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Acknowledgment

While this manual is (in at least the legal sense) my own work, I am very much indebted to my predecessors and colleagues, particularly Ken Shriver, Richard Helms, and Sherry Thompson. I am also grateful for much valuable feedback from my Teaching Assistants. About the only parts of the manual which are completely my own are the many mistakes. As you discover these errors, please point them out to your TA.

Oh, and I am also indebted to Randall Munroe, Wikipedia, and all who contribute to the Creative Commons. I plan to join them as soon as the lawyers let me.

Forrest T. Charnock

Cover Illustration: Diagram of a temperature compensating gridiron pendulum, invented by John Harrison, 1726. By combining metal rods of with different coefficients of thermal expansion, the length of the pendulum is kept constant over a wide range of temperatures. If you have never heard of John Harrison, you should. Google him.
General Physics Laboratory I

PHYS 1601L

(Prior to Fall 2015, this lab was known as PHYS 116A.)

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Useful Physical Constants

Universal gravitational constant: \[ G = 6.67408(31) \times 10^{-11} \frac{m^3}{kg \ s^2} \]

Speed of light (exact by definition) \[ c \equiv 2.99792458 \times 10^8 \frac{m}{s} \]

Avogadro’s Constant (exact)\(^\dagger\) \[ N_A \equiv 6.02214076 \times 10^{23} \text{ mol}^{-1} \]

Boltzmann’s Constant (exact)\(^\dagger\) \[ k \equiv 1.380649 \times 10^{-23} \text{ J/K} \]

Universal gas constant \[ R = 8.3144598(48) \frac{J}{\text{mol} \ K} \]

Acceleration due to gravity at Vanderbilt\(^\ddagger\): \[ g = 9.7943(32) \frac{m}{s^2} \]

Standard atmospheric pressure \[ 1 \text{ atm} \equiv 1.01325 \times 10^5 \text{ Pa} \]

Absolute zero \[ 0 \text{ K} = -273.15 ^\circ \text{C} \]

---

\(^\ast\) Values of fundamental constants are from NIST (physics.nist.gov/cuu). The number in parentheses is the standard uncertainty of the final digits of the main number. For example: 6.67408 ± 0.00031 = 6.67408 (31)

\(^\dagger\) As of May 20, 2019, this exact value is adopted by the General Conference on Weights and Measures (GCWM)

\(^\ddagger\) Dr. Medford Webster, Vanderbilt University
Introduction

The Sermon

The speed of light is $2.99792458 \times 10^8$ m/s. This is not science.

The Wikipedia entry on Newton’s 2nd law of motion is not science.

Nor is the periodic table of the elements.

Science is not a collection of facts. (Not even true facts!) Rather, science is a process for figuring out what is really going on. What is the underlying principle here? How does this relate to some other observation? If you are not involved in such a process, you are not doing science. A brilliant, dedicated, A+ student memorizing a list of equations is not doing science. A baby dropping peas on the floor to see what happens: now that’s science!! (Does oatmeal fall too? Let’s find out!!)

This is a science lab. I expect you to do some science in it.

“Yeah, yeah, I’ve heard this sermon before.”

Perhaps so, but I have seen too many brilliant and dedicated students who have learned to succeed in their other science classes by learning lots of stuff. So, they come into physics planning to memorize every equation they encounter and are completely overwhelmed. You cannot succeed in physics by learning lots of stuff. There are simply too many physics problems in the world; you cannot learn them all.

Instead, you should learn as little as possible! More than any other science, physics is about fundamental principles, and those few principles† must be the focus of your attention. Identify and learn those fundamental principles and how to use them. Then you can derive whatever solution that you need. And that process of derivation is the process of science.

“OK, thanks for the advice for the class, but this is a lab!”

It’s still about fundamental principles. Look, each week you will come to lab and do lots of stuff. By following the instructions and copying (...oh, I mean sharing...) a few answers from your lab partners, you can blunder through each lab just fine. The problem is that the following week you will have a quiz, and you will not remember everything you did in that lab the week before.

When you are doing each lab, consciously relate your experiments to the underlying principles.

How did I measure this? Which principle am I applying? Why are we doing this?

On the subsequent quiz, instead of having to remember what you did, you can apply the principles to figure out what you did. Trust me. It really is easier this way.

†... but not less.
‡ $F = ma$, conservation of energy and momentum, oscillations and waves, trigonometry. You will learn a few more in the second semester.
GOALS AND OBJECTIVES

Physics is about the real world, not some idealized Platonic world that only exists in your head.³ The purpose of this lab is to relate the theories and equations you are learning in the classroom to reality. Hopefully, we’ll convince you that all that physics stuff actually does work. Of course, reality can be messy, and along the way you will learn to deal with experimental uncertainty, loose cables, bad sensors, sticky wheels, temperamental software, temperamental lab partners, your own awful handwriting, and the typos in this lab book.

Welcome to experimental physics!

CORRELATION WITH LECTURE

Most of the topics covered in the lab will also be covered in your lecture, although not necessarily in the same sequence or at the same time during the semester. Given the scheduling (and re-scheduling) of the different lecture sections (some are MWF and some are TR), and the different lab sections (the first lab is Monday at 1 PM, the last is Thursday at 4 PM), perfect correlation of lecture and lab topics is not possible. The TA will provide a brief overview of the physics concept being explored in the lab during the first part of each lab section.

Occasionally, to improve the correlation with the lecture, the order of the labs may be changed from the sequence in this lab book. If so, you will be informed by your TA. Check your email regularly.

³ That’s the Mathematics Dept. Walk over to Bldg. 3.
PREPARATION

Prior to coming to lab, you should read over each experiment. Furthermore, for each laboratory, you must complete a pre-lab activity printed at the beginning of each lab in this manual. The pre-lab should be completed before the lab and turned in at the beginning of the lab. See the course syllabus for more details. For some labs, you may also be required to complete experimental predictions and enter them in your lab manual before you come to lab. Your TA will discuss this with you when necessary. Bring the following to each lab:

- Your complete lab manual secured in a 3-ring binder, including your previous graded labs.
- Your completed pre-lab.
- A scientific calculator. Graphing calculators are nice but not necessary. For some calculations, you may find a spreadsheet (such as Excel), Matlab, or some other computer based tools more appropriate. You are welcomed and encouraged to use such tools, but you still need a calculator.
- A pen, pencil and an eraser.

Often, the pre-lab includes online media for you to watch. Direct URL links are printed in the text, but clickable links may be found here:

https://my.vanderbilt.edu/physicslabs/videos/

PROCEDURE IN THE LABORATORY

In the laboratory, you will need to be efficient in the use of your time. We encourage a free exchange of ideas between group members and among students in the section, and we expect you to share both in taking data and in operating the computer, but you should do your own work (using your own words) in answering questions in the lab manual and on the review questions handed out in lab.
HONOR CODE

The Vanderbilt Honor Code applies to all work done in this course. Violations of the Honor Code include, but are not limited to:

- Copying another student’s answers on a pre-lab, lab questions, review questions, or quiz;
- Submitting data as your own when you were not involved in the acquisition of that data; and
- Copying data or answers from a prior term’s lab (even from your own, in the event that you are repeating the course).

GRADING

Your lab reports will be graded each week and returned to you the following week. Grades (including lab and quiz grades) will be posted on Brightspace.

- **Mistakes happen!** Check that the scores on Brightspace are correct. If you don’t do this, *no one will.*
- Retain you lab reports so that any such errors can be verified and corrected.
- Details of grading may be found on the online syllabus.

MAKING UP MISSED LABS

For details, I refer you to the syllabus (see below), but the main points are . . .

- **All** labs must be completed.
- If you know ahead of time that you will miss a lab, you must email both Dr. Charnock and your TA no later than the Friday before you will miss the lab.

    forrest.t.charnock@vanderbilt.edu

In that email, include

- Your lab (1601L, 1602L, 1501L, or 1502L)
- Section number
- TA name
- A brief explanation of why you are missing lab.

- If arranging a make-up ahead of time is not possible, email us as soon as possible.
  - If you are abducted by aliens, whip out your phone and compose an email describing your predicament while the tractor beam is lifting you into the air. Make sure to hit SEND before the iris door closes or the message won’t go out.** Update us on your situation as soon as you are returned to Devil’s Tower.
- You must be pro-active in making up labs.
  - Do NOT passively wait for someone to tell you what to do.

---

** Aliens rarely share their Wi-Fi passwords.
- If you do not receive a reply from Dr. Charnock within 24 hrs, **email him again**. Repeat as necessary.\(^{11}\)

\[\text{xkcd.com}\]

**SYLLABUS**: available online

https://my.vanderbilt.edu/physicslabs/documents/

\(^{11}\) Luke 18:1-5
How to Count Significant Figures

For all measured quantities (excepting counted quantities\textsuperscript{\textsuperscript{\textsuperscript{III}}}), there will always be an associated uncertainty. For example,

\begin{equation*}
\text{height of Mt. Everest} = 8844.43 \text{ m} \pm 0.21 \text{ m}
\end{equation*}

Understanding the uncertainty is crucial to understanding the quantity. Explicitly stating the value of your uncertainty (\pm 0.21 m) is the ideal. However, it is not always necessary to provide a precise uncertainty range as shown above. The crudest way to represent uncertainty is the method significant figures. Here, the \pm is dropped and the uncertainty is implied by the figures that are shown. An individual digit is usually considered significant if its uncertainty is less than \pm 5. In the case of Mt. Everest, the uncertainty is greater than 0.05 m; thus making the "3" uncertain. Rounding to the nearest 0.1 meter, we can write

\begin{equation*}
\text{height of Mt. Everest} = 8844.4 \text{ m}.
\end{equation*}

This quantity has five significant figures. Notice that a digit does not need to be precisely known to be significant. Maybe the actual height is 8844.2 m. Maybe it is 8844.6 m. But the Chinese Academy of Sciences is confident that it is NOT 8844.7 m. Hence, that final "4" is worth recording.

Please note: \textbf{This method of accounting for uncertainty is literally an order of approximation.} But it is quick and can help you develop an intuition for the quality of your results. In general, the rules for interpreting a value written this way are . . .

- All non-zero digits are significant
- All zeros written between non-zero digits are significant
- All zeros right of the decimal AND right of the number are significant
- Unless otherwise indicated, all other zeros are implied to be mere place-holders and are not significant.

Consider the following examples. The significant digits are underlined

\begin{equation*}
\begin{array}{c}
1023 \\
102300 \\
102300.00
\end{array}
\end{equation*}

\begin{equation*}
\begin{array}{c}
001023.450 \\
0.0010230
\end{array}
\end{equation*}

\textsuperscript{\textsuperscript{III}} Even if you think you understand significant figures, read this anyway. Some of what you think you know may be wrong.

\textsuperscript{\textsuperscript{III}} For example: "There are \textbf{exactly} 12 eggs in that carton."

Occasionally, a zero that appears to be a mere place-holder is actually significant. For example, the length of a road may be measured as 15000 m ± 25 m. The second zero is significant. There are two common ways two write this.

- Use scientific notation (preferable): \( 1.500 \times 10^4 \text{ m} \)
- Use a bar to indicate the least significant figure: \( 1\,\overline{5}\,0\,0\,0 \text{ m} \) or \( 1\,5\,0\,0\,0 \text{ m} \)

**Addition and Subtraction**

If several quantities are added or subtracted, the result will be limited by the number with the largest uncertain decimal position. Consider the sum below:

\[
\begin{align*}
123.4500 \\
+ 12.20 \\
+ 0.00023 \\
\hline
135.65023 \\
\hline
135.65
\end{align*}
\]

This sum is limited by 12.20; the result should be rounded down to the nearest hundredth. Again, consider another example:

\[
\begin{align*}
321000 \\
+ 12.30 \\
\hline
32060.3 \\
\hline
320680
\end{align*}
\]

In 321000, the last zero is not significant. The final answer is rounded up to the ten’s position.

**Multiplication and Division**

When multiplying or dividing quantities, the quantity with the fewest significant figures will determine the number of significant figures in the answer.

\[
\begin{align*}
\frac{123.45 \times 0.0555}{22.22} &= 0.30834721 = 0.308
\end{align*}
\]

0.0555 has the fewest significant figures with three. Thus, the answer must have three significant figures.

To ensure that round off errors do not accumulate, it is good practice to keep at least one digit more than is warranted by significant figures during intermediate calculations. Do the final round off at the end. But, **counting significant figures is literally an order-of-magnitude approximation**, so rounding up or down does not really matter that much.
How Do I Round a Number Like 7.5? It is exactly in the middle of 7 and 8. Do I round up or down?

For a fraction of 0.5, I always round up*(for example, 7.5 → 8), but others have different opinions†. Counting significant figures is literally an order-of-magnitude approximation, so rounding up or down does not really matter that much.

How Can This Break Down?
Remember, counting significant figures is NOT a perfect way of accounting for uncertainty. It is only a first approximation that is easy to implement.

For transcendental functions (sines, cosines, exponentials, etc.) these rules simply don’t apply. When doing calculations with these, I usually keep one extra digit to avoid throwing away resolution.

However, even with simple arithmetic, naively applying the above rules can cause you to needlessly loose resolution.

Suppose you are given two measurements 10m and 9s. You are asked to calculate the speed.

With 10m I will assume an uncertainty of about 0.5 out of 10 or about 5%.‡

With 9s you have almost the same uncertainty (0.5 out of 9), but technically we only have one significant digit instead of two.

If I naively apply the rules . . .

\[ \frac{10m}{9s} = 1.1111 \frac{m}{s} \]

. . . my answer has an uncertainty of 0.5 out of 1!!! 50%!!!

This is what I call the odometer problem: When you move from numbers that are close to rolling over to the next digit (for example, 98) to numbers that have just barely rolled over (say, 103), the practical uncertainty is almost unchanged (about 1 part in 100). But having an extra significant digit implies 10× greater precision, which is clearly absurd.§ Counting significant figures is literally an order of magnitude approximation. Here, we really need to keep a second digit in the answer.

\[ \frac{10m}{9s} = 1.11 \frac{m}{s} \]

---

* . . . , and for good mathematical reasons, mind you. But still, it does not really matter that much.
† Google it, if you want to waste an hour of your life.
‡ Of course, I don’t really know what the uncertainty is. It could be much larger, but bear with me anyway.
§ . . . and, vice-versa.
Notice: In the problem above, if the numbers are flipped, the odometer problem goes away:

\[
\frac{9m}{10s} = 0.9000 \frac{m}{s} = 0.9 \frac{m}{s}
\]

Oh Great! I thought this was supposed to be easy.

Well . . . , it is! But, you still have to use your head!

- Apply the rules.
- Look out for the odometer problem.
- If warranted, keep an extra digit.
- Simple!

Remember: counting significant figures is literally an order-of-magnitude approximation. So, don’t get too uptight about it. We will learn a more sophisticated way of dealing with uncertainty in Lab 1.

What you should never do is willy-nilly copy down every digit from your calculator. If you are in the habit of doing that, STOP IT. You are wasting your time and lying to yourself. If you ever claim that your cart was traveling at 1.35967494 m/s, expect your TA to slap you down. That is just wrong!

What makes a result scientific is honesty, not precision! To remain honest, you must never imply that you know something that you don’t!

To say that the altitude of Mt. Everest is about 9000 m is perfectly true. To say that the altitude of Mt. Everest is 8844.4324 m is a lie.

![Image of XKCD comic about significant figures](xkcd.com)
The **Système International d'Unités**, aka SI units

In physics, we make our measurements using **SI units**. This is also called the **mks system** (for *meters, kilograms, seconds*).

For first semester physics, our base units are

- **Length**: meter (m)
- **Time**: second (s)
- **Mass**: kilogram (kg)
- **Temperature**: kelvin (K)

For second semester physics, we will add a unit for electric charge, the **coulomb** (C).

That is it!! No more! SI units have no more base units than are absolutely necessary. For any other quantities that we need to express, we will combine the units that we already have. For example:

- **Speed**: m/s
- **Area**: m²
- **Acceleration**: m/s²
- **Volume**: m³

Notice that the base unit for volume is not liters! The **liter is not an SI unit**! Of course, it’s ok to measure things with liters, but it is understood that 1 liter is shorthand for 0.001 m³. And, it’s ok to measure things with kilometers, but it is understood that 1 km is shorthand for 1000 m.

There are other things that we will need to measure, such as **force** and **energy**.

Force has units of mass × acceleration. Thus, the SI unit of force will be

- **Force**: kg m/s²

However, we measure force often, and it gets tiresome to say “kg m/s²” over and over. So, we have given this parameter a name, the **newton (N)**.

\[ 1 \text{ N} = 1 \text{ kg m/s}² \]

Energy has units of force × distance, or N m. We call this a **joule (J)**

- **Energy**: 1 N m = 1 J

So, whenever we decide to bless a parameter with a name in SI units, that parameter will be built up from the base units: m, kg, s, K.

---

* The base SI units also include the **candela (cd)** (luminous intensity) and **mole (mol)** (amount of substance).
Deviant Measurement Systems

The Système International is not the metric system. Rather, it is a metric system. There are other metric systems. I mention these only to warn you of their existence.

The most notorious is a system maintained by a cabal of benighted scientists known as chemists. Instead of basing their units on the noble and righteous standard defined by the General Conference on Weights and Measures using kilograms, meters, and seconds (mks), this cabal clings to a deviant set of units based on centimeters, grams, and seconds. This is called the cgs system or Gaussian units. As with the SI, other units can be built up from these. For example:

\[
\begin{align*}
\text{Speed: } & \text{ cm } / \text{ s} & \text{Area: } & \text{ cm}^2 \\
\text{Acceleration: } & \text{ cm } / \text{ s}^2 & \text{Volume: } & \text{ cm}^3
\end{align*}
\]

The basic unit of force in the cgs system is called the dyne (dyn)

\[1 \text{ dyn} = 1 \text{ g cm } / \text{ s}^2 = 10^{-5} \text{ N}\]

The basic unit of energy is called the erg (erg).

\[1 \text{ erg} = 1 \text{ dyn cm} = 10^{-7} \text{ J}\]

The close resemblance of the cgs system to the SI makes it only more devious and worthy of your attentive derision.

Even more ridiculous, there exists a truly demented sect of technologists who persist in using medieval units. We'll discuss them no further.

---

* There is also a small group of benighted theoretical physicists who persist in using the cgs system, but we don’t talk about them.

* . . . which sounds like something you’d groan while straining on a toilet.

† OK, there actually are some virtues to British Imperial units. But that is a long, nerdy discussion we can have at our inconvenience. Make no mistake, they are terrible.
The Greek Alphabet

The 26 letters of the Standard English alphabet do not supply enough variables for our algebraic needs. So, the sciences have adopted the Greek alphabet as well. You will have to learn it eventually, so go ahead and learn it now, particularly the lower case letters.

<table>
<thead>
<tr>
<th>Greek Letter</th>
<th>Lower Case</th>
<th>Upper Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha (α)</td>
<td>A</td>
<td>Α</td>
</tr>
<tr>
<td>Beta (β)</td>
<td>B</td>
<td>Β</td>
</tr>
<tr>
<td>Gamma (γ)</td>
<td>Γ</td>
<td>Γ</td>
</tr>
<tr>
<td>Delta (δ)</td>
<td>Δ</td>
<td>Δ</td>
</tr>
<tr>
<td>Epsilon (ε)</td>
<td>E</td>
<td>Ε</td>
</tr>
<tr>
<td>Zeta (ζ)</td>
<td>Ζ</td>
<td>Ζ</td>
</tr>
<tr>
<td>Eta (η)</td>
<td>H</td>
<td>Η</td>
</tr>
<tr>
<td>Theta (θ)</td>
<td>Θ</td>
<td>Θ</td>
</tr>
<tr>
<td>Kappa (κ)</td>
<td>K</td>
<td>Κ</td>
</tr>
<tr>
<td>Lambda (λ)</td>
<td>Λ</td>
<td>Λ</td>
</tr>
<tr>
<td>Mu (μ)</td>
<td>M</td>
<td>Μ</td>
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<tr>
<td>Nu (ν)</td>
<td>N</td>
<td>Ν</td>
</tr>
<tr>
<td>Xi (ξ)</td>
<td>Ξ</td>
<td>Ξ</td>
</tr>
<tr>
<td>Omicron (ο)</td>
<td>O</td>
<td>Ο</td>
</tr>
<tr>
<td>Pi (π)</td>
<td>Π</td>
<td>Π</td>
</tr>
<tr>
<td>Rho (ρ)</td>
<td>R</td>
<td>Ρ</td>
</tr>
<tr>
<td>Sigma (σ)</td>
<td>Σ</td>
<td>Σ</td>
</tr>
<tr>
<td>Tau (τ)</td>
<td>T</td>
<td>Τ</td>
</tr>
<tr>
<td>Upsilon (υ)</td>
<td>Υ</td>
<td>Υ</td>
</tr>
<tr>
<td>Phi (φ)</td>
<td>Φ</td>
<td>Φ or φ</td>
</tr>
<tr>
<td>Chi (χ)</td>
<td>Χ</td>
<td>Χ</td>
</tr>
<tr>
<td>Psi (ψ)</td>
<td>Ψ</td>
<td>Ψ</td>
</tr>
<tr>
<td>Omega (ω)</td>
<td>Ω</td>
<td>Ω</td>
</tr>
</tbody>
</table>

**Important:** ω is NOT w! Note how the Greek ω is curvy, the Latin w is pointy. Please do not call ω 'double-u'; it is 'omega'.

And ρ is a Greek rho, NOT a Latin p!
Pre-Lab Preparation Sheet for Lab 1:
Problem Solving

(Due at the Beginning of Lab)

Directions:
Before attending your lab session, do the following:

Watch the following videos:
https://www.youtube.com/watch?v=INlPbfBGPtw
https://www.youtube.com/watch?v=2s1qTUqOv88

Read the essay How to Count Significant Figures, then read over the following lab. Applying the rules of significant figures, calculate the following:

\[123.4 + 120 + 4.822 - 21 =\]

\[\frac{185.643 \times 0.0034}{3022} =\]

\[(523400 \times 0.0032) + 253 =\]
Lab 1: Estimation and Problem Solving

“Imagination is more important than knowledge.” -- Albert Einstein

“Well, you have to know these things when you’re a king, you know.” --Arthur, King of the Britons

Objective: To learn how to approach a problem by asking the right questions.

Equipment: meter stick, ruler, caliper, stopwatch, standard masses (5 kg, 1 kg, 0.1 kg, 0.01 kg), your brain

Metric Intuition

First, I want you to develop an intuition for the SI values of various quantities.

1. On your table you will find several standard masses. Hold each in your hand to get a sense of its mass.
2. With the 2m stick, measure your height.
3. With the bath scale, measure your mass.
4. Quickly determine the following values in SI units (meters, kilograms, seconds):

<table>
<thead>
<tr>
<th>How tall are you?</th>
</tr>
</thead>
<tbody>
<tr>
<td>How wide is your thumb?</td>
</tr>
<tr>
<td>What is the width of a sheet of paper?</td>
</tr>
<tr>
<td>What is the thickness of a sheet of paper?</td>
</tr>
<tr>
<td>What is the height of this room?</td>
</tr>
<tr>
<td>What is your mass?</td>
</tr>
<tr>
<td>What is the mass of your phone?</td>
</tr>
<tr>
<td>What is the mass of a penny?</td>
</tr>
<tr>
<td>What is the mass of a sheet of paper?</td>
</tr>
<tr>
<td>What is the duration of a relaxed breath?</td>
</tr>
<tr>
<td>What is the duration of <em>Happy Birthday</em>?</td>
</tr>
<tr>
<td>What is the allotted time of this laboratory?</td>
</tr>
</tbody>
</table>
Estimation and Extrapolation

Now that you have a base set of measured values, you can make some reasonable guesses to extrapolate from these values to very different quantities. Consult with your lab partners and answer the following questions. Don’t worry about making exact measurements. The point is to quickly get a rough idea of the quantity. (Stick to SI units.)

What is the volume of this room?

What is the height of this building?

What is the diameter of a fully dilated pupil?

What is LeBron James’ hang time?

What is the mass of a golden retriever?

Fermi Problems

Consider an apparently intractable problem:

How many salt grains are there in a salt shaker?

Fermi’s method is to break this apparently intractable question into simpler questions for which you can make reasonable guesses.

What is the volume of a single salt grain?

What are the dimensions of a salt shaker?

You may need to narrow the scope of a vague question.

What are the dimensions of a single grain of Morton’s salt? (0.3 mm)$^3$

What are the dimensions of the table shakers used in the cafeteria? (25 × 25 × 40) mm$^3$

From your everyday experience, you can make reasonable guesses for these values.
\[
\text{number of grains} = \frac{\text{volume of shaker}}{\text{volume of a single grain}} = \frac{(25\times25\times40)\text{mm}^3}{(0.5 \text{ mm})^3} = 200000
\]

A quick check of the units indicates a coherent answer (unit-less, in this case). Now, this result may be off by quite a bit, perhaps a factor of 10. But you now know that it is more than thousands and less than billions. You did not know that before.

Consider the following questions?

*What is the mass of a postage stamp?*

*How long would it take to walk around a football field?*

*How long would it take to walk to Memphis?*

*What is the wing velocity of an unladen sparrow?*

Your TA will ask you several other questions.
Name_____________________________ Section ______ Date__________

Pre-Lab Preparation Sheet for Lab 2: Measurement, Uncertainty, and Uncertainty Propagation

(Due at the Beginning of Lab)

Carefully, read over the lab, then answer the following questions.

Consider two quantities:

\[ A = (10 \text{ m} \pm 1 \text{ m}) \quad B = (100 \text{ m} \pm 5 \text{ m}) \]

1. What are the relative uncertainties of A and B?

2. What is the uncertainty of \((A + B)\)?

3. What is the uncertainty of \((A - B)\)?


\[
\begin{align*}
\text{PRECISE} + \text{PRECISE} &= \text{SLIGHTLY LESS PRECISE NUMBER} \\
\text{PRECISE} \times \text{PRECISE} &= \text{SLIGHTLY LESS PRECISE NUMBER} \\
\text{PRECISE NUMBER} + \text{GARBAGE} &= \text{GARBAGE} \\
\text{PRECISE NUMBER} \times \text{GARBAGE} &= \text{GARBAGE} \\
\sqrt{\text{GARBAGE}} &= \text{LESS BAD GARBAGE} \\
(\text{GARBAGE})^2 &= \text{WORSE GARBAGE} \\
\frac{1}{N} \sum \left( N \text{ PIECES OF STATISTICALLY INDEPENDENT GARBAGE} \right) &= \text{BETTER GARBAGE} \\
\left(\text{PRECISE NUMBER}\right)^{\text{GARBAGE}} &= \text{MUCH WORSE GARBAGE} \\
\text{GARBAGE} - \text{GARBAGE} &= \text{MUCH WORSE GARBAGE} \\
\frac{\text{PRECISE NUMBER}}{\text{GARBAGE} - \text{GARBAGE}} &= \text{MUCH WORSE GARBAGE, POSSIBLE DIVISION BY ZERO} \\
\text{GARBAGE} \times 0 &= \text{PRECISE NUMBER}
\end{align*}
\]

xkcd.com
Lab 2: Measurement, Uncertainty, and Uncertainty Propagation

"The first principle is that you must not fool yourself – and you are the easiest person to fool."

--Richard Feynman

Objective: To understand how to report both a measurement and its uncertainty.

Learn how to propagate uncertainties through calculations

Define mean, standard deviation, and standard deviation of the mean.

Equipment: meter stick, 1 kg mass, ruler, caliper, short wooden plank, graduated cylinder

Discussion
Before you can really know anything, you have to measure something, be it distance, time, acidity, or social status. However, measurements cannot be exact. Rather, all measurements have some uncertainty associated with them.† Ideally, all measurements should consist of two numbers: the value of the measured quantity \( x \) and its uncertainty\( ^\dagger \) \( \Delta x \). The uncertainty reflects the reliability of the measurement. The range of measurement uncertainties varies widely. Some quantities, such as the mass of the electron \( m_e = (9.1093897 \pm 0.00000054) \times 10^{-31} \) kg, are known to better than one part per million. Other quantities are only loosely bounded: There are 100 to 400 billion stars in the Milky Way.

Note that we are not talking about "human error"! We are not talking about mistakes! Rather, uncertainty is inherent in the instruments and methods that we use even when perfectly applied. The goddess Athena cannot not read a digital scale any better than you.

† The only exceptions are counted quantities. "There are exactly 12 eggs in that carton."
\( ^\dagger \) Sometimes this is called the error of the measurement, but uncertainty is the better term. Error implies a variance from the true value:

\[
\text{error} = x_{\text{measured}} - x_{\text{true}}
\]

But, of course, we don’t know what the true value is. If we did, we would not need to make the measurement in the first place. In fact, we cannot know the error in principle! But we can measure the uncertainty.
RECORDING UNCERTAINTY

In general, uncertainties are usually quoted with no more precision than the measured result; and the last significant figure of a result should match that of the uncertainty. For example, a measurement of the acceleration due to gravity on the surface of the Earth might be given as

\[ g = 9.7 \pm 1.2 \text{ m/s}^2 \quad \text{or} \quad g = 9.9 \pm 0.5 \text{ m/s}^2 \]

But you should never write

\[ g = 9.7 \pm 1.25 \text{ m/s}^2 \quad \text{or} \quad g = 9.92 \pm 0.5 \text{ m/s}^2. \]

In the last two cases, the precision of the result and uncertainty do not match.

Uncertainty is an inherently fuzzy thing to measure; so, it makes little sense to present the uncertainty of your measurement with extraordinary precision. It would be silly to say that I am \((1.823643 \pm 0.124992)\) m tall. Therefore, the stated uncertainty will usually have only one significant digit. For example

\[ 23.5 \pm 0.4 \quad \text{or} \quad 13600 \pm 700 \]

However, if the uncertainty is between 1.0 and 2.9 (or 10 and 29, or 0.0010 and 0.0029, etc.) it may be better to have two significant digits. For example,

\[ 124.5 \pm 1.2 \]

There is a big difference between saying \(\pm 1\) and \(\pm 1.4\). There is not a big difference between \(\pm 7\) and \(\pm 7.4\). (This is related to the odometer problem. See the essay *How to Count Significant Figures*.)

VOCABULARY

Here we define some useful terms (with examples) and discuss how uncertainties are reported in the lab.

**Absolute uncertainty**: This is the magnitude of the uncertainty assigned to a measured physical quantity. It has the same units as the measured quantity.

Example 1. Again, consider the example above:

\[ L = (1.56 \pm 0.03) \text{ cm} \]

Here, the uncertainty is given in units of length: 0.03 cm. When the uncertainty has the same dimension as the measurement, this is an *absolute uncertainty*. 
**Relative uncertainty:** This is the ratio of the absolute uncertainty and the value of the measured quantity. It has no units; that is, it is dimensionless. It is also called the *fractional uncertainty* or, when appropriate, the *percent uncertainty.*

Example 2. In the example above the *fractional uncertainty* is

\[
\frac{\Delta V}{V} = \frac{0.03 \text{ cm}}{1.56 \text{ cm}} = 0.019
\]

The *percent uncertainty* would be 1.9%.

**Types of uncertainties**

*Random uncertainties* occur when the results of repeated measurements vary due to truly random processes. For example, random uncertainties may arise from small fluctuations in experimental conditions or due to variations in the stability of measurement equipment. These uncertainties can be estimated by repeating the measurement many times.

*A systematic uncertainty* occurs when all of the individual measurements of a quantity are biased by the same amount. These uncertainties can arise from the calibration of instruments or by experimental conditions. For example, slow reflexes while operating a stopwatch would systematically yield longer measurements than the true time duration.

![Diagram showing random and systematic uncertainties](random.png)

*Mistakes* can be made in any experiment, either in making the measurements or in calculating the results. However, by definition, mistakes can also be avoided. Such blunders and major systematic errors can only be avoided by a thoughtful and careful approach to the experiment.

**Estimating uncertainty**

*Repeated observation:* Suppose you make repeated measurements of something: say with a stopwatch you time the fall of a ball. Due to random variations, each measurement will be a little different. From the spread of the measurements, you can calculate the uncertainty of your results.
Shortly, we will describe the formal procedure to do this calculation. (Oddly enough, truly random uncertainties are the easiest to deal with.)

*By eye or reason:* Sometimes, repeated measurements are not useful. Suppose you measure the length of something with a meter stick. Meter sticks are typically ruled to the mm; however, we can often read them more precisely than that.

Consider the figure above. Measuring from the left side of each mark and considering the position uncertainties of both ends of the bar, I can confidently say that the bar is $(1.56 \pm 0.03)$ cm. Perhaps your younger eyes could read it with more confidence, but when in doubt it is better to overestimate uncertainty.

Could I do a better job by measuring several times? Not always. Sometimes with repeated measurements, it still comes down to “Looks like $(1.56 \pm 0.03)$ cm to me.” But that’s ok. Your reasoned judgment is sufficient. **Science is defined by rigorous honesty, not rigorous precision!**

**Exercise 1**

1. Measure the height of one member of your group.
2. Estimate the uncertainty of this measurement.
3. Now, have another group measure the height of the same student.

<table>
<thead>
<tr>
<th>Name</th>
<th>height</th>
<th>uncertainty</th>
</tr>
</thead>
<tbody>
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<td></td>
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</tbody>
</table>
Since there are always measurement uncertainties, your numbers are probably different. But these two measurements may still be consistent with each other.

4. Are these different measurements consistent with each other?
5. What do you mean when you say they are or are not consistent?

Reducing random uncertainty by repeated observation

By taking a large number of individual measurements, we can use statistics to reduce the random uncertainty of a quantity. For instance, suppose we want to determine the mass of a standard U.S. penny. We measure the mass of a single penny many times using a balance. The results of 17 measurements on the same penny are summarized in Table 1.

Table 1. Data recorded measuring the mass of a US penny.

<table>
<thead>
<tr>
<th></th>
<th>mass (g)</th>
<th>deviation (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.43</td>
<td>-0.088</td>
</tr>
<tr>
<td>2</td>
<td>2.49</td>
<td>-0.028</td>
</tr>
<tr>
<td>3</td>
<td>2.49</td>
<td>-0.028</td>
</tr>
<tr>
<td>4</td>
<td>2.58</td>
<td>0.062</td>
</tr>
<tr>
<td>5</td>
<td>2.52</td>
<td>0.002</td>
</tr>
<tr>
<td>6</td>
<td>2.55</td>
<td>0.032</td>
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<tr>
<td>7</td>
<td>2.52</td>
<td>0.002</td>
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<tr>
<td>8</td>
<td>2.64</td>
<td>0.122</td>
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<td>9</td>
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<td>10</td>
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<td>-0.058</td>
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<td>11</td>
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<td>13</td>
<td>2.58</td>
<td>0.062</td>
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<tr>
<td>14</td>
<td>2.61</td>
<td>0.092</td>
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<tr>
<td>15</td>
<td>2.49</td>
<td>-0.028</td>
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<tr>
<td>16</td>
<td>2.52</td>
<td>0.002</td>
</tr>
<tr>
<td>17</td>
<td>2.46</td>
<td>-0.058</td>
</tr>
</tbody>
</table>

The mean value \( \bar{m} \) (that is, the average) of the measurements is defined to be

\[
\bar{m} = \frac{1}{N} \sum_{i=1}^{N} m_i = \frac{1}{17} (m_1 + m_2 + \ldots + m_{17}) = 2.518 \text{g}
\]  

(1)

The deviation \( d_i \) of the \( i \)th measurement \( m_i \) from the mean value \( \bar{m} \) is defined to be

\[
d_i = m_i - \bar{m}
\]  

(2)

Fig. 1 shows a histogram plot of the data on the mass of a US penny. Also on the graph is a plot of the smooth bell curve (that is a normal distribution) that represents what the distribution of measured values would look like if we took many, many measurements. The result of a large set

** If the numbers are the same, you could probably add another digit of precision to your measurement.
of repeated measurements (when subject only to random uncertainties) will always approach a normal distribution which is symmetrical about \( \bar{m} \).

![Mass of a US Penny](image)

Figure 1. The Gaussian or normal distribution for the mass of a penny \( N=17, \bar{m} =2.518 \text{ g}, \Delta m=0.063 \text{ g} \).

**OK, now I have all of these measurements. How accurate is any one of these measurements?**

For this, we now define the **standard deviation** \( \Delta m \) as

\[
\Delta m = \sqrt{\frac{\sum_{i=1}^{N} (m_i - \bar{m})^2}{(N-1)}} = \sqrt{\frac{\sum_{i=1}^{N} (m_i - \bar{m})^2}{16}} = 0.063 \text{ g}
\] (3)

For normal distributions, 68% of the time the result of an individual measurement would be within \( \pm \Delta m \) of the mean value \( \bar{m} \). Thus, \( \Delta m \) is the experimental uncertainty for an individual measurement of \( m \).

The mean \( \bar{m} \) should have less uncertainty than any individual measurement. What is that uncertainty?

The uncertainty of the final average is called the **standard deviation of the mean**. It is given by

\[
\Delta M = \frac{\Delta m}{\sqrt{N}}
\] (4)

With a set of \( N=17 \) measurements, our result is
mass of a penny \( \bar{m} \pm \Delta M \) = \( \bar{m} \pm \frac{\Delta m}{\sqrt{N}} \)

\[ = 2.518 \text{ g} \pm \frac{0.063 \text{ g}}{\sqrt{17}} \]

\[ = (2.518 \pm 0.015) \text{ g} \] (5)

Thus, if our experiment is only subject to random uncertainties in the individual measurements, we can improve the precision of that measurement by doing it repeatedly and finding the average. Note, however, that the precision improves only as \( \frac{1}{\sqrt{N}} \). To reduce the uncertainty by a factor of 10, we have to make 100 times as many measurements. We also have to be careful in trying to get better results by letting \( N \to \infty \), because the overall accuracy of our measurements will eventually be limited by systematic errors, which do not cancel out like random errors do.

Exercise 2:

a. With a caliper, measure the width and thickness of the plank. Make at least five measurements of each dimension and enter the result into Table 2.

b. With a ruler, measure the length of the wooden plank on your table as precisely as possible. Estimate the uncertainty, and enter the result below.

c. For both the width and thickness, calculate your final result (the mean) and the uncertainty (the standard deviation of the mean). Enter the final results below the table.
Table 2

<table>
<thead>
<tr>
<th>width</th>
<th>deviation</th>
<th>thickness</th>
<th>deviation</th>
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</tbody>
</table>

width = 

thickness = 

length = 

Propagation of uncertainties

Usually, to obtain a final result, we have to measure a variety of quantities (say, length and time) and mathematically combine them to obtain a final result (speed). How the uncertainties in individual quantities combine to produce the uncertainty in the final result is called the propagation of uncertainty.

Here we summarize a number of common cases. For the most part these should take care of what you need to know about how to combine uncertainties.††

†† These expressions for the propagation of uncertainty are an upper limit to the resulting uncertainty. In this case, you could actually do better. See Appendix C for details.
**Uncertainties in sums and differences:**

If several quantities \( x_1, x_2, x_3 \) are measured with absolute uncertainties \( \Delta x_1, \Delta x_2, \Delta x_3 \) then the absolute uncertainty in \( Q \) (where \( Q = x_1 \pm x_2 \pm x_3 \)) is

\[
\Delta Q = |\Delta x_1| + |\Delta x_2| + |\Delta x_3|
\]

(6)

In other words, *for sums and differences, add the absolute uncertainties.*

**Uncertainties in products and quotients:**

Several quantities \( x, y, z \) (with uncertainties \( \Delta x, \Delta y, \Delta z \)) combine to form \( Q \), where

\[
Q = \frac{x \cdot y}{z}
\]

(or any other combination of multiplication and division). Then the fractional uncertainty in \( Q \) will be

\[
\frac{\Delta Q}{|Q|} = \left| \frac{\Delta x}{x} \right| + \left| \frac{\Delta y}{y} \right| + \left| \frac{\Delta z}{z} \right|
\]

(7)

In other words, *for products and quotients, add the fractional uncertainties.*

**Exercise 3:**

d. Calculate the total volume of the block and the associated uncertainty. Show your math below.
Exercise 4:

You can also find the volume of an object by measuring the volume of water displaced by the object when it is submerged.

e. Measure the volume of water in the graduated cylinder and the associated uncertainty.
f. Submerge the block (holding it under the surface with a pen or pencil), and find the resulting volume and the associated uncertainty.
g. Calculate the volume of the plank and the associated uncertainty. Show your math below.
h. Is this answer consistent with your result from Exercise 2? Explain.
Exercise 5:

a. Calculate the total surface area of the block and the associated uncertainty. Show your math below.
Exercise 6

a. Measure the volume of a single sheet of paper and the associated uncertainty. Which uncertainties are negligible?

b. Measure the surface area of a single sheet of paper. Which uncertainties are negligible?