2022 Shanks Workshop on

Mathematical Aspects of Fluid Dynamics

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Organizers: Leo Abbrescia, Marcelo Disconzi, Gieri Simonett, Jared Speck

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Mathematical wave turbulence and propagation of chaos

Abstract: The theory of wave turbulence can be traced back to the 1920s and has played significant roles in many different areas of physics. However, for a long time the mathematical foundation of the theory has not been established. The central topics here are the wave kinetic equation, which describes the thermodynamic limit of interacting wave systems, and the propagation of chaos, which is a fundamental physical assumption in this field that lacks mathematical justification. In this talk, I will present recent results with Zaher Hani (University of Michigan), where we provide the first rigorous derivation of the wave kinetic equation, and also justify the propagation of chaos assumption in the same setting.

Vera Mikyoung Hur
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Stokes waves are unstable

Abstract: I will discuss a new periodic Evans function approach for cylindrical domains, and its application to the spectral instability of Stokes waves in the finite depth. Numerical evidence suggests instability whenever the unperturbed wave is 'resonant' with its infinitesimal perturbation waves. This has not been studied analytically except the Benjamin-Feir instability, near the origin of the spectral plane, when (wave number)x(depth) $> 1.3627...$ for small amplitude. I will discuss an alternative proof of the Benjamin-Feir instability and, also, the first proof of spectral instability away from the origin, when (wave number)x(depth) is not 1.8494..., thereby all Stokes waves are unstable. Joint work with Z. Yang.

Mihaela Ifrim
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The time-like minimal surface equation in Minkowski space: low regularity solutions

Abstract: It has long been conjectured that for nonlinear wave equations which satisfy a nonlinear form of the null condition, the low regularity well-posedness theory can be significantly improved compared to the sharp results of Smith-Tataru for the generic case. The aim of this article is to prove the first result in this direction, namely for the time-like minimal surface equation in the Minkowski space-time. Further, our improvement is substantial, namely by $3/8$ derivatives in two space dimensions and by $1/4$ derivatives in higher dimensions. This work is joint with Albert Ai and Daniel Tataru.

Mihaela Ignatova
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Electroconvection in fluids

Abstract: We consider models of electroconvection in fluids, in which a fluid interacts with an electrical charge density. The fluid obeys Navier-Stokes equations driven by the electrical and body forces. The charge density is carried by the fluid and electrical fluxes. I will discuss results regarding the regularity of solutions and their long-time dynamics. I will also discuss the active scalar equation obtained when the Navier-Stokes equations are replaced by Darcy's law.

Maxim Olshanskii
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Modeling and simulation of surface fluids

Abstract: We briefly review mathematical models of viscous deformable interfaces (such as plasma membranes) leading to fluid equations posed on evolving 2D surfaces embedded in $\mathbb{R}^3$. We further report on some recent advances in understanding and numerical simulation of the resulting fluid systems.

Sohrab Shahshahani
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The hard phase fluid with free boundary in relativity

Abstract: The hard phase model is an idealized model for a relativistic fluid where the sound speed approaches the speed of light. In this talk I will describe a free boundary problem corresponding to this model, and present recent progress on the well-posedness for this problem. This talk is based on joint works with Shuang Miao and Sijue Wu.

Konstantina Trivisa
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Invariant measures for the stochastic Navier-Stokes Equations for compressible flows and the problem of turbulence

Abstract: In this talk I will present results on the long-time behavior of solutions to a stochastically forced Navier-Stokes system, describing the motion of a compressible viscous fluid. In the one dimensional case, the existence of an invariant measure for the Markov process generated by strong solutions was established in collaboration with Michele Coti-Zelati and Nathan Glatt-Holtz. In that work, we overcome the difficulties of working with non-Feller Markov semigroups on non-complete metric spaces by generalizing the classical Krylov-Bogoliubov method, and by providing suitable polynomial and exponential moment bounds on the solution, together with pathwise estimates. The talk will conclude with a discussion on some recent developments on the multi-dimensional case for related models.