# International Conference on $L$-Functions and Automorphic Forms

Titles and Abstracts

May 13, 2024 – May 16, 2024

- **Nick Andersen** — *A generalization of Kohnen's identity for Kloosterman sums*
  Kohnen's identity for Kloosterman sums allows for passage of information from spaces of modular forms of half-integral weight to those of integral weight. Notably, it is used in Kohnen's proof of the Kohnen–Zagier formula for level $N$, as well as the more recent results of Duke, Imamoglu, and Toth on cycle integrals of the $j$-function and geometric invariants for real quadratic fields. We prove a generalization of Kohnen's identity in the setting of Kloosterman sums for the Weil representation. As a corollary, we obtain a new Weil-type bound for these sums. This is joint with Gradin Anderson and Amy Woodall.

- **Olivia Beckwith** — *A modular framework for generalized Hurwitz class numbers*
  We discover a simple relation between the mock modular generating functions of the level 1 and level $N$ Hurwitz class numbers. This relation gives rise to a holomorphic modular form of weight $3/2$ and level $4N$, for odd square free $N > 1$. We extend this observation to a non-holomorphic framework and obtain a higher level analogue of Zagier's weight $3/2$ Eisenstein series as well as a preimage under the Bruinier–Funke operator. All of these observations are deduced from a more general inspection of a certain weight $1/2$ Maass–Eisenstein series of level $4N$ at its spectral point $s = 3/4$. This idea goes back to Duke, Imamoglu and Toth in level 4 and relies on the theory of so-called sesquiharmonic Maass forms. This is joint work with Andreas Mono.

- **Farrell Brumley** — *Some new equidistribution results for tori in* $\mathrm{GSp}(4)$
  Duke's theorem on the equidistribution of CM points of large discriminant on the modular curve was generalized to the case of anisotropic tori in $\mathrm{GL}(3)$ by Einsiedler, Lindenstrauss, Michel, and Venkatesh, using a beautiful mix of ergodic and automorphic methods. We will describe ongoing work with Jasmin Matz, in which we extend some of their methods to the setting of $\mathrm{GSp}(4)$.

- **Claire Burrin** — *Rational points on the sphere*
  I will discuss my current favorite illustration of the 'unreasonable effectiveness of modular forms' at the hand of the problem of understanding the distribution of rational points on the sphere. Based on joint work with Matthias Gröbner.

- **Vorrapan Chandee** — *The eighth moment of* $\Gamma_1(q)$ *$L$-functions*
  I will discuss my on-going joint work with Alexander Dunn, Xiannan Li and Joshua Stucky on an unconditional asymptotic formula for the eighth moment of $\Gamma_1(q)$ $L$-functions, which are associated with eigenforms for the congruence subgroups $\Gamma_1(q)$. Similar to a large family of Dirichlet $L$-functions, the family of $\Gamma_1(q)$ $L$-functions has a size around $q^2$ while the conductor is of size $q$. An asymptotic large sieve of the family is available by the work of Iwaniec and Xiaoqing Li, and they observed that this family of harmonics is not perfectly orthogonal. This introduces certain subtleties in our work, which do not appear in the large family of Dirichlet $L$-functions.

- **Chantal David** — *Moment of cubic L-functions over* $\mathbb{F}_q(t)$ *at* $s = 1/3$
  We compute the first moment of Dirichlet $L$-functions attached to cubic characters over $\mathbb{F}_q(t)$, evaluated at an arbitrary $s \in (0, 1)$. We find a transition at the point $s = 1/3$, using the deep connections between Dirichlet series of cubic Gauss sums and metaplectic Eisenstein series first introduced by Kubota to obtain cancellation between the principal sum and the dual sum at $s = 1/3$. We also explain how at $s = 1/3$, the first moment matches corresponding statistics of the group of unitary matrices multiplied by a weight function at the $q$-limit, which was our original motivation to study this question. This is joint work with P. Meisner (Gothenburg).

- **Alex Dunn** — *Quartic Gauss sums over primes and metaplectic theta functions*
  We improve 1987 estimates of Patterson for sums of quartic Gauss sums over primes. Our Type-I and Type-II estimates feature new ideas, including use of the quadratic large sieve over the Gaussian quadratic field, and Suzuki's evaluation of the Fourier–Whittaker coefficients of quartic theta functions at squares. We also conjecture asymptotics for certain moments of quartic Gauss sums over primes. This is a joint work with C.David, A.Hamieh, and H.Lin.

- **Alexandre de Faveri** — *Non-escape of mass for automorphic forms in hyperbolic 4-manifolds*
  The arithmetic quantum unique ergodicity (AQUE) conjecture predicts that the $L^2$ mass of Hecke–Maass cusp forms on an arithmetic hyperbolic manifold becomes equidistributed as the Laplace eigenvalue grows. If the underlying manifold is non-compact, mass could "escape to infinity", and it can be a delicate matter to rule out such a possibility. This was achieved by Soundararajan for arithmetic surfaces, which when combined with celebrated work of Lindenstrauss completed the proof of AQUE for surfaces. We establish non-escape of mass for Hecke-Mass cusp forms on a congruence quotient of hyperbolic 4-space. Unlike in the setting of hyperbolic 2- or 3-manifolds (for which AQUE has been proved), the number of terms in the Hecke relations is unbounded, which prevents us from naively applying Cauchy–Schwarz. We instead view the isometry group as a group of quaternionic matrices, and rely on non-commutative unique factorization along with certain structural features of the Hecke action. Joint work with Zvi Shem-Tov.

- **Solomon Friedberg** — *Towards a New Shimura Lift*
  The classical Shimura lift sends modular forms of half-integral weight to modular forms of integral weight. It was generalized to number fields many years ago, lifting automorphic forms on the double cover of $\mathrm{SL}_2$ to automorphic forms on $\mathrm{PGL}_2$. Efforts have been made for roughly 40 years to generalize the Shimura map further, to higher rank groups and to higher degree covers, but our knowledge is limited. In this talk I will explain the meaning of an "automorphic form on a higher degree covering group" in terms of classical number theory, and give an account of what is known about Shimura lifts in general. Then I will describe my recent joint work with Omer Offen, which points to a new Shimura lift that lifts automorphic forms on the triple cover of $\mathrm{SL}_3$. We establish the Fundamental Lemma for a new relative trace formula. Moreover, this project should characterize the image of the lift by means of the nonvanishing of an exotic period, one involving a theta function on the orthogonal group $\mathrm{SO}_8$, consistent with a 2001 conjecture of Bump, Friedberg and Ginzburg.

- **Dorian Goldfeld** — *The functional equation of Langland's Eisenstein series for* $\mathrm{SL}(n, \mathbb{Z})$
  I shall present a simple explicit description of the general Langland's Eisenstein series for $\mathrm{SL}(n, \mathbb{Z})$. It can then be shown that the functional equations of these Eisenstein series can be derived from the functional equations of certain divisor sums and Whittaker functions which appear in the Fourier coefficients of the Eisenstein series. We conjecture that these functional equations are unique assuming they take the form of a real affine transformation of the $s$ variables defining the

Eisenstein series. We can prove the uniqueness conjecture in certain special cases. This is joint work with Eric State and Michael Woodbury.

- **Alia Hamieh** — *Moments of Rankin-Selberg L-functions in the prime-power level aspect*
  In this talk we discuss joint work with Fatma Çiçek on establishing asymptotic formulae with power saving error term for first and second moments of Rankin-Selberg $L$-functions $L(1/2 + i\mu, f \otimes g)$ where $f$ varies over newforms of conductor $p^n$ with $p$ being a fixed prime and $n \to \infty$.

- **Gergely Harcos** — *The prime geodesic theorem in arithmetic progressions*
  One can count hyperbolic conjugacy classes in $\mathrm{SL}_2(\mathbb{Z})$ according to their traces. The result is the prime geodesic theorem, which bears a close similarity with the prime number theorem. As primes are equidistributed in reduced residue classes, the natural question arises if the same is true of the traces mentioned above. It turns out that the answer is no, and the corresponding non-uniform distribution can be determined explicitly. This confirms a conjecture of Golovchanskiĭ–Smotrov (1999). Based on joint work with Dimitrios Chatzakos and Ikuya Kaneko.

- **Peter Humphries** — *Restricted Arithmetic Quantum Unique Ergodicity*
  The quantum unique ergodicity conjecture of Rudnick and Sarnak concerns the mass equidistribution in the large eigenvalue limit of Laplacian eigenfunctions on negatively curved manifolds. This conjecture has been resolved by Lindenstrauss when this manifold is the modular surface assuming these eigenfunctions are additionally Hecke eigenfunctions, namely Hecke–Maass cusp forms. I will discuss a variant of this problem in this arithmetic setting concerning the mass equidistribution of Hecke–Maass cusp forms on submanifolds of the modular surface, along with connections to period integrals of automorphic forms.

- **Subhajit Jana** — *Motohashi formula and its generalization in a non-split case*
  We will start with the classical Motohashi formula that relates a fourth moment of the Riemann zeta function and a cubic moment of GL(2)-Hecke $L$-functions. We will describe a generalization of this formula to a non-split case, namely when the fourth moment of the Riemann zeta function is replaced by a second moment of GL(2)-Hecke $L$-functions. Moreover, we will explain how the integral transforms of the involving test functions arise from a purely local representation theoretic viewpoint. This is a joint work with Valentin Blomer and Paul Nelson.

- **Winnie Li** — *Hypergeometric functions, Galois representations, and modular forms*
  Historically, the hypergeometric functions were studied through the lens of differential equations. When the parameters are rational numbers, Katz introduced Galois representations and developed a theory parallel to that in the classical setting. These Katz representations can be realized on cohomological groups of algebraic varieties, and hence they are expected to be automorphic according to Langlands' philosophy. In this talk we will showcase the interconnections among hypergeometric functions, Galois representations, and modular forms by exploring the analytic and algebraic aspects of hypergeometric functions. This is based on joint works with (1) Tong Liu and Ling Long, (2) Ling Long and Fang-Ting Tu, and (3) Bill Hoffman, Ling Long and Fang-Ting Tu.

- **Xiannan Li** — *One level density for a large orthogonal family of L-functions*
  I will describe recent joint work with Baluyot and Chandee, where we study a new orthogonal family of $L$-functions associated with holomorphic Hecke newforms of level $q$, averaged over $q \asymp Q$. To illustrate our methods, we prove a one level density result assuming GRH with the support of the Fourier transform of the test function being extended to be inside $(-4, 4)$.

- **Eugenia Rosu** — *A higher degree Weierstrass function*
  The Weierstrass $\wp$-function plays a great role in the classical theory of complex elliptic curves. A related function, the Weierstrass $\zeta$-function, is used by Guerzhoy to construct preimages under the $\xi$-operator of newforms of weight 2, corresponding to elliptic curves. In this talk, I will discuss a generalization of the Weierstrass $\zeta$-function and an application to harmonic Maass forms. More precisely, I will describe a construction of a preimage of the $\xi$-operator of a newform of weight $k$ for $k > 2$. This is based on joint work with C. Alfes-Neumann, J. Funke and M. Mertens.

- **Kannan Soundararajan** — *Covering integers by quadratic forms*
  How large must $\Delta$ be so that we can cover a substantial proportion of the integers below $X$ using the binary quadratic forms $x^2 + dy^2$ with $d$ below $\Delta$? Problems involving representations by binary quadratic forms have a long history, going back to Fermat. The particular problem mentioned here was recently considered by Hanson and Vaughan, and Y. Diao. In ongoing work with Ben Green, we resolve this problem, and identify a sharp phase transition: If $\Delta$ is below $(\log X)^{\log 2-\epsilon}$ then zero percent of the integers below $X$ are represented, whereas if $\Delta$ is above $(\log X)^{\log 2+\epsilon}$ then 100 percent of the integers below $X$ are represented.

- **Caroline Turnage-Butterbaugh** — *Averages of Long Dirichlet Polynomial Approximations of Primitive Dirichlet L-functions*
  In recent decades there has been much interest and measured progress in the study of moments of the Riemann zeta-function and, more generally, of various $L$-functions. Despite a great deal of effort spanning over a century, asymptotic formulas for moments of $L$-functions remain stubbornly out of reach in all but a few cases. In this talk, we consider the problem for the family of all Dirichlet $L$-functions of even primitive characters of bounded conductor. I will outline how to harness the asymptotic large sieve to prove an asymptotic formula for the general $2k$th moment of an approximation to this family. The result, which assumes the generalized Lindelöf hypothesis for large values of $k$, agrees with the prediction of Conrey, Farmer, Keating, Rubenstein, and Snaith. Moreover, it provides the first rigorous evidence beyond the so-called "diagonal terms" in their conjectured asymptotic formula for this family of $L$-functions. This is joint work with Siegfred Baluyot.

- **Liyang Yang** — *Harmonic Analysis on* $\mathrm{GL}(n+1)$ *and Rankin-Selberg L-Functions*
  In this talk, we will introduce an application of harmonic analysis on $\mathrm{GL}(n+1)$ to study Rankin-Selberg $L$-functions for $\mathrm{GL}(n+1) \times \mathrm{GL}(n)$. Specifically, this will yield bounds for standard $L$-functions on $\mathrm{GL}(n)$, showcasing the effectiveness of higher rank tools. We will present several applications concerning the subconvexity problem and nonvanishing problem, while also comparing this approach with classical methods in lower rank cases.