Homework 1

1 Alex and Bobby

1. Alex and Bobby two POW's in a British camp. Each gets the same endowment of coffee and tea per month:

$$\overline{C}_A = \overline{C}_B = \frac{1}{2},\ \overline{T}_A = \overline{T}_B = 10.$$

Alex and Bobby can exchange these endowments for currency at market prices, which are exogenous to them, in a central marketplace. Let P_C and P_T be the currency price of coffee and tea, respectively, and let $p \equiv \frac{P_C}{P_T}$ be the relative price of coffee.

(a) If the price of coffee is 12 units of currency per unit of coffee, and the price of tea is 2 units of currency per unit of tea, what is Alex's (and Bobby's) income/month measured in currency?

A:

$$\overbrace{\frac{1}{2}}^{\overline{C}_i} \times \overbrace{\$12/C}^{P_C} + \overbrace{10}^{\overline{T}_I} \times \overbrace{\$2/T}^{P_T} = \$26/Month$$

(b) What is Alex's (and Bobby's) income/month measured in units of coffee? In units of tea?

A: This question asks what is the maximum amount of coffee (tea) Alex (and Bobby) could buy given their endowments and these prices. If Alex consumes no tea, he could spend all \$26 on coffee. Coffee costs \$12/unit of coffee, so (similar reasoning for tea)

$$\frac{\$26}{\$12/C} = 2\frac{1}{6}\ units\ of\ coffee/month \qquad \text{(C/month)}$$

$$\frac{\$26}{\$2/C} = 13\ units\ of\ tea/month \qquad \text{(T/month)}$$

(c) What is the relative price of coffee? What are the units of this price?

A: Relative price of coffee is how many units of tea it takes to purchase a unit of coffee. Hence

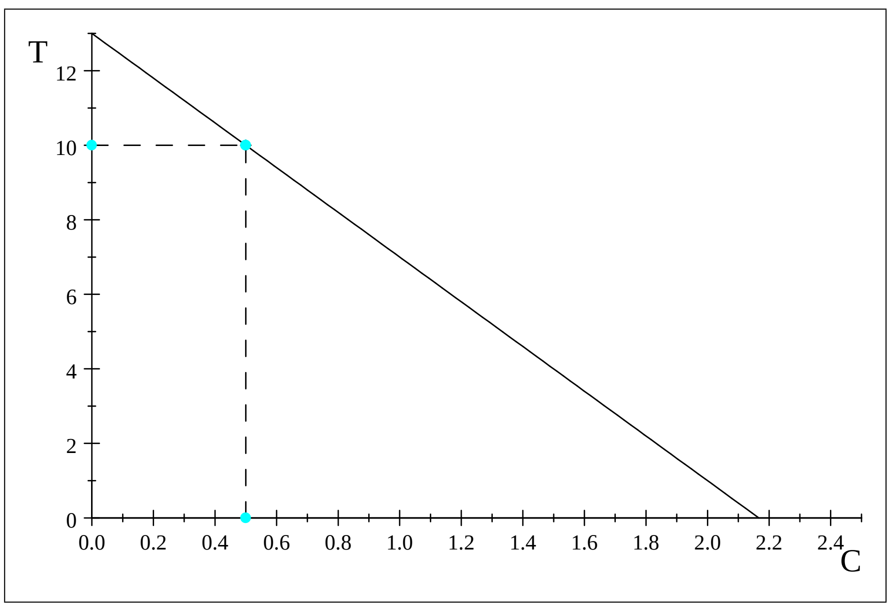
$$\frac{P_C}{P_T} = \frac{12}{2} = 6.$$

(d) For these endowments and prices, write Alex's (and Bobby's) budget constraint in standard slope-intercept form with consumption of tea/month on the left-hand-side of the equality sign.

A: Start with income equals expenditure, rearrange to put $P_T T_i$ on the lhs, then divide by P_T:

$$\begin{aligned} P_C\overline{C}_i + P_T\overline{T}_i &= P_C C_i + P_T T_i \ , i = A, B; \\ P_T T_i &= P_C\overline{C}_i + P_T\overline{T}_i - P_C C_i; \\ T_i &= \overline{T}_i + p\overline{C}_i - pC_i; \\ T_i &= \overbrace{\overbrace{10}^{\overline{T}_i} + 6 \times \overbrace{\frac{1}{2}}^{\overline{C}_i}}^{R_i^T} - 6C_i \\ &= 13 - 6C_i. \end{aligned}$$

(e) With tea on the vertical axis and coffee on the horizontal, draw a schematic diagram of his budget constraint, making sure you identify all relevant features, i.e., slope, intercepts, and endowment point.
Intercepts: $(0, 13)$, $(\frac{13}{6} \approx 2.166, 0)$
Slope: -6



(f) What would happen to this schematic diagram if both P_C and P_T were to double? Triple? Be cut in half?
A. Nothing

2. Their preferences are represented by the following utility functions:

$$\begin{aligned} U_A &= T_A + 4\ln C_A; \\ U_B &= T_B + 2\ln C_B. \end{aligned}$$

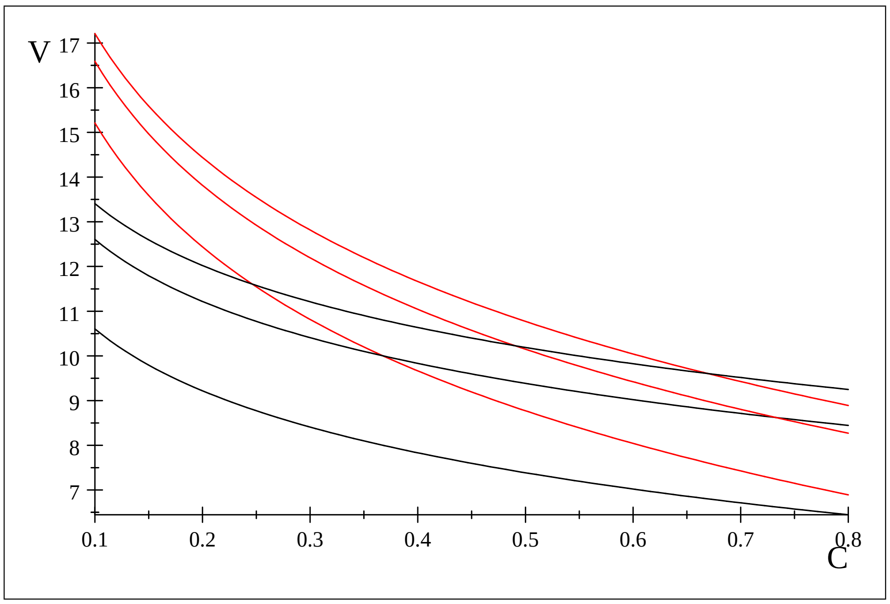
Note: The parametric form of these utility functions are:

$$U_A = T_A + \gamma_A \ln C_A;$$
$$U_B = T_B + \gamma_B \ln C_B.$$

Their indifference maps are thus described by the family of curves found by rearranging the above equations as

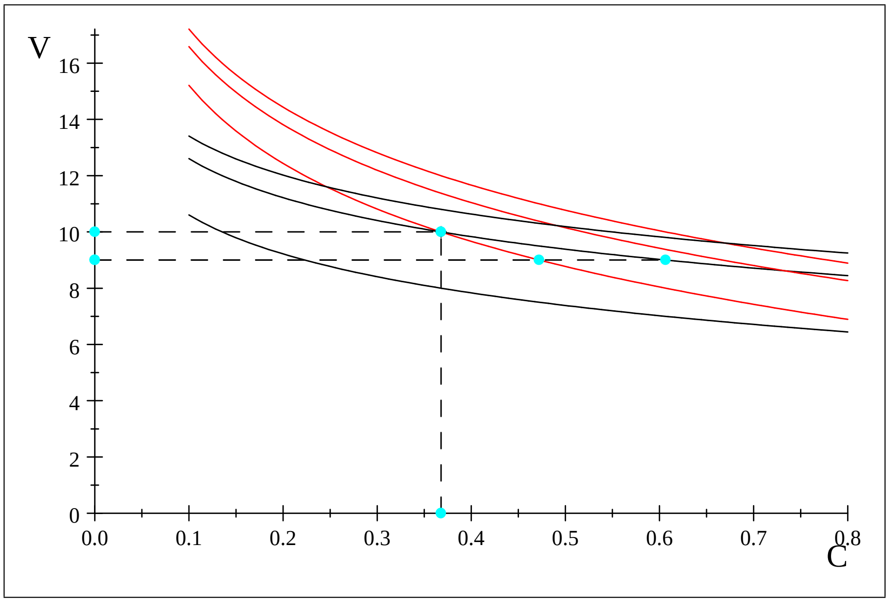
$$T_A = U_A - 4 \ln C_A;$$
$$T_B = U_B - 2 \ln C_B.$$

They look like the following, where Alex's indifference curves are red and Bobby's are black:



$\gamma_A = 4, \gamma_B = 2$

(a) Who would you describe as the "coffee-lover" and explain why.



$\gamma_A = 4, \gamma_B = 2$

Consider a point where an indifference curve for Alex crosses an indifference curve for Bobby, as illustrated above. Imagine taking one unit (to be exact, an infinitesimally small amount) of V away from each of them. To make them just as well off as they were at the initial point, you would have to give Bobby more C. Bobby loves tea so much relative to Alex that if you take away a little tea from him you need to give him more coffee to compensate him, i.e., make him as well off as before you took away the tea, than you have to give Alex. Bobby is (relatively speaking) the "tea-lover" and Alex is (relatively speaking (the "coffee-lover).

(b) Derive the demand curves and the inverse demand curves for coffee for Alex and for Bobby.

A. I reproduce the utility functions (rearranged with T on the lhs of the equations)

$$\begin{aligned} T_A &= U_A - 4 \ln C_A; \\ T_B &= U_B - 2 \ln C_B, \end{aligned}$$

The tangency condition requires that the $MRS = -p:$

$$\frac{dT_A}{dC_A} = \frac{-4}{C_A} = -p; \tag{TC}$$

$$C_A^d = \frac{4}{p}; \tag{C_A^d}$$

$$p = \frac{4}{C_A}. \tag{invrse}$$

For Bobby:

$$
\begin{aligned}
\frac{dT_B}{dC_B} &= \frac{-2}{C_B} = -p; && \text{(TC)}\\
C_B^d &= \frac{2}{p}; && (C_B^d)\\
p &= \frac{2}{C_B}. && \text{(invrse)}
\end{aligned}
$$

(c) Derive the demand curves for both Alex and for Bobby for tea.

A. We need the budget constraints:

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i,\ i = A, B.$$

We know what pC_i is from the coffee demand curves:

$$
\begin{aligned}
pC_A &= 4;\\
pC_B &= 2.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
T_A &= \overbrace{10}^{\overline{T}_A} + p \times \overbrace{\frac{1}{2}}^{\overline{C}_A} - \overbrace{4}^{pC_A} = 6 + \frac{1}{2}p;\\
T_B &= 10 + p \times \frac{1}{2} - 2 = 4 + \frac{1}{2}p.
\end{aligned}
$$

(d) Assume Alex and Bobby are the only two POW's in the British camp. Derive the autarkic equilibrium for their camp. That is, put numbers in for the following variables:

$$
\begin{aligned}
p_a &= \_\_\_;\\
C_A &= \_\_\_;\\
T_A &= \_\_\_;\\
C_B &= \_\_\_;\\
T_B &= \_\_\_;\\
U_A &= \_\_\_;\\
U_B &= \_\_\_.
\end{aligned}
$$

Answer: First find the market demand curve for coffee:

$$C_A^d + C_B^d = \frac{2}{p} + \frac{4}{p} = \frac{6}{p}.$$

Market supply is

$$\overline{C}_A + \overline{C}_B = \frac{1}{2} + \frac{1}{2} = 1.$$

Equilibrium is thus

$$\frac{6}{p} = 1;\ p_a = 6.$$

From the demand functions:

$$\begin{aligned} C_A &= \frac{4}{p_a} = \frac{2}{3};\ T_A = 6 + \frac{1}{2}p_a = 9; \\ C_B &= \frac{2}{p_a} = \frac{1}{3};\ T_B = 8 + \frac{1}{2}p_a = 11. \end{aligned}$$

Utility levels?

$$\begin{aligned} U_A &= T_A + 4\ln C_A = 9 + 4\ln\frac{2}{3} = 7.3781; \\ U_B &= T_B + 2\ln C_B = 11 + 2\ln\frac{1}{3} = 8.8028; \end{aligned}$$

$10 + 2\ln\frac{1}{2} = 8.6137$
$10 + 4\ln\frac{1}{2} = 7.2274$
$11 + 2\ln\frac{1}{3} = 8.8028$
$9 + 4\ln\frac{2}{3} = 7.3781$

(e) If Alex and Bobby were forced to consume their own endowment without trading with each other, what would their utility levels have been?

A:

$$\begin{aligned} U_A &= T_A + 4\ln C_A = 10 + 4\ln\frac{1}{2} = 7.2274; \\ U_B &= T_B + 2\ln C_B = 10 + 2\ln\frac{1}{2} = 8.6137. \end{aligned}$$

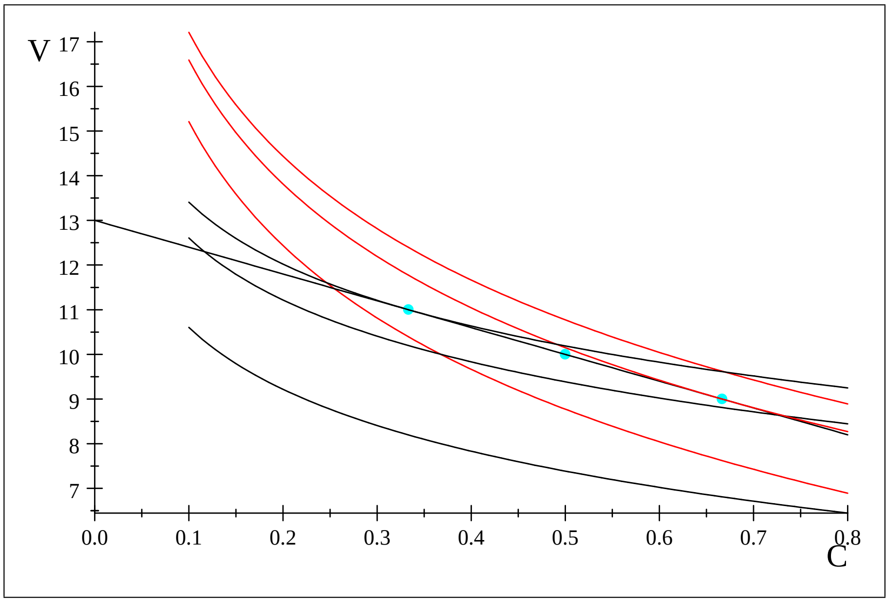
(f) In the situation where they both consumed their respective endowments, would you say Bobby is better off than Alex?

A: No. Monotonic transformations.

(g) By being able to trade with each other instead of having to consume their endowments, Alex's utility function changed by +.1507, while Bobby's changed by +.1891. Would you say Bobby gained more from being able to trade with Alex than did Alex?

No. Once again, no interpersonal comparisons of utility! New Jersey Turnpike exit numbering!

Below is a depiction of autarkic equilibrium.



$p_a = 6, \gamma_A = 4, \gamma_B = 2$

2 Antoine and Baptiste

The French camp has two POW's as well, Antoine and Baptiste. Each has the same endowment as the English POW's, namely

$$\overline{C}_A^* = \overline{C}_B^* = \frac{1}{2},\ \overline{T}_A^* = \overline{T}_B^* = 10.$$