Econ 3600 International Trade

Introduction

1. Syllabus highlights (why only highlights?)

a. Exam times: **Quiz #1 Wednesday Sept. 12**; **Exam #2 Monday Nov. 12**

b. Class procedures

- **i**. Synchronize watches.
- **ii**. Pay attention.

2. Intro

a. Issues:

- **i**. Trump!
  - **A**. "Buy American, hire American."
  - **B**. "American steel, aluminum."
  - **C**. Out of TPP, renegotiate NAFTA (https://nyti.ms/2uXVCox)
  - **D**. Tariff wars, trade deficits, currency manipulation, immigration.
- **ii**. Who said this, and when?

  “When you invest in education and health care and benefits for working Americans, it pays dividends throughout every level of our economy. . . . I think that if you polled many of the people in this room, most of us are strong free traders and most of us believe in markets. Bob [Rubin] and I have had a running debate now for about a year about how do we, in fact, deal with the losers in a globalized economy. There has been a tendency in the past for us to say, well, look, we have got to grow the pie, and we will retrain those who need retraining. But, in fact, we have never taken that side of the equation as seriously as we need to take it. . . . Just remember . . . [t] here are people in places like Decatur, Illinois, or Galesburg, Illinois, who have seen their jobs eliminated. They have lost their health care. They have lost their retirement security. . . . They believe that this may be the first generation in which their children do worse than they do.”

b. Purview: interactions between economic units (people, firms, gov’ts, orgs.) located in different sovereign nations

- **i**. Not *Nations* that "decide" to have trade def’s, etc. but the decision-making entities within these nations, e.g., people, firms, gov’ts, orgs.
- **ii**. Why countries as line of demarcation?
  - **A**. Traditionally, factor mobility: "Protectionism for Liberals" by Robert Skidelsky, Aug 14, 2018, *Project Syndicate:*

    “Experience ... shows,” Ricardo wrote, “that the fancied or real insecurity of capital, when not under the immediate control of its owner, together with the natural disinclination which every man has to quit the country of his birth and connexions, and intrust himself, with all his habits fixed, to a strange government and new laws, check the emigration of capital. These feelings, which I should be sorry to see weakened, induce most men of property to be satisfied with a low rate of profits in their own country, rather than seek a more advantageous employment for their wealth in foreign nations.”
  - **B**. "This prudential barrier to capital export fell as secure conditions emerged in more parts of the world. In our own time, the emigration of capital has led to the emigration of jobs, as technology transfer has made possible the reallocation of domestic production to foreign locations – thus compounding the potential for job losses."
  - **C**. Sovereign policies: among other things, in US, no barriers to interstate trade; only dollars as legal tender.

c. Themes.

- **i**. Pattern of trade
- **ii**. Effects of trade, i.e., "Gains from trade"

d. Basic reason for trade:

- **i**. Proximate cause: differences in prices across locations (POW camp example; Seinfeld)
  - **A**. Goods and services
  - **B**. Trading current for future consumption.

ii. Peeling the onion: what causes these different *autarkic* prices?

e. Effects of trade:

i. Trade changes *relative* prices vis a vis their autarkic values

ii. We need a framework for thinking about how changes in relative prices affect individuals

3. Is this class for you?

a. Subject matter

b. Approach: involves solving numerical (and more general) problems. See one, do one, teach one.

c. The Bard of Baltimore, H. L. Mencken:

i. "For every complex problem there is an answer that is clear, simple, and wrong."

LAGNIAPPE ("a little extra;" for this class, it means not required or necessarily covered in class but perhaps useful, or sometimes just fun)

ii. Lagniappe: "Every normal man must be tempted, at times, to spit on his hands, hoist the black flag, and begin slitting throats."

iii. Lagniappe: "No one ever went broke underestimating the taste of the American public."

iv. Lagniappe: "On some great and glorious day the plain folks of the land will reach their heart's desire at last, and the White House will be adorned by a downright moron."

LAGNIAPPE: Models

1. Definition: logical representation of a priori or theoretical knowledge economic analysis suggests is most relevant for treating a particular problem

2. Elements:

a. Variables: exog and endog

b. Equations: logical structuring and representation of basic interrelationships ("systematic relationships")

i. Structural equations, structural model.

ii. Depictions

A. words, mathematical notation

B. Graphs

iii. Parameters

3. Solving:

a. Canonical question

b. Strategies

i. sub-models

ii. math

iii. Graphs

iv. Dimensionality

Need something as a break from all this theory stuff? Read about "Chicken Wars." https://en.wikipedia.org/wiki/Chicken_tax

The Endowment Economy

Introduction

1. Purpose: develop simplest GE model that helps us understand some aspects of trade: pattern, effects.

a. Why general equilibrium?

i. Key insights are fundamentally GE (appeal to my authority)

ii. Analogy: tariffs (effect exports!); electric cars, solar panels, recycling

b. Why two agents?

2. Goals

a. Understand the determinants of AERP (interplay of tastes, resources)

b. Understand how price affects each individual's well-being.

3. What is hard? **Relative prices instead of nominal prices**.

The model

1. POW camp, Halloween

2. Two sub-models: demand (consumer) and supply (simple)

3. For model as a whole:

a. Exogenous: tastes (preferences), resources (endowments).

b. Endogenous: quantities consumed, relative price, levels of well-being (on which of their I-curves individuals find themselves).

4. For sub-model of demand:

a. Exogenous: tastes (preferences), resources (endowments), **relative prices**

b. Endogenous: most-preferred choices of commodities (most-preferred pair for two-good case).

5. For sub-model of supply: endowments.

Strategy

1. Read for an overview.

2. Solving problems, aka "toy models," with specific functional forms: (p. 59, "Why we use functional forms").

a. With numbers reinforces the idea of what things are exogenous, what are endogenous, and is less abstract than a parametric approach.

b. With parameters, lets us have a more generic understanding.

c. Makes models and their implications more concrete, easier to remember..

d. Makes it easy to know what it means to "solve" a model.

e. If you have read the text beforehand, these examples will be easier to follow.

3. In this outline, mathematical steps are outlined; we go quickly, and they are there for you as needed by you. At a minimum, what you need to understand and appreciate is:

a. Behind every individual demand curve is an individual preference relationship, an individual budget constraint, and a most-preferred pair of goods, e.g., a most-preferred pair (C_i, T_i).

b. Intersections of curves in diagrams represent pairs of numbers;

c. what determines the placement of curves in the Cartesian (x, y) plane and what determines how they shift when there are changes in variables that are not x or y

d. For the "A" and "B" students: How to derive individual and market (equivalently, aggregate) demand curves from a Cobb-Douglas utility function and from two specific no-income-effect utility functions that we will introduce.

Demand sub-model

1. Endowments for each person

a. Exogenous

b. Notation (e.g., for two goods and two people: Alex, Bob)

$$\overline{C}_A, \overline{T}_A, \overline{C}_B, \overline{T}_B,$$

2. Budget constraints for Alex (Analogous expression for Bob):

a. Equality of income and expenditure

$$P_C\overline{C}_A + P_T\overline{T}_A = P_C C_A + P_T T_A;$$

b. In graph-friendly form:

$$T_A = \overline{T}_A + p\overline{C}_A - pC_A$$

c. $p \equiv \frac{P_C}{P_T}$ **exogenous**.

- **i**. Reflects Opp. cost as Alex contemplates choosing different bundles of (C_A, T_A).
- **ii**. We consistently refer to it as *the relative price of coffee*.
- **iii**. To help remember: ordinary (currency) prices are dollars/unit of thing, relative price of coffee is tea/unit of coffee.
- **iv**. What is relative price of tea? units of coffee/unit of tea, i.e., $\frac{1}{p}$.

d. **Homework 1 Part 1** (**see one**, **do one**, ...), **in groups**.

- **i**. 5 points total. Imagine Alex (A) works as an Uber driver and earns $\$12.00$ per day. Alex only is interested in consuming two commodities: coffee and tea. Exogenous to him are his income of $\$12/day$, and the prices for coffee and tea: $P_C = \$6$/unit of coffee, $P_T = \$2$/unit of tea. Alex constructs a budget for himself. He contemplates buying differing quantities of coffee, namely either one unit, two units, or three units. Given his income of $\$12/day$, and the prices he faces for coffee and tea, he figures out how much tea he can consume for those three (3) different choices of amounts of coffee.
  - **A**. 2 points. Fill in those possible feasible choices of tea, and the associated expenditures on tea, in

the following chart:

coffee quantities	0	1	2
coffee price	$6	$6	$6
coffee expenditure	$0	$6	$12
tea quantities	?	?	?
tea price	$2	$2	$2
tea expenditure	?	?	?
total expenditure	$12	$12	$12

A:

coffee quantities	0	1	2
coffee price	$6	$6	$6
coffee expenditure	$0	$6	$12
tea quantities	6	3	0
tea price	$2	$2	$2
tea expenditure	$12	$6	$0
total expenditure	$12	$12	$12

B. 2 points. Fill in the missing numbers in the following sentences: This chart shows that if you contemplate an initial purchase of no coffee (0 units) and six (6) units of tea, then to move from this initial point to a purchase of one unit of coffee, you must forgo __ units of tea. Likewise, starting from a point of 1 unit of coffee and 3 units of tea, if you purchased one more unit of coffee, you would have to forego another ___ units of tea.

A: three (3) units of tea; three (3) units of tea.

C. 1 point. This suggests that an alternative way of thinking about a relative price is to contemplate spending a fixed amount of dollars (your nominal income) on coffee and tea. To buy more coffee, you necessarily must buy less tea. What is the relative price of coffee in this example?

A: Three (3) units of tea per unit of coffee. Note:

$$\frac{P_C}{P_T} = \frac{\$6/coffee}{\$2/tea} = 3 units\ tea/unit\ coffee.$$

ii. 10 points total. Assume two POW's, Alex (A) and Bobby (B), each get (exogenous) endowments of one (1) unit of coffee and one (1) unit of tea per month. That is,

$$\overline{C}_A = 1;\ \overline{C}_B = 1;$$

$$\overline{T}_A = 1;\ \overline{T}_B = 1.$$

Consider four different (exogenous to Alex and Bob) sets of *nominal* prices for coffee and tea:

$$Set\ 1.\ \left(P_C = \$10/lb\ ,\ P_T = \$20/lb\right);$$

$$Set\ 2.\ \left(P_C = \$10/lb,\ P_T = \$10/lb\right);$$

$$Set\ 3.\ \left(P_C = \$10/lb,\ P_T = \$5/lb\right);$$

$$Set\ 4.\ \left(P_C = \$20/lb,\ P_T = \$20/lb\right).$$

A. 5 points. Define the relative price of coffee as the units of tea that exchange for one unit of coffee. What is the relative price of coffee associated with each of the above sets of prices?

Answer: $p(1) = \frac{1}{2}$, $p(2) = 1$, $p(3) = 2$, $p(4) = 1$.

B. 5 points. Write the equation that describes all the ordered pairs of coffee and tea that could be consumed by Alex and Bobby for each set of prices. That is, you are looking for a function of the form $T_i = f(C_i; P_C, P_T, \overline{C}_i, \overline{T}_i)$. Your equation must have consumption of tea ($T_i,\ i = A, B$) as the only variable on the left-hand-side. Depict these four lines all in one *schematic* diagram, i.e., in a diagram with consumption of tea on the vertical axis and consumption of coffee on the horizontal axis in which all lines have slopes and vertical intercepts identified and all relative qualitative properties among the lines are correct.

Answer: (parametric form first to easily check answers)

$$\overbrace{P_T T_i + P_C C_i}^{Exp} = \overbrace{P_T \overline{T}_i + P_C \overline{C}_i}^{Income};$$

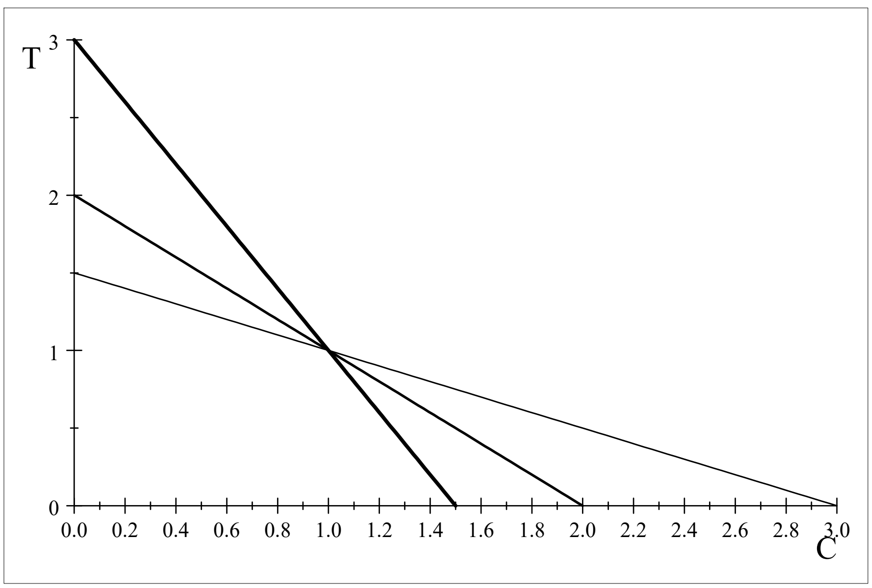
$$P_T T_i = P_T \overline{T}_i + P_C \overline{C}_i - P_C C_i;$$

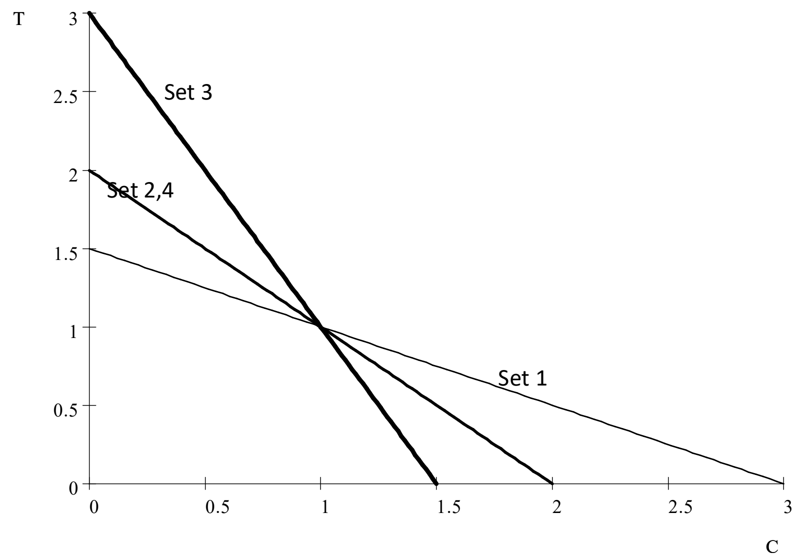
$$T_i = \overline{T}_i + \frac{P_C}{P_T}\overline{C}_i - \frac{P_C}{P_T}C_i.$$

$$\overline{T}_i = \overline{C}_i = 1;\ T_i = \overbrace{1}^{\overline{T}_i} + \frac{P_C}{P_T} \times \overbrace{1}^{\overline{C}_i} - \frac{P_C}{P_T}C_i;$$

$$T_i = 1 + \frac{P_C}{P_T} - \frac{P_C}{P_T} \times C_i.$$

1. $\left(P_C = \$10/lb\ ,\ P_T = \$20/lb\right);\ T_i = \frac{3}{2} - \frac{1}{2}C_i$
2. $\left(P_C = \$10/lb,\ P_T = \$10/lb\right);\ T_i = 2 - C_i;$
3. $\left(P_C = \$10/lb,\ P_T = \$5/lb\right);\ T_i = 3 - 2C_i.$
4. $\left(P_C = \$20/lb,\ P_T = \$20/lb\right);\ T_i = 2 - C_i.$





Slopes should be added by students:
$-\frac{1}{2},\ -1,\ -2$

iii. 35 points. From the specific to the general. Suppose Alex grows 80 kilos of coffee per year, and coffee exchanges in the market place for $2.00/kilo. Tea exchanges in the market place for $4.00/kilo. Let C_A symbolize the variable that measures the amount of coffee Alex consumes per year, and T_A symbolize the variable that measures the amount of tea Alex consumes per year.

A. 5 points. Describe in an equation with only T_A on the left-hand-side of the equality sign all those pairs of kilos of coffee/yr. and kilos of tea/year that Alex could consume at these prices, assuming he spent all of his income.

Answer: Let me put the budget constraint in parametric form. First I express in an equation the equality of income and expenditure:

$$\overbrace{P_C C_A + P_T T_A}^{Expenditure} = \overbrace{P_T \overline{T}_A + P_C \overline{C}_A}^{Income}.$$

I then rearrange to isolate T_A on the l.h.s. of the equality sign:

$$T_A = \frac{P_C \overline{C}_A}{P_T} + \overline{T}_A - \frac{P_C}{P_T} C_A.$$

Upon substitution of the given values of the exogenous variables:

$$\frac{P_C}{P_T} = \frac{2}{4} = .5; \overline{C}_A = 80;$$

$$T_A = .5 \times 80 - .5 C_A;$$

$$T_A = 40 - .5 C_A$$

B. 5 points. What is Alex's income per year measured in units of dollars/year?

A:

$$\overbrace{80}^{\bar{C}_A} \times \overbrace{2}^{P_C} = 160\$/year.$$

C. 5 points. Let us generalize from specific numerical values for the dollar prices of coffee and tea. Let P_C symbolize the variable that describes the market price of coffee in terms of dollars/kilo of coffee, and let P_T symbolize the variable that describes the market price of tea in terms of dollars/kilo of tea. Given that Alex produces 80 kilos of coffee per year, describe in an equation using the above symbols all those pairs of kilos of coffee/year (C_A)and kilos of tea/year (T_A) that Alex could consume for arbitrary values of P_C and P_T , assuming he spent all of his income. That is, write down Alex's budget constraint. In this equation, put T_A as the only variable on the left-hand-side of the equality sign.

Answer:

$$T_A = \frac{80P_C}{P_T} - \frac{P_C}{P_T}C_A.$$

D. 5 points. Let us generalize further. Let $\overline{C}_A$ symbolize the variable that describes Alex's production of kilos. of coffee per year. For arbitrary values of Alex's production of kilos. of coffee/yr. and arbitrary values of the price of coffee and the price of tea, describe in an equation all those pairs of kilos of coffee/yr. and kilos of tea/yr. that Alex could consume if he spent all of his income. Again, write this equation with T_A as the only variable on the left-hand-side of the equality sign.

Answer:

$$T_A = \frac{P_C}{P_T}\overline{C}_A - \frac{P_C}{P_T}C_A.$$

E. 5 points. Draw a **schematic** diagram of the above equation in the coffee-tea plane. By coffee-tea plane, we mean the standard picture in which tea/year is measured on the vertical axis and coffee/year on the horizontal axis. By **schematic** we mean that the key qualitative features of the equation, namely the slope and the intercepts, are depicted and identified, although not necessarily to scale.

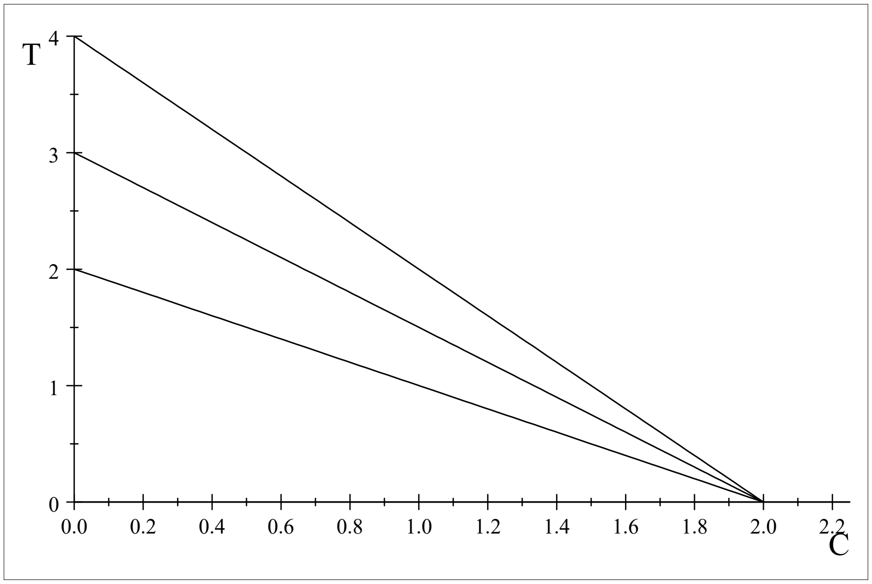
Answer: slope and intercepts identified, and the line connecting the intercepts arguably a straight line.

F. 5 points. What would happen to this schematic diagram if both P_C and P_T were to double? Triple? Be cut in half?

A: nothing. Both slope and intercept remain unchanged.

G. 5 points. Draw a schematic diagram of this budget constraint that demonstrates that the budget constraint line rotates clockwise through the endowment point as $\frac{P_C}{P_T}$ takes on larger and larger values.

$y = 2 - x$



$p = 1, 1.5, 2$

3. Tastes, aka preferences.

a. Exogenous

b. Represented as a family of indifference curves, which can be represented as utility function.

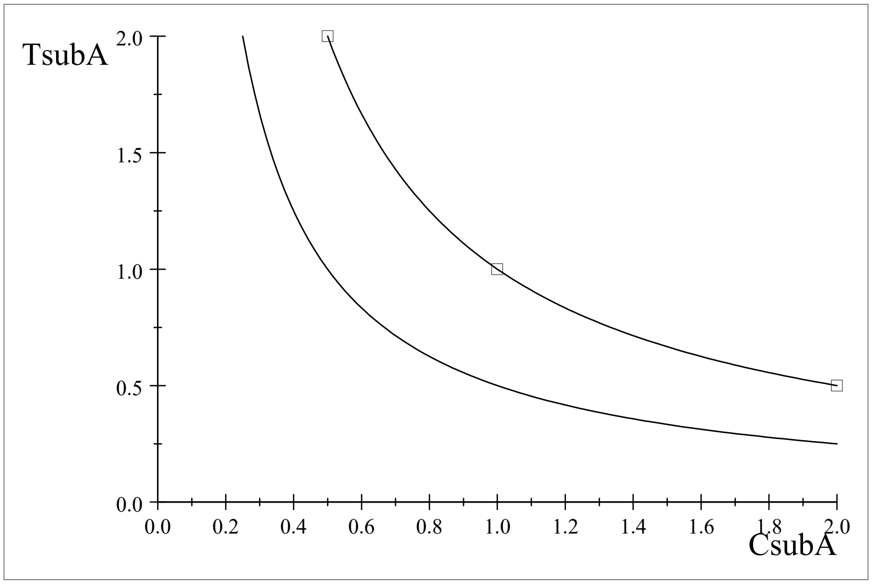
c. Classic example of intermediate micro is Cobb-Douglas, for which we put in numbers for parameters:

$$U_A = (C_A)^{\frac{1}{2}} (T_A)^{\frac{1}{2}}; \qquad \text{U-function}$$

$$(T_A)^{\frac{1}{2}} = \frac{U_A}{(C_A)^{\frac{1}{2}}} \qquad \text{rearrange}$$

$$\left[(T_A)^{\frac{1}{2}} \right]^2 = \left[\frac{U_A}{(C_A)^{\frac{1}{2}}} \right]^2 \qquad \text{fun w exp.}$$

$$T_A = \frac{(U_A)^2}{C_A}; \qquad \text{useful form}$$

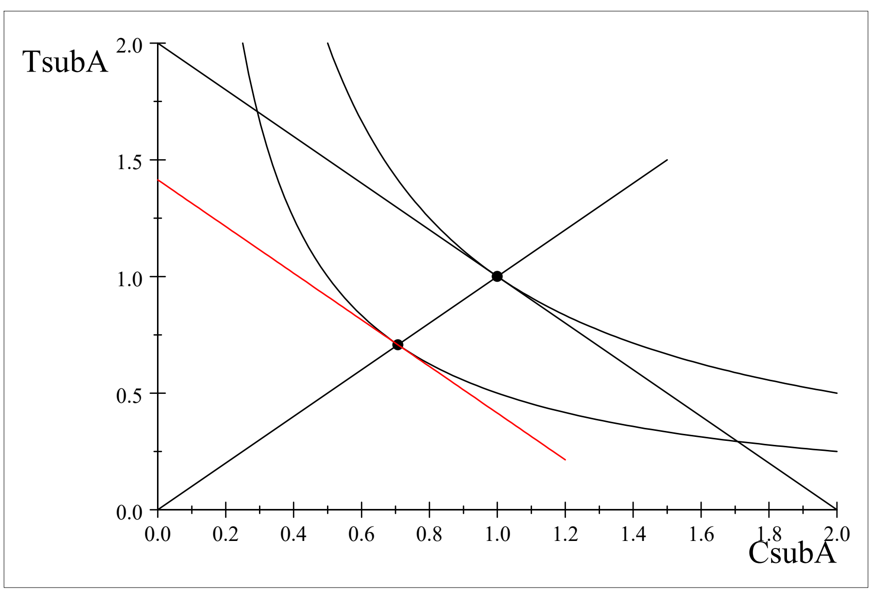


$$U_A = 1, U_A = \sqrt{\tfrac{1}{2}}$$

d. A key component: marginal rate of substitution(-slope of IC)

$$\frac{dT_A}{dC_A} = -\frac{(U_A)^2}{(C_A)^2} = \frac{\left[\overbrace{(C_A)^{\frac{1}{2}}(T_A)^{\frac{1}{2}}}^{U_A}\right]^2}{(C_A)^2}$$

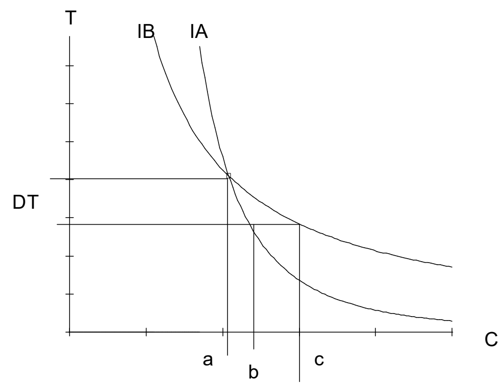
$$= \frac{C_A T_A}{(C_A)^2} = \frac{T_A}{C_A}.$$



$$U_A = 1, U_A = \sqrt{\tfrac{1}{2}}$$

e. NB: why didn't we leave the MRS function with U_A in the rhs?

f. Different tastes?



4. Why MRS so important?

a. Describes different tastes.

b. NJT (rank-order) versus Interstate 65 (cardinal).

5. Three parametric examples

a. Cobb-Douglas example

$$U_A = (C_A)^{\gamma_A}(T_A)^{1-\gamma_A}; 0 < \gamma_A < 1. \qquad \text{U-fnctn}$$

$$\left[(T_A)^{1-\gamma_A}\right]^{\frac{1}{1-\gamma_A}} = \left[\frac{U_A}{(C_A)^{\gamma_A}}\right]^{\frac{1}{1-\gamma_A}} = \frac{(U_A)^{\frac{1}{1-\gamma_A}}}{(C_A)^{\frac{\gamma_A}{1-\gamma_A}}}; \qquad \text{rearrange}$$

$$\frac{dT_A}{dC_A} = -\left(\frac{\gamma_A}{1-\gamma_A}\right)\left(\frac{(U_A)^{\frac{1}{1-\gamma_A}}}{(C_A)^{\frac{\gamma_A}{1-\gamma_A}+1}}\right) \qquad \text{ruls diffr.}$$

$$= -\left(\frac{\gamma_A}{1-\gamma_A}\right)\frac{\left(\overbrace{(C_A)^{\gamma_A}(T_A)^{1-\gamma_A}}^{U_A}\right)^{\frac{1}{1-\gamma_A}}}{(C_A)^{\frac{1}{1-\gamma_A}}} \qquad \text{sb}$$

$$= -\left(\frac{\gamma_A}{1-\gamma_A}\right)\left(\frac{(C_A)^{\frac{\gamma_A}{1-\gamma_A}}(T_A)}{(C_A)^{\frac{1}{1-\gamma_A}}}\right) \qquad \text{smp}$$

$$= -\left(\frac{\gamma_A}{1-\gamma_A}\right)\left((C_A)^{\frac{\gamma_A}{1-\gamma_A}-\frac{1}{1-\gamma_A}}(T_A)\right) \qquad \text{exprl}$$

$$= -\left(\frac{\gamma_A}{1-\gamma_A}\right)\left(\frac{T_A}{C_A}\right). \qquad \text{Miller Time!}$$

i. γ_A is a *parameter:* an exogenous number, e.g., $.6$

ii. Any monotonic transformation represents same family of indifference curves:

$$U_A = \phi_A(C_A)^{\gamma_A}(T_A)^{1-\gamma_A};$$

$$U_A = \left[(C_A)^{\gamma_A}(T_A)^{1-\gamma_A}\right]^{\phi_A};$$

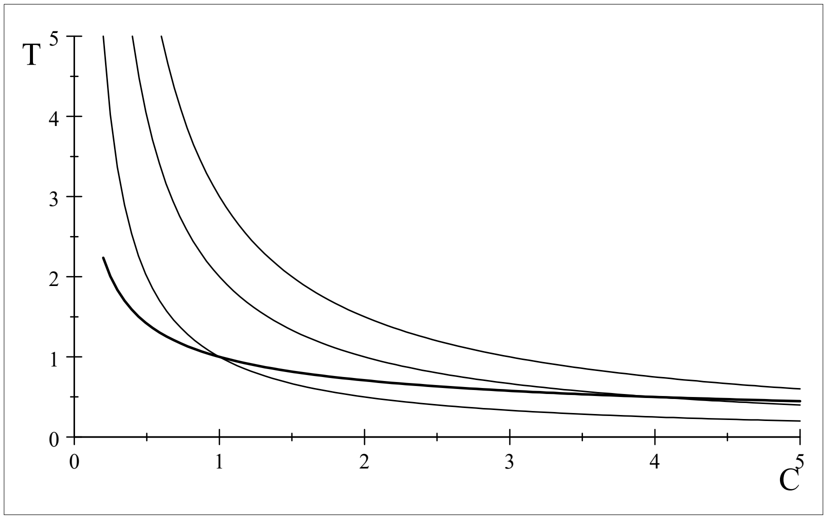
$$U_A = (C_A)^{\gamma_A}(T_A)^{1-\gamma_A} + \phi_A;$$

$$\frac{dT_A}{dC_A} = -\left(\frac{\gamma_A}{1-\gamma_A}\right)\left(\frac{T_A}{C_A}\right).$$

where ϕ_A is some number, any number whatsoever. (JBB says ...)

iii. Analogous specifications for Bob; example:

$$U_A = (C_A)^{\frac{1}{2}}(T_A)^{\frac{1}{2}}; U_B = (C_B)^{\frac{1}{3}}(T_B)^{\frac{2}{3}}$$



IA (Thin); IB (Thick)

b. No income effects on coffee with power function:

$$U_A = T_A + \frac{\gamma_A}{\phi_A}(C_A)^{\phi_A}; \gamma_A > 0; \phi_A < 1; \qquad \text{U fnctn}$$

$$T_A = U_A - \frac{\gamma_A}{\phi_A}(C_A)^{\phi_A}; \qquad \text{Rearrange}$$

$$\frac{dT_A}{dC_A} = -\gamma_A(C_A)^{\phi_A - 1} \qquad \text{-MRS}$$

i. Again, γ_A and ϕ_A are *parameters*, e.g., $\gamma_A = .6$, $\phi_A = \frac{1}{2}$.

ii. Useful when we want to abstract from complications of income effects.

iii. Again, as always, any monotonic transformation represents same family of indifference curves.

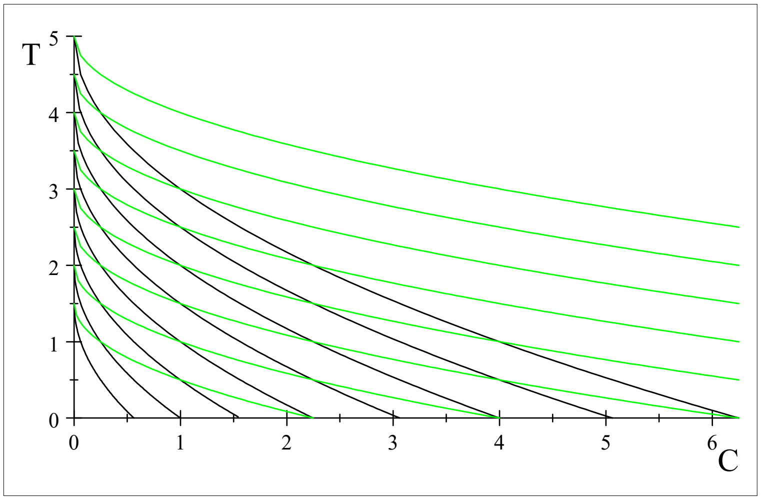
iv. If $\phi_i = 0$, we should write our preference function as

$$U_A = T_A + \gamma_A \ln C_A.$$

v. Analogous specifications for Bob

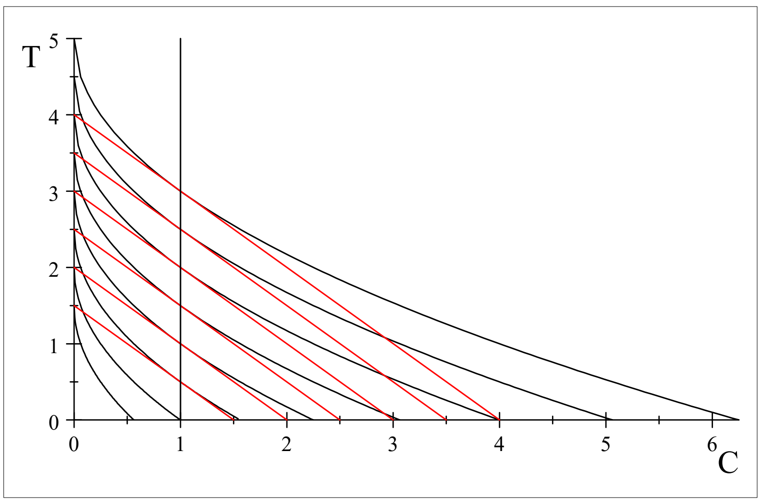
vi. Useful because it generates constant elasticity demand functions.

vii. Picture:



$\gamma = 1, (bl),\ \gamma = \frac{1}{2}(gr);\ \phi = \frac{1}{2}$

viii. Picture with set of budget constraints (each with same slope, $-\frac{P_C}{P_T}$):



$\gamma = 1,\ \phi = \frac{1}{2}$

c. NIE with quadratic

$$U_A = T_A + \alpha_{0A}C_A - \frac{\alpha_A}{2}(C_A)^2 \textit{ for } C_A \leq \frac{\alpha_{0A}}{\alpha_A}; \qquad \text{Ufnctn}$$

$$U_A = T_A + \frac{\alpha_{0A}^2}{2\alpha_A} \textit{ for } C_A \geq \frac{\alpha_{0A}}{\alpha_A}; \qquad \text{U-fnctn}$$

$$T_A = U_A - \alpha_{0A}C_A + \frac{\alpha_A}{2}(C_A)^2 \textit{ for } C_A \leq \frac{\alpha_{0A}}{\alpha_A}; \qquad \text{rerrng}$$

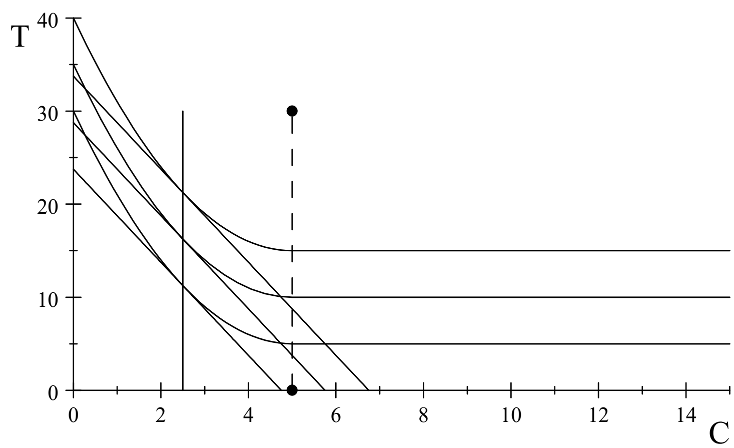
$$\frac{dT_A}{dC_A} = -\alpha_{0A} + \alpha_A C_A. \qquad \text{-MRS}$$

i. Again, α_A and α_{0A} are parameters, e.g.,

$$U_A = \begin{cases} T_A + 10C_A - (C_A)^2 & if \quad 0 \le C_A \le 5 \\ T_A + 25 & if \quad C_A > 5 \end{cases}$$

ii. Useful because it generates linear demand functions.

iii. Picture with set of equal-slope BC's:



$$T = U - 10C_A + C_A^2\ ;\ (\alpha_0 = 10,\ \alpha = 2)$$

6. **Most-preferred points and demand functions**, **aka finding consumer equilibrium**: Cobb-Douglas example

a. Tangency condition

$$\overbrace{\frac{-dT_i}{dC_i}}^{MRS_i(C_i,T_i)} = p; \qquad \text{generic statement}$$

$$-\frac{dT_i}{dC_i} = -\left(\frac{\gamma_i}{1-\gamma_i}\right)\left(\frac{T_i}{C_i}\right) = p; \qquad \text{CD example}$$

$$T_i = \frac{1-\gamma_i}{\gamma_i} pC_i \qquad \text{graph-friendly}$$

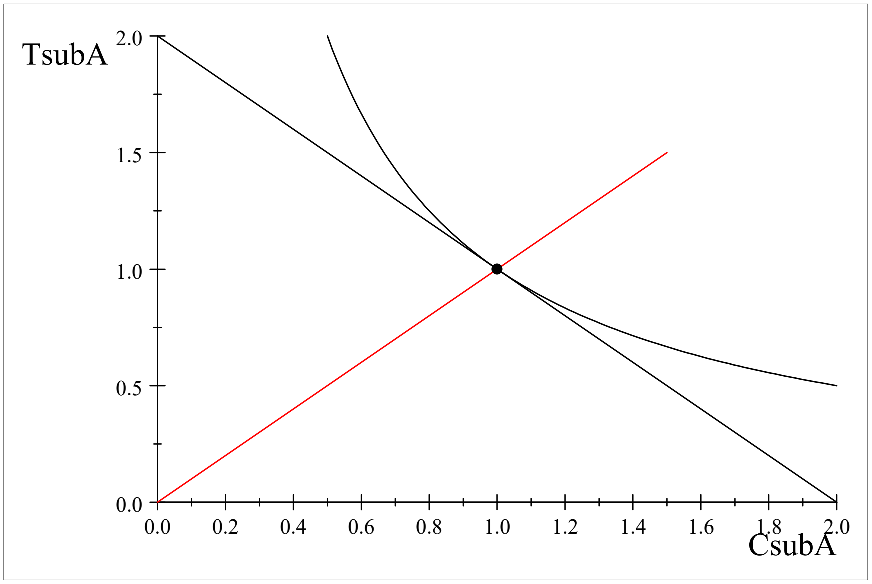
b. BC:

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i$$

c. Two equations, two unknowns (C_i, T_i) :

$$T_i = \frac{1-\gamma_i}{\gamma_i} pC_i;$$

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i$$



$BC(bl),\ TC(red)$

d. Solution:

$$T_i = \frac{1-\gamma_i}{\gamma_i} pC_i; \qquad \text{TC}$$

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i; \qquad \text{BC}$$

$$\overbrace{\frac{1-\gamma_i}{\gamma_i} pC_i}^{T_i} = \overline{T}_i + p\overline{C}_i - pC_i; \qquad \text{subst.}$$

$$pC_i\left[\frac{1-\gamma_i}{\gamma_i} + 1\right] = \overline{T}_i + p\overline{C}_i; \qquad \text{rearrange}$$

$$pC_i\left[\frac{1-\gamma_i}{\gamma_i} + \frac{\gamma_i}{\gamma_i}\right] = \overline{T}_i + p\overline{C}_i; \qquad \text{rearrange}$$

$$pC_i\left[\frac{1-\gamma_i+\gamma_i}{\gamma_i}\right] = \overline{T}_i + p\overline{C}_i; \qquad \text{rearrange}$$

$$pC_i = \gamma_i[\overline{T}_i + p\overline{C}_i]; \qquad \text{rearr gt exp shre}$$

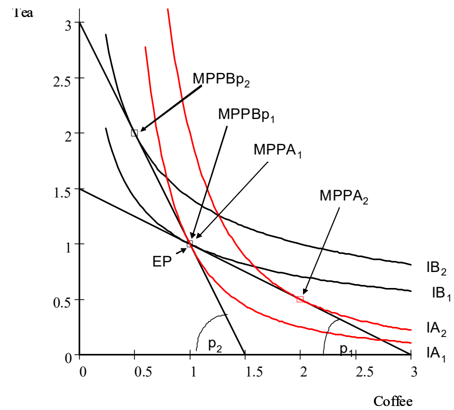
$$C_i^d = \gamma_i\left[\frac{\overline{T}_i + p\overline{C}_i}{p}\right]; \qquad \text{Miller time!}$$

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i \qquad \text{BC again}$$

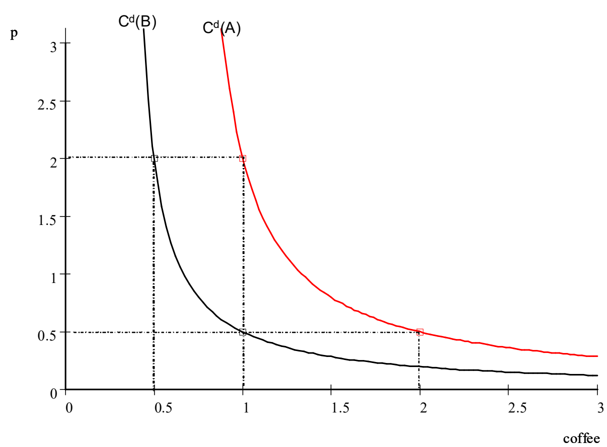
$$= \overline{T}_i + p\overline{C}_i - \overbrace{\gamma_i[\overline{T}_i + p\overline{C}_i]}^{pC_i} \qquad \text{sub}$$

$$T_i^d = (1-\gamma_i)[\overline{T}_i + p\overline{C}_i] \qquad \text{Tea time!}$$

e. Depiction



f. Ind. demand curve, function:



g. Aggregate or market demand curve:

i. Add up individual d-curves to create market demand curves, e.g.,

$$C^d \equiv C_A^d(p; \overline{C}_A, \overline{T}_A) + C_B^d(p; \overline{C}_B, \overline{T}_B)$$

ii. This is a curve in coffee-p plane.

iii. Usually assumed downward-sloping (an empirical assumption!)

iv. In general, placement in plane depends on all four endowments.

v. Example:

$$C^d = \frac{\gamma_A}{p}(\overline{T}_A + p\overline{C}_A) + \frac{\gamma_B}{p}(\overline{T}_B + p\overline{C}_B)$$

vi. With $\overline{T}_i = \overline{C}_i = 1,\ i = A,B,\ \gamma_A = \frac{1}{2},\ \gamma_B = \frac{1}{4}$

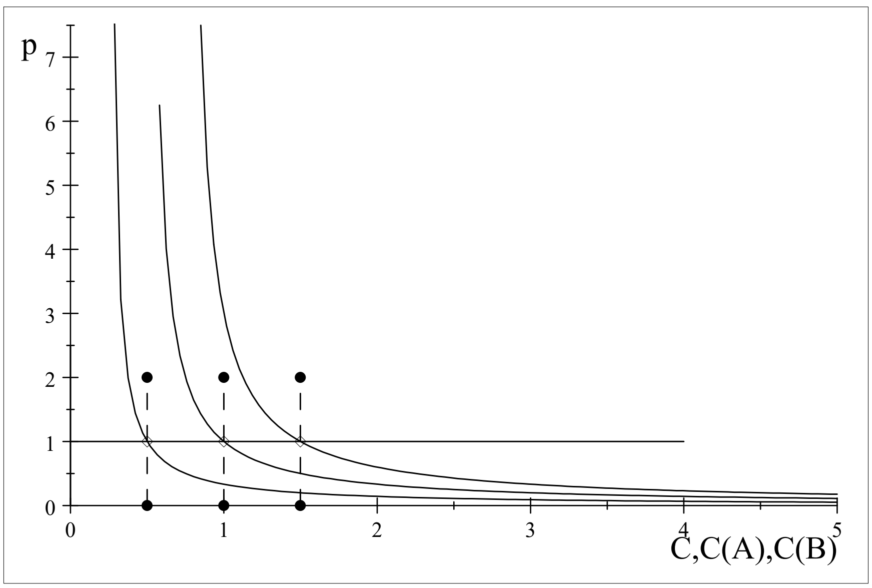
$$C_A^d = \frac{1+p}{2p};\ 2pC_A^d = 1+p;\ p(2C_A^d - 1) = 1;\ p = \frac{1}{2C_A^d - 1} = \frac{.5}{C_A^d - .5}$$

$$C_B^d = \frac{1+p}{4p}; 4pC_B^d = 1+p;\ p(4C_B^d - 1) = 1;\ p = \frac{1}{4C_B^d - 1} = \frac{.25}{C_B^d - .25}$$

$$C^d = \frac{.75}{p}(1+p).$$

vii. Inverse:

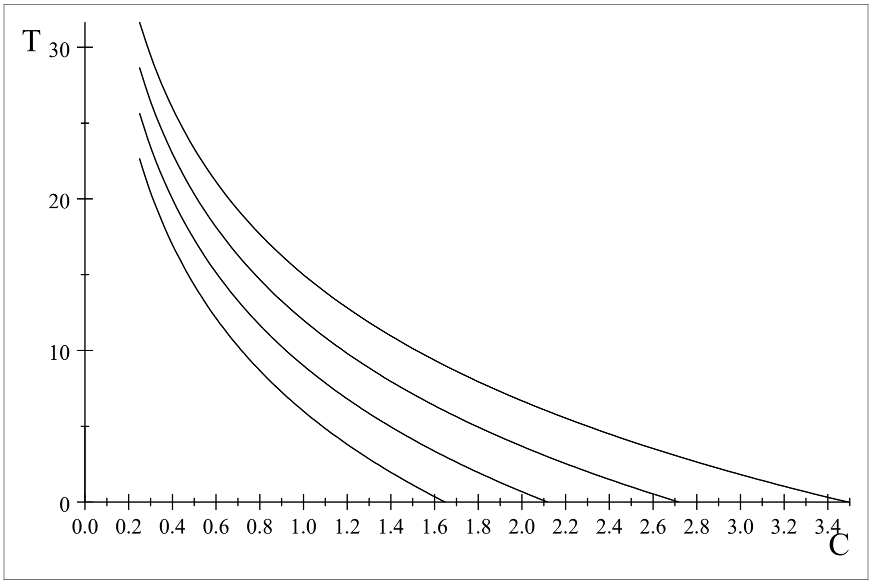
$$pC^d = .75 + .75p;$$
$$p(C^d - .75) = .75;$$
$$p = \frac{.75}{(C^d - .75)}$$



7. Most-preferred points (consumer equilibrium): NIE with ln function example.

$$U_i = T_i + \gamma_i \ln C_i; \qquad \text{U-fnctn}$$

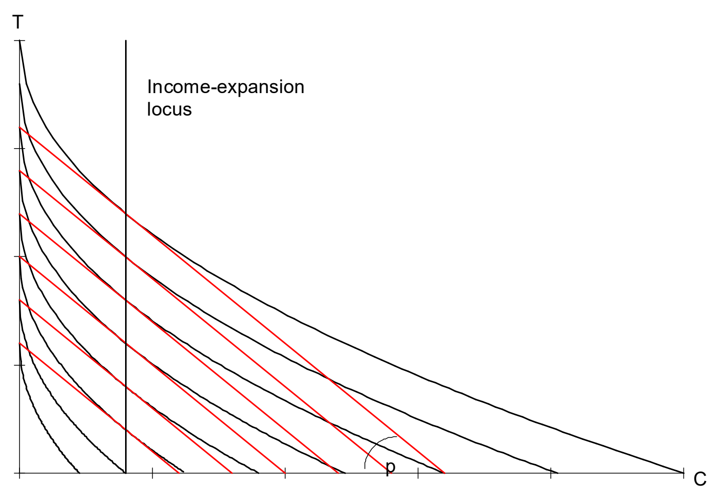
$$T_i = U_i - \gamma_i \ln C_i. \qquad \text{rearrange}$$



$$U_A = T_A + \gamma_A \ln C_A; \gamma_A = 12$$

a. Tangency condition:

$$T_i = U_i - \gamma_i \ln C_i;$$
$$\frac{dT_i}{dC_i} = \frac{-\gamma_i}{C_i} = -p;$$
$$C_i^d = \frac{\gamma_i}{p}.$$



Quasilinear log function

This is a demand curve! It answers the question: what happens to the value of C_i when p changes?

b. Budget constraint:

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i;$$

c. Demand for T_i :

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i; \qquad \text{BC}$$

$$pC_i = \gamma_i; \qquad \text{from TC}$$

$$T_i = \overline{T}_i + p\overline{C}_i - \gamma_i. \qquad T_i^d$$

d. Solution is thus

$$C_i^d = \frac{\gamma_i}{p}; T_i^d = \overline{T}_i + p\overline{C}_i - \gamma_i.$$

Note: need

$$\overline{T}_i + p\overline{C}_i - \gamma_i > 0.$$

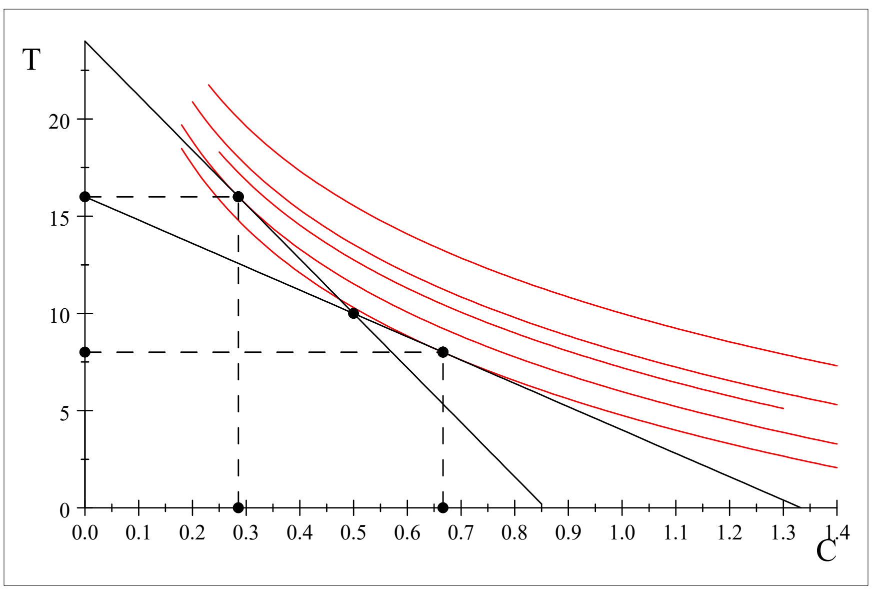
e. Aggregate or market demand curve:

$$C^d = \frac{1}{p}\sum_i \gamma_i.$$

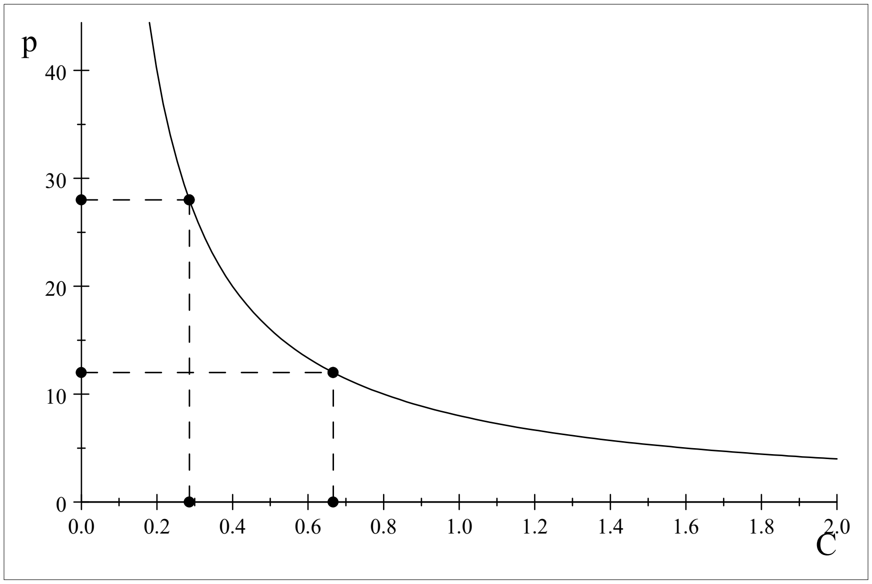
f. Inverse market demand curve:

$$p^d = \frac{\gamma}{p},\ \gamma \equiv \sum_i \gamma_i.$$

g. Depiction: same concept as with Cobb-Douglas diagrams.

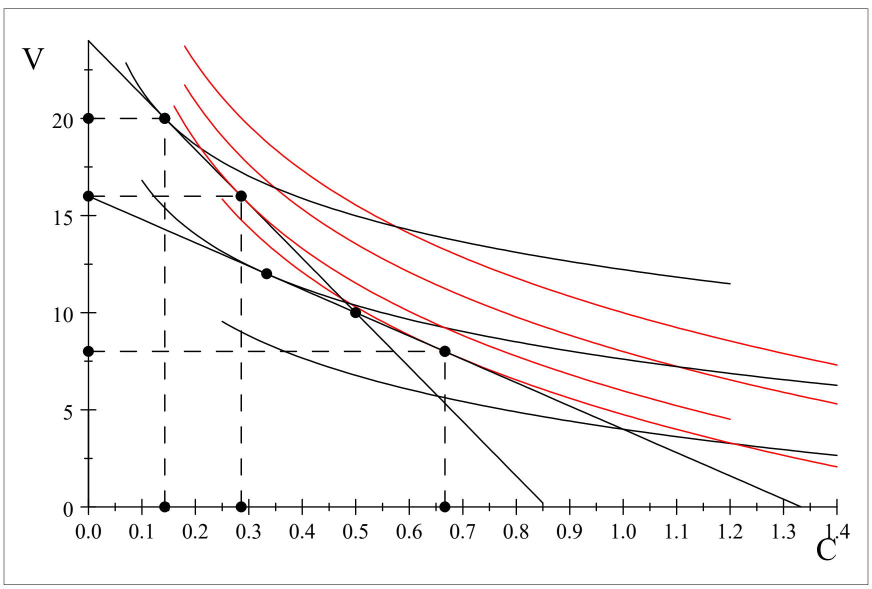


$$p = 12,\ 28,\ \gamma_A^* = 8, C_A = \tfrac{2}{7}, \tfrac{2}{3},\ T_A = 16, 12$$

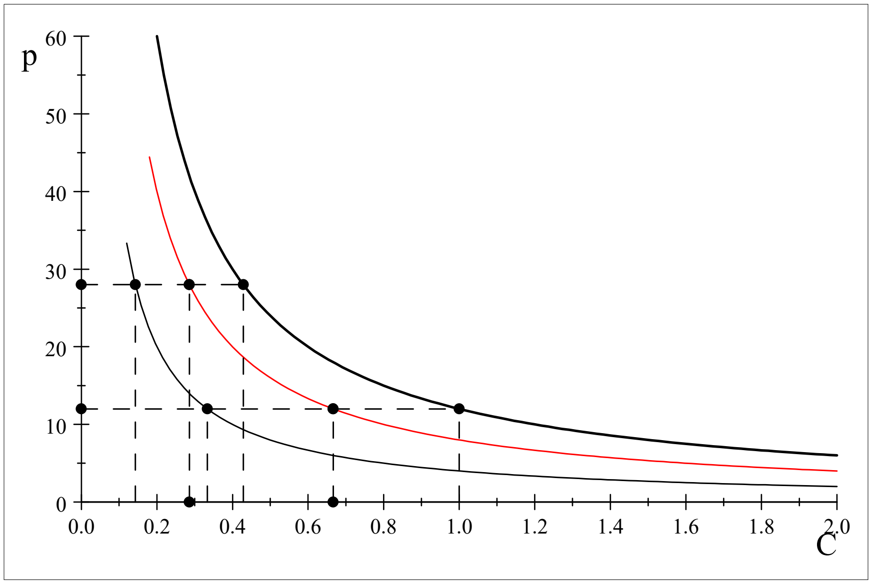


$$p = 12,\ 28;\ C = \frac{2}{7},\ \frac{2}{3};\ p = \frac{8}{C}$$

We add Bobby:



$$p_a^* = 12,\ 28; \gamma_A^* = 8, \gamma_B^* = 4$$



$$p = 12,\ 28;\ C_A = \tfrac{2}{7},\ \tfrac{2}{3};\ p = \tfrac{8}{C_A}, p = \tfrac{4}{C_B}$$

8. Most-preferred pair (consumer equilibrium) for quadratic NIE

$$U_A = T_A + \alpha_{0A} C_A - \frac{\alpha_A}{2}(C_A)^2 \text{ for } C_A \leq \frac{\alpha_{0A}}{\alpha_A};$$

$$U_A = T_A + \frac{\alpha_{0A}^2}{2\alpha_A} \text{ for } C_A \geq \frac{\alpha_{0A}}{\alpha_A};$$

a. TC (and demand for C):

$$T_A = U_A - \alpha_{0A} C_A + \frac{\alpha_A}{2}(C_A)^2 \text{ for } C_A \leq \frac{\alpha_{0A}}{\alpha_A};$$ rerrng

$$\frac{dT_A}{dC_A} = -\alpha_{0A} + \alpha_A C_A = -p;$$ -MRS

$$C_A^d = \frac{-1}{\alpha_A} p + \frac{\alpha_{0A}}{\alpha_A}.$$ Demand

b. BC

$$T_i = \overline{T}_i + p\overline{C}_i - pC_i;$$

c. Demand for T_A :

$$T_i = \overline{T}_i + p\overline{C}_i - \overbrace{\frac{-1}{\alpha_A} p^2 + \frac{\alpha_{0A}}{\alpha_A} p}^{pC_i}.$$

d. Inverse demand for coffee:

$$\alpha_{0i} - \alpha_i C_i^d = p.$$

e. Market demand curve

$$C_i^d = \frac{a_{0i}}{\alpha_i} - \frac{1}{\alpha_i}p;$$
$$C^d = \sum \frac{a_{0i}}{\alpha_i} - \sum \frac{1}{\alpha_i}p;$$
$$C^d = a_0 - \alpha p;\ a_0 \equiv \sum \frac{a_{0i}}{\alpha_i},\ \alpha \equiv \sum \frac{1}{\alpha_i}.$$

f. Inverse market demand curve

$$p = \frac{-1}{\alpha}C^d + \frac{\alpha_0}{\alpha}.$$

g. Depiction: similar (but demand curves are linear)

Supply sub-model

1. Simple: the benefit of the endowment economy specification

2. Market (aggregate) supply just the sum of individual endowments

$$C^S = \overline{C}_A + \overline{C}_B;$$
$$T^S = \overline{T}_A + \overline{T}_B.$$

3. Coffee supply (inverse) is depicted as vertical line in coffee-p plane.

Equilibrium: solving the model

1. Equilibrium: demand equals supply

$$\overline{C}_A + \overline{C}_B = C_A^d(p;\overline{C}_A,\overline{T}_A) + C_B^d(p;\overline{C}_B,\overline{T}_B).$$

2. This is one equation with 1 endogenous variable–p–and 4 exog.variables.

3. Supply only depends on *sum* of coffee endowments, demand on all of individual endowments.

4. Examples:

a. Cobb-Douglas

i. Solve for p_a (Autarkic Equilibrium Relative Price):

$$\overbrace{\overline{C}_A + \overline{C}_B}^{C^s} = \overbrace{\frac{\gamma_A}{p}(\overline{T}_A + p\overline{C}_A) + \frac{\gamma_B}{p}(\overline{T}_B + p\overline{C}_B)}^{C^d};$$
$$p(\overline{C}_A + \overline{C}_B) = \gamma_A\overline{T}_A + \gamma_A p\overline{C}_A + \gamma_B\overline{T}_B + \gamma_B p\overline{C}_B;$$
$$p(\overline{C}_A[1-\gamma_A] + \overline{C}_B[1-\gamma_B]) = \gamma_A\overline{T}_A + \gamma_B\overline{T}_B;$$
$$p_a = \frac{(\gamma_A\overline{T}_A + \gamma_B\overline{T}_B)}{(\overline{C}_A(1-\gamma_A) + \overline{C}_B(1-\gamma_B))}$$

A. Numerical example of solution for p_a :

$$\overline{T}_i = \overline{C}_i = 1,\ i = A,B,\ \gamma_A = \frac{1}{2},\ \gamma_B = \frac{1}{4}$$

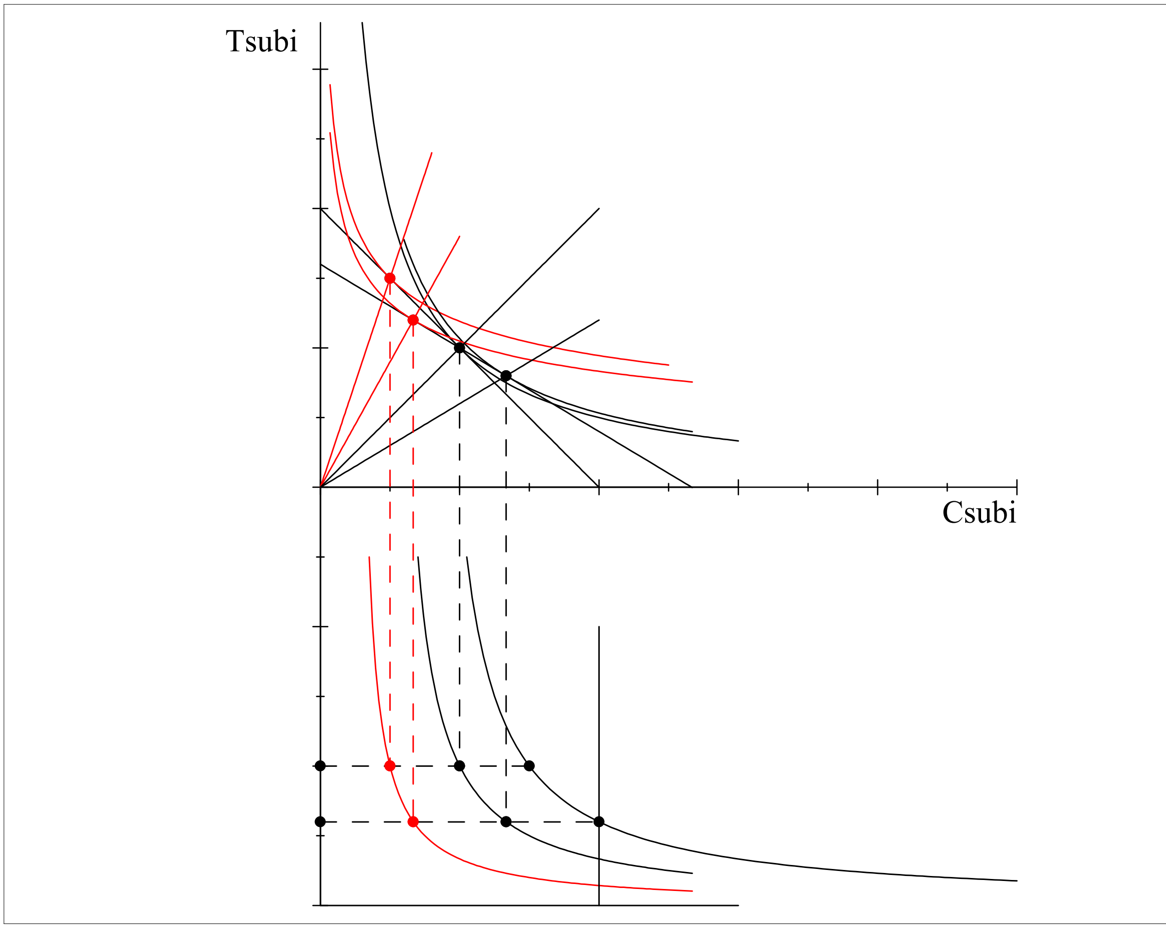
$$\frac{\overbrace{.75}^{\gamma_A+\gamma_B}}{p}\overbrace{(1+p)}^{\overline{T}_A+p\overline{C}_A} = \overbrace{2}^{\overline{C}_A+\overline{C}_B};$$

$$2p = (1+p)(.75);$$

$$p(2-.75) = .75$$

$$p_a = \frac{.75}{(1.25)} = \frac{3}{5}.$$

B. Depiction



Andy (r), Bob (b); $p = 1, p = .6$

ii. Substitute the AERP value of p, i.e., p_a, into individual demand curves to determine equilibrium consumptions of each individual.

$$C_A^d = \frac{\gamma_A\overline{T}_A(\overline{C}_A+\overline{C}_B)+\gamma_A\gamma_B(\overline{T}_B\overline{C}_A-\overline{T}_A\overline{C}_B)}{(\gamma_A\overline{T}_A+\gamma_B\overline{T}_B)}$$

iii. Substitute this value of p and C_i^d into BC or TC to obtain equilibrium quantity of other good consumed for each individual.

$$T_A^d = (1-\gamma_A)(\overline{T}_A + p\overline{C}_A)$$
$$= (1-\gamma_A)\frac{\overline{T}_A(\overline{C}_A+\overline{C}_B)+\gamma_B(\overline{T}_B\overline{C}_A-\overline{T}_A\overline{C}_B)}{(\overline{C}_A(1-\gamma_A)+\overline{C}_B(1-\gamma_B))}.$$

iv. Levels of well-being (cardinal):

$$U_A = (C_A^d)^\gamma (T_A^d)^{1-\gamma}$$

v. Example:

$$\overline{T}_i = \overline{C}_i = 1,\ i = A,B,\ \gamma_A = \frac{1}{2},\ \gamma_B = \frac{1}{4};$$
$$p_a = \frac{3}{5}; C_A = 1\frac{1}{3}, T_A = \frac{4}{5}, C_B = \frac{2}{3}, T_B = \frac{6}{5};$$
$$U_A = \left(\frac{8}{6}\right)^{.5}\left(\frac{4}{5}\right)^{.5} = 1.0328;$$
$$U_B = \left(\frac{2}{3}\right)^{.25}\left(\frac{6}{5}\right)^{.75} = 1.036.$$

vi. Compare with no trade:

$$U_A = 1;\ U_B = 1.$$

vii. Question: True or false: compared with not being able to trade with each other, Bob benefitted more than Alex from their joint trading ability.

b. NIE with natural log:

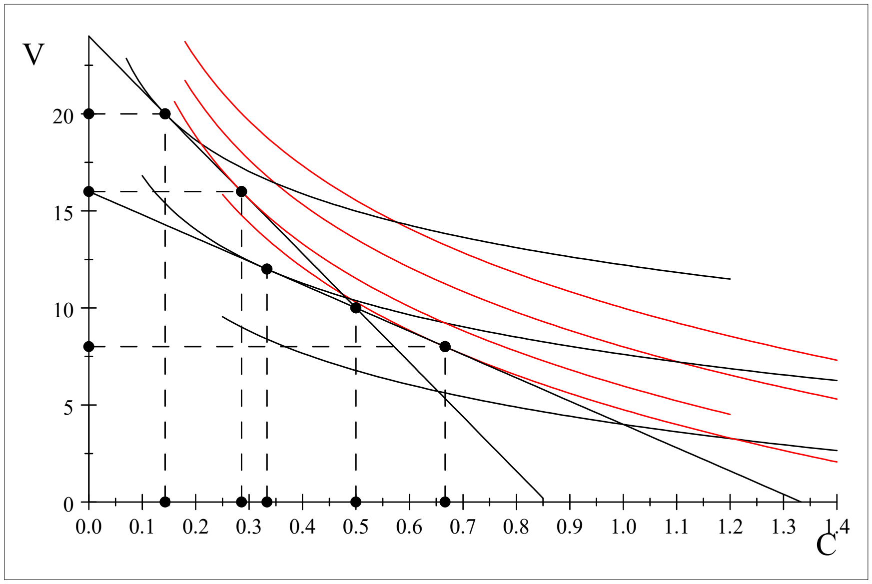
i. Solve for p_a :

$$C^d \equiv C_A^d + C_B^d = \frac{\gamma_A+\gamma_B}{p} = \overline{C} \equiv \overline{C}_A + \overline{C}_B;$$
$$p_a = \frac{\gamma_A+\gamma_B}{\overline{C}_A+\overline{C}_B} = \frac{\gamma}{\overline{C}},\ \gamma \equiv \gamma_A+\gamma_B.$$

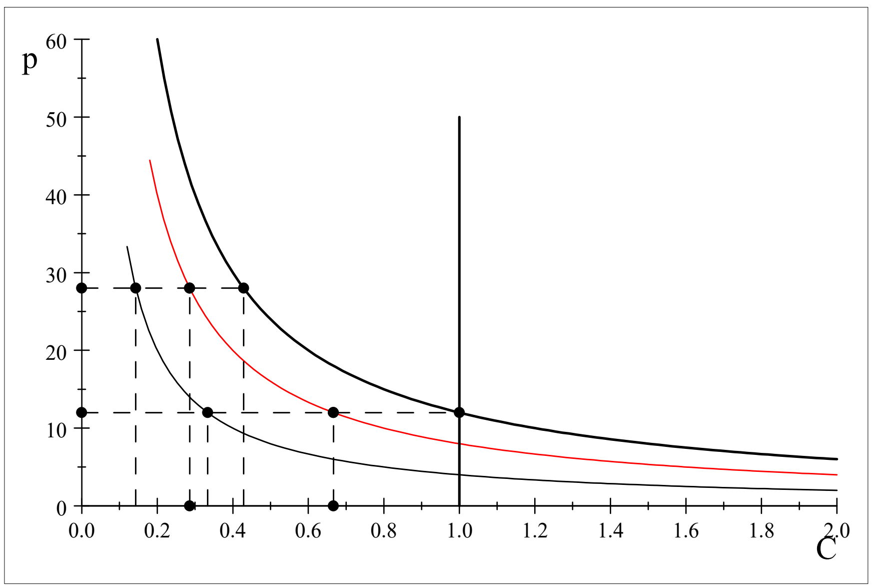
A. With the following numerical example:

$$U_i = T_i + \gamma_i \ln C_i;\ \overline{C}_A = \overline{C}_B = \frac{1}{2};$$
$$\overline{T}_A = \overline{T}_B = 10.$$
$$\gamma_A = 8,\ \gamma_B = 4,\ \gamma = 12;$$
$$p_a = \frac{8+4}{\frac{1}{2}+\frac{1}{2}} = 12.$$

B. Depiction



$$p_a^* = 12,\ 28; \gamma_A^* = 8, \gamma_B^* = 4$$



$$p = 12,\ 28;\ C_A = \frac{2}{7},\ \frac{2}{3};\ p = \frac{8}{C_A}, p = \frac{4}{C_B}$$

ii. Sub p_a back into individual demand functions to get equilibrium C_i^d, T_i^d.

$$C_A = \frac{\gamma_A}{\frac{\gamma}{\overline{C}}},\ C_B = \frac{\gamma_B}{\frac{\gamma}{\overline{C}}}$$

$$T_A = \overline{T}_A + p_a\overline{C}_A - p_aC_A;$$

$$T_B = \overline{T}_B + p_a\overline{C}_B - p_aC_B.$$

With our numbers:

$$C_A = \frac{\overbrace{8}^{\gamma_A}}{\frac{\overbrace{12}^{\gamma}}{\underbrace{1}_{\overline{C}}}} = \frac{2}{3}; C_B = \frac{4}{12} = \frac{1}{3};$$

$$T_A = \overbrace{10}^{\overline{T}_A} + \overbrace{12}^{p_a} \times \overbrace{\frac{1}{2}}^{\overline{C}_A} - \overbrace{12}^{p_a} \times \overbrace{\frac{2}{3}}^{C_A} = 8;$$

$$T_B = 10 + 12 \times \frac{1}{2} - 12 \times \frac{1}{3} = 12.$$

iii. Levels of well-being (cardinal):

$$U_A = 8 + 8\ln\left(\frac{2}{3}\right) = 4.7563$$

$$U_B = 12 + 4\ln\left(\frac{1}{3}\right) = 7.6056$$

$8 + 8\ln\left(\frac{2}{3}\right) = 4.7563$

iv. Example all in one place:

$$\overline{T}_i = 10,\ \overline{C}_i = \frac{1}{2},\ i = A,B,\ \gamma_A = 8,\ \gamma_B = 4;$$

$$p_a = 12; C_A = \frac{2}{3}, T_A = 8, C_B = \frac{1}{3}, T_B = 12;$$

$$U_A = 4.7563;$$

$$U_B = 7.6056.$$

v. Compare with no trade: $10 + 4\ln(\frac{1}{2}) = 7.2274$

$$U_A = 10 + 8\ln\left(\frac{1}{2}\right) = 4.4548;$$

$$U_B = 10 + 4\ln\left(\frac{1}{2}\right) = 7.2274.$$

vi. Question: True or false: compared with not being able to trade with each other, Bobby benefitted more than Alex from their joint trading ability.

5. Depiction vs solution (analytic) vs solution (in actual economies).

6. Walras Law: *if* the market for coffee is in equilibrium, *then* the market for tea must also be in equilibrium. (p. 115-116 CD)