Computing Multi-Digit Numbers

- Students are now required to learn step-by-step procedures for solving problems.
- Although using a step-by-step procedure, students must understand when, why, and how to use different methods.
- Conceptual understanding of basic facts and place value are necessary for success in computing multi-digit problems.
- Hierarchy of skills must be followed - scope and sequence for addition, subtraction, multiplication, and division is provided later in the presentation.
Computing Multi-Digit Numbers

• When beginning to learn computation skills, problems should be related to real-life situations/word problems.

• It is important to teach students to begin on the right side of the problems in addition, subtraction, and multiplication. If students begin on the left side, errors will not occur when regrouping is not required, but will once regrouping is required.
Methods for Multi-Digit Computation

- Algorithms
- Calculators/Technology
- Estimation
- Mental Computation
Algorithms

• Step-by-step procedures used to solve problems.
• Used when an exact answer is needed.
• Algorithms should be taught at first by using proportional concrete objects.
Calculators/Technology

• Used when an exact answer is needed.
• Often used when large quantities of data are being used.
• Students must have explicit instruction in how to use a calculator - this is not something that we should assume students know how to do.
Estimation

• Used when an exact answer is not needed, a reasonable estimation will work.
• Estimation skills are used more often in everyday life experiences than are paper and pencil exercises.
• Students must have opportunities to practice using the estimation skills they have taught.
Mental Math

• Problems students can figure out in their heads.
• Students must have strategies for doing this. Many students will make up their own strategies, but should be exposed to some examples.

Example: 34 + 28  Student may think
  • 30 + 20 = 50
  • 4 + 8 = 12
  • 50 + 12 = 62

– Example: 62 + 13 + 24
  • 62 + 13 = 75
  • 75 + 24 = 99
Mental Math

• When students are developing their own strategies teachers should ask them to explain their thinking so it can be ensured that the strategy will work with all problems, not just the one being presented.
Regrouping

• The concept of regrouping is covered when teaching place value, but will need to be emphasized again when beginning to compute multi-digit problems.

• Once students learn to regroup, many will believe they always have to regroup. It is important to provide students with mixed problems so they know when and when not to regroup.
General Strategies

• Graph Paper - one of the common errors in computing multi-digit numbers is column alignment. The use of graph paper will assist students in correctly lining up the numbers and also cue them that only one digit can go in each block.

• Concrete Objects - proportional concrete objects will assist students in understanding when they need to regroup and to see how the process is completed. It is not necessary to spend a great deal of time with these objects, just enough to make sure students are understanding.
General Strategies

- Place Value Mats - may assist students - making sure they understand no more than 10 objects can be in each column.
- Partial Products - breaking numbers down into smaller components - hundreds, tens, ones. Specific examples will be given for each area.
Sequence of Addition

- Adding 2-digit plus 1-digit without regrouping
- Adding 2-digit plus 2-digit without regrouping
- Adding 2-digit plus 1-digit with regrouping
- Adding 2-digit plus 2-digit with regrouping
- Adding 3-digit plus 2-digit without regrouping
- Adding 3-digit plus 2-digit with regrouping
- Adding 3-digit plus 3-digit without regrouping
- Adding 3-digit plus 3-digit with regrouping
- Adding multi-digit plus multi-digit without regrouping
- Adding multi-digit plus multi-digit with regrouping
Addition Word Problems

• Carrie has 13 shirts. Her mother bought her 5 new shirts for her birthday. How many shirts does she have all together?

• Jeff has 64 baseball cards. He buys a new package with 48 baseball cards. How many baseball cards does he have now?

• Sally has 345 stamps in her stamp collection. Her brother Steve has 487 stamps in his collection. How many stamps do they have together?
Addition Algorithms

• 13 + 6
  – Add the ones place (3 + 6)
  – Record the 9 in the ones place
  – Bring down the tens
  – Read completed problem: 13 + 6 = 19

• 17 + 9
  – Add the ones place (7+9)
  – Record the 6 in the ones place
  – Regroup the 1 to the tens place
  – Add the tens place (1 + 1)
  – Record 2 in the tens place
  – Read completed problem: 17 + 9 = 26
Addition Algorithms

• $23 + 43$
  – Add the ones place ($3 + 3$)
  – Record 6 in the ones place
  – Add the tens place ($2 + 4$)
  – Record the 6 in the tens place
  – Read completed problem: $23 + 43 = 66$

• $48 + 38$
  – Add the ones place ($8 + 8$)
  – Record 6 in the ones place
  – Regroup 1 to the tens place
  – Add the tens place ($4 + 3$)
  – Add the regrouped tens ($7 + 1$)
  – Record the 8 in the tens place
  – Read completed problem: $48 + 38 = 86$
Addition Algorithms

• Algorithms for larger numbers will follow the same sequence.
CRA Method for Teaching Addition

• Concrete (43 + 38)
  – Show 4 tens, 3 ones and 3 tens, 8 ones
  – Combine the ones to show 11
  – Regroup 10 ones for one ten - one left over
  – Add 4 + 3 + 1
  – Write answer 43 + 38 = 81

• Representational
  – Show pictures of representing 43 and 38
  – Circle 10 ones and draw arrow to tens place - one left in the ones place
  – Add groups of ten
  – Write answer 43 + 38 = 81

• Abstract
  – Provide problem with no concrete objects or pictures
  – Student answers problem
Teaching Addition

- Partial Sums - rather than regrouping, students add the ones and tens. Example:

  \[
  \begin{array}{ccc}
  54 & 68 & 436 \\
  + 8 & + 24 & + 359 \\
  12 & 12 & 15 \\
  50 & 80 & 80 \\
  62 & 92 & + 700 \\
  \end{array}
  \]

  \[
  \text{795}
  \]

- This should only be used to assist students who need help, as it is not the most efficient way of computing the problem.
Teaching Addition

• Estimation - students must know if their answer is reasonable.
  – Example: $56 + 9 = 515$. Students should know that 9 more than 56 is not going to be 515, and it should cue them to compute their answer again.
Teaching Addition

• Mental Math - students will need practice with this concept.
  
  \[58 + 32\]
  \[60 + 30 = 90\]

  \[298 = 298 + 2 = 300\]
  \[+ 127 = 127 - 2 = 125\]
  \[\text{_________________________} 425\]
Sequence for Teaching Multi-Digit Subtraction

• 2-digit minus 1-digit without regrouping
• 2-digit minus 1-digit with regrouping
• 2-digit minus 2-digit without regrouping
• 3-digit minus 2-digit without regrouping
• Multi-digit minus multi-digit without regrouping
• 2-digit minus 2-digit with regrouping
• 3-digit minus 2-digit with one regrouping
• 3-digit minus 2-digit with two regroupings
• 3-digit minus 3-digit with one regrouping
• 3-digit minus 3-digit with two regroupings
• Multi-digit minus multi-digit with regroupings
• Computing differences with zeros in the minuend
Subtraction Word Problems

• Kevin’s family has 27 horses. They sell 14 of them. How many horses do they still have?
• Pat wants to collect 500 baseball cards. If he has 374 cards, how many more does he need to reach his goal?
• The Johnson’s are on a 1,745 mile trip. If they traveled 576 miles on the first day, how many miles do they have left?
Subtraction Algorithms

• 28 - 5
  – Subtract the ones place (8 - 5)
  – Record 3 in the ones place
  – Bring down the number in the tens place (2)
  – Read completed problem: 28 - 5 = 23

• 32 - 9
  – Subtract the ones place (2 - 9)
  – Realize you cannot take 9 away from 2
  – Regroup 1 ten to make 10 ones
  – Subtract ones place (12 - 9)
  – Record 3 in the ones place
  – Bring down number in tens place (2)
  – Read completed problem: 32 - 9 = 23
Subtraction Algorithms

• \(304 - 159\)
  – Subtract the ones place - notice you cannot take 9 away from 4
  – Regroup tens place - notice you cannot take 1 away from 0
  – Take 1 hundreds away and regroup to 10 tens
  – Take 1 tens away and regroup to 10 ones
  – Subtract ones place (14 - 9)
  – Record 5 in the ones place
  – Subtract tens place (9 - 5)
  – Record 4 in the tens place
  – Subtract hundreds place (2 - 1)
  – Record 1 in the hundreds place
  – Read completed answer: \(304 - 159 = 145\)
Subtraction Algorithms

• There are several examples of subtraction algorithms. The steps will remain the same for larger numbers. However, it is extremely important that students have ample opportunities to practice with numbers that include 0 and numbers that require regrouping across 2 or more places.
CRA Method for Subtraction

• Concrete (64 - 35)
  – Show 6 tens, 4 ones
  – Regroup 1 ten for 10 ones
  – Take away 5 ones
  – Take away 3 tens
  – Write answer 64 - 35 = 29

• Representational
  – Show picture representing 64
  – Cross out 1 ten and draw 10 ones
  – Cross out 5 tens
  – Write answer 64 - 35 = 29

• Abstract
  – Provide problem with no concrete objects or pictures
  – Student answers problem
Teaching Subtraction

• Use of Graph Paper will again assist students in column alignment - students only put one number in each column, will help them focus on regrouping questions.

• Students may have difficulty with 0 in numbers. It is important to provide them with ample opportunities to practice this and observe strategies they are using.

• Double regrouping across tens and hundreds may cause some students difficulty.
Teaching Subtraction

• Estimation - it is important to ask students if estimation is all right to use. How close does the answer need to be? The larger the numbers, the larger the estimate can be.

\[
\begin{array}{c@{}c}
67 & 70 \\
- 49 & - 50 \\
\hline
18 & 20 \\
\end{array}
\]

\[
\begin{array}{c@{}c}
307 & 300 \\
- 173 & - 175 \\
\hline
134 & 125 \\
\end{array}
\]
References

