# XXVII Southeast Geometry Seminar

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### Cohomogeneity one Ricci Flow and Nonnegative Sectional Curvature

Abstract: We show that some compact 4-manifolds, including $S^4$ and $CP^2$, admit metrics with nonnegative sectional curvature that immediately lose this property when evolved via Ricci flow. This behavior, which indicates certain limitations of Ricci flow beyond dimension 3 (where nonnegative sectional curvature is preserved), was previously known to happen only in dimensions $> 5$ or in noncompact manifolds. Such new examples stem from studying this evolution equation on manifolds with isometric cohomogeneity one actions, where it reduces to a system of PDEs in 2 variables. This is based on joint work with A. Krishnan.

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### Valuation theory of the Heisenberg algebra and contact manifolds

Abstract: Valuation theory lies between differential and integral geometry, bringing together such fundamental results as the Chern-Gauss-Bonnet theorem and the Blaschke-Chern-Federer-Santalo kinematic formulas. The Euclidean intrinsic volumes (volume, surface area, Euler characterisic,...) are a pivotal example of valuations that led to numerous central notions in geometry. Analogous families of intrinsic volumes were constructed fairly recently in other spaces, such as complex Hermitian space, pseudo-Euclidean space and a few others. Such intrinsic volumes can often be conveniently given by Crofton formulas. Following a brief introduction, we shall study the valuation theory of spaces with symplectic flavor. We first consider the Heisenberg algebra. There we classify the invariant valuations analogous to the intrinsic volumes; we apply them to construct new invariants of submanifolds in contact geometry, as well as to obtain a Crofton formula for the gaussian curvature of hypersurfaces. We will also produce a Crofton formula for the symplectic areas in a linear symplectic space, where they replace the Euclidean intrinsic volumes.

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## Asymptotically Locally Euclidean Kahler Manifolds

Abstract: The study of asymptotically locally Euclidean Kahler manifolds had a tremendous development in the last few years. This talk presents a survey of the main results and the open problems in this area. When the manifolds support an ALE Ricci flat Kahler metric the complex surfaces and their metric structures are well understood. The remaining case to be studied is that of ALE scalar flat Kahler manifolds. In this direction, the underlying complex manifold is described.

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## Dual curvature measures and their Minkowski problems

Abstract: The classical Minkowski problem asks for necessary and sufficient conditions in order to construct a convex body in the Euclidean space by prescribing surface area measure (or Gauss curvature in the smooth case). We discuss the Minkowski problem for the recently discovered geometric measures — dual curvature measures. The associated equations are Monge-Ampère type equations with measure data. Existence of solutions depends on measure concentration conditions.