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OPTIMAL CONTROL OF PHASE-FIELD SEPARATIONS BY THE CAHN-HILLIARD NAVIER-STOKES  
MODEL

Abstract: Phase separation is considerably involved with a lot of important applications in industry. The main equation that describes the phase separation of a binary fluid mixture is the Cahn-Hilliard equation. In this paper, we consider the phase-field model by the coupled Cahn-Hilliard Navier-Stokes system. Our main objective is to control the separation of the two fluids by the boundary control of the fluid velocity field. Our control of the velocity field enters to the boundary where the flow is governed by the Navier-Stokes equation, and enters to the Cahn-Hilliard system as a transport of the concentrations. Also, the fluid structure interaction enters into the Navier-Stokes equation as a force. We use the Lagrange Calculus in the analysis of this highly non-linear and non-smooth control problem. First, we introduce the differential form of the Cahn-Hilliard Navier-Stokes system, where the separation of the two fluids is governed by Cahn-Hilliard equations based on Ginzburg-Landau Potential coupled with the incompressible flow governed by Navier-Stokes equation with velocity boundary control. Then, we formulate the optimal control problem for the phase separation in terms of concentration of the two fluids. We use a tracking type cost functional and show the well-posedness of this control coupled nonlinear PDE's system with boundary velocity control for the fluid dynamics. We show the existence of optimal control and obtain necessary optimality conditions using Lagrange Calculus. After that, we develop numerical analysis results by the so called sequential programming method.

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LOCAL EXISTENCE OF CLASSICAL SOLUTIONS TO SEMI-GEOSTROPHIC SYSTEM WITH VARIABLE  
CORIOLIS PARAMETER

Abstract: The semi-geostrophic system (abbreviated as SG) is a model of large-scale atmospheric/ocean flows. Previous works about the SG system have been restricted to the case of constant Coriolis force, where we write the equation in “dual coordinates” and solve. This method does not apply for variable Coriolis parameter case. We develop a time-stepping procedure to overcome this difficulty and prove local existence and uniqueness of smooth solutions to SG system. This is joint work with Michael Cullen and Mikhail Feldman.

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AN ONSAGER SINGULARITY THEOREM FOR THE COMPRESSIBLE EULER EQUATIONS

Abstract: We prove that bounded weak solutions of the compressible Euler equations will conserve thermodynamic entropy, unless the solution fields have sufficiently low space-time Besov regularity. A quantity measuring kinetic energy cascade will also vanish for such Euler solutions, unless the same singularity conditions are satisfied. It is shown furthermore that strong limits of solutions of compressible Navier-Stokes equations that are bounded and exhibit anomalous dissipation are weak Euler solutions. These inviscid limit solutions have non-negative anomalous entropy production and kinetic energy dissipation, with both vanishing when solutions are above the critical degree of Besov regularity. This talk is based on joint work with G. Eyink.

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