Fragments of residuated lattices axiomatized by simple equations and decidability

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The decidability for the universal and (quasi-)equational theories for the varieties of residuated lattices RL extended by equations in the ordered monoid signature $\{\leq, \cdot, 1\}$ has been a topic richly studied. In particular, commutativity, weakened variants of commutativity, and knotted equations (such as *contraction* $x \leq x^2$). By the work of van Alten [1999], subvarieties of commutative residuated lattices defined by such equations actually have the finite embeddabillity property and hence are decidable. On the other hand, Horčík [2015] demonstrated the undecidability of the word problem for RL with contraction (and, in fact, equations from a much broader class), which was further bootstrapped by Chvalovský and Horčík [2016] to show that its equational theory is actually undecidable.

Following this programme, we investigate decidability questions for subvarieties of RL defined from a broader class of equations, called *simple*, in the idempotent semiring signature $\{\vee, \cdot, 1\}$, as well as compare and contrast the decidability of their various fragments. We show how previous techniques to handle the non-commutative, as well as the commutative, cases can be adapted and improved upon in this broader context and prove new undecidability results for so-called *spineless* equations. We also show that such subvarieties of residuated lattices are conservative extensions of their corresponding idempotent semirings fragments, which will shed light onto the boundary between decidable and undecidable theories in this context.