the lower price of coffee and Andy is worse off seems intuitively correct. This observation that different values of p have different effects on

[9] One of Andy's (and Bob's) most - preferred bundles turns out to be his endowment point in this diagram. There is no special significance to this.

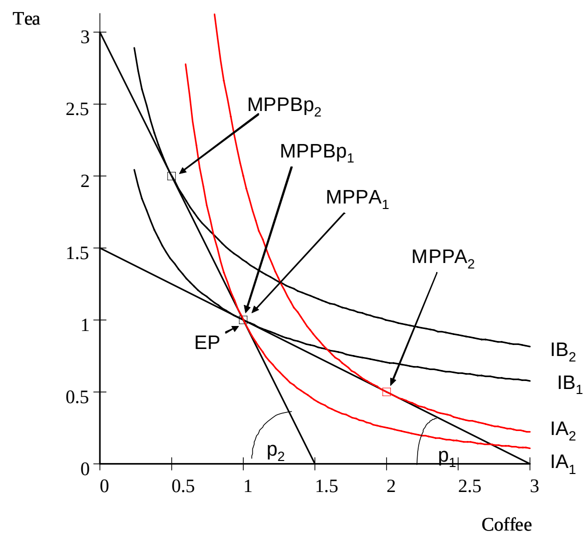

Figure 9: Andy and Bob: equilibria

different individual's well-being will be important in understanding the effects on individuals of free trade between different POW camps, the topic that we take up in the next chapter.

By varying p over all possible values between zero (0) and infinity (∞), we could imagine finding the associated most-preferred pairs for Andy and for Bob. We could then depict the information about the relation between the equilibrium quantities of coffee chosen by Andy and Bob at varying prices by plotting this relationship in the $p - C^d$ plane. These would be Andy and Bob's general equilibrium demand functions. In keeping with the tradition in economics, we of course will graph the exogenous variable p on the vertical axis, and hence we actually graph Andy's and Bob's *inverse* demand functions. Hypothetical inverse demand functions are depicted in Figure 10. In keeping with our above example in which Andy was the coffee-lover and Bob the tea-lover, we depict Andy's inverse demand curve ($\mathrm{C}^d(A)$)as farther away from the $p-$axis than Bob's ($\mathrm{C}^d(B)$. The inverse demand curves illustrate that at any price, Andy demands more coffee than Bob.[10]

These general - equilibrium demand curves *depict* the reduced-form coffee demand functions for Andy and Bob. Notice that we could also depict the graph of the optimal tea choices of Andy and/or Bob as a function of p, the

[10] There is no particular significance to the fact that the coffee demanded by Bob at the lower illustrated value of p is equal to the same quantity demanded by Andy at the higher illustrated value of p, although as the daugram shows it is possible.

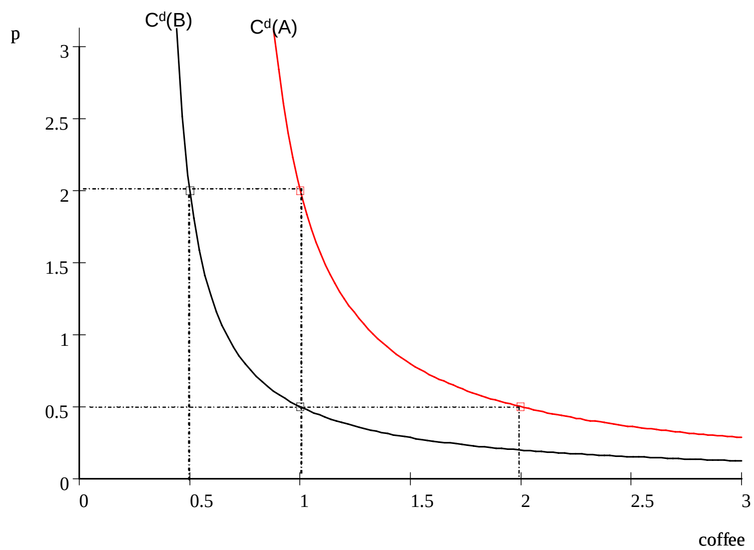

Figure 10: Andy and Bob' inverse demand functions

relative price of coffee. These diagrams would be the depictions of the reduced-form solution equations for the optimal choices of tea of Andy and Bob. It turns out we don't need these depictions to solve the entire model in terms of graphs, so we don't depict them.

Our hypothetical example depicts a *downward - sloping* inverse demand function for both Andy and Bob. Is this a necessary feature of inverse demand curves? If you have studied intermediate microeconomics, you may recall that demand curves need not slope down. Demand curves that do slope down are described as depicting a situation in which **substitution effects** outweigh **income effects**. For expositional ease, we assume that any situation we are concerned with is one in which substitution effects dominate income effects, which implies that our demand curves (and, by implication, inverse demand curves) slope down.

Demand curve "shifters" An individual's demand curve is a function of *both* the relative price p and the endowments, $\overline{C}_i$ and $\overline{T}_i$. The two-dimensional "picture-friendly" depiction of a demand curve thus makes the ceterus paribus assumption concerning the endowments: the placement of the curve in the (p, C_i) plane is made based on an assumption of constant values for $\overline{C}_i$ and $\overline{T}_i$. If we were to contemplate different values for these endowments, the placement of the demand curve in the (p, C_i) plane would be different.

Economic theory in its most general form tells us little more than that:

changes in endowments would change the placement of the demand curve. But empirical observation suggests that increases in endowments, which we know *shift out* the budget constraint in the (C, T) plane, usually lead to an *increase* in the most-preferred choice of *both* coffee and tea, or at worst *no decrease* in the most-preferred choice of either coffee or tea. This means that an increase in an endowment or endowments usually leads to a placement of the demand curve farther out along the horizontal axis than the initial position, or at worst to no change in placement.

For the mathematically inclined: a parametric example of an analytic solution.

Consider the following structural (sub)model of a consumer (say POW Andy):

$$p = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{T_A}{C_A}\right) \qquad (p = mrs_A)$$

$$T_A = p\overline{T}_A + \overline{C}_A - pC_A. \qquad \text{(Bud. constraint)}$$

where γ is a parameter that can take on any value between zero (0) and one (1), such as one-half of two-thirds. Notice that the tangency condition can be rewritten as

$$T_A = p\left(\frac{1-\gamma}{\gamma}\right)C_A.$$

The diagrammatic depiction of this tangency condition is thus a straight line through the origin of the tea-coffee plane with slope $p\left(\frac{1-\gamma}{\gamma}\right)$. After substitution of this expression for T_A into the budget constraint, the resulting equation can be rearranged as the demand curve for coffee:

$$C_A^d = (\gamma)\left(\frac{p\overline{T}_A + \overline{C}_A}{p}\right).$$

If, for example, $\gamma = \frac{1}{3}$, and $\overline{T}_A = \overline{C}_A = 1$, Andy's demand curve for coffee would be

$$C_A^d = \frac{1+p}{3p}.$$

Note that if $p = 2$ for this example, then $C_A^d = \frac{1}{2}$, and if $p = \frac{1}{2}$, then $C_A^d = 1$.

Substitution of the coffee demand function into the tangency condition (or into the budget constraint: either will do) yields the reduced form for the quantity of tea chosen by Andy as a function of endowments and relative price p:

$$T_A^d = (1-\gamma)\left(p\overline{T}_A + \overline{C}_A\right).$$

Again, if for example $\gamma = \frac{1}{3}$ and $\overline{T}_A = \overline{C}_A = 1$, then

$$T_A^d = \frac{2}{3}(1+p).$$

To complete solution of the reduced form, we need a structural equation that assigns order-preserving numbers to indifference curves. One such function that is consistent with preferences represented by the above marginal rate of substitution function is:

$$U_A = (C_A)^{\gamma}(T_A)^{1-\gamma}$$

The *indirect utility function* is found by substituting the reduced-form equations for coffee and tea into this utility function:

$$U_A = \left((\gamma)\left(\frac{p\overline{T}_A + \overline{C}_A}{p}\right)\right)^{\gamma}\left((1-\gamma)\left(p\overline{T}_A + \overline{C}_A\right)\right)^{1-\gamma}$$

4.5 Market Equilibrium: solving the complete autarkic model.

Having solved the sub-model of the individual consumer, we can now use this to make a sub-model of the entire demand side, and combine this with a sub-model of supply to get a solution to the complete endowment economy model. The first step is to complete our model of demand. We then specify the (extremely simple) supply sub-model. We connect these two sub-models with an equilibrium condition, and then solve the model.

4.5.1 Modeling a market

First, we should note the difference between how economists think about trade between just two or just a few individuals, and how they model more organized markets where many people are involved. When just a few people trade among themselves, economists call this is bargaining situation. For example, when my sister and I traded Halloween candy, we were striking a bargain in which we agreed to exchange a certain amount of one kind of candy for another. The determination of the actual rate of exchange, that is, how much hard candy I got in exchange for one piece of chewy candy, probably depended not just on the differences in tastes between the two of us, but also on our bargaining skills. My older sister had traits that I'm sure younger siblings everywhere recognize as older sibling traits: she could persuade me that her more numerous life experiences made her much more knowledgeable about what the appropriate rate if exchange should be, and she could beat me up and appropriate my candy if we failed to reach a bargain that satisfied her. In the parlance of economists, she had more bargaining power than did I.

While many exchanges in life are best viewed as the outcome of a bargaining process, the trading situations of most interest to the study of international economics are those in which buyers and sellers don't bargain in small groups but rather buy and sell in organized institutions such as markets. Most of us have experiences with what we might think of as the exemplar of a competitive market: some of us have attended or participated in auctions in which an auctioneer asks a large number of participants to bid on an item; some of us have been to a farmer's market in a city center, where many buyers and sellers are congregated in a small place, allowing people to easily compare prices among vendors; or perhaps some have observed a financial exchange such as the Chicago Board of Trade, in which buyers and sellers are again arranged in close proximity (a "pit"). In all of these situations, competition between the many buyers and sellers makes it clear that prices - the rate of exchange between one good and another at which transactions are carried out - are not dependent on any one person's bargaining ability, but rather are by and large exogenous to the individual participants and close to identical for everyone. Furthermore, these prices are approximately identical for each and every market participant.

What might seem odd to non-economists is that many less-centralized arrangements are also modeled by economists as if they took place in a centralized market or at an auction. For example, many purchases and sales such as take place in the retail book market, where someone may purchase a book at a large store such as Barnes and Noble, or such as take place in the retail grocery market, where people frequently do "one-stop" shopping at a large supermarket, are obviously not carried out in an auction or centralized market setting. Economists, though, model them as if they were. For economists, the crucial question that determines whether to treat a market as if it is like an open-cry auction or a centralized exchange as opposed to a bargaining situation is whether the prices and exchanges that take place can be viewed as exogenous by market participants, and whether these prices are close to the same for all participants.

The economist's basic model of a competitive marketplace is one in which suppliers and demanders come together in a one central location at which an auctioneer starts things off by "crying out" or posting a price. At this price, all demanders submit to the auctioneer truthful responses about how much they would demand at this price. Suppliers also submit truthful responses to the auctioneer about how much they would supply at this price. The auctioneer tallies up the total quantities that would be demanded at this price and the total quantity that would be supplied at this price. If total demand equals total supply, he collects the quantities promised from the suppliers and allocates them to demanders as indicated by their orders. We describe this as the market having achieved equilibrium.

If, on the other hand, at the initial price called out or posted by the auctioneer there is either excess demand - the sum of all the submitted responses by demanders is greater than the sum of responses from suppliers - or excess supply, then no trades are consummated, and the auctioneer tries another price. This process continues until the market is in equilibrium.

Of course, most markets don't work like this. In the grocery example, the store posts a price, and people buy, i.e., trades are consummated, at that price. If the store finds that inventories are falling, it may raise prices the following day, and if inventories are rising, it may lower prices. What economists have found is that their model of a metaphorical market, though it ignores the richness and complexity of real-world situations, does a good job of capturing how prices and quantities actually behave.[11]

Given that economists think that trading situations with only a few participants are different than situations with many participants, we should note that for pedagogical reasons we frequently create models in which we assume there are only a few participants but in which we also assume economic interactions between these individuals are governed by the above-described market mechanism. The assumption of only a few participants is a simplifying abstraction that lets us capture in the most clear possible manner the workings of a complicated system.

We now move on to develop the sub-models that we use to construct the demand and supply schedules that we assume people bring with them to the

[11] For a fascinating description of a variety of real-world markets, see *Reinventing the Bazaar: a Natural History of Markets* by John McMillan, 2002, W. W. Norton & Co., NY, NY.

marketplace.

4.5.2 Market demand curves

We have just constructed individual demand curves that answer the question: for any permissible value of p, what is the quantity per unit of time demanded by the individual in question? A **market**, or, in equivalent language, an **aggregate** demand curve answers the similar question: for any permissible value of p, what is the total quantity per unit of time demanded by all individuals in the market?

To answer this question, all we need do is add up all the individual quantities per unit of time demanded at some permissible value of p. This means that if we have demand curves for each individual, then for any value of p, say some particular value denoted by p_0, we add up the associated individual demands at that value of p, denoted by $(C_i^d)_0$, and create the new ordered pair $(p_0, \sum_{i=1}^{i=l}(C_i^d)_0$ where the symbol $\sum_{i=1}^{i=l}$ tells us to add up all the individual $(C_i^d)_0$ values, and where we symbolize the number of people in the economy by l. That is, if there are, for example, two (2) consumers in the economy, i.e., $l = 2$, we would add up $(C_1^d)_0$ and $(C_2^d)_0$.

To be more concrete, consider again Andy and Bob. We could substitute various values of p into their demand curves for coffee and we could construct the following hypothetical chart:

p	C_A^d	C_B^d	$C_A^d + C_B^d$
2	$\frac{1}{2}$	1	$1\frac{1}{2}$
$\frac{1}{2}$	1	2	3

All those ordered pairs $(p, C_A^d + C_B^d)$, i.e., $(2, 1\frac{1}{2})$ and $(\frac{1}{2}, 3)$, would be members of the market demand function. We could plot these points in the $C^d - p$ plane, and add other points for other values of p, and construct a graph of the inverse market demand curve. As one can see, to graphically construct an inverse demand curve we simply plot the ordered pairs which consist of any particular value of p and the associated sum of the individual quantities demanded at that value of p. This works for any market, no matter how many individuals there are in such a market. In Figure 11, Andy's ($\mathrm{C}^d(A)$) and Bob's ($\mathrm{C}^d(B)$) inverse demand curves are depicted, along with the market demand curve (C^d).

Mathematically, the procedure described above of adding all the individual quantities demanded at any particular price to form the market demand curve is written as:

$$C_i^d = f_i(p); i = 1, 2, ...l. \tag{14.i}$$

$$C^d \equiv \sum_{i=1}^{i=l} f_i(p) \equiv F(p) \tag{14.ii}$$

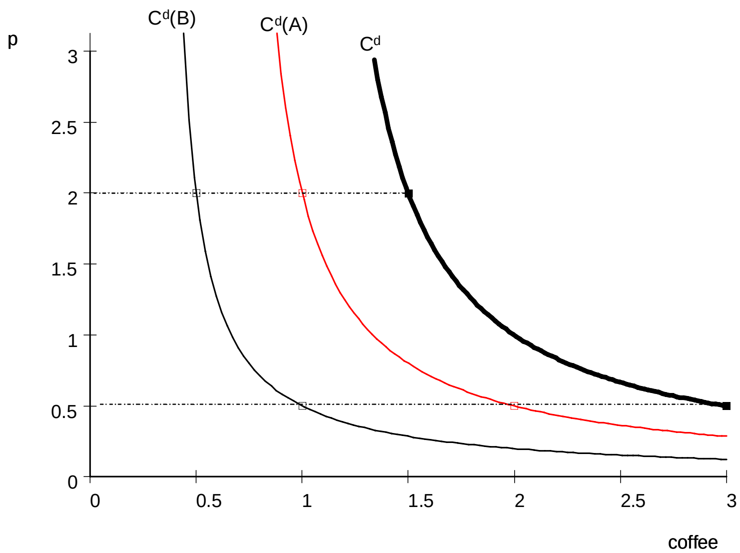

Figure 11: Aggregate inverse demand curve

where we use an un-subscripted. C^d to symbolize the sum of all individual quantities demanded, and we denote the market demand function as $F(p)$.

Note that we could carry out the same analysis with respect to tea. For reasons we will explain later, it is only necessary to analyze one of two markets in a two-good economy. Hence we focus on just coffee in terms of deriving a market demand function.

4.5.3 Market demand curve "shifters"

Just as with an individual demand curve, we can ask about the market demand curve: what happens to its placement in the (p, C) plane when endowments change?

Because the market demand curve is constructed by "adding up" individual demand curves, it follows that anything that shifts individual demand curves may shift the market demand curve. Hence, any change in individual endowments may change the market demand curve.

We say changes in individual endowments "may" change the market demand curve because the concept of a "change in individual endowments" is too broad to allow us to make concrete predictions. For example, we could imagine an increase in Andy's endowment of coffee and a decrease in Bob's. This would shift out Andy's demand curve, but shift Bob's *in*. The qualitative effects on the market demand curve, which reflect the changes in both Andy's and Bob's endowment change, are thus unknown without exact knowledge of the sizes of the changes in endowments and the exact specifications of preferences for both individuals.

An unambiguous thought experiment, though, is one in which endowments for all individuals move in the same direction. In such a case, the market demand curve shifts in the same direction as each of the individual demand curves: if all endowments increase, then the market demand curve shifts out.

The market demand function forms "one blade of the scissors" necessary to compute the equilibrium values of price, and the equilibrium values of the quantities per unit of time bought, sold, and consumed by each individual . We now construct the other blade, the market supply function.

4.5.4 Market supply curves

The endowment economy has the virtue of having the simplest possible market supply function. Market supply is just the sum of all the individual supplies. In the endowment case, this means that market supply is just the sum of the individual endowments. Symbolically, we denote market, or equivalently, aggregate, supply, as an un-subscripted. C^s. Hence,

$$C^s = \sum_{i=1}^{i=l} \overline{C}_i; i = 1, 2, ..., l. \tag{15}$$

For our two-person economy consisting of Andy and Bob, each of whom receives one (1) unit of coffee per unit of time, aggregate supply would thus be two (2) units of coffee:

$$C^s = 1 + 1 = 2.$$

This would be depicted in the $C-p$ plane as a vertical line at $C = 2$,as depicted in Figure 12

4.5.5 Market equilibrium: the equilibrium condition and the solution of the model

We are now ready to determine the equilibrium values of all of the endogenous variables in this model of an endowment economy. The assumption we make is that individuals bring to market their endowments of coffee and tea, and an auctioneer calls out different values of the relative price of coffee until market demand equals market supply:

$$C^d = C^s \tag{16}$$

This equilibrium condition determines the equilibrium relative price of coffee, which we will denote by $\widehat{p}_a$. The "hat" signifies that this is a *particular* value of p, and the subscript a helps us remember that this particular value is for the equilibrium of an autarkic economy. To emphasize that this condition is sufficient to determine the equilibrium price, rewrite equation (16) with the functions $F(p)$ and the number $\overline{C}$ substituted for C^d and C^s, respectively:

$$\overbrace{F(p)}^{C^d} = \overbrace{\overline{C}}^{C^s}.$$

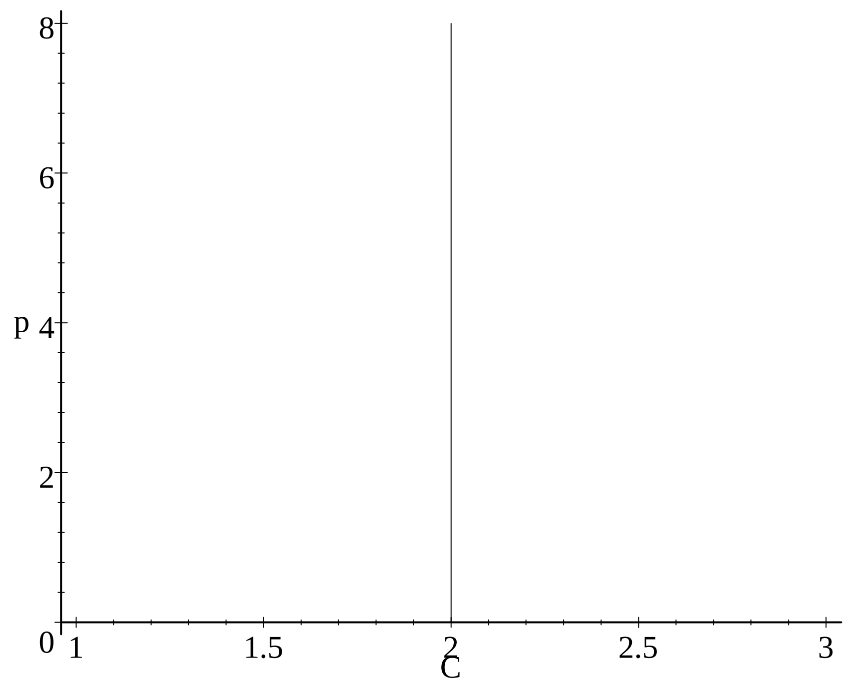

Figure 12: Market supply

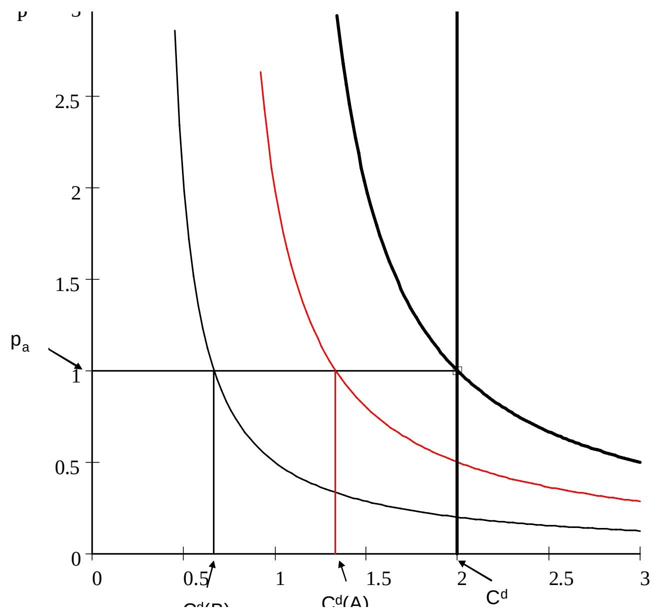

Figure 13: Market equilibrium

This emphasizes that the equilibrium condition can be reduced to one equation in the one unknown p. The diagrammatic depiction of the solution to this equation is the intersection of the market inverse demand function with the market inverse supply function. For the Andy and Bob autarkic economy we have been working with and which has its market inverse demand function depicted in Figure 11 and its inverse market supply function depicted in Figure 12, the graphical depiction of equilibrium is found by superimposing the market supply and demand functions on the same $C - p$ plane, as in Figure 13.

The equilibrium pair $(\widehat{C}, \widehat{p}_a)$ is depicted as the box around the intersection of the market demand and supply curves. Obviously, the equilibrium value of C is the sum of the endowments of the two individuals in this economy. This simplicity on the market supply side is one of the endowment economy's virtues. The equilibrium quantities consumed by each individual can be found by locating on the individual demand curves the quantities demanded at the equilibrium price.

For this example, we now move on to determining the equilibrium values of the rest of the endogenous variables in this model, namely the equilibrium quantities of coffee and tea consumed by each individual in the economy. This is a straightforward task: we simply substitute the equilibrium value $\widehat{p}_a$ into each individual's demand functions for both coffee and tea. Because these functions are just a function of p, this fully determines the equilibrium values of coffee and tea consumed by each individual. We denote symbolically these values for each individual i as $\widehat{C}_i^d$ and $\widehat{T}_i^d$, respectively. Diagrammatically, the equilibrium values $\widehat{C}_i^d$ have already been found from the intersection of each individual's demand curve with a horizontal line of height $\widehat{p}_a$.

4.5.6 Determinants of the autarkic equilibrium value of p

As noted in the introduction to this chapter, a key reason for studying this simple model is to understand how the interplay of tastes and resources determines the autarkic equilibrium relative price, $\widehat{p}_a$. One way to address this question is to ask: how does the relative autarkic equilibrium price *differ* in economies that have *different* tastes and/or resources?

Again, the concept of "different tastes and/or different resources" is too broad to allow us to say anything precise. Hence, we must consider some specific "ceterus paribus" thought experiments to help us understand the answer to this question.

For example, we have considered the model British POW camp as consisting of two POW's: Andy and Bob. In our discussion of this model, we have assumed Andy is the "tea-lover" *relative* to Bob. What would happen if in our economy Andy was "replaced" by a different individual, say "Alphonse," who was, relatively speaking, more of a coffee-lover than was Andy?

Concept check: if Alphonse is more of a coffee-lover than Andy, is Alphonse less of a tea-lover relative to Andy?

Andy's individual demand curve would, in this thought experiment, be replaced by Alphonse's individual demand curve. Because by assumption Alphonse is more of a coffee-lover relative to Andy, his individual coffee demand curve specifies a greater quantity of coffee demanded at every relative price p than does Andy's, ceterus paribus.[12] Thus, the market demand curve for the economy of Alphonse and Bob specifies a greater quantity of coffee demanded at every relative price p than does the market demand curve for the economy of Andy and Bob. That is, the Alphonse/Bob market demand curve is "shifted out" in the (C, p) plane relative to the Andy/Bob market demand curve.

What is the effect on the autarkic equilibrium relative price? Because market supply (by assumption) has not changed, the intersection of market demand and market supply will occur at a lower value of p, the relative price of coffee. This example shows how a clear-cut thought experiment about "different tastes" leads to a clear-cut conclusion about what would be the effect on the autarkic equilibrium relative price.

Now consider another thought experiment. Imagine a scenario in which either Andy or Bob or both had an increase in their endowments of tea. What would happen to the autarkic equilibrium relative price? The increase in the tea endowment for each individual shifts out their budget constraint and thus shifts out their individual demand curves. This implies that the market demand curve for coffee has shifted out, and must intersect the (unchanged by assumption) market supply curve at a lower relative price of coffee.

[12] The "ceterus paribus" here refers to endowments: Andy and Alphonse are assumed to have the same endowments.

Of course, other less clear-cut thought experiments could be carried out, such as simultaneous changes in both tastes and endowments. The point to remember here is that the determination of the autarkic equilibrium value of p depends on the interplay of supply and demand, and supply and demand depend on tastes and endowments.

Recap of the market equilibrium solution So far, we have seen that there are five (5) endogenous variables whose values we can conceptually solve once we are given the values taken by the exogenous variables $\overline{C}_i$ and $\overline{T}_i.(i = A, B)$. We denote these *equilibrium*, or in equivalent language, *solution* values as $\widehat{C}_A^d$, $\widehat{T}_A^d$, $\widehat{C}_B^d$, $\widehat{T}_B^d$ and $\widehat{p}$. The "hat" over the variable is used to indicate it is a specific value, the one that, in conjunction with the other equilibrium values of the other endogenous variables, solves the model.

Notice that we haven't explicitly solved for the value of the other endogenous variables, U_A^d and U_B^d. The recursive structure of our model makes this something easy to do: simply substitute the equilibrium values $\widehat{C}_i^d$ and $\widehat{T}_i^d$ into these functions. The interesting results from solving for the equilibrium values of the rank - orderings of individuals in this economy are best seen graphically, though, and are presented in the following section in which we *depict* rather than compute the equilibrium.

Knowing that we have computed the equilibrium of this economy, we can now depict this equilibrium schematically (diagrammatically) in a way that gives us insight into why, in a two-person economy in which individuals have a motive to trade, both individuals will gain.

5 Depicting individual equilibrium in the complete model

Our two-person prisoner-of-war camp economy is constructed with both individuals receiving the identical endowment package from the Red Cross. Furthermore, in equilibrium, they both face the same relative price of coffee that prevails in the marketplace. Hence, they have identical budget constraints, and the same budget line depicted in the coffee-tea plane applies to both of them. This budget line has slope equal to the negative of the equilibrium relative price, goes through the endowment point $(\overline{C}_A, \overline{T}_A)$ −which is the same point as $(\overline{C}_B, \overline{T}_B)$−and has the identical vertical intercept equal to $(\overline{T}_i + \widehat{p}\overline{C}_i)$, $i = A, B$.

Now, in equilibrium, if Andy and Bob have different tastes, that is, if their indifference curves at any and all points in the $C - T$ plane have different slopes, then their demand curves will differ, and in equilibrium they will consume amounts of coffee and tea different from each other[13]. This, of course,

[13]We have adopted a strong definition of what it means to have different tastes: their indifference curves have different slopes at *any and all* points in the $C - T$ plane. In fact, they could have indifference curves with different slopes only at points "close" to the points of equilibrium, and nothing of substance would change. We use the more restrictive definition

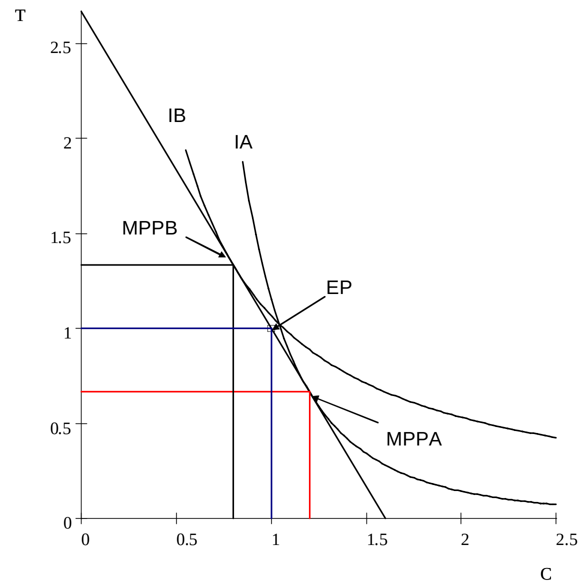

Figure 14: Depicting Andy and Bob's equilibrium

implies that they don't consume amounts of coffee and tea just equal to their endowments: if their endowments are identical, and they don't consume the same amounts, they must not be consuming amounts identical to their endowments.

Also because total demand must equal total supply, one of them must be consuming less coffee than his endowment and the other must be consuming more coffee than his endowment. And because of each individual's budget constraint, the one that consumes more coffee than his endowment must consume less tea than his endowment, and vice-versa for the other individual. The schematic drawing of this is shown in Figure 14. In the drawing, Bob's indifference curves are flatter than Andy's. Line segments connect the endowment point to the vertical and horizontal axes at $(0,1)$ and $(1,0)$ respectively. The budget constraint goes through the endowment point and has slope $\widehat{p}_a = \frac{5}{3}$. Finally, dashed vertical and horizontal line segments for Bob and for Andy connect their respective equilibrium points to the vertical and horizontal axes.

6 Gains from trade in the two-person case

Are Andy and Bob better off in this trading equilibrium than they would be if they could only consume their endowment? The answer is yes. To see this, remember that through every point in quadrant I of the $C-T$ plane goes

simply to keep the idea clear.

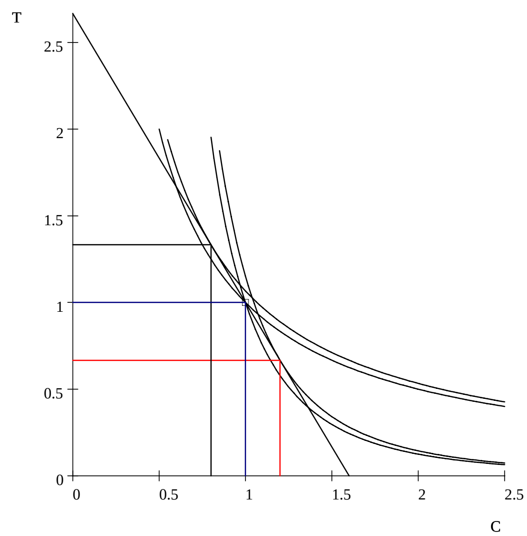

Figure 15: Depicting Andy and Bob's gains from trade

one and only one indifference curve per individual. Hence, one of Andy's and one of Bob's indifference curves must go through the endowment point. For expositional ease, call such curves an individual's *endowment-point indifference curve*. Because indifference curves for an individual cannot intersect, as long as the endowment point is not the equilibrium point for Andy or Bob, the indifference curves through the endowment point depict pairs of coffee and tea that are less preferred than the equilibrium pair. This is depicted in Figure 15.

Another way of thinking about this result is to remember that Andy and Bob participate in the market freely. They could if they so desired not trade, and instead simply consume their endowment. Hence, given our assumptions that people know their own preferences and act rationally in their own self-interest, if two people voluntarily trade with each other, we can infer they must be better off if they trade than if they don't trade.

This result does not depend on any special assumptions we've made about Andy's and Bob's preferences, and it doesn't depend on them having identical endowments (although the identical endowments case is easiest to depict diagrammatically because in it both individuals have the identical budget constraint). The logic of the general case follows closely the key features of the identical-endowments case. First, through any individual's endowment point runs one and only one indifference curve. At the endowment point, this curve has a particular slope. If, in equilibrium, the relative price at which a person trades differs from the slope of that individual's endowment-point indifference curve *evaluated at the endowment point*, then the endowment point cannot be optimal. Remember, an optimal point must be one for which the indifference curve that runs through that point has a slope at that point equal to the slope of the budget constraint, namely $-\widehat{p}_a$.

This two-person result is important because it highlights that trade can be mutually beneficial. Non-economists going back to the Mercantilists in the time of Adam Smith often have argued that in any exchange there must be one winner and one loser. This is clearly not the case.

The logic of the two-person result also suggests another general conclusion: any individual who can trade in any environment, whether a two-person or many-person market or even a non-market situation, in which the relative price differs from the slope of his or her endowment-point indifference curve *evaluated at the endowment point,* will be better off.

This result perhaps requires a little elaboration. Consider an arbitrary endowment point $(\overline{C}_i, \overline{T}_i)$. The slope of the endowment-point indifference curve at that point, and hence the marginal rate of substitution at that point, is thus a function of, i.e., depends upon, the values of the endowments. For notational convenience, denote this value as $p_{i,a}$. That is, the marginal rate of substitution of the endowment-point indifference curve evaluated at $(\overline{C}_i, \overline{T}_i)$ is denoted as $p_{i,a}$. Symbolically, we write this as:

$$mrs_i(\overline{C}_i, \overline{T}_i) = p_{i,a}$$

The subscript i indicates that this value will in general differ from individual to individual. The subscript a is a mnemonic device to remind us that, as we will shortly demonstrate, this number will be associated with what we will call *individual autarky.*

To make our point about the gains from trade, note that if the price at which an individual can trade is $p_{i,a}$, then the endowment point is also the most-preferred consumption point: the individual would consume his or her endowment even in the presence of trading opportunities. Now consider, though, the implications of being able to trade at any price other than $p_{i,a}$. If such a price were greater than $p_{i,a}$, then the individual's budget constraint would have, in comparison to the situation when $\widehat{p}_a = p_{i,a}$, rotated around the endowment point in a counterclockwise manner. If such a price were smaller than $p_{i,a}$, then the individual's budget constraint would have, in comparison to the situation when $\widehat{p}_a = p_{i,a}$, rotated around the endowment point in a clockwise manner. For either case, the budget constraint will no longer be tangent to the endowment-point indifference curve but must "cut" the curve both at the endowment point and at some other point. This is because the indifference curve is "bowed in" to the origin, i.e., is convex to the origin, reflecting our assumption of a diminishing rate of marginal substitution. Hence, there are points on these new budget constraints that are associated with higher levels of satisfaction than are associated with the endowment point. This is depicted in Figure 16.

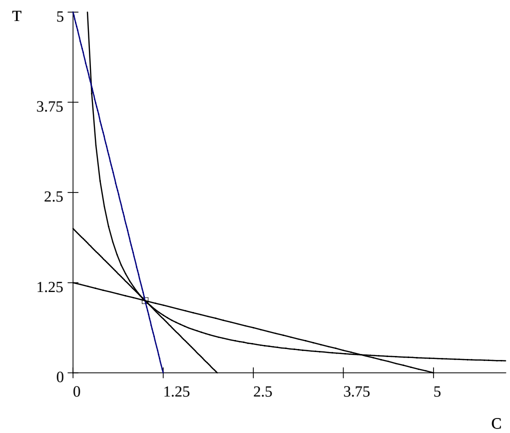

Figure 16: Gains from trade

6.1 A caveat

What one must remember, though, is that this result *does not prove* that free trade between countries is better *for every member of each country* than is autarky for *every member of each country.* As we will shortly see, a change from autarky to free trade may harm some people in each economy relative to their previous situation. Put another way, any trading opportunity for an individual must be at least as good and perhaps better than *individual* autarky, but an ability of people within a country to trade with members of another country might not make every member of the country better off, and indeed may hurt some members.

7 The Edgeworth Box

One advantage of the endowment economy model in which all individuals receive the same endowment is that every person's budget constraint is identical. This allows us to represent individual autarkic equilibrium of a two-person economy on the same diagram: one line represents both the budget constraints (because both individual's have the same endowment point and both face the same relative price) and the indifference curves that go through each individual's optimal point, i.e., that are just tangent to the budget constraint, can be plotted so as to make excess supply of coffee for one individual just equal excess demand for the other (the same is true of tea).

What happens to our model if we generalize it to allow individuals to receive

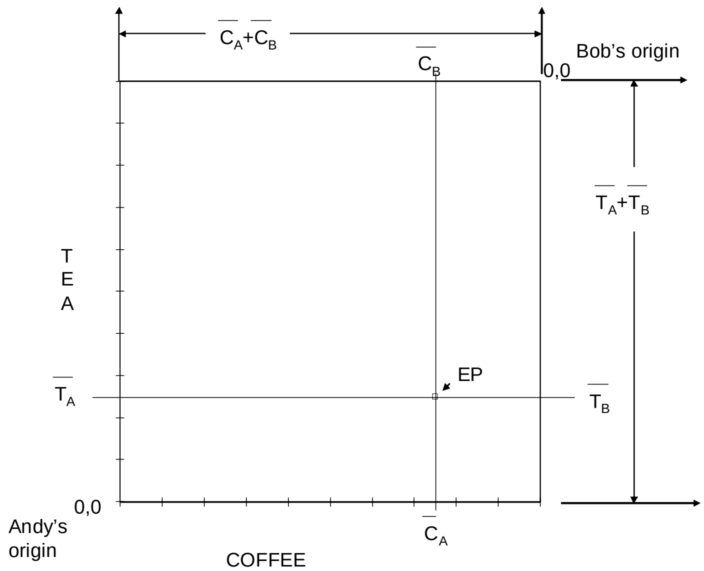

Figure 17:

arbitrary, perhaps different, endowments of coffee and tea? Can we continue to make the same inferences about the gains from trade?

We can. The device we use to do this is known as an Edgeworth Box (sometimes called an Edgeworth-Bowley Box) in honor of the economist(s) who first developed it. (Edgeworth is also the same 19th century economist who believed economists would someday develop a "hedometer" to measure utility). The generalization we carry out with this device also demonstrates how economists "relax" restrictive assumptions (in this case the assumption of identical endowments) so as to draw more universal conclusions.

An Edgeworth Box is a box with vertical dimension equal to the sum of two individuals' endowments of one good (tea, for example) and horizontal dimension equal to the sum of the two individuals' endowments of the other good (coffee, for example). Any point within such a box thus can represent an allocation between the two individuals of the same aggregate economy-wide amount of goods. Such a box is depicted in Figure 17. The point EP represents the endowment point.

One individual's family of indifference curves is plotted within the box taking the Southwest corner of the box as the origin, while the second individual's family is plotted taking the Northeast corner as the origin. An example is depicted in Figure 18. In this figure, Andy's origin (at which Andy has zero of both coffee and tea) is the southwest corner of the box (point A), while Bob's origin is the northeast corner of the box (point C). Indifference curves for Andy

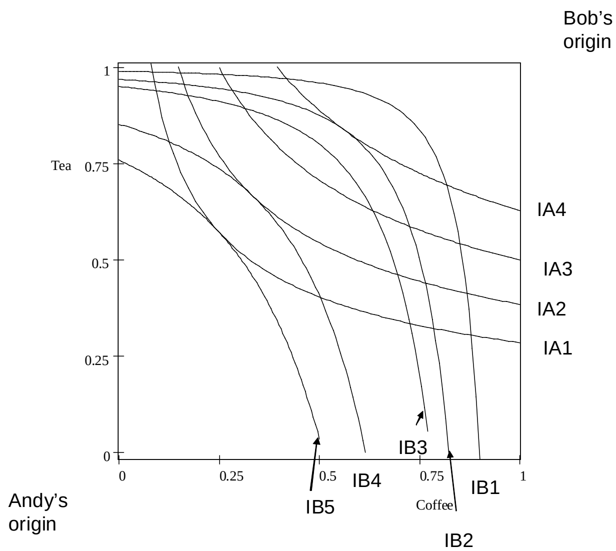

Figure 18: Depicting preferences

are labelled IA1, IA2, IA3, and IA4, with higher numbers associated with higher levels of well-being. Indifference curves for Bob are labelled IB1, IB2, IB3, IB4, and IB5 (there is no significance attached to having more indifference curves displayed for Bob than for Andy).

Notice that some of Andy's and Bob's indifference curves are tangent to each other in the box. If we were to put all the family members of Andy's and Bob's indifference maps in the Edgeworth Box and connect all points of tangency, we would generate what we call the **contract curve**. This curve is superimposed on the displayed indifference curves in Figure 19 and is identified as CC.

By virtue of its construction, any point on the contract curve is one where the quantities of coffee and tea consumed by Andy and Bob are such that at that point both Andy and Bob have the same marginal rates of substitution.

We are now ready to complete our depiction of equilibrium. In equilibrium, both Andy and Bob face the same relative price of coffee, $\widehat{p}_a$, and have a budget constraint that goes through their endowment point. Thus, a straight line through the endowment point in an Edgeworth box with slope $-\widehat{p}_a$ serves as the depiction of both Andy's and Bob's budget constraint. This is depicted in Figure 20.

Furthermore, in autarkic equilibrium we must have that total consumption of coffee and tea by Andy and Bob equal the total amount available, which amounts make up the dimensions of the Edgeworth box. Notice that any point inside the box would satisfy this requirement. But in addition, in equilibrium

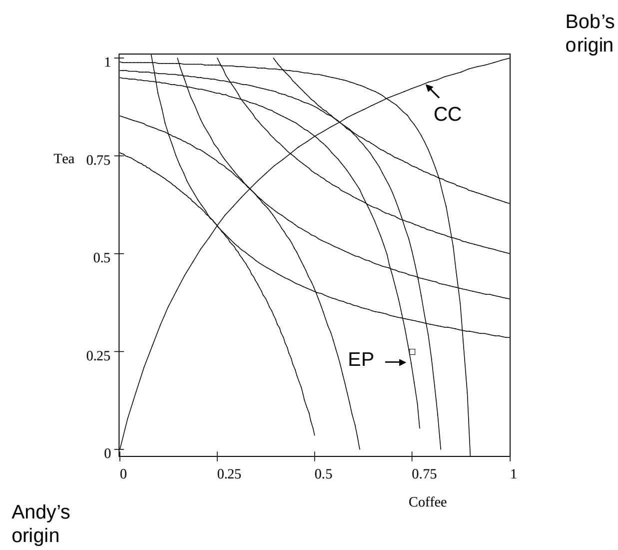

Figure 19: Contract Curve

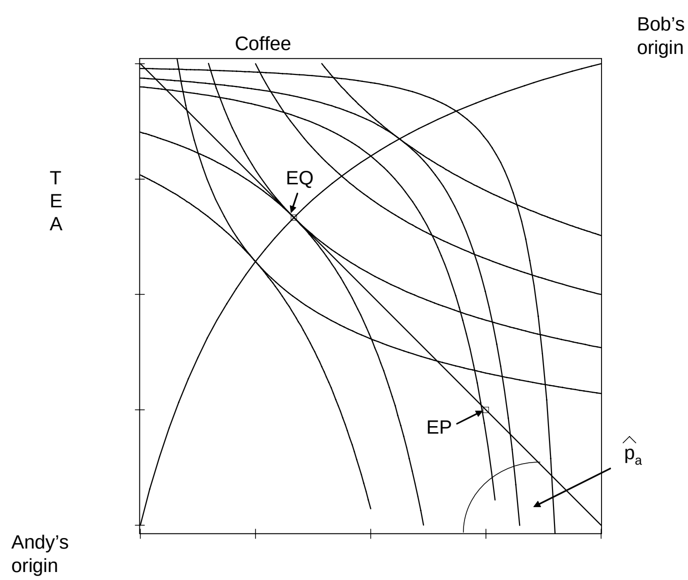

Figure 20: Equilibrium in the Edgeworth box

both Andy and Bob must be choosing a coffee-tea pair for which the marginal rate of substitution equals $-\widehat{p}_a$.

Which point in the box satisfies these requirements? It must be the point that lies on both the budget constraint and the contract curve which is identified as EQ in Figure 21. Notice that the diagram illustrates the gains from trade: for both Andy and Bob, the indifference curves through the equilibrium point are farther away from their respective origins than the curves through the endowment point.

7.0.1 What about the market for tea? Walras' law

In the preceding section, we solved the endowment economy model by making use of the equilibrium condition that market demand per unit of time of coffee equaled market supply per unit of time of coffee. One might ask, what about the market for tea? Why did we not have to use the condition of market equilibrium for tea to be able to solve our model? Could we have solved the model using the equilibrium condition in the tea market as an equation instead of equilibrium in the market for coffee?

Up until now, we have been thinking about the market environment of the POW endowment economy as one in which individuals are given their endowments, and they then congregate in a marketplace. In that marketplace, people are assumed to contract with an auctioneer in the following way: for any relative price of coffee the auctioneer calls out, the individual would be willing, if asked, to give his endowment of coffee and tea to the auctioneer and in exchange collect his desired demands of coffee and tea from the auctioneer. The auctioneer is assumed to "grope" -the so-called "tatonnment" process-by calling out different prices until he calls one at which the total quantity of coffee demanded just equals the total quantity available, namely the sum of everyone's coffee endowment. How do we know that when the auctioneer has called out the equilibrium price at which coffee demand equals coffee supply, he or she has also found a price at which total tea demand equals total tea supply? To make it clear that this must be so, we develop a concept known as Walras' Law. In an appendix, we reinforce this idea by developing a technique known as offer curve analysis.

Walras' Law One way to illustrate that the relative price that clears the market for coffee also clears the market for tea is to look at the implications of an economy's *aggregate* budget constraint for the resolution of this question. The aggregate budget constraint is just the sum of all the individual budget constraints. For example, for our two-good, two-person economy, the aggregate budget constraint would be:

$$\overline{T}_A + \overline{T}_B + p\overline{C}_A + p\overline{C}_B = T_A^d + T_B^d + pC_A^d + pC_B^d.$$

We can regroup this as

$$\overline{T}_A + \overline{T}_B - (T_A^d + T_B^d) = p[(\overline{C}_A + \overline{C}_B) - (C_A^d + C_B^d)]. \tag{17}$$

Now consider the equilibrium condition in the market for coffee:

$$\overline{C}_A + \overline{C}_B = C_A^d + C_B^d. \tag{18}$$

If this is satisfied, then the right-hand-side of the aggregate budget constraint as expressed in (17) is zero. Hence,

$$\overline{T}_A + \overline{T}_B = (T_A^d + T_B^d) \tag{19}$$

That is, *if* the market for coffee is in equilibrium, *then* the market for tea must also be in equilibrium. This is an example of what is known as Walras' Law: in an economy with n markets, if $n-1$ of those markets are in equilibrium, then the other market must also be in equilibrium. This law allows us to focus attention in our analyses upon one less market than there are in the total economy. In the present case, it allows us to remain "two-dimensional" friendly by only having to analyze and depict the market for coffee (or the market for tea, but not both). The law reflects an underlying feature of how we model consumer behavior: each consumer's optimal choice of a bundle of goods and services is a simultaneous choice constrained by his or her budget constraint. Hence, for each individual, a demand for some things is a simultaneous offer of another thing in exchange. Put in other language, a desired choice of one good implies a *reciprocal* exchange of some other good or goods as payment.

Most of us, in everyday economic life, lose sight of the reciprocal nature of demand because our exchanges are generally intermediated by money. We are, though, exchanging something, namely money, for something else, the good in question. And money is simply a claim on other goods and services. Hence, an exchange of money for one thing is effectively an exchange of some goods for other goods.

7.1 Autarkic Real Economy Summary

1. Endowment economies are economies in which people are given exogenously determined quantities of goods and services. That is, in such economies people make no production decisions. The simplification introduced by eliminating production decisions makes such an economy a good vehicle for introducing the concepts of a general equilibrium model and illustrating the concept of "gains from trade."

2. Economists model a "market" by assuming every market participant faces the same price, a price determined by the intersection of demand and supply curves.

3. People's preferences (assumed exogenous) can be represented by families of downward-sloping, concave-to-the-origin, non-intersecting indifference curves. Ordinal rankings of well-being are higher for indifference curves relatively farther away from the origin than other indifference curves.

4. People's choices of a most-preferred consumption bundle are characterized as the bundle on their budget constraint that is tangent to an indifference curve.

5. The systematic relationship between each commodity of most-preferred consumption bundles and relative prices is described as a demand function for that commodity.

6. Market demand functions consist of the sum of individual demand functions.

7. The equilibrium relative price of a commodity is determined by the intersection of demand and supply. Once this equilibrium price is determined, the rest of the solution to the model can be determined by substituting this price into the individual demand functions for the various commodities.

8. The intersection of demand and supply can be thought of as the interplay of tastes and resources, which are the fundamental determinants of demand and supply functions.

9. In a two-person economy there are "gains from trade:" each individual is never worse off if allowed to trade with the other person.

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