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Organizers: Marcelo Disconzi, Chenyun Luo, Giusy Mazzone, Gieri Simonett

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Low Regularity Solutions for Gravity Water Waves

Abstract: We consider Strichartz estimates for gravity water waves without surface tension, which have applications to low regularity well-posedness. As the equations in Eulerian coordinates are dispersive of of order 1/2 but also possess a transport term of higher order, a key step is to analyze the change of variables to Lagrangian coordinates.

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Relativistic Beltrami Flows in Ideal Fluids and Plasmas

Abstract: In this talk, we overview the development of a relativistic generalization of Beltrami flows. We review basic mathematical properties of Beltrami fields and show that Beltrami field configurations are the lowest energy states of a fluid system of a given helicity. We conclude by proving that relativistic Beltrami fields can only occur when the heat differential associated with the fluid is vanishing.

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Shocks and Conservation Laws with Random Initial Conditions

Abstract: We consider a nonlinear conservation laws with random initial conditions. As shocks are produced even with smooth data, the introduction of random initial conditions such as Brownian motion presents a challenging problem in terms of the density of shocks at arbitrary x and t. I consider a minimization approach (Hopf-Lax) which avoids tracking individual shocks and relates the problem to Hamilton-Jacobi equations. By discretizing the Brownian motion, one can obtain a closed-form expression in terms of n nested Gaussian integrals for the discretized in initial condition problem. One can show convergence in a probabilistic sense to the continuous problem. Some properties of the variance of the solution are also obtained.

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VANISHING RELAXATION TIME DYNAMICS OF THE MOORE-GIBSON- THOMPSON (MGT) EQUATION ARISING IN HIGH-FREQUENCY ULTRASOUND (HFU)

Abstract: The MGT equation is a model describing acoustic wave propagation in gasses and liquids. This third-order- in-time equation of hyperbolic type arises as a model of high-frequency ultrasound (HFU) waves. One application of HFU is extracorporeal shock wave lithotripsy, which is a noninvasive treatment of kidney stones. The dynamic response of MGT depends on the relaxation parameter which accounts for the finite speed of propagation -thus eliminating the so called infinite speed of propagation paradox. Since is relatively small in applications, it is important to trace the dynamics with vanishing 0. In the limit, the MGT system becomes parabolic-like with drastically different PDE characteristics. It is shown that the decay rates for the finite energy of MGT equation are preserved uniformly with vanishing parameter 0. The corresponding result provides not only a robust stabilizing mechanism for HFU waves, but also leads to a new "higher energy" stability estimates valid for the limit problem. The latter is of independent mathematical interest.

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SPHERE INTERACTION IN BOUNDED SHEAR FLOW OF OLDROYD-B FLUIDS

Abstract: In the bounded shear flow of a Newtonian fluid, it is well-known that binary encounters of spheres can have either pass or return trajectories under creeping flow conditions (Zurita-gotor et al., J. Fluid Mech. 592 (2007) 447-469). The motion of dilute sphere suspensions in bounded shear flow of Oldroyd-B fluids at zero Reynolds number has been studied. The pass and return trajectories of the two ball mass centers in a two wall driven shear flow are similar to those in a Newtonian fluid but a new motion called tumbling chain of two balls (a dipole) has been observed, depending on the value of the Weissenberg number and the initial vertical displacement of the ball mass center to the middle plane between two walls. The two ball tumbling motion has also been compared with that of an ellipsoid in bounded shear flow Oldroyd-B fluids. This work was supported by NSF (grant DMS 1418308).

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GEOMETRIC APPROACH ON THE GLOBAL CONSERVATIVE SOLUTIONS OF THE CAMASSA-HOLM EQUATION

Abstract: We construct global weak conservative solutions of the Camassa-Holm equation on the periodic domain. We rst express the equation in Lagrangian ow variable η and then transform it using a simple change of variable $\rho = \sqrt{\eta_x}$. The new variable removes the singularity of the Camassa-Holm equation, and we obtain both global weak conservative solutions and global spatial smoothness of the Lagrangian trajectories, which were originally discovered by Bressan-Constantin and McKean, respectively. This work is inspired by J. Lenells who proved similar results for the Hunter-Saxton equation using the geometric interpretation.

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Turbulent Flow Statistics

Abstract: Because of the limitations of computers, we are forced to struggle with the meaning of the under-resolved flow simulation. Turbulence models are introduced to account for sub-mesh scale effects, when solving flow problem numerically on an under-resolved spatial mesh size h. One key in getting a good approximation for a turbulence model is to correctly calibrate the energy dissipation $E(u)$ in the model on the under-resolved mesh. Energy dissipation rates of various turbulence models have been analyzed assuming infinite resolution, but not coarse resolution. The unexplored question is: What is the time-averaged energy dissipation rate $E(u^h)$ when u^h is an approximation of u on a coarse mesh h, associated with the computational cost?

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Stabilizing the Turbulence of Navier-Stokes Equations in Besov and Lp-based Sobolev spaces

Abstract: We consider the 2- or 3-dimensional Navier-Stokes equations defined on a bounded domain, initially with no-slip boundary conditions and subject to a force (f_e). Such forcing term may cause turbulence. We then seek to uniformly stabilize the N-S system, either by an internal localized control or else possibly by a localized boundary tangential control. In either case, we aimed an explicitly constructed finite-dimensional control. Prior results in the Hilbert (L_2-based Sobolev) spaces. In the 3D boundary tangential case, the finite dimensionality of the stabilizing control in the Hilbert setting is generally not possible (unless the I.C. vanishes on the boundary), as the N-S non-linearity for 3D forces the necessity of imposing compatibility conditions. Then we seek to solve the boundary tangential case in the context of L_p-based Sobolev spaces and Besov spaces, which don't impose c. c. We begin with the interior stabilization to test the technique. The proof relies on many technical ingredients including: 1. recently developed maximal regularity theory for Stokes operators acting on L_p-based Sobolev and Besov spaces, 2. and a unique continuation theorem developed for Oseen operators via appropriate Carleman-type estimates.

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Modeling Fluid-structure interaction between blood flow and stented coronary artery

Abstract: This work is motivated by fluid-structure interaction between a coronary artery treated with a stent, pulsatile blood flow, and heart contractions. The fluid is modeled by the incompressible, viscous Navier-Stokes equations, the vascular walls are modeled as multi-layered composite elastic stuctures, and the presence of a stent is modeled by changing the density and elasticity coefficients of the structure in the inner-most layer, corresponding to the intima, where the stent is located. To include the effects of heart contractions, external force was applied to model the change in artery curvature. The fluid and composite structure are fully coupled via kinematic and dynamic coupling

conditions, which describe continuity of velocity and balance of contact forces. The contractions of the heart are modeled by adding a periodic external force to the structures outer surface. A loosely coupled partitioned scheme combined with an ALE approach is used to solve this nonlinear FSI problem. Four types of commercially available coronary stents are considered, and their performance compared and quantified. Based on the simulations, two out of four stents are considered to be more suitable for coronary angioplasty with stenting in curved, or tortuous arteries. Our results correlate well with clinical results. This is a joint work with Sunčica Čanić, Martina Bukač and Josip Tambača.

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Oscillations and integrability of the vorticity in the 3D NS flows

Abstract: An L^1 bound on the evolution of the vorticity in the 3D Navier-Stokes equations (NSE) was obtained by Constantin (Navier-Stokes equations and area of interfaces); this is an analogue of Leray's L^2 bound on the velocity. Until now there has been no priori estimate on the weak solutions to the 3D NSE breaking such scaling. In 2014, Bradshaw and Grujic proved that an assumption such as the vorticity direction is uniformly-in-time bounded in the logarithmically weighted BMO space yields a uniformly-in-time $L \log L$ bound on the vorticity, breaking the energy level scaling L^1. A remarkable feature of this problem is that such type of weighted BMO spaces allows for discontinuities. Then it follows an interesting question: Is it possible to break energy level scaling in vorticity bounds under a weaker assumption on the vorticity direction.

In a recent paper we attempt to generalize the result (Bradshaw and Grujic) by studying $L \log^k L$ bound for the vorticity. The main challenge is to prove the k-th iterated logarithm of vorticity has bounded weighted BMO norms with some reasonable assumption. One way is to make a restriction on the time development of vorticity near the blow-up time. Another way is to assume the singular profile has a regular shape or has "mild"' oscillation and the convergence is well-behaved at the blow-up time. Additional motivation for generalizing the theorem is to allow weaker assumption on the oscillation of vorticity direction (i.e. the vorticity direction is in a larger weighted BMO space allowing stronger discontinuities) but which can still exclude the blow-up scenario according to the article by Bradshaw and Grujic – Blow-up scenarios for 3D NSE exhibiting sub-criticality with respect to the scaling of 1D local sparseness and the article by Dascaliuc and Grujic – Vortex stretching and criticality for the 3D NSE.