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31st Annual Shanks Lecture

Hidden convexity for nonlinear PDE

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I will briefly discuss how important convexity conditions are for nonlinear partial differential equations and will then illustrate by a series of examples from recent research how seemingly non convex problems may in fact have a “hidden convex structure”.

Keynote speakers

Singularity formation in black hole interiors

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The prediction that solutions of the Einstein equations in the interior of black holes must always terminate at a singularity was originally conceived by Penrose in 1969, under the name of “weak cosmic censorship hypothesis”. The nature of this break-down (i.e. the asymptotic properties of the space-time metric as one approaches the terminal singularity) is not predicted, and remains a very hotly debated question to this day. One key question is the causal nature of the singularity (space-like, vs null for example). Another is the rate of blow-up of natural physical/geometric quantities at the singularity. Mutually contradicting predictions abound in this topic. Much work has been done under the assumption of spherical symmetry (for various matter models). We present recent developments (partly due to the speaker and G. Fournodavlos) which go well beyond this restrictive class, in vacuum.

Geometric graph-based methods for high dimensional data

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I will talk about a new class of problems in machine learning: segmenting large datasets using penalized graph cuts. One class of methods is based on interface models in PDE such as motion by mean curvature, the Allen-Cahn equation, and the MBO scheme, which already have been used for low dimensional image processing problems. Instead we consider large discrete datasets with a similarity graph connecting the discrete pieces of information. I will review both analytical results for these discrete problems, such as Gamma convergence, and show the behavior of the methods on real datasets.

Multilinear Restriction Theory

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I will introduce the linear and multilinear restriction theory and their relations with various areas of mathematics, in particular PDE and Harmonic Analysis. I will talk in more detail about the effect of the underlying geometry in the context of multilinear theory.

On mass-transfer across clean / contaminated fluid interfaces

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The local rate of mass transfer across a fluid interface, say the transfer of a gaseous component from rising gas bubbles, is strongly influenced by the presence of surface active substances, so-called surfactants. This is due to both a change of the local hydrodynamics due to Marangoni stresses because of inhomogeneous surface tension and a barrier effect resulting from the coverage of the interface by surfactant molecules. Starting from the case of mass transfer across deformable clean fluid interfaces, we introduce a sharp interface continuum thermodynamical model and a numerical method based on an interface tracking approach which is able to account for such surfactant effects on mass transfer.

Nonlocal evolution equations, electroconvection

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Coauthors: T. Elgindi, M. Ignatova, V. Vicol

I will describe results concerning nonlinear lower bounds and commutators involving the square root of the Dirichlet Laplacian in bounded domains. These arise in connection with the analysis of a model for electroconvection, which will be presented as well.

Geodesic flows on Fréchet–Lie groups

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Coauthors: Martin Bauer, University of Vienna; Boris Kolev, University of Aix-Marseille

The aim of the talk is to expose that various model equations appearing in mathematical hydrodynamics can be recast as geodesic flows of right-invariant weak Riemannian metrics induced by Fourier multiplication operators on the Fréchet-Lie group of all diffeomorphisms of the n-dimensional torus or the Euclidean n-space. Given mild structural conditions of the symbol of the inertia operator, it is shown that the corresponding initial value problem is well-posed in the smooth category and that the Riemannian exponential mapping is a smooth local diffeomorphism. This approach covers in particular the case of right-invariant metrics induced by Sobolev norms of any fractional order larger than 1/2. In addition, if the order of the inertia operator is larger than 3 then it is exposed that the corresponding weak Riemannian metric is geodesically complete. Applications to the equations of Camassa–Holm, Degasperis–Procesi, and Constantin–Lax–Majda are also discussed.

J. Escher, Non-metric two-component Euler equations on the circle. *Monatsh. Math.* **167** (2012), 449–459.

J. Escher and B. Kolev, Geodesic completeness for Sobolev H^s-metrics on the diffeomorphism group of the circle. *J. Evol. Equ.* **14** (2014), 949–968.

M. Bauer, J. Escher, and B. Kolev, Local and global well-posedness of the fractional order EPDiff equation on $\mathbb{R}^d$. *J. Differential Equations* **258** (2015), 2010–2053.

High frequency dynamics for NLS on a torus

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I will present the derivation of a new equation describing the dynamics of the nonlinear Schrodinger equation (NLS) set on a d-dimensional torus, in the high frequency, weakly nonlinear regime. This equation informs on the large time behavior of NLS, and is related to the theory of weak turbulence; it also has very intriguing properties.

Dynamics of the Ericksen-Leslie Model for Nematic Liquid Crystal Flow

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Coauthors: Jan Prüss

Consider the Ericksen-Leslie model describing the flow of nematic liquid crystals with general Leslie stress. We discuss recent local and global well-posedness results in the strong sense for this system and describe in addition the dynamical behaviour of its solutions. It is remarkable that for these results no structural assumptions on the Leslie coefficients are being imposed. Ths is joint work with Jan Prüss.

How to stabilize a 3-D Navier Stokes equation with a finite dimensional boundary feedback controller?

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Solutions to a 3 -D Navier Stokes equations in the vicinity of unstable equilibrium will be considered. The system is acted upon by external forces which render the flow turbulent. The corresponding steady state equation contains multiple unstable equilibria. Uniform stabilization near a priori selected equilibrium will be discussed. Several results leading to various forms of feedback control with constrained support (including boundary control) will be reviewed and discussed. In particular, it will be shown that *finite dimensional boundary feedback control* can effectively stabilize locally the system also in a 3-D case. This result hinges upon maximal parabolic regularity of Stokes operator in L_p settings along with new unique continuation results obtained for Oseen's operator.

Heat kernels, maximal regularity, and semi-linear parabolic equations on non-compact manifolds.

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Using finite speed of propagation, we investigate properties of operators of the form $f(\sqrt{L})$, where L is second-order, uniformly elliptic, positive semi-definite differential operator on a complete Riemannian manifold of bounded geometry M, acting between sections of a vector bundle with bounded geometry E over M. In particular, we establish results on the distribution kernels and mapping properties of e^{-tL} and $(\mu + L)^s$. We show that L generates a holomorphic semigroup on $W^{s,p}$-Sobolev spaces on M and E. We also prove that L satisfies maximal L^pL^q regularity for $1 < p, q < \infty$. We apply these results to study parabolic systems of semi-linear equations of the form $\partial_t u + Lu = F(t, x, u, \nabla u)$.

Long term dynamics of nonlinear dispersive evolution equations

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Coauthors: Nicolas Burq, Genevieve Raugel

We will review some of the work on the nonlinear version of the asymptotic completeness problem for dispersive wave equations. In particular, we will describe recent work with Burq and Raugel (Orsay, France) on the case of a damped Klein-Gordon equation.

Fluid-fluid and fluid-solid interfaces and a model for Rayleigh-Taylor instability

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I will describe certain conditions on initial data which prevent both fluid-fluid and fluid-solid interfaces from either self-intersecting or colliding with fixed boundaries in finite time. I will then present a new model for Rayleigh-Taylor interface growth and instability.

Shock formation in nearly plane symmetric solutions to quasilinear wave equations
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Coauthors: Gustav Holzegel, Jonathan Luk, Willie Wong

I will discuss some new results on the formation of shocks in solutions to various quasilinear wave equations in two space dimensions, including the compressible Euler equations in the absence of vorticity. The results apply to initial data with approximate plane symmetry. In contrast to previous works on shock formation, the solutions do not experience wave dispersion, which necessitates a new methodology for treating error terms. The results may be viewed as an extension of P. Lax's famous blow-up result for 2x2 strictly hyperbolic genuinely nonlinear systems, where the new feature is that we can allow for perturbations that break the one-spatial-dimensional symmetry. I will highlight some of the main ideas behind the analysis including the critical role played by geometric decompositions based on true characteristic hypersurfaces and the surprisingly good null structure of the equations. The geometric framework allows for a complete description of the shape of the boundary of the region of classical existence and the mechanism that drives the singularity formation.

Conservation laws in completely integrable pde's
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One of the well known properties of completely integrable integrable pde's is that they admit an infinite number of conservation laws. In this work we show how the integrable properties generate in effect a continuum of conservation laws. Examples to be discussed include NLS, mKdV and KdV.

Is Dispersion a Stabilizing or Destabilizing Mechanism?
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In this talk I will present a unified approach for the effect of fast rotation and dispersion as an averaging mechanism for, on the one hand, regularizing and stabilizing certain evolution equations, such as the Navier-Stokes and Burgers equations. On the other hand, I will also present some results in which large dispersion acts as a destabilizing mechanism for the long-time dynamics of certain dissipative evolution equations, such as the Kuramoto-Sivashinsky equation.

On the vanishing viscosity limit for the Navier-Stokes equations
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Coauthors: Joint work with: Peter Constantin, Tarek Elgindi, Mihaela Ignatova; and also Igor Kukavica and Fei Wang

We consider the vanishing viscosity limit of the Navier-Stokes equations in a half space, with Dirichlet boundary conditions. We prove that the inviscid limit holds in the energy norm if the Navier-Stokes solutions remain bounded in $L_t^2 L_x^\infty$ independently of the kinematic viscosity, and if they are equicontinuous at $x_2 = 0$. These conditions imply that there is no boundary layer separation: the Lagrangian paths originating in a boundary layer, stay in a proportional boundary layer during the time interval considered. We then give a proof of the conjecture of vanDommelen and Shen (1980) which predicts the finite time blowup of the displacement thickness in the Prandtl boundary layer equations. This shows that the Prandtl layer exhibits dynamic separation in finite time.

Stability of the 3D Couette Flow
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We discuss the dynamics of small perturbations of the plane, periodic Couette flow in the 3D incompressible Navier-Stokes equations at high Reynolds number. For sufficiently regular initial data, we determine the stability threshold for small perturbations and characterize the long time dynamics of solutions near this threshold. For rougher data, we obtain an estimate of the stability threshold which agrees closely with numerical experiments. The primary linear stability mechanism is an anisotropic enhanced dissipation resulting from the mixing caused by the large mean shear; the main linear instability is a non-normal instability known as the lift-up effect. Understanding the variety of nonlinear resonances and devising the correct norms to estimate them form the core of the analysis we undertake. Joint work with Pierre Germain and Jacob Bedrossian

Celebrating the work of Jan Prüss (Simonett and Hieber)

Domain variations and the linearization of moving boundary problems.
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In this talk we shall present a novel approach to the linearization of moving boundary problems which allows for the linearization to be computed with respect to the moving boundary itself and not via a parametrization over a smooth reference manifold as has become customary to do. The upshot is a simpler and insightful representation of the linearization itself.

The Hartman-Grobman Theorem for Semilinear Hyperbolic Evolution Equations
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We extend the famous Hartman-Grobman Theorem (also known as *linearisation theorem*) for ordinary differential equations to abstract semilinear hyperbolic evolution equations in Banach spaces. Common proofs of this theorem for ordinary differential equations use topological arguments like Brouwers fixed point theorem to show the surjectivity of the linearising map. Our proof avoids any such additional topological arguments, which are not available in the infinite dimensional case, and thus also gives a new method of proof in the finite dimensional case. This is done by deriving an injective left inverse for the linearising map. With this direct approach we show that the linearising map is a homeomorphism.

Furthermore, we show Hölder continuity of the linearizing map and its inverse and conclude the stability of these maps w. r. t. perturbations of the data.

References

M.-L. Hein, J. Prüss. The Hartman-Grobman Theorem for Semilinear Hyperbolic Evolution Equations. Submitted 2016.

Global Strong L^p Well-Posedness of the 3D Primitive Equations

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Primitive Equations are considered to be a fundamental model for geophysical flows. Here, the L^p theory for the full primitive equations, i.e. the three dimensional primitive equations coupled to the equation for temperature and salinity, is developed. This set of equations is globally strongly well-posed for arbitrary large initial data lying in certain interpolation spaces, which are explicitly characterized as subspaces of $H^{2/p,p}$, $1 < p < \infty$, satisfying certain boundary conditions. Thus, the general L^p setting admits rougher data than the usual L^2 theory with initial data in H^1.

In this study, the linearized Stokes type problem plays a prominent role, and it turns out that it can be treated efficiently using perturbation methods for H^∞-calculus due to Jan Prüss.

Optimal Estimates for Multiplication and Analytic Nemytskij Operators in Anisotropic Function Spaces

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We study multiplication as well as Nemytskij operators in anisotropic vector-valued Besov spaces $B_p^{s,\omega}$, Bessel potential spaces $H_p^{s,\omega}$, and Sobolev-Slobodeckij spaces $W_p^{s,\omega}$. Concerning multiplication we obtain optimal estimates, which constitute generalizations and improvements of known estimates in the isotropic/scalar-valued case. Concerning Nemytskij operators we consider the acting of analytic functions on supercritial anisotropic vector-valued function spaces of the above type. Moreover, we show how the given estimates may be used in order to improve results on quasi-linear evolution equations as well as their proofs.

Stability of cylinders in surface diffusion flow under general perturbations

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The surface diffusion flow is a geometric evolution equation which prescribes the normal velocity of points on immersed, oriented manifolds to equal the Laplace-Beltrami operator acting on the mean curvature at the point. Given a parameterization for the manifold, the morphological evolution of the parameterization is expressed by a fourth-order, quasilinear, parabolic pde. In this talk, I will discuss results regarding the stability of unbounded cylinders (as stationary solutions to surface diffusion flow) under general perturbations with periodicity along the cylindrical axis.

Regularity of stochastic integral equations driven by Poisson random measures
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We examine a stochastic integral equation of Volterra type with singular kernels driven by a measurable process ξ measured against a compensated Poisson random measure. The increase or decrease of the regularity of the solution (taking values in a Banach space) as compared to that of ξ is examined as a function of the parameters of the singular kernels. The regularity is measured in real interpolation spaces determined by the operator A occurring in the equation. The results generalize maximal regularity results obtained by Brzezniak and Hausenblas. Our estimates utilize properties of the resolvent kernels and L^p-estimates obtained by Dirksen, Maas, and v.Neerven.

Evolution systems of measures and asymptotic behaviour in linear non-autonomous Kolmogorov equations.
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Coauthors: Joint works with L. Angiuli, A. Lunardi, R. Schnaubelt

In this talk, I will discuss about evolution systems of measures associated with Kolmogorov operators $\{\mathcal{A}(t) : t \in I\}$ (I being a right-halfine, possibly $I = \mathbb{R}$) and show the role played by the tight system in the asymptotic analysis at infinity of the evolution operator $G(t, s)$ associated with $\mathcal{A}(t)$.

On the motions of a liquid-filled rigid body around a fixed point
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We consider the motions around a fixed point of a coupled system $\mathcal{S}$ constituted by a heavy rigid body $\mathcal{B}$ with an interior cavity $\mathcal{C}$ completely filled by an incompressible viscous fluid, and whose motion is governed by the Navier-Stokes equations. We give a complete description of the steady state solutions to the relevant equations of motion. For what concerns the time-dependent problem, for a large class of liquid-solid configurations, we show that weak solutions corresponding to initial data of (arbitrary) finite total energy converge to a steady state as time tends to infinity. Specifically, the velocity of the liquid relative to the solid tends to zero, and the ultimate motion of the coupled system $\mathcal{S}$ is either a permanent rotation or a steady precession. Moreover, we answer some questions regarding attainability and nonlinear stability of the relevant steady states.

Interior local null controllability of one-dimensional compressible flow near a constant steady state

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We consider the one-dimensional compressible Navier-Stokes equations with periodic boundary conditions, with initial conditions in a small neighborhood of a state of uniform density and uniform nonzero velocity. We prove that, with a control given by a body force localized in a subinterval, we can steer the system to uniform density and velocity.

Linearized stability for nonlinear Volterra equations

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In the context of the nonlinear Volterra equation $u(t) + \int_0^t b(t-s)Au(s)ds \ni u_0$, $t \geq 0$, with $A \subset X \times X$ an m$-\alpha-$accretive operator in a Banach space X, and b a completely positive kernel, we [1] establish a principle of linearized stability of an equilibrium solution u_e under the assumption of the existence of a resolvent differential $\tilde{A} \subset X \times X$ of A at u_e with the property that $(\tilde{A} - \omega I)$ is accretive for some $\omega > 0$. For the special semilinear case $A = (-B + F)$, with $B : D(B) \subset X \to X$ the infinitesimal generator of a C_0-semigroup on X, and F with a kind of 'relative F-differential' $F'[u_e]$ at the equilibrium, a resolvent differential of A is given by $\tilde{A} = (-B + F'[u_e])$ ([2]).

References

[1] S.-O. Londen, and W. M. Ruess, Linearized stability for nonlinear Volterra equations, preprint 2015
[2] W.M. Ruess, Linearized stability for nonlinear evolution equations, J. Evol. Equ. 3 (2003), 361-373

Analysis of a Living Fluid Continuum Model

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We present an analytical approach to a generalized Navier-Stokes system which was proposed recently to describe active turbulence in living fluids. Results on wellposedness and stability are derived. Due to the presence of a Swift-Hohenberg term global wellposedness in a strong setting for arbitrary square integrable initial data is available. Based on the existence of global strong solutions, results on linear and nonlinear (in-) stability for the disordered steady state and the manifold of ordered polar steady states are derived, depending on the involved parameters.

Exponential stability of a thermoviscoelastic mixture with second sound

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In this talk we consider a one-dimensional model of a thermoviscoelastic mixture with second sound. Under suitable assumptions on the constitutive constants of the system we prove, using a characterization of the exponential stability of semigroups obtained by Jan Prüss, that the damping effect through heat conduction given by Cattaneos law is strong enough to stabilize the system.

Error analysis of the ADI Splitting method for the Maxwell equations

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The ADI scheme is a very efficient explicit-implicit time integration method for the linear Maxwell equations in certain situations. It is based on a splitting of the curl operators, which allows to decouple implicit steps into one dimensional sub problems. We establish error bounds for this scheme and show near preservation of the divergence conditions. We also treat the inhomogeneous system with currents and charges, but we do not consider the space discretization. Our results are based on wellposedness and regularity results for the full and the split systems.

Wellposedness of a nonlocal nonlinear diffusion equation of image processing

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In this talk, we will establish the wellposedness of a degenerate regularization of the well-known Perona-Malik equation in noise reduction for discontinuous initial data. We will also show the (exponential) asymptotic stability of stationary solutions.

Two-phase flows with phase transitions

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We consider models for compressible and incompressible two-phase flows with phase transitions. These are based on first principles, i.e., balance of mass, momentum, and energy. Performing a Hanzawa transform, the problem is transformed to a quasilinear parabolic two-phase problem with complicated transmission conditions on the interface in a fixed domain. We prove maximal Lp-regularity of the corresponding linearized problem, and then by a fixed point argument in a suitable space, we obtain local existence for the isothermal, compressible model with phase transitions. This is a joint work with Prof. Jan Prüss (Halle, Germany).

Heat-Structure interaction:optimal rational decay rate by microlocal analysis

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In this paper we consider a heat-structure interaction model (first step toward a fluid-structure interaction model) in 2 or 3 dimensions. It consists of a heat equation defined on an external domain coupled at the interface with a wave equation defined on an internal domain. Boundary coupling involves matching of the velocity of the wave and of the fluid and matching the normal stresses at the interface between the two domains. We take Initial Conditions in the domain of the generator of the strongly continuous contraction semi-group that models the entire coupled system. We then shows that the solutions decay as 1/t, asymptotically, thereby establishing the conjectured optimal decay rate. This improvement of past results is established via two main technical approaches: (i) a recent frequency domain characterization of the rational decay in terms of the resolvent operator evaluated on the imaginary axis; (ii) a microlocal analysis treatment to estimate a critical term involving two problematic boundary traces. (Received April 25, 2016)

A Free Boundary Problem Modeling MEMS

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Idealized electrostatic microelectromechanical systems (MEMS) consist of a fixed ground plate above which an elastic plate is suspended. The elastic plate deforms due to a Coulomb force induced by a voltage difference that is applied between the two components. The corresponding mathematical model involves the harmonic electrostatic potential in the free domain between the two plates along with an evolution equation for the displacement of the elastic plate. Of particular interest is the dynamics in dependence of the voltage difference applied between the plates. In this talk we discuss some existence results and results on the singularity that possibly occurs in the evolution when the elastic plate touches down on the ground plate.

On the Westervelt equation with absorbing boundary conditions

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We investigate the Westervelt equation from nonlinear acoustics, subject to nonlinear absorbing boundary conditions, which were recently proposed by Kaltenbacher & Shevchenko. We apply the concept of maximal regularity of type L_p to prove global well-posedness for small initial data. Moreover, we show that the solutions regularize instantaneously which means that they are C^∞; with respect to time t as soon as $t > 0$. Finally, we show that each equilibrium is stable and each solution which starts sufficiently close to an equilibrium converges at an exponential rate to a possibly different equilibrium.

Stability, instability, and blowup for non-local in time reaction-diffusion equations

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I will consider a class of semilinear reaction-diffusion equations with an integro-differential operator in time. The important case of a fractional dynamics of order in between 0 and 1 is included. I will present recent results on the well-posedness in a setting of bounded weak solutions, stability and instability of equilibria, as well as on the blowup of solutions.

Qualitative properties of solutions to free boundary problems (Escher)

Weak solutions to a two-phase thin film equation with insoluble surfactant

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The study of thin film equations constitutes a rich and complex area of research with a long list of contribution. Considering the influence of surface active agents (short *surfactants*) on the dynamics of thin liquid films finds applications in multiple industrial and biochemical fields. Although, during the last decades there has been various modeling and numerical treatment of several aspects of the surfactant induced evolution of thin films, only recently analytical investigations have started.

In this talk, we consider a model describing the spreading of an insoluble surfactant on the upper surface of a viscous complete wetting two-phase thin film. Taking capillary effects as the only driving force, the system of evolution equations consists of two strongly coupled degenerated equations of fourth order describing the film heights of the fluids, which are additionally strongly coupled to a second-order transport equation for the surfactant concentration. Owing to the degeneracy, it is in general not clear whether one can prove the existence of global solutions in a classical sense, which motivates the study of weak solutions.

On periodic traveling waves of the Camassa-Holm equation

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In this talk we are concerned with the wave length λ of smooth periodic traveling wave solutions of the CamassaHolm equation. The set of these solutions can be parametrized using the wave height a. Our main result establishes monotonicity properties of the map $\lambda(a)$ i.e., the wave length as a function of the wave height. We obtain the explicit bifurcation values, in terms of the parameters associated with the equation, which distinguish between the two possible qualitative behaviours of $\lambda(a)$, namely monotonicity and unimodality. The key point is to relate $\lambda(a)$ to the period function of a planar differential system with a quadratic-like first integral, and to apply a criterion which bounds the number of critical periods for this type of systems.

On Qualitative Properties of Solutions to Microelectromechanical Systems with General Permittivity

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Qualitative properties of solutions to an evolution problem modelling microelectromechanical systems with general permittivity profile are investigated. The system couples a parabolic evolution problem for the displacement of a membrane with an elliptic free boundary value problem for the electrostatic potential in the region between the membrane and a rigid ground plate. A topic is addressed that is of note not till non-constant permittivity profiles are taken into account – the direction of the membrane's deflection or, in mathematical parlance, the sign of the solution to the evolution problem. By means of the parabolic comparison principle structural conditions on the potential and on the permittivity profile are specified which guarantee non-positivity of the membrane's displacement. Finally, numerical results on the coupled system are presented which in particular justify the consideration of the full coupled problem by revealing a substantial qualitative difference between the solutions to the widely-used small-aspect ratio model and the full coupled problem.

A domain of parabolicity for the Muskat problem

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The Muskat problem is a moving boundary problem describing the evolution of two immiscible layers of Newtonian fluids with different densities and viscosities, for example oil and water, in a porous medium. This problem has been studied in the last two decades by many mathematicians, the methods employed being various. We show that in the absence of surface tension effects the Rayleigh-Taylor sign condition identifies a domain of parabolicity for the Muskat problem. When allowing for surface tension effects, the Muskat problem is of parabolic type for general initial and boundary data. The parabolicity property is used to establish the well-posed of the problem and to study the stability properties of equilibria.

Singular problems in fluid mechanics (Mazzucato)

Boundary layer analysis of the linearized Navier-Stokes equations

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In this talk, we study the asymptotic behavior at small viscosity of solutions to the linearized Navier-Stokes equations. By constructing a boundary layer corrector, which approximate the difference of the Navier-Stokes and Euler solutions, we validate the smallness of our asymptotic expansion with respect to the viscosity parameter and prove the vanishing viscosity limit with a rate of convergence.

Asymtotic expansion for solutions of Navier-Stokes equations with a non-pontential body force

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We study the asymptotic behavior of solutions to the three-dimensional incompressible Navier-Stokes equations (NSE) with periodic boundary conditions and non-potential body forces. We establish a Foias-Saut-type asymptotic expansion for weak solutions of the NSE in Gevrey classes. This extends the previous results for the case of potential forces. The proof uses the Gevrey-norm technique recently adapted by the author and V.R.Martinez, which makes it short and applicable to other types of forces.

The 3D steady and unsteady Primitive Equations of the Ocean

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In this article, the convergence of the solution for the 3D viscous primitive equations (PEs for brevity) of the ocean under two physically relevant boundary conditions when t towards to infinity is studied. Also, the existence and uniqueness of the solution for the 3D steady viscous PEs of the ocean are analyzed. Finally, the H^2-regularity results of the solution for the 3D steady viscous PEs is provided.

On Gevrey regularity of the supercritical SQG equation

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The 2D surface quasi-geostrophic (SQG) equation is an important model in geophysical fluid dynamics and from the mathematical point of view as well, since it shares strong analogies to the 3D Euler equation. Our paper explores the smoothing effect in the dissipative SQG equation in the critical Besov spaces, in the case of supercritical diffusion. Here, "supercritical" refers to the range of dissipation powers in which the order of the nonlinearity dominates that of the dissipation term. Given arbitrary initial data belonging to the critical Besov spaces, we show that solutions to the SQG equation instantaneously enter a subanalytic Gevrey class at least for short time, and that the regularity propagates for all time provided that the data is sufficiently small. To overcome the issue of low diffusion, we establish commutator estimates in a Gevrey class by viewing the commutator as a bilinear multiplier operator. Although the derivative estimates for the corresponding symbol do not imply boundedness of the operator in general, we observe that with the additional localizations from working in Besov spaces they are indeed sufficient to deduce the requisite boundedness.

Euler equation and related models (Masmoudi)

Onsager's conjecture

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In 1949, Lars Onsager in his famous note on statistical hydrodynamics conjectured that weak solutions to the Euler equation belonging to Hölder spaces with Hölder exponent greater than 1/3 conserve energy; conversely, he conjectured the existence of solutions belonging to any Hölder space with exponent less than 1/3 which dissipate energy.

The first part of this conjecture has since been confirmed (cf. Eyink 1994, Constantin, E and Titi 1994). During this talk we will discuss recent work related to resolving the second component of Onsager's conjecture.

Illposedness Results for the Incompressible Euler Equations in Critical Spaces

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We will discuss a few recent results on illposedness for the incompressible Euler equations in critical Sobolev spaces.

The lifespan of small solutions to the KP-I

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Coauthors: Mihaela Ifrim, Daniel Tataru

We shall address in this talk the problem of the incompressible limit for the 2D isentropic fluids associated to ill-prepared initial data and for which the vortices are not necessarily bounded and belong to some weighted BMO spaces. The proof is mainly based on two ingredients: On one hand, the Strichartz estimates to control the acoustic part and prove that it does not contribute for low Mach number. On the other hand, we use the transport compressible structure of the vorticity and we establish a linear propagation estimate in the weighted BMO spaces.

On the 2D isentropic Euler system with unbounded initial vorticity

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We shall address in this talk the problem of the incompressible limit for the 2D isentropic fluids associated to ill-prepared initial data and for which the vortices are not necessarily bounded and belong to some weighted BMO spaces. The proof is mainly based on two ingredients: On one hand, the Strichartz estimates to control the acoustic part and prove that it does not contribute for low Mach number. On the other hand, we use the transport compressible structure of the vorticity and we establish a linear propagation estimate in the weighted BMO spaces.

Fluids and geometry (Disconzi)

Two Dimensional gravity water waves with constant vorticity I: cubic lifespan

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I will talk about the incompressible, infinite depth water wave equations in two space dimensions, with gravity and constant vorticity but with no surface tension. We consider this problem expressed in position-velocity potential holomorphic coordinates, and prove local well-posedness for large data, as well as cubic lifespan bounds for small data solutions.

Normal Forms for the L^2 Riemannian Exponential Map on Diffeomorphism Groups.

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I will describe some results on the structure of the conjugate locus and some local properties of the L^2 exponential map on the diffeomorphism group of a two-dimensional compact manifold. As a Corollary, we obtain an L^2 version of the classical Morse-Littauer theorem in Riemannian geometry.

Euler-Arnold equations and Teichmüller theory

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From the Arnold point of view, the Euler equations of fluid mechanics may be viewed as the geodesic equation on the group of volumorphisms. There are several one-dimensional models of the 3D Euler equation, but only one (the Wunsch equation) has the property that it is also an Euler-Arnold equation on a diffeomorphism group. I will discuss the geometric properties of this equation and blowup results, along with how it arises from the geometry of the universal Teichmüller space. A similar equation (called Euler-Weil-Petersson) was derived by Gay-Balmaz and Ratiu, and I will discuss a global existence result for it and how it relates to the Teichmüller theory. More generally I will discuss ways to view both the Wunsch equation and the surface quasigeostrophic equation as boundary values of simpler Euler-Arnold equations in higher dimensions.

Front propagation and symmetrization for the fractional Fisher-KPP equation

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We prove strong gradient decay estimates for solutions to the multi- dimensional Fisher-KPP equation with fractional diffusion. It is known that this equation exhibits exponentially advancing level sets with strong qualitative upper and lower bounds on the solution. However, little has been shown concerning the gradient of the solution. We prove that, under mild conditions on the initial data, the first and second derivatives of the solution obey a comparative exponential decay in time. We then use this estimate to prove a symmetrization result, which shows that the reaction front flattens and quantifiably circularizes, losing its initial structure.

Mathematical general relativity (Disconzi)

On Gravitational Collapse in General Relativity

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In the process of gravitational collapse, singularities may form, which are either covered by trapped surfaces (black holes) or visible to faraway observers (naked singularities).

In this talk, I will present three results with regard to gravitational collapse for Einstein vacuum equation.

The first is a simplified approach to Christodoulous monumental result which showed that trapped surfaces can form dynamically by the focusing of gravitational radiation from past null infinity. We extend the methods of Klainerman-Rodnianski, who gave a simplified proof of this result in a finite region.

The second result extends the theorem of Christodoulou by allowing for weaker initial data but still guaranteeing that a trapped surface forms in the causal domain. In particular, we show that a trapped surface can form dynamically from initial data which is merely large in a scale-invariant way. The second result is obtained jointly with Luk.

The third result extends Christodoulous famous example on formation of naked singularity for Einstein-scalar field system under spherical symmetry. With numerical and analytic tools, we generalize Christodoulous result and construct an example of naked singularity formation for Einstein vacuum equation in higher dimension. The third result is obtained jointly with Zhang.

Is Spacetime Locally Inertial for General Relativistic Shock Wave Solutions?

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It is an open question whether solutions of the coupled Einstein Euler equations are smooth enough to admit locally inertial coordinates at points of shock wave interaction, or whether "regularity singularities" exist. A *regularity singularity* is a point where the gravitational metric tensor would be no smoother than Lipschitz continuous in any coordinate system. Such a metric regularity would be too low for spacetime to be locally inertial, which raises the possibility of new gravitational effects to exist. As shown in Israel's famous 1966 paper, a metric tensor Lipschitz continuous across a *single* shock surface can be mapped by a coordinate transformation to a metric tensor with sufficient regularity for locally inertial coordinates to exist. However, generalizing Israel's method of proof to the case of even the simplest shock wave interaction seems hopeless. In our recent RSPA-paper, we develop a new method to address basic shock wave interactions and prove that the metric regularity can be lifted sufficiently for locally inertial coordinates to exist. Our result generalizes Israel's result to shock interactions and shows that regularity singularities do not exist in these basic cases. Whether such singularities exist in more complicated shock wave solutions of the Einstein Euler equations remains an open problem.

Pointwise decay for the Maxwell system on black holes

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We consider both the inhomogeneous wave and the inhomogeneous Maxwell system on nonstationary black hole space times. Under the assumption that uniform energy bounds and a weak form of local energy decay hold forward in time, we establish sharp pointwise decay rates for the solutions. In the case of the wave equation, the rate of decay is $t^{-1}(t-r)^{-2}$, and the result is sharp. For the Maxwell system, we recover peeling estimates that are direction dependent near the light cone, as well as a t^{-4} rate of decay on compact regions for all the components of the Maxwell tensor.

A choice-free black box for obtaining the maximal globally hyperbolic Cauchy development

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Choquet-Bruhat and Geroch, in their 1969 paper asserting the existence of the maximal globally hyperbolic Cauchy development, appealed to the Axiom of Choice in their construction. In 2013, the speaker and Jan Sbierski independently obtained proofs which do not require the Axiom of Choice. In this talk, the black box construction central to the speaker's proof will be discussed. The black box extracts out the algebraic portion of the construction of the MGHCD, and is thus largely agnostic to the underlying physical theory. By applying the black box, any geometric physical theory admitting a sufficiently strong "local-in-space, local-in-time" well-posedness result of the initial value problem will automatically possess a notion of the MGHCD.

Nonlinear waves (Tataru and Bejenaru)

Type one and type two blowup for some dispersive equations

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In this talk we will discuss the well - posedness problem for some nonlinear wave and Schrodinger equations. We will discuss scattering and blowup, type one and type two.

An introduction to the finite depth gravity water waves in holomorphic coordinates

Benjamin Harrop-Griffiths
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Coauthors: Mihaela Ifrim, Daniel Tataru

We consider irrotational gravity water waves with finite bottom in 2d. We discuss the the set-up of the problem in holomorphic coordinates and some of the key differences from the infinite depth setting

Finite depth gravity water waves in holomorphic coordinates

Mihaela Ifrim
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Coauthors: Benjamin Harrop-Griffiths and Daniel Tataru

We consider irrotational gravity water waves with finite bottom. Our goal is two fold: -first we express the equations in holomorphic coordinates and discuss the well-posedness of the problem -and secondly, we consider the small data problem and establish cubic lifespan bounds for the solutions.

Ground States and Bifurcation for NLS on a closed graph

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We consider standing waves in the focusing nonlinear Schrodinger (NLS) equation on a dumbbell graph (two rings attached to a central line segment subject to the Kirchhoff boundary conditions at the junctions). In the limit of small mass, the ground state (the orbitally stable standing wave of the smallest energy at a fixed mass) is represented by a constant solution. However, when the mass is increased, this constant solution undertakes two bifurcations, where the first is the pitchfork (symmetry breaking) bifurcation and the second one is the symmetry preserving bifurcation. As a result of the first symmetry breaking bifurcation, the standing wave becomes more localized in one of the two rings. As a result of the second symmetry preserving bifurcation, the standing wave becomes localized in the central line segment. In the limit of large norm solutions, both standing waves are represented by a truncated solitary wave localized in either the ring or the central line segment.

Random data Cauchy theory for some nonlinear wave equations

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I will discuss some recent work on the random data Cauchy theory for energy critical nonlinear wave equations with null form nonlinearities in both the periodic and the Euclidean setting. In certain cases, there are counterexamples which yield a gap between the regularity of the initial data required in order to obtain local well-posedness, and the desired result in the critical energy space. We will show that for suitably chosen random initial data, we are able to exploit the null structure and prove estimates for solutions arising from initial data of supercritical regularity, in spite of the counterexamples. Consequently, we produce subsets of close to full measure in those spaces on which these equations are locally well-posed. This is part of a project in progress with Chanillo, Czubak, Nahmod and Staffilani.

Local well-posedness for quasilinear Schrodinger equations

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I will speak on a recent joint study with J. Marzuola and D. Tataru which proves low regularity local well-posedness for quasilinear Schroedinger equations. Similar results were previously proved by Kenig, Ponce, and Vega in much higher regularity spaces using an artificial viscosity method. Our techniques, and in particular the spaces in which we work, are motivated by those used by Bejenaru and Tataru for semilinear equations.

Global existence for quasilinear wave equations close to Schwarzschild

Mihai Tohaneanu
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Coauthors: Hans Lindblad

We study the quasilinear wave equation $\Box_g u = 0$, where the metric g depends on u and equals the Schwarzschild metric when u is identically 0. Under a couple of extra assumptions on the metric g near the trapped set and the light cone, we prove global existence of solutions.

Stability of Solutions to a Beta-Plane Equation

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We consider the so-called beta-plane model for a two-dimensional rotating and inviscid fluid. In such a setup the Coriolis effect is seen to introduce a linear dispersive term into the Euler equations. This allows for a perturbative approach to the question of stability of solutions. However, the anisotropy of this mechanism complicates standard techniques.

Analysis and control of PDE evolutions with an interface (Lasiecka)

Control and Sensitivity Analysis in Fluid-Elasticity Interactions

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Free and moving boundary fluid-structure interactions are ubiquitous in nature, with most known examples coming from industrial processes, aero-elasticity, and biomechanics. We consider optimal control problems subject to interactions between viscous, incompressible fluid and nonlinear elasticity. We discuss existence of optimal controls, sensitivity equations, and optimality conditions. One of the main challenges of applying optimization tools to free and moving boundary interactions is the proper derivation of the adjoint sensitivity information with correct adjoint balancing conditions on the common interface. As the coupled fluid-structure state is the solution of a system of PDEs that are coupled through continuity relations defined on the free interface, sensitivity of the fluid state, which is an Eulerian quantity, with respect to the motion of the solid, which is a Lagrangian quantity, falls into moving shape analysis framework.

Attractors for Strongly Damped Wave Equations with Nonlinear Hyperbolic Dynamic Boundary Conditions

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We consider a nonlinear coupled PDE system consisting of a strongly damped wave equation on a bounded domain with hyperbolic dynamic boundary conditions. The coupling occurs on the boundary interface. Both the regularity of the semiflow and the existence of a global attractor are studied. In particular, we use the extra Gevrey class regularity of the underlying linear semigroup to prove that solutions to the nonlinear system enjoy parabolic-like smoothing in time.

Boundary control of Schrödinger and heat equations with point-mass interfaces

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I'll describe some boundary control results for a Schrödinger equation on the domain $(-1,0)\cup(0,1)$ with a singular transmission condition at $x=0$:

$$u_t + iu_{xx} = 0, x \in (-1,0)\cup(0,1),\ \ t>0$$

$$u(0^-,t) = u(0+,t), t>0$$

$$\frac{d}{dt}u(0,t) + i[u_x(0^+,t) - u_x(0^-,t)] = 0, t>0$$

$$u(-1,t) = 0, \quad t>0$$

with either Dirichlet control : $u(1,t)=f(t)$, or Neumann control: $u_x(1,t)=f(t)$. In the Neumann case, we find results analogous to known results for the wave equation, in which exact controllability holds on a space with differing regularities on each side of the interface. This is not the case however in the case of Dirichlet control, where the controllability space is the same on each side. Some general results (for non-symmetric domains) will also be described. Related null-controllability results for the heat equation and controllability results for the Euler-Bernoulli equation will also be discussed.

Mathematical Challenges Arising from the Questions of Controllability for Mixed Structures

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In the study of control and stabilization of dynamic systems, a significant challenge is the ability to rigorously address whether linked dynamic structures can be controlled using boundary feedback alone. Structures composed of interconnected elements or components of different dimensions or states, such as fluid versus matter, pose serious challenges because the energy transferred through the interfaces between components can lead to uncontrollable behavior. This talk focuses on issues that arise when attempting to understand the control and stability of such complex systems.

On the local existence of the free surface Euler equation with surface tension

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We revisit the Lagrangian approach to the free surface rotational Euler equation with surface tension in 3D and provide a priori estimates leading to local existence in low regularity Sobolev spaces.

Global solutions in quasilinear Thermoelasticity

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Full von Karman thermoelastic system will be considered with free boundary conditions and partial boundary dissipation imposed only on in plane displacement. It will be shown that the corresponding solutions are exponentially stable, though there is no mechanical dissipation on vertical displacements. The main tools uses are: (i) partial analyticity of linearized semigroup and (ii) new trace estimates which exploit hidden regularity harvested from partial analyticity.

Modeling biofilm evolution with a variational inequality

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In this talk we discuss our analysis and computational results for a model of biofilm at porescale coupled to a viscous flow model in the surrounding pore space. Biofilm is a collection of microbial cells which adhere to each other and to fluid and fluid-solid interfaces. Its growth depends on the presence of nutrient(s) and on the environmental factors such as flow rates. In particular, the growth may be inhibited at low flow rates when the nutrient supply is not sufficient, and in the presence of high flow rates which may cause mechanical deformation and local destruction of the biofilm. The most interesting and challenging part of any biofilm model is how to account for the constraint on the maximum density of biofilm that can be present, and for the associated notion of growth through interfaces.

Inspired by available micro-imaging data, we proposed a new model for biofilm growth using a variational inequality approach, and we model the fluid-biofilm interface similarly to that in one-phase Stefan problem. While in other existing models the growth through interface is realized through degenerate and singular diffusivities and/or phase-field like constructs, these approaches are intractable in practical computations involving porous media. Under realistic (Neumann) external boundary conditions, the fully coupled biofilm-nutrient-hydrodynamics model is not yet tractable, but in the talk we present our recent results on the simplified model. These include the analysis and numerical analysis of the model with Dirichlet boundary conditions, as well as upscaling, and can be directly related to the experimental results.

Single-phase Flow in Thermo-Poro-Elastic Media

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The mathematical formulation and analysis of thermo-poro-elastic models are discussed. These describe consolidation and heat tranport in a fluid-saturated porous medium. We shall focus on the dependence of regularizing effects of the evolution system on the modeling assumptions.

Stabilization of a transmission system involving thermoelasticity

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The feedback control of interacting waves through the usual transmission conditions are known to be quite challenging especially when the two wave equations involved have different speeds of propagation. In this contribution, we consider a thermoelasticity system composed of a wave equation and a parabolic equation in a bounded domain surrounding another domain modeled by a wave equation. The thermoelasticity system and the other wave equation are coupled at the interface through the usual transmission conditions. Under a geometric constraint on the interface, and assuming that the speed of the thermoelastic wave is smaller than, or equal to, that of the undamped wave, we prove that the transmission system is exponentially stable. If the speed of the undamped wave is smaller instead, then we prove that the transmission system is polynomially stable. Our proofs rely on a combination of multiplier techniques and the frequency domain method.

The discrete inf-sup inequality for finite-element solutions of fluid-structure interaction problems

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Coauthors: George Avalos (University of Nebraska-Lincoln)

A seminal result concerning mixed finite element approximations of the Stokes equation was the discrete inf-sup inequality, *uniform* with respect to (small) discretization mesh parameter h, which ultimately leads to the error estimates for the convergence of the FEM scheme. An essential requisite ingredient for this result was the no-slip condition imposed on the entire boundary of the fluid domain. On the other hand, in fluid-structure interaction problems the interface between the fluid and the solid is subject to velocity and stress matching constraints which do not enforce the no-slip condition. Accordingly, we establish the discrete inf-sup estimate for the case when the no-slip condition holds only on a portion of the boundary, thus paving the way to FEM error estimates for fluid-structure interaction problems.

An Effective Decomposition of Flow-Plate Dynamics for Strong Stability

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In this talk we examine a recent decomposition technique for a flow-plate dynamics which models the panel flutter phenomenon. It is known that in the absence of imposed structural damping the (nonlinear—Berger) plate dynamics converge (in time) to a uniformly attracting set. Based on experimental results—which assert that the end behavior of a panel in subsonic flows should be static—we investigate the subsonic model.

We will show that in the presence of arbitrarily small structural damping, smooth solutions (both plate and flow) converge to an equilibria set. For finite energy solutions, "large" damping is necessary to obtain the same strong convergence to the equilibria set. Our proof here relies on a decomposition of the nonlinear structural dynamics into an exponentially stable component and a "smooth" component; we then decompose the flow dynamics accordingly (via the boundary-to-interior Neumann map) into corresponding components, and analyze the fully decomposed flow-plate systems. We apply different techniques on each piece of the decomposed dynamics. Our approach does not obtain for supersonic flows, and this is in agreement with observed fact that panels in supersonic flow may flutter (i.e., exhibit non-stationary end behavior).

Qualitative study of the Navier-Stokes, Euler and related geophysical systems (Titi)

Determining modes for the statistical solutions of the Navier-Stokes equations

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Coauthors: C. Foias, C. Mondaini and B. Zhang

Starting with the work of Foias and Prodi (1967), and subsequently by others (e.g. Foias and Temam (1984), Jones and Titi (1992, 1992, 1993)), it is well known that dissipative dynamical systems possess finite determining parameters such as determining modes, nodes and volume elements. In this work, we extend the notion of determining modes for dissipative systems to the framework of the Foias-Prodi statistical solutions.

Sharp L^p estimates for singular transport equations

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We will discuss sharp L^p estimates for equations of the following type:

$$\partial_t f + u \cdot \nabla f = R(f)$$

with u a given divergence-free velocity field and R a singular integral operator. Such models show up readily in incompressible fluid dynamics.

The space $B^{-1}_{\infty,\infty}$, volumetric sparseness, and 3D NSE

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In the context of the L^{∞}-theory of the 3D NSE, it is shown that suitable smallness of a solution in Besov space $B^{-1}_{\infty,\infty}$ suffices to prevent a possible blow-up.

The Weak Sigma-Attractor For the Semi-Diffusive 2D Boussinesq Equations

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The Boussinesq equations arise in the study of ocean flows. Recently, there has been much work on of the equations in the partial absence of dissipative terms, but certain restrictions on the assumptions resisted very sophisticated mathematics. I will discuss an approach which does not require these restrictions, and furthermore is done with more elementary techniques. Furthermore, the large-time behavior of a dissipative system can often be understood by studying its global attractor, which can contain deep information about its underlying structure. I will show that the notion of attractor can be extended to the Boussinesq system with only partial dissipation. We will see that this generalized attractor not only has a rich structure, but also encodes a wealth of turbulent phenomena in a single object.

Recent advances on the primitive equations of oceanic and atmospheric dynamics

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Coauthors: Chongsheng Cao, Edriss S. Titi

The motion of the large-scale atmospheric and oceanic flows is governed by the primitive equations (PEs), which are derived from the Navier-Stokes equations by using the Boussinesq and hydrostatic approximations. The strong horizontal turbulent mixing creates the horizontal eddy viscosity. Mathematically, this leads us to consider the PEs with horizontal viscosity. In this talk, we will present several recent advances on the PEs, with full or partial viscosity. Specifically, it will be shown that the 3D PEs with horizontal viscosity admits a unique global strong solution, for arbitrary sufficient smooth initial data, as long as one still has the horizontal or vertical thermal diffusivity. These are joint works with Chongsheng Cao and Edriss S. Titi.

A spatio-temporal discrete data assimilation algorithm for the 2D Navier-Stokes equations and statistics

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Recently, A. Azouani, E. Olson and E. Titi introduced a new continuous in time data assimilation algorithm inspired by ideas from control theory that is applicable to a wide range of dissipative evolution equations. In this talk, I will present a data assimilation algorithm for the 2D Navier-Stokes equations by employing, in a more realistic setting, observational measurements that are discrete in space and time and that may also be contaminated by systematic errors. We will see that, under suitable conditions on the relaxation parameter, the spatial resolution of the mesh and the time step between successive measurements, an asymptotic in time estimate of the difference between the approximating solution and the reference solution can be obtained, in an appropriate norm, which shows exponential convergence up to a term which depends on the size of the errors. A stationary statistical analysis of this spatio-temporal discrete data assimilation algorithm will also be provided. This is a joint work with C. Foias and E. Titi.

Recent Developments on the Magnetohydrodynamics and Related Systems
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Magnetohydrodynamics (MHD) system is obtained through a coupling of the Navier-Stokes and the Maxwell's equations and describes the motion of electrically conducting fluids.

We discuss, depending on time, recent developments on the mathematical analysis of this system from both deterministic and stochastic perspectives. The results include regularity criteria in the deterministic case, well-posedness, path-wise uniqueness, ergodicity and large deviation principle in the stochastic case.

Related systems which we discuss include the nonhomogeneous MHD system, Hall-MHD system, Boussinesq system, Benard and magnetic Benard problems, micropolar and magneto-micropolar fluid systems.

Enhanced dissipation, hypoellipticity and mixing in shear flows
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Coauthors: Jacob Bedrossian

We analyze the decay and instant regularization properties of the evolution semigroups generated by two-dimensional drift-diffusion equations in which the scalar is advected by a shear flow and dissipated by full or partial diffusion. We consider both the case of space-periodic and the case of a bounded channel with no-flux boundary conditions. In the infinite Pclet number limit, our work quantifies the enhanced dissipation effect due to the shear. We also obtain hypoelliptic regularization, showing that solutions are instantly Gevrey regular even with only partial diffusion.

Analysis of fluid flow (Shkoller and Vicol)

The Inverse Laplacian on Bounded Functions
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Consider the problem of finding u which solves

$$\Delta u = f,$$

for a given locally bounded function f. It is well known that D^2u locally belongs to L^p for any $p < \infty$. However, there are numerous examples where $D^2u \notin L^\infty$. We discuss a few non-trivial scenarios where one is able to derive L^∞ estimates on D^2u when f satisfies suitable symmetry assumptions. Some applications to incompressible fluid flow will be discussed as well.

The inhomogeneous Muskat problem

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In this talk we will review some results on the Muskat problem (or two phase Hele-Shaw with gravity). We will also present some new results concerning the case where the permeability is a non-negative step function and the initial data is merely H^2. This problem is known in the literature as the inhomogeneous Muskat problem.

Almost global existence of the Prandtl equations

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Coauthors: Vlad Vicol, Princeton University

We address the Prandtl equations on the half space with real-analytic initial datum with respect to the tangential variable. The boundary traces of the horizontal Euler flow and pressure are taken to be constants. We establish that if the initial datum lies within ϵ of a stable pro

file, then the time of existence for the solution is at least $\exp(\epsilon^{-1}/\log(\epsilon^{-1}))$. This is a joint work with V. Vicol.

Sobolev stability threshold for 2D shear flows near Couette

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We consider the 2D Navier-Stokes equation on $\mathbb{T} \times \mathbb{R}$, with initial datum that is ϵ-close in H^N to a shear flow $(U(y), 0)$, where $\|U(y) - y\|_{H^{N+4}} \ll 1$ and $N > 1$. We prove that if $\epsilon \ll \nu^{1/2}$, where ν denotes the inverse Reynolds number, then the solution of the Navier-Stokes equation remains ϵ-close in H^1 to $(e^{t\nu\partial_{yy}}U(y), 0)$ for all $t > 0$. Moreover, the solution converges to a decaying shear flow for times $t \gg \nu^{-1/3}$ by a mixing-enhanced dissipation effect, and experiences a transient growth of gradients. In particular, this shows that the stability threshold in finite regularity scales as $\nu^{1/2}$ for 2D shear flows close to the Couette flow.

Convergence to Stratified Flow for an Inviscid 3D Boussinesq System

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We study the stability of special, stratified solutions of a 3d Boussinesq system describing an incompressible, inviscid 3d fluid with variable density (or temperature, depending on the context) under the effect of a uni-directional gravitational force. The behavior is shown to depend on the properties of an anisotropic dispersive operator with weak decay in time. However, the dispersive decay also depends on the strength of the gravity in the system and on the profile of the stratified solution, whose stability we study. We show that as the strength of the dispersion in the system tends to infinity, the 3d system of equations tends to a stratified system of 2d Euler equations with stratified density.

Stochastic perturbations of passive scalars and small noise inviscid limits
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We consider a class of invariant measures for a passive scalar driven by an incompressible velocity u field on a periodic domain. The measures are obtained as limits of stochastic viscous perturbations. We prove that the span of the H^1 eigenfunctions of the operator $u \cdot \nabla$ contains the support of these measures, and apply the result to a number of examples in which explicit computations are possible (relaxation enhancing, shear, cellular flows). In the case of shear flows, anomalous scalings can be handled in view of a precise quantification of the enhanced dissipation effects due to the flow.

Contributed talks

Exponentially Harmonic Maps and Their Properties
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Exponentially harmonic maps were first introduced by James Eells and Luc Lemaire in 1990. In this paper, we show that the associated quadratic differentials of exponentially harmonic maps are holomorphic under certain circumstance. We study the sufficient and necessary conditions for axially symmetric maps which are exponentially harmonic. We obtain exponentially harmonic equations for rotationally symmetric maps between rotationally symmetric manifolds of low dimensions. This paper has appeared in Mathematische Nachrichten by Wiley-VCH.

Regularization techniques and numerical schemes for a non-linear ill-posed evolution equation
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Regularization techniques including quasi-reversibility methods have been widely studied for ill-posed evolution equations such as the backward heat equation. In this talk, we consider a non-linear ill-posed problem $du/dt = Au + h(t, u(t))$, $0 < t < T$, $u(0) = x$, where A is a certain unbounded linear operator in a Banach space X, and u is an X-valued function. Boussetila and Rebbani apply a modified quasi-reversibility method for the problem which involves a logarithmic approximation of the operator A. Using the same approximation, we directly prove continuous dependence on modeling. Also, numerical experiments and linear recursive schemes are considered in order to scrutinize concrete examples in L^p spaces.

Asymptotic behavior of a viscous flow past a body
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The steady incompressible Navier-Stokes equations in a two-dimensional exterior domain are considered with a nonzero constant velocity at infinity. It is known that the asymptotic behavior of the velocity field is described by the linearization around the velocity at infinity. I will present the asymptotic behavior of the vorticity and explain why surprisingly it is not characterized by the previous linearization. More precisely, the vorticity has a power of decay at infinity which depends continuously on the data.

TBA
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TBA.

On the compressible water-wave with vorticity
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In this talk, we study a priori estimates for the compressible Euler equations with moving physical vacuum boundary and unbounded initial domain, with an equation of state given by $p(\rho) = \mathfrak{K}(\rho^\gamma - 1)$, which is in the case of a liquid, and $\mathfrak{K}$ denotes the sound speed of the fluid. Our estimates are uniform in the sound speed.

Sensitivity analysis in poro-visco-elasticity.
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The lamina cribrosa is a thin, porous tissue at the base of the optic nerve head which stabilizes the pressure difference between the intraocular pressure (IOP) inside the eye globe and the retrolaminar tissue pressure (RLTp) in the optic nerve head. It is believed that the biomechanics of the lamina cribrosa plays an important role in the retinal ganglion cell loss in glaucoma. Therefore, understanding the deformation and perfusion of the lamina cribrosa as a function of the difference between IOP and RLTp is of utmost importance in unlocking the cause and progression of glaucoma. Using a poro-visco-elastic model for the lamina cribrosa in which the IOP and RLTp appear as boundary conditions, we aim to perform a sensitivity analysis with respect to important biological parameters (such as the IOP and RLTp) in order to determine which parameters are most influential and need to be controlled in order to prevent the development of glaucoma. Nonlinear coupling in the model causes difficulty when implementing traditional methods for sensitivity analysis (such as sensitivity equations). For this reason, we use instead the lesser known complex step method which uses the Cauchy-Riemann equations to compute sensitivities.

Global well-posedness and asymptotic behavior of solutions to a reaction-convection-diffusion cholera epidemic model

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Cholera is an infectious disease caused by the bacterium Vibrio cholerae, and transmitted through indirect environment-to-person contact or direct person-to-person contact. The health issue concerning this disease worldwide is well-known.

We study the initial boundary value problem of a reaction-convection-diffusion epidemic model for cholera dynamics, which was developed in [X. Wang and J. Wang, Analysis of cholera epidemics with bacterial growth and spatial movement, J. Biol. Dyn., 9 (2015), 233-261], named susceptible-infected-recovered-susceptible-bacteria (SIRS-B) epidemic PDE model. First, a local well-posedness result relying on the theory of cooperative dynamics systems is obtained. Via a priori estimates making use of the special structure of the system and continuation of local theory argument, in fact this problem is globally well-posed. Secondly, we analyze the local asymptotic stability of the solutions based on the basic reproduction number associated with this model.

We also report new results in which we extend the local stability to global. Specifically, we show that when the basic reproduction number is beneath one, the disease-free-equilibrium is globally attractive, whereas when the basic reproduction number exceeds one, if the infectious and recovered hosts as well as the concentration of bacteria in the contaminated water are not initially identically zero, there exists at least one positive steady state, and we also prove uniform persistence result.

An explicit estimate on the Kolmogorov entropy of the weak global attractor of 3D Navier-Stokes equations

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In this short talk, I will present an estimate of the Kolomogorov entropy for the weak global attractor of three dimensional Navier-Stokes Equations with periodic boundary condition. The estimates are explicitly expressed in terms of the Grashof number, a physically non-dimensional parameter.